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A MODIFICATION OF THE NEWTONIAN DYNAMICS: IMPLICATIONS FOR GALAXY SYSTEMS[1]

M. MILGROM
Department of Physics, Weizmann Institute, Rehovot, Israel; and The Institute for Advanced Study
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ABSTRACT

I consider the implications of a modification of the Newtonian dynamics to galaxy systems. Masses and mass-to-light ratios are rederived, on the basis of existing data, for binary galaxies, small groups, clusters of galaxies, and the Virgo Supercluster. For each type of galaxy system, the average M/L values come out to be a few solar units. These results eliminate the need to assume large amounts of hidden mass in galaxy systems, if the modified dynamics applies.

Subject headings: galaxies: clusters of — stars: stellar dynamics

I. INTRODUCTION

This is the third in a series of papers in which I consider the possibility that the dynamics of galaxies and systems of galaxies are governed by a modified version of the Newtonian dynamics (second law + gravity) for small accelerations. The basic ingredient of the modified dynamics is that the acceleration, a, of a particle in the gravitational field of some mass M is not given by the conventional dynamics in the limit when this acceleration is much smaller than some acceleration constant a_0. Instead, we have in this limit:

$$a^2/a_0 \approx g_N, \tag{1}$$

where g_N is the standard gravitational acceleration.

In Milgrom (1983*a*, hereafter Paper I), I discuss matters of principle concerning this modification. In Milgrom (1983*b*, hereafter Paper II), I consider the implications for galaxies. In particular I find in Paper II

$$a_0 \approx 2\times 10^{-8} h_{50}^2 (P/P_0)^{-1}\ \mathrm{cm\ s^{-2}}, \tag{2}$$

where $h_{50} = H_0/50$ km s^{-1} Mpc^{-1}, P is the M/L value for the standard matter in galaxies, and P_0 is some theoretical value I used for P. This value of a_0 is good within a factor of two.

In the present paper I consider the implications of the modified dynamics for systems of galaxies, i.e. binary galaxies, small groups, galaxy clusters, and the Virgo Supercluster. My main aim is to rederive masses and mass-to-light ratios for these systems, using the modified dynamics, on the basis of existing data. As will turn out, the results are consistent with all of the mass, in these systems, being in observable forms, i.e. in stars, interstellar gas, and intergalactic gas.

Present analyses, using the conventional dynamics, indicate very strongly that M/L values increase as we go up in the galaxy grouping hierarchy (Faber and Gallagher (1979, hereafter FG), Davis *et al.* (1980), and Rood (1981).

In each particular case of mass determination, there are very large uncertainties involved (see, for example, FG and Rood 1981 and references therein for an extensive discussion of these uncertainties). In rederiving mass-to-light ratios, I do not attempt to suggest any new remedies for these uncertainties. I shall go over the various analyses discussed in FG, adopting in each case the assumptions concerning the data, made by the respective authors.

The accelerations involved in all the systems discussed in this paper, are much smaller than a_0, and I shall use the small acceleration asymptotic law, equation (1). The value of a_0 from equation (2) will be used throughout. Since masses determined through equation (1) scale like V^4/a_0 (see below), they will scale as h_{50}^{-2}, like the luminosities. The M/L ratios I get in this paper are thus independent of the assumed value of H_0. They scale like the model M/L values assumed in Paper II for the standard matter in galaxies (which go into the value of a_0 in eq. [2]).

An assumption which is made throughout this paper is that the acceleration of the systems considered, in an external field, can be ignored when discussing their internal dynamics. I also assume that accelerations within the accelerated bodies do not affect their center of mass motion. These two important assumptions are discussed in Paper I.

In § II, I rederive masses and M/L values of galaxies in binaries. In § III, I do the same for galaxies in small groups. Section IV deals with galaxy clusters, and § V with the infall of the Local Group within the Local Supercluster. Section VI summarizes the results.

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II. BINARY GALAXIES

Analyses of binary galaxy samples (Turner 1976*a*, *b*; Peterson 1979*a*, *b*; and Karachentsev 1978), are summarized by FG who give values of M/L_B (in solar units) corrected to the standard system of FG, of between 20 and 40 for the Turner and Peterson samples. For the small separations sample of Karachentsev, FG give $M/L_B = 5.9 \pm 2.7$. (An important reevaluation and reanalysis of the Turner catalog is to be found in White *et al.* 1983. Their corrections have not been included in my analysis.)

To estimate masses on the basis of the modified dynamics, I assume that the binaries contain galaxies of equal mass M. I assume circular orbits. I also assume that the acceleration of the galaxies within the binary is large compared with the acceleration of the binary in an external field (say of a group the binary may belong to). This assumption breaks down at large separations.

If V is the velocity difference and R, the true separation, the mass M is given approximately by

$$M = V^4/4a_0G. \tag{3}$$

This expression is to be compared with $M = V^2R/2G$ for the mass M based on the conventional dynamics. Note that equation (3) is not, strictly speaking, valid even in the phenomenological framework in which I am working, as each of the two galaxies cannot be assumed to move in a static mean field. I shall nevertheless use this approximation.

Define (as in Peterson 1979*b*) the projection angles φ, between the line of sight and the separation vector and ψ, between V and the plane defined by these two directions. Then the observed quantities v (absolute value of the projection of V along the line of sight) and r_p (projection of R on the plane of the sky, in absolute value) are given by $v = V\cos\varphi\cos\psi$ and $r_p = R\cos\varphi$. One thus has

$$M = v^4/4a_0G\cos^4\varphi\cos^4\psi. \tag{4}$$

I define various measures of the average mass of the sample by

$$M_n \equiv [\langle v^n\rangle/\langle\cos^n\varphi\rangle\langle\cos^n\psi\rangle]^{4/n}(4a_0G)^{-1}, \tag{5}$$

for $n = 1,2,4$, based on different moments of the v distribution. For averaging $\cos^n\varphi$ and $\cos^n\psi$ I use Peterson's procedure.

If the probabilities for finding certain values of the angles φ are independent on that for finding a certain M, we can write

$$M_4 = \langle M\rangle, \qquad M_2 = \langle M^{1/2}\rangle^2, \qquad \text{and } M_1 = \langle M^{1/4}\rangle^4. \tag{6}$$

I have calculated $\langle v^n\rangle$ from the published histograms of v for the samples of Turner 1976*b* (his Fig. 1), Peterson 1979*b* (his Fig. 1) and Karachentsev (1978) (his Fig. 1). I give in Table 1 the average mass measures M_n for $n = 1,2,4$, for the three samples (the velocity cutoffs which were applied are given in parentheses). Peterson gives the v distributions for both his full sample and the subsample of pairs not containing early type galaxies (E or S0's) (his S and O pairs). I give the results for both samples. I also give in Table 1 (in parentheses) the values of M_n for the Turner and Peterson samples with all pairs containing ellipticals removed (using Fig. 4 of FG). The deduced masses in Table 1 are smaller, by a factor of 10–20, than those which Turner and Peterson obtain using the conventional dynamics.

I have also estimated the average M/L values for binaries in the following way. If $\tilde{M}/L$ is the value for an

TABLE 1

AVERAGE GALAXY MASSES M_n (IN UNITS OF $10^{10}\ M_\odot$) DEDUCED FROM BINARY DYNAMICS

n	TURNER (450)[a] 59 (52)[b] pairs	PETERSON (450)[a] 86 (73)[b] pairs	PETERSON (450)[a] 60 pairs (only S and O pairs)	KARACHENTSEV (500)[a] 236 pairs
1	17.1 (7.1)[b]	8 (3.6)[b]	4.1	7.4
2	20.9 (10.2)[b]	12.2 (6.1)[b]	6.4	10.5
4	26.4 (15.6)[b]	19.5 (10.9)[b]	11.7	16.6

[a]Number in parentheses: velocity cutoff in km s^{-1}.
[b]Number in parentheses: for samples with all pairs containing ellipticals removed.

individual binary, deduced with the conventional dynamics, the corrected value is given by (eq. [4])

$$M/L = (\tilde{M}/L)V^2/2a_0R$$
$$= (\tilde{M}/L)v^2/\cos\varphi\cos^2\psi 2a_0r_p. \quad (7)$$

Defining an effective correction factor $C = \langle v^2\rangle/2a_0\langle\cos\varphi\cos^2\psi\rangle\langle r_p\rangle$, I get C (Turner) = 0.122; C (Peterson) = 0.082, both for the samples which include all types. Using the average values of $\tilde{M}/L$ for the two samples corrected to the standard system of FG (for circular orbits), I get (M/L_B) (Turner) = 2.1 $(M/L_B)_\odot$ and (M/L_B) (Peterson) = 2.6 $(M/L_B)_\odot$.

Rivolo and Yahil (1981) have recently published an analysis which sheds doubt on the interpretation of galaxy pairs as isolated bound states (see also Yahil 1977).

Finally note that the picture advocated here does not require (White and Sharp 1977) that binaries spiral down and merge in a small number of revolutions, since galaxies do not have extensive halos which interact effectively.

III. SMALL GROUPS OF GALAXIES

To estimate M/L values for small groups of galaxies I make use of the data collected by Rood and Dickel (1978). They, in turn, use two samples of galaxy groups listed by Sandage and Tammann (1975), de Vaucouleurs (1975), and Sandage (1975) (put together in one sample named the STV sample) and the sample of Turner and Gott (1976) (named the TG sample). A recent analysis of these samples is described by Mezzetti, Giuricin, and Mardirossian (1982). I calculate M/L for each group. I do not have an analog of the virial theorem for an N-body system. I use a relation of the form:

$$MGa_0 \approx \bar{V}^4, \quad (8)$$

where $\bar{V}^2 = 3\langle v^2\rangle$, v is the radial velocity of a galaxy relative to the center. I use for $\bar{V}$ the quantity defined by Rood and Dickel as the virial velocity of the group. To get the results in FG's standard system, the values of M/L, I get from equation (8) with the group total luminosity L_T estimated by Rood and Dickel, have to be multiplied by 1.25 (see FG, p. 169).

The average and median values of M/L (in solar units), for the two samples, are given in Table 2. Values are given both for the full samples and for the subsamples of groups with, at least, 10 measured redshifts. As can be seen in Table 2, $\langle M/L\rangle$ and $(M/L)_{\rm med}$ are not stable to changes of the sample. Nevertheless, it is clear that all estimates of M/L are much smaller than those obtained with the conventional dynamics. Within the uncertainties, they are consistent with the possibility that no hidden mass is present in appreciable quantities.

For the sample of small groups published recently by Huchra and Geller (1982) I find $(M/L)_{\rm med} \approx 4$, for groups with at least seven measured redshifts (compared with their value of 170). I assumed that Huchra and Geller used $h_{50} = 2$.

FG discuss at length the many sources of uncertainty which plague the analysis of small groups. All these uncertainties apply, of course, to my results, too. In addition, there is an important source of uncertainty, which is specific to the present analysis. It has to do with the fact that the groups themselves are not isolated but, in general, accelerated in external gravitational fields. If the acceleration of the group as a whole cannot be ignored and if the external acceleration, g, is larger than the internal ones, $a_{\rm in} \sim \bar{V}^2/r$, equation (8) is not valid. Instead one has approximately

$$M \approx G^{-1}ga_0^{-1}\bar{V}^2r. \quad (9)$$

The use of equation (8) in this case thus leads to an underestimate of M.

IV. CLUSTERS OF GALAXIES

The analyses of Zwicky (1933) and Smith (1936), of radial velocities data of the Virgo Cluster, first pointed out the discrepancy between dynamically determined masses and the mass indicated by the luminosity. Their method, which employs the virial theorem, remains, with improved data and refined analysis, the major way of obtaining cluster masses. Bahcall (1977), FG, and Rood (1981) summarize the results for the M/L values deduced for clusters. With large uncertainties and scatter, the M/L_B values are typically between 100 and 300 solar units.

TABLE 2

M/L_B VALUES FOR SMALL GROUPS OF GALAXIES

	STV		TG	
PARAMETER	All 63 groups	$(N \geq 10)$ 8 groups	All 29 groups	$(N \geq 10)$ 5 groups
$\langle M/L\rangle$	14.5	1.2	4.7	9.5
$(M/L)_{\rm med}$ ...	0.6	1.0	0.12	5.2

TABLE 3

MASSES AND M/L_V VALUES FOR GALAXY CLUSTERS

Cluster	Cz (km s^{-1})	σ (km s^{-1})	M ($10^{12}\,M_\odot$)	L_V ($10^{12}\,L_{V,\odot}$)	M/L_V (s.u)
A98	31168[a]	887[a]	141	21[b]	6.8
A154	19746[a]	904[a]	175	11[b]	15.4
A168	13557[a]	605[a]	38	12[b]	3.1
A194	5312[a]	414[c]	9	1.8[d]	5.0
A401	22379[e]	1390[c,e]; 967[c,e]	944; 221	18[b]	52; 12.3
A1314	10215[a]	701[a]	71	3.1[d]	23.0
A1367	6552[a]	634[a]	50	3.3[d]	15.0
Coma	6821[a]	975[a]	279	9.4[d]; 30[j]	30; 9.3
A1940	41686[a]	616[a]	29	17[b]	1.7
A2029	23000[f]	1540[f]	1413	38[b]	37
Hercules ...	10775[g]	652[g]	53	3.6[d]	14.7
A2197	9082[a]	352[a]	5	4.2[d]	1.1
A2199	9250[a]	887[a]; 541[h]	185; 26	5.7[d]	32; 4.5
A2256	18069[a]	1357[a]	905	23[b]	39
A2670	22590[i]	890[i]	158	10[b]	15.8

[a]Yahil and Vidal 1977.
[b]Dressler 1978.
[c]This paper.
[d]Oemler 1974.
[e]Hintzen *et al.* 1977.
[f]Dressler 1981.
[g]Burbidge and Burbidge 1959.
[h]Abell 1977.
[i]Oemler 1973.
[j]Rood *et al.* 1972.

I derive cluster masses using a relation of the form

$$M = \lambda (a_0 G)^{-1} \sigma_i^4. \tag{10}$$

Here σ_i is the line-of-sight (l.o.s.) intrinsic velocity dispersion for the whole cluster, and λ is a factor between 1 and 10 which depends on the mass distribution within the cluster and on the shapes of orbits which galaxies have in the cluster, and which also contains the projection correction (to correct l.o.s. velocities to space velocities). The factor λ probably varies from cluster to cluster. For test particles in isotropic circular orbits around a point mass M, $\lambda = 9$ which I shall use. Isothermal spheres, in the modified dynamics, satisfy equation (10) with a value of λ of that order (see Milgrom 1983*c*).

I have collected all clusters for which I could find data on both velocity dispersions and luminosities. Observed velocity dispersions (σ) are taken mainly from Yahil and Vidal (1977, hereafter YV) and luminosities (L) are taken mainly from Oemler (1974) and Dressler (1978) with a few exceptions. The results for 15 such clusters are given in Table 3.

The luminosities given are corrected to the V band ($H_0 = 50$ km s^{-1} Mpc^{-1}). Those given by Oemler (1974) are already in the V band. Those from Dressler (1978) I divided by 1.15 (a correction factor suggested by Dressler) to get the V band values.

In some cases YV give σ both for the full sample and for the sample with suspected field galaxies removed (a small fraction of the galaxies, in all cases). I have always taken σ for the second case. For A401 I use the sample of Hintzen, Scott, and Tarenghi (1977). I get for the full sample $\sigma = 1390$ km s^{-1}. Two galaxies in the sample (of 14) lie far on the tail of the velocity distribution. If the average and σ are calculated for the sample without these two, these galaxies lie 3.4 σ and 2.6 σ away from the average. Removing these two I get $\sigma = 967$ km s^{-1}. I give the results for both values of σ.

The values of σ which I adopted are given in Table 3, together with the values of Cz for the cluster (mostly from YV). The values of σ are corrected for red shift by a factor $(1+z)^{-1}$ to obtain σ_i. The masses are thus calculated as

$$M = 9\,\sigma^4 (1+z)^{-4} a_0^{-1} G^{-1}, \tag{11}$$

and are given in Table 3 together with the M/L_V values in solar units. The systematic errors I can think of (contamination by field galaxies, nonsphericity of some clusters, and the use of dispersions which come mostly from the core regions) tend to produce overestimates of the masses. If, in addition, we allow for intergalactic gas (as evidenced by X-ray emission), the results are consistent with most of the dynamic mass in clusters being in conventional forms.

V. THE VIRGO INFALL

It is observed that the local group has a peculiar velocity (v_p) with respect to the Hubble flow. One usually assumes that this peculiar velocity is produced by the acceleration due to the mass excess within the

Local Supercluster over the ambient density. The measured value of v_p can then be used to deduce the excess mass. If the density contrast can be obtained observationally, one can also deduce the ambient density itself (see, for example, Silk 1974, Peebles 1976, Gunn 1977, Davis *et al.* 1980, Yahil, Sandage, and Tammann 1980, Hoffman, Olson, and Salpeter 1980, Tonry and Davis 1981, and a review by Yahil 1981). The various analyses yield large M/L values for the Local Supercluster, of the order of those for galaxy clusters.

The procedure by which one relates the peculiar velocity to the mass density in the standard dynamics is described, for example, by Peebles (1976) and Gunn (1977) who give analytic results for the perturbative case which, to a good approximation, can be written as (Davis *et al.* 1980)

$$v_p/v_H \approx \bar{\delta}\Omega^{2/3}/3. \qquad (12)$$

Here v_H is the Hubble velocity which corresponds to the distance between the Local Group and the Local Supercluster's center, $1+\bar{\delta}$ is the average density within the position of the Local Group in units of the ambient density, and Ω is the ratio of ambient to closure densities.

As I explain in Paper I, the modified dynamics implies a change in our view of cosmology. In particular, the possibility that a_0 varies on the Hubble time scale has to be considered. Since I do not have the alternative cosmology yet, I cannot calculate the relation between mass excess and peculiar velocity exactly. I approximate the expected peculiar velocity by that which would result from the present acceleration by the mass excess, during the Hubble time.

With the unmodified law this estimate gives: $v_p \sim \delta M G r_G^{-2} H_0^{-1} = \bar{\delta}\Omega v_H/2$. Here δM is the mass excess within the position radius r_G of the Local Group. For $0.1 < \Omega < 1$, v_p derived from this expression does not differ from that given by equation (12) by more than 50%.

Using the modified dynamics, however, I get $v_p \sim (\delta M G r_G^{-2} a_0)^{1/2} H_0^{-1}$, which can also be written as

$$v_p^2 \sim \bar{\delta}\Omega v_H a_0 H_0^{-1}/2. \qquad (13)$$

Thus, the mass excess (proportional to $\bar{\delta}\Omega r_G^3$) is smaller by a factor of about $v_p H_0/a_0$ than what one gets with the conventional dynamics.

Using, for example, the results of Davis *et al.* (1980) I get $M/L_{BT} \sim 1.0 \times (v_p/440 \text{ km s}^{-1})^2 (M/L_{BT})_\odot$. This value of M/L must be considered only a rough estimate.

VI. CONCLUSIONS

I have rederived masses and mass-to-light ratios for galaxy systems using a modified form of the nonrelativistic dynamics in the limit of small accelerations. I considered binary galaxies, small groups, galaxy clusters, and the Local Supercluster. In each particular case I have used published data for samples of galaxy systems which were used before to derive M/L values with the unmodified law. I have adopted in each case the astrophysical assumptions about the samples made by the respective authors.

The masses and M/L values I obtain are much smaller than what one gets with the unmodified law. In fact, for each type of galaxy system, the average value of (M/L) is a few solar units. I conclude then that if the modified dynamics hold, there is no need to assume hidden mass, in appreciable quantities, in galaxy systems.

Note that the results of this paper do not depend on the exact formulation of the modified dynamics. All that went into the analysis is the assumption that Newtonian dynamics in a gravitational field breaks down at accelerations much smaller than a_0, and that in this limit $a^2/a_0 \sim MGr^{-2}$.

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M. MILGROM: Department of Physics, Weizmann Institute of Science, Rehovot 76100, Israel