PUBLICATIONS OF THE ASTRONOMICAL SOCIETY OF THE PACIFIC
94:627-633, August 1982

PUBLICATIONS OF THE
ASTRONOMICAL SOCIETY OF THE PACIFIC

Vol. 94 August 1982 No. 560

THE BIERMANN MECHANISM AND SPONTANEOUS MAGNETIC FIELD GENERATION IN STARS*

JAMES C. KEMP

Department of Physics, 122 Science I, University of Oregon, Eugene, Oregon 97403

Received 1982 March 22

How magnetic fields can arise spontaneously in stars and nebular clouds, without the initial presence of primordial or seed fields, is a topic which has been somewhat bypassed in the solar and astrophysical literature. It is known that at least one efficient spontaneous process exists, the one proposed by L. Biermann in 1950. In this paper I attempt to furnish particularly to nonspecialists a clear, physical explanation of this interesting mechanism. Certain other mechanisms will also be mentioned.

Key words: magnetic fields—stellar rotations

I. Background

The genesis of magnetic fields in the sun and in some kinds of stars has occupied many astrophysical theorists. The point of view is often taken that relatively weak "seed" fields existed in the primordial nebulae. In protostars, these fields could have been amplified and modified by hydromagnetic action, particularly by turbulence, wherein mechanical energy can be converted to magnetic energy, essentially through the force J $\times$ B which acts on the body of the plasma. If the plasma velocity is locally antiparallel to J $\times$ B, then the plasma is decelerated, and the absorbed mechanical energy can augment the magnetic energy, thus the field strength. However if at the beginning J and B were zero, dynamos of this kind would not work.

It is pleasing to know that at least one totally spontaneous dynamo exists, not requiring seed fields. This is the Biermann mechanism, which depends on differential rotation in stars (Biermann 1950). A brief description in English of Biermann's work is given in Biermann and Schlüter (1951). In a discursive paper by Layzer, Rosner, and Doyle (1979) on solar dynamo theories, it is stated that: "Biermann's mechanism is the only known process that spontaneously generates large-scale magnetic fields under conditions of astronomical interest." A careful picture of this intriguing and rather subtle mechanism is aimed at here. The discussion is directed toward observational stellar and solar astronomers interested in a qualitative understanding, and not toward the theorists in this field. Certain other spontaneous mechanisms, much less efficient ones, will also be briefly examined. Any elementary process such as Biermann's may, as Layzer et al. (1979) point out, be considered as a source of seed fields, which may then be modified hydromagnetically.

In this paper I will use MKS units, but with the magnetic fields also given in gauss.

II. Mechanical and Electrostatic Equilibrium in a Star

It is often overlooked that hydrostatic equilibrium in a star involves strong electrostatic fields. In Figure 1 is pictured the outer portion, let us say the outer third, of a nonrotating star. For simplicity I suppose total ionization, $N_i = N_e = N$, where N_i and N_e are the number densities of ions and electrons. The ions and electrons constitute coexisting particle gases, coupled statically by electrical forces. Hydrostatic equilibrium of the two gases, separately, is governed by the equations

$$-\nabla P + N m_i \mathrm{g} + N e \mathrm{E} = 0 \qquad (1)$$

$$-\nabla P + N m_e \mathrm{g} + N e \mathrm{E} = 0 \quad , \qquad (2)$$

where ∇P is the pressure gradient, of either ions or electrons. The electron and ion pressures are assumed to be essentially equal, $P_e = P_i$. The total pressure is $2P$. The pressures exert "mass-independent" forces, while the gravity forces m_ig and m_eg are "mass-proportional", where $g = G\mathfrak{M}/R^2$ is the local gravitational acceleration.

The gravitational force on the *electrons* is negligible compared to the pressure-gradient force. For the ions, these two forces are comparable. A small charge separa-

*One in a series of review articles currently appearing in these *Publications.*

tion comes about, producing an electron skin, in a manner of speaking, on the star's surface. This sets up an electric field E which holds the electron gas to the star. Aproximately, eE equals half the gravitational force $m_i g$ on an ion, as can be seen from equations (1,2) wherein $m_e/m_i << 1$.

To estimate the radial charge separation, I show in the lower part of Figure 1 a radial column of pressure scale height h, containing ions and electrons. (In an isothermal approximation, assuming only a small change of g with r, $P \propto \exp(-r/h)$, where $h = kT/(m_i g)$.) The ion and electron densities are supposed constant within the column of height h, but the electron gas extends outward to $h + \Delta h$. Thus there is a negative charge density $-Ne$ in the skin of thickness Δh, while the column of height h has a net positive charge density of magnitude $(\Delta h/h)\, Ne$. The E field in the column is essentially that created by a (negative) surface charge density $\Delta h N e$, thus $E \sim (1/\epsilon_0)(\Delta h)Ne$. From equations (1) and (2), approximately, $eE = m_i\, g/2$. Since $g = G\mathfrak{M}/R^2$, if we set $h \sim R$ (the star's radius), we find for the fractional charge separation

$$\frac{\Delta h}{h} \sim \frac{6\, \epsilon_0\, m_i^2\, G}{e^2} \sim 10^{-36} \quad .$$

The charge separation is utterly negligible, even though the E fields are substantial. The presence of electrical fields for maintaining charge neutrality is critical for the Biermann mechanism.

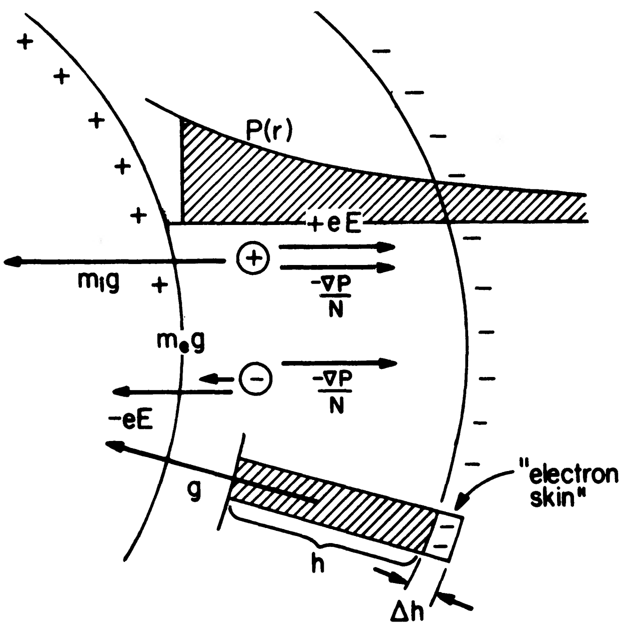

FIG. 1—Cross section of a nonrotating star, illustrating the role played by a radial electrostatic field in hydrostatic equilibrium.

III. Differential Rotation and the Biermann Magnetic-Field Generator

A differentially rotating star is one in which the angular rotation velocity ω varies with z, the spin-axis coordinate. In the sun, well-known surface observations indicate that ω decreases with the absolute latitude; ω is greatest at the equator. The situation is pictured in Figure 2. The gravitational force g is partially opposed by a centrifugal force ω^2 R, where R is the vector distance from the spin axis. Now if ω varies with z, Biermann found that a purely *static* electric field cannot prevent the occurrence of pressure gradients which cause a differential motion of the electron gas, relative to the ions. Currents—and thus a magnetic field—then arise.

To see how this works, it is easiest to deal not with a spherical star, but with a "cylindrical" star. That this is meaningful is suggested in Figure 2, in which the radial gravitational force in a spherical star is seen, at least near the equator, to be directed largely inward in the $-R$ direction. We now imagine a cylindrical star, having a constant axial mass concentration extending indefinitely along the z axis. We suppose the star to have a differential rotation $\omega(z)$, with for example the values $\omega(z_1)$ and

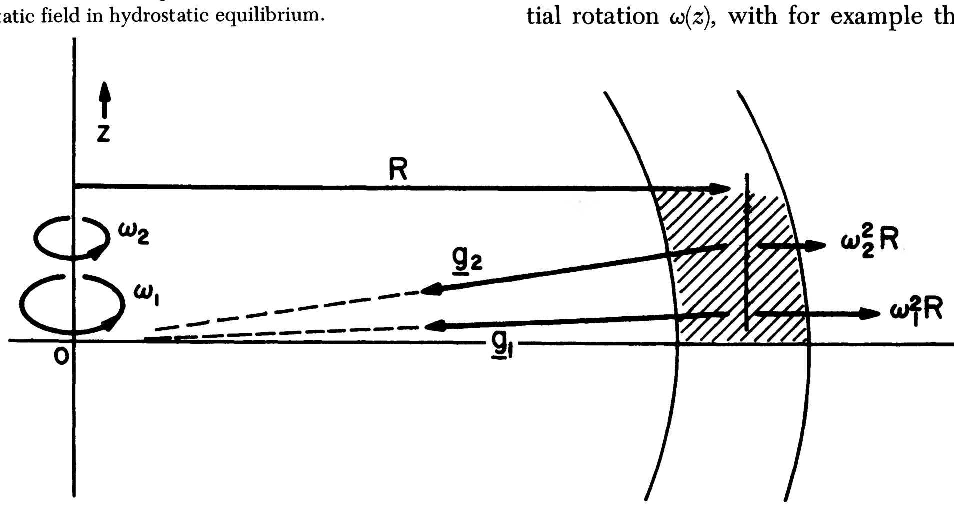

FIG. 2—Force relationships in a differentially rotating star, e.g., one with ω largest at the equator and decreasing with $|z|$, such as the sun.

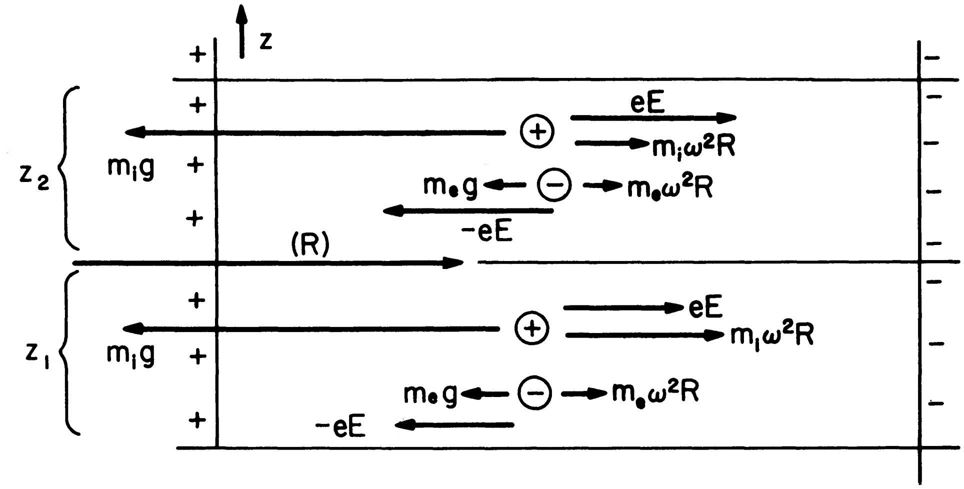

FIG. 3—Force relationships in two cross-sectional slabs (cut parallel to the spin axis) in a differentially rotating "cylindrical" star, in which ω increases with z. Outer boundary of the star is to the right.

ωz_2) as in Figure 3. Motions in the Φ direction about the z axis will be disposed of by invoking centrifugal forces. We thus define an effective gravity by:

$$g' = g - \omega^2 R \quad .$$

Consistent with this, all *velocities* henceforth will be only the vector components in the meridional plane.

In Figure 3 is shown a meridional, cross-sectional view. The effective gravities differ between the slabs at heights z_1 and z_2. In equations (1) and (2) we replace g by $g'(z)$. Then stability would seem to call for an electrostatic field varying with z, with surface charges varying with z (see Fig. 3).

However, even apart from the electrical forces, the density-pressure situation in Figure 3 is unstable, if the *temperature* is independent of z. In a scale-height approximation applied at each level z (which assumes that T is independent of r or R), the scale height is $h = kT/(m_i g')$. If conditions were vertically isothermal, $T(z_1) = T(z_2)$, then we would have $h(z_2) < h(z_1)$, since $g'(z_1) < g'(z_2)$. The pressure distributions $P(r)$, which are drawn in Figure 4 as exponentials, would vary with z. Vertical mass diffusion would occur, in a circulating pattern as indicated in Figure 4. To obtain mechanical equilibrium, we must require that T varies with z; in the case of Figure 4, we must have $T(z_2) > T(z_1)$. With a correct $T(z)$, the pressures all across the contact plane between the horizontal slabs at z_1 and z_2 could be equalized. Then, however, the surfaces of constant pressure, which are concentric cylinders, would no longer be parallel to the surfaces of constant temperature. Mechanical stability thus requires a violation of the conditions assumed in von Zeipel's theorem, in which the constant-T and constant-P surfaces coincide. The violation is of course due to the fact that here $\nabla \times g' = \nabla \times (\omega^2 R) \neq 0$, i.e., the centrifugal forces are not derivable from a potential.

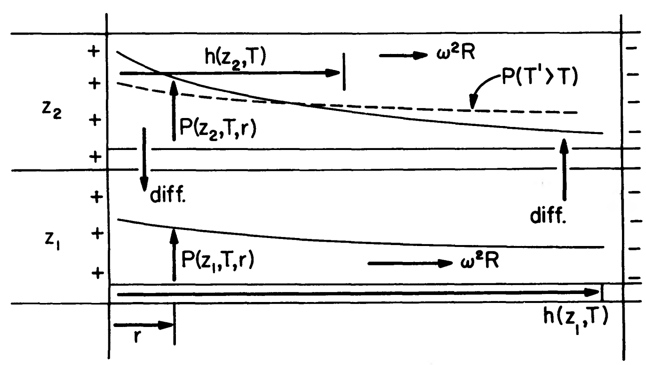

FIG. 4—Pressure distributions in the meridional cross section of a "cylindrical" star as in Figure 3, at two z levels. If constant temperature were assumed, or merely that T were not to vary with z, the pressure curves would not be the same (solid curves) at z_1 and z_2; diffusion, i.e., circulation, would ensue as per the arrows labeled "diff.". Stability against such circulation would require that $T(z_2)$ exceed $T(z_1)$, such that the scale heights and pressure distributions would match up.

If we allow now for possible accelerations of the electron and ion gases and replace g with the effective gravity g', we write equations (1) and (2) as:

$$- \nabla P/N + m_i \, g' + e \, \mathbf{E} = m_i \, (d \, v_i/dt) \tag{3}$$

$$- \nabla P/N + m_e \, g' - e \, \mathbf{E} = m_e \, (d \, v_e/dt) \quad . \tag{4}$$

Here, having divided by the number density N, we consider now the individual-particle forces and accelerations, rather than (as in eqs. (1) and (2)) the forces per unit volume.

Collision terms, which account for ohmic resistance between the electron and ion gases, are not included in equations (3) and (4); a resistance term will be added later.

The condition for static mechanical equilibrium is that the center-of-mass velocity v of the plasma be zero, in which case:

$$(m_i + m_e)(d\mathbf{v}/dt) = m_i(d\mathbf{v}_i/dt) + m_e(d\mathbf{v}_e/dt) = 0 \quad (5)$$

With this condition, which is in fact attainable only with a proper temperature variation $T(z)$, we find from equations (3) and (4) an expression for the *relative* acceleration of ions and electrons, namely:

$$\frac{d}{dt}(\mathbf{v}_i - \mathbf{v}_e) = \left(\frac{1}{m_i} + \frac{1}{m_e}\right) e\mathbf{E} + \left(\frac{1}{m_e} - \frac{1}{m_i}\right)\left(\frac{m_i + m_e}{2}\right) \mathbf{g}' \quad . \tag{6}$$

The left-hand side here, if we multiply if by Ne, is a possible electric current build-up. If we take the curl of both sides of equation (6), we see that since $\nabla \times \mathbf{g}' \neq 0$, the curl of either the relative velocity field $(\mathbf{v}_i - \mathbf{v}_e)$, or of $\mathbf{E}$, must not vanish. If we suppose that $\nabla \times \mathbf{E} = 0$, then the relative velocity field $(\mathbf{v}_i - \mathbf{v}_e)$ cannot remain zero, thus currents flow. But if currents build up, they generate increasing magnetic fields $\mathbf{B}(t)$, producing then "rotational" electric fields since $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$. Actually neither the current-derivative term nor the E-field term in equation (6) vanishes; they are proportional to each other.

The engine for these currents is basically an uncompensated pressure gradient which, mainly, accelerates the electrons. The driving term appearing in equation (6), the term containing $\mathbf{g}'$ on the right, should more physically be written in the following form, which is easily derived from equations (3), (4), and (5):

$$\left(\frac{1}{m_e} - \frac{1}{m_i}\right)\left(\frac{m_i + m_e}{2}\right) \mathbf{g}' = \left(\frac{1}{m_e} - \frac{1}{m_i}\right) \times \frac{1}{N}\nabla P \cong \frac{1}{m_e}\frac{1}{N}\nabla P \quad . \tag{7}$$

It is important to see that the centrifugal forces, themselves, are not the direct agents behind the currents. The forces $m_i\mathbf{g}'$ and $m_e\mathbf{g}'$, being mass-proportional, cause identical and collinear accelerations of the electrons and ions. It is the fact that $\nabla \times (\nabla P/N) \neq 0$ that currents arise. While the curl of ∇P is zero, that of $(\nabla P/N)$ is not:

$$\nabla \times \left(\frac{1}{N}\nabla P\right) = \frac{1}{NT}(\nabla T) \times (\nabla P) \quad .$$

This is not zero because the constant-T and constant-P (also constant-N) surfaces are not parallel.

Before going on to estimates of the magnetic field and to a discussion of a magnetic steady state, I must comment on simplifications which I have made compared to the original treatment by Biermann. The reader may notice that I have omitted a $\mathbf{J} \times \mathbf{B}$ term, a magnetoresistance term, and a "motional" electric-field term $e\mathbf{v} \times \mathbf{B}$. For understanding a spontaneous buildup of magnetic fields from zero, those terms are inconsequential; they are initially zero since $\mathbf{B}(0) = 0$. The first two of these terms are effectively quadratic in B, since the B field is proportional to J through $\nabla \times \mathbf{H} = \mathbf{J}$. The field strengths which we will estimate with these terms neglected are basically the same as in Biermann's results.

Secondly, for the Biermann mechanism to work it is not necessary to assume strict mechanical equilibrium, i.e., zero plasma velocity. Roxburgh (1966) has discussed cases with meridional mass circulation. In general neither v nor dv/dt need be zero. In place of our equation (5) we may write:

$$-2\frac{\nabla P}{N} + m\mathbf{g}' = m\frac{D\mathbf{v}}{Dt} = m\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\times\nabla\,\mathbf{v}\right) \quad , \tag{5a}$$

where here $m = m_i + m_e$. This is the equation of motion for the plasma center of mass allowing for mass flow and acceleration, neglecting a $\mathbf{J} \times \mathbf{B}$ term which as noted is zero before any field buildup. (Viscosity is omitted too, but it seems that that plays no essential role.) Here $D\mathbf{v}/Dt$ means the acceleration as seen in the moving frame of the plasma, which properly equals the applied force over the mass. Also $D\mathbf{v}/Dt$ equals identically the sum of the two terms $\partial\mathbf{v}/\partial t$, the acceleration seen at any point by the stationary observer, and a velocity-gradient term which expresses the fact that nearby plasma elements with differing velocities "drift into" a given region. It is interesting to consider a steady meridional circulation, wherein $\partial\mathbf{v}/\partial t = 0$. For that case let us take a line integral along a stream line of the circulation:

$$\oint_{\text{stream}} (-2\nabla P/N + m\mathbf{g}') \cdot d\mathbf{s} = m\oint_{\text{stream}} (\mathbf{v}\cdot\nabla\mathbf{v}) \cdot d\mathbf{s} \quad .$$

The right-hand integral vanishes, because $\mathbf{v} \cdot \nabla \mathbf{v} = (1/2)\nabla(v^2) - \mathbf{v} \times (\nabla \times \mathbf{v})$; the first term of this gives zero since $\nabla(v^2)$ is a gradient, the second one gives zero since $d\mathbf{s}$ is parallel to $\mathbf{v}$ and is thus everywhere normal to $\mathbf{v} \times (\nabla \times \mathbf{v})$. Now the left side can be changed by Stokes' theorem to an area integral in the meridional plane, enclosed by the streamline:

$$-2\int_A [\nabla \times (\nabla P/N)] \cdot d\mathbf{A} + m\int_A (\nabla\times\mathbf{g}') \cdot d\mathbf{A} = 0 \quad .$$

Clearly since $\nabla \times \mathbf{g}' \neq 0$ with differential rotation, the second integral doesn't vanish, thus the first one cannot vanish either. But then the constant-P and constant-T (also constant-N) surfaces must not be parallel, so that $\nabla \times (\nabla P/N) \neq 0$, just as in the case of no circulation. At least in an average or integral sense (for the purpose of integrals of quantities over large areas of the meridional plane), the velocity-gradient term on the right in equation (5a) can be assumed zero, or neglected. That term expresses alternating speed changes along the steady-state stream lines, plus a centripetal acceleration directed, crudely speaking, toward the center of the meridional quadrant. The Biermann process is not interfered with, since its functioning requires only that $\nabla \times (\nabla P/N)$ be nonzero.

In the general case of time-varying circulation, it is thought that again $\nabla \times (\nabla P/N)$ would not usually vanish.

Let us return to the matter of estimating the Biermann field strength. We take the curl of equation (6), making also a minor simplification in view of $m_e/m_i << 1$:

$$m_e \nabla \times [d(\mathbf{v}_i - \mathbf{v}_e)/dt] = e \nabla \times \mathbf{E} + (m_i/2) \nabla \times \mathbf{g}' \quad . \tag{8}$$

We now think of currents $\mathbf{J} = Ne(\mathbf{v}_i - \mathbf{v}_e)$ which are started from zero. These currents build up a magnetic field, and thus an electric reaction field which opposes the currents, in accord with $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$. Since the current and the B field are essentially proportional to each other through $\nabla \times \mathbf{H} = \mathbf{J}$, the current being the source of $\mathbf{B} = \mu_0 \mathbf{H}$, the left-hand side of equation (8) and the $\nabla \times \mathbf{E}$ term are proportional to each other; but the latter is far larger. Roughly, $|\nabla \times \mathbf{H}| = |\mathbf{J}| \sim H/R$, and $|\nabla \times \mathbf{E}| \sim E/R \sim \partial B/\partial t$, where R is a stellar radius. It follows that:

$$|m_e \nabla \times d(\mathbf{v}_i - \mathbf{v}_e)/dt| \sim \left(\frac{m_e m_i R}{\mu_0 M e}\right) |e \nabla \times \mathbf{E}| \sim 10^{-34}\, e\, |\nabla \times \mathbf{E}| \quad .$$

The "inertial" term on the left in equation (8) is then a small uncompensated difference between the driving term, the second term on the right, and the inductive $\nabla \times \mathbf{E}$ term. The latter represents a collective "electromagnetic inertia" of the whole electron gas, due to Faraday induction, while the left-hand side of equation (8) pertains to the inertia of only one electron. Thus effectively we have:

$$\nabla \times \mathbf{E} = -(\partial \mathbf{B}/\partial t) = -(m_i/2e) \nabla \times \mathbf{g}' = -(m_i/2e) \nabla \times (\omega^2 \mathbf{R}) \quad .$$

In order to describe a steady state for the magnetic fields, we may appeal to electrical resistance. Electron-ion collisions can be accounted for by adding a relaxation term to the right-hand side of equation (8), i.e.,

$$-(m_e/\tau) \nabla \times (\mathbf{v}_i - \mathbf{v}_e),$$

where τ is the electron-ion intercollision time. With the single-electron inertial term on the left in equation (8) neglected in favor of the inductive $\partial B/\partial t$ term, as discussed above, we then have:

$$-(m_e/\tau) \nabla \times (\mathbf{v}_i - \mathbf{v}_e) - e(\partial \mathbf{B}/\partial t) = -(m_i/2) \nabla \times \mathbf{g}' \quad . \tag{9}$$

This equation relates the current $\mathbf{J} = N e(\mathbf{v}_i - \mathbf{v}_e)$ to the field B at any point in the meridional plane; it is a three-dimensional, partial vector differential equation including the time dependence plus two dimensions in the meridional plane. (See Fig. 5. It can be assumed that the B field is always normal to the plane while J lies in the plane.) The J and B fields build up and diffuse spatially. However, to simplify things we can ignore the spatial diffusion and presume the currents and fields to build up with time-invariant spatial distributions, such that the time dependence factors out. Then for example, $\mathbf{B}(x,z,t) = \mathbf{F}(x,z)\, B(t)$. In this case, we may speak of a simple circuit or one-turn current loop around the meridional quadrant. The circuit current is roughly $I = J A \sim J(\pi R^2/2) \sim N e(\pi R^2/2)(v_i - v_e)$, where $A = \pi R^2/2$ is an appropriate mean area for the current, which flows parallel to meridian planes but is distributed around the star (see Fig. 5). The field B is proportional to this circuit current I: The one-turn current loop has a mean radius of about $R/4$, which gives a field $B \sim 2\mu_0 I/R$ in the center of the meridional quadrant.

Since the current and magnetic field vary spatially from zero to maximum over a mean distance $R/2$, we replace the curl operators in equation (9) with $2/R$, remembering that only the rotational parts of $(\mathbf{v}_i - \mathbf{v}_e)$ and $\mathbf{g}'$ are involved. Then equation (9) becomes:

$$\left(\frac{m_e}{\tau} \frac{4}{Ne\pi R^2}\right) I + \left(\frac{2\mu_0 e}{R}\right) \frac{dI}{dt} = \left(\frac{m_i}{R}\right) g'_{\text{rot}} \,. \tag{10}$$

This is tantamount to a lumped-parameter circuit equation of the type:

$$\mathscr{R} I + L\,(dI/dt) = V \quad , \tag{10$'$}$$

where $\mathscr{R}$ is the resistance, L is the inductance, and V is an applied constant voltage, proportional to g'_{rot}, which can be presumed to be "turned on" at $t = 0$. The current and magnetic field in this IR circuit build up merely as $(1 - e^{-t/T})$, where $T = L/\mathscr{R}$ is the time constant. In the final state, $dI/dt \to 0$, and the ultimate field is determined by

$$I_{\max} = \frac{R}{2\mu_0} B_{\max} = \left(\frac{\pi N e R^3 \tau}{2m_e}\right) \frac{m_i}{2R} g'_{\text{rot}} \quad ,$$

or

$$B_{\max} = \left(\frac{Ne^2\tau}{m_e}\right) \cdot \left(\frac{\pi \mu_0 m_i R}{2e}\right) \cdot g'_{\text{rot}} \quad . \tag{11}$$

Here $N e^2 \tau/m_e$ is the plasma conductivity. As for the driving voltage and g'_{rot}, we note that if ω varies simply in the z direction we can approximate:

$$|\nabla \times \mathbf{g}'| = |\nabla \times (\omega^2 R)| = 2\omega R\,(d\omega/dz) \simeq 2\omega^2\,(\Delta\omega/\omega) \quad ,$$

where $\nabla\omega/\omega$ is the fractional change in ω over $z = 0$ to $z = R$. Thus $g'_{rot} = 2\,R\,\omega^2\,(\Delta\omega/\omega)$, and generally,

$$B_{max} \simeq \sigma \left(\frac{\pi\mu_0 m_i R^2}{e} \right) \omega^2 \left(\frac{\Delta\omega}{\omega} \right) \quad .$$

For the sun, a representative conductivity is $\sigma \sim 10^6$ mhos/meter, and we have $r \sim 10^9$ meters and $\omega \sim 3 \times 10^{-6}$ sec^{-1}. Recalling that $\mu_0 = 1.2 \times 10^{-6}$ henry/meter, we deduce that

$$B_{max} \sim 0.05\,(\Delta\omega/\omega)\ \text{weber}/m^2 = 5000\,(\Delta\omega/\omega)\ \text{gauss} \quad . \tag{12}$$

Thus if $\Delta\omega/\omega$ is of the order of 0.2, as surface observations of the sun's differential rotation would suggest, *the ultimate Biermann fields approach a kilogauss in strength.*

The build-up time in the RL circuit is $T = L/\mathscr{R}$. From equation (10), and with $\sigma = N\,e^2\tau/m_e$, we have

$$T \sim (\pi/2)\,\mu_0\,\sigma\,R^2 \quad . \tag{13}$$

For solar parameters as above, this T would be $\sim 5 \times 10^{10}$ years, or ten times the sun's assumed age. Of course our grossly simplified model neglects the details of the diffusion problem and other aspects, and the correct mean conductivity may be somewhat smaller than 10^6 mhos/meter. However, it is clear from detailed treatments that the build-up time cannot be less than about the sun's age. I should add that the diffusion time for currents and fields in a plasma is given by $T_d = \mu_0\,\sigma\,d^2$, for a distance d. This is similar to T above, but is perhaps shorter if the appropriate d is $\sim R/2$ rather than R. The lumped-parameter circuit approximation probably overestimates the buildup time, since in fact the fields "begin" in a restricted area (with a smaller R, as it were), then diffuse throughout the star.

More explicit treatments by Biermann (1950) and Roxburgh (1966) lead again to ultimate fields of the order of a kilogauss, for the sun. Roxburgh finds an equilibration time of the order of the sun's age. He also notes that in massive early stars (but not in middle-late stars), the equilibrium state might be controlled not by ohmic disspitation—as I have assumed—but by the magnetoresistance. The latter leads to a term which is effectively quadratic in H (or B), and I have neglected all such effects here.

IV. Other Spontaneous Magnetic Batteries or Generators

One relatively trivial source of a spontaneous magnetic field is simple stellar rotation, combined with the electron-skin effect described in section II above. The rotating charge skin produces a central, dipolar magnetic field of the order of $H = (\omega/R)\,(\Delta h/h)\,N_t e$, where $N_t = M/m_i$ is the total number of atoms in the star. For the sun with $\omega \sim 10^{-6}$ sec^{-1} and $\Delta h/h \sim 10^{-36}$ as found in section II, we get $B = \mu_0\,H \sim 10^{-19}$ web/m^2 $= 10^{-15}$ gauss. For a neutron star (assuming it has an electron skin), with $\omega \sim 100$ sec^{-1} we find $B \sim 10^{-2}$ gauss. A rather insignificant magnetic-field source, even for seed fields.

What makes the Biermann process work is the difference in accelerations imparted to the electrons, as compared to the ions, by unbalanced forces (in this case differential pressures) which are not simply proportional to the particle masses. Differential accelerations and currents thus result. Are there other kinds of such forces? The answer is yes, but the others seem much less effective.

Viscosity is an example, as has been discussed by Browne (1968), specifically what is called molecular or particle viscosity. A velocity gradient dv/dy at right angles to the gas velocity v, as in a differentially rotating star or in a Keplerian rotating gas disk, produces a shearing force $\eta\ dv_x/dy$, where η is the viscosity (force per unit area per unit velocity gradient). If, further, there is a second derivative d^2v_x/dy^2, a net viscous force is exerted which accelerates local slabs of material relative to adjacent slabs. In a plasma, in a two-fluid picture we can suppose that the electron gas has its own viscosity, the ion gas its own. The separate viscosities are proportional to $m^{1/2}$ where m is the particle mass involved. But the accelerations are proportional to m^{-1}, thus the electron-gas acceleration exceeds that of the ions by $(m_i/m_e)^{1/2} \simeq 40$. Currents and therefore magnetic fields thus arise. The steady state is determined by resistivity, i.e., by electron-ion collisions, and/or magneto-resistance. But because of the smallness of the particle viscosity (see for example Kemp 1980), the ensuing fields and currents are miniscule. Nevertheless Browne (1968) has proposed that predicted fields of the order of 10^{-4} gauss in stars, caused by this mechanism, are adequate candidates for seed fields which may then be amplified hydromagnetically to the kilogauss level.

I have made an estimate of the fields expected in a Keplerian gas disk of diameter 10^5 km around a collapsed object such as a neutron star, as generated by the differential particle viscosity, and the field strengths seem not to exceed 10^{-10} gauss even in the center. (Such fields protrude normally from the disk.)

Might there still be other efficient spontaneous magnetic-field generators, besides Biermann's process? I wonder whether some combination of thermal and pressure gradients (which basically power the Biermann mechanism) with *turbulent* motion might produce a battery-like process. Or also, in the macroscopic motions of convective cells there may be transient pressure/temperature gradients which could act as batteries. It is not clear however how large-scale, organized fields could result in such cases.

Mention should be made of certain other processes. Cattani and Sacchi (1966) proposed the Poynting-Robertson effect as a "battery" mechanism, which accelerates or decelerates electrons relative to ions as they orbit about a strong radiation source (e.g., a star or perhaps a galactic nucleus). Owing to differential radiation pressures coupled with abberation, the electrons are preferentially braked, and a net circulating current around the radiation source develops, thus a magnetic field. (Harwit (1973) gives an excellent description.) Separately Leahy and Valenkin (1981) have proposed that neutrinos, asymmetrically emitted by black holes, may produce proton currents and therefore magnetic fields.

V. Remarks

There are indeed spontaneous sources of magnetic fields; by far the most efficient one known is Biermann's. That process leads to a distinctive magnetic-field geometry in a differentially-rotating star, namely a double toroidal field, the fields being oppositely circulating in the upper and lower hemispheres. This is sketched in Figure 5. The characteristic pattern of sunspot-pair magnetic polarities has long been interpreted in terms of toroidal fields buried deep in the sun, the spot pairs being produced by convective tearing out of the field lines. Direct sensing of the toroidal fields is difficult, but there may be evidence from disk-sector magnetic polarities measured by Duvall et al. (1979); see also Kemp (1981). Certainly we do not have the simple case of an essentially constant double-toroidal pattern of the Biermann type, because the spot-pair polarity reversals which accompany the 22-year solar cycle could not be explained. The sense of the differential rotation determines the Biermann-field directions, and in general the rotation pattern (as seen of course only on the surface) does not change or show reversals; always, ω is largest at the equator.

It is not clear whether the Biermann mechanism really plays a role in sustaining the sun's magnetic fields at the present stage. Mestel and Roxburgh (1962) argued that the Biermann process is thwarted right from the beginning if a star has an initial, even quite small, poloidal field. In any event, double-toroidal fields must play a role. The reversal of sunspot-pair polarities through the 22-year cycle has been seen as a torsional oscillation, in which the toroidal fields are alternately unwound and rewound. An interesting question is whether there is a small DC "bias" in the 22-year magnetic cycle, which would underlie a controversial even-odd difference as between alternate 11-year half cycles. One wonders whether the underlying bias, if it is real, has the senses of magnetic-field directions expected from the Biermann process. A readable discussion of aspects of this matter is to be found in the review by Layzer et al. (1979).

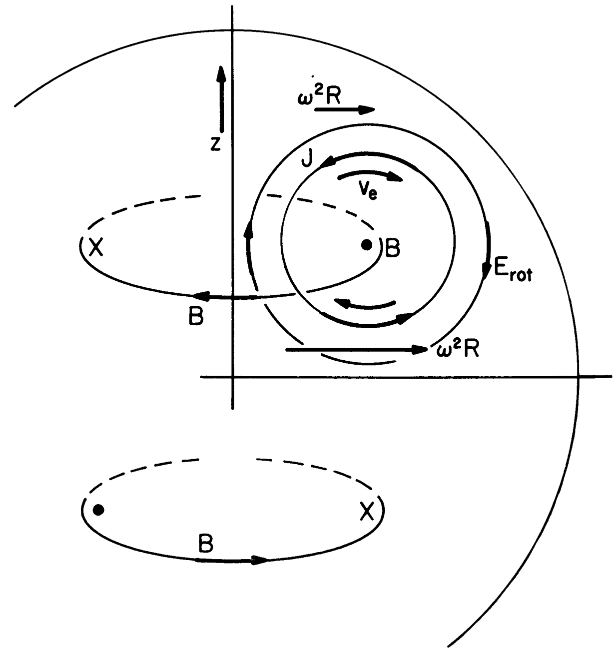

FIG. 5—The Biermann currents and double-toroidal magnetic fields in a differentially rotating star, schematically.

I am grateful for a tutorial given me by Leon Mestel, some years back, on the basics of the Biermann mechanism; and I am further grateful to him for his careful reading of this paper and for suggestions.

References

Biermann, L. 1950, *Zs. Naturforschung* 5a, 65.
Biermann, L., and Schlüter, A. 1951, *Phys. Rev.* 82, 863.
Browne, P. F. 1968, *Ap. Letters* 2, 217.
Cattani, D., and Sacchi, S. 1966, *Nuovo Cimento* 46 B, 8046.
Duvall, T. L., Scherrer, P. H., Svalgaard, L., and Wilcox, J. M. 1979, *Solar Physics* 61, 233.
Harwit, M. 1973, *Astrophysical Concepts* (New York: John Wiley & Sons), pp. 413–17.
Kemp, J. C. 1980, *Ap. J.* 235, 595.
——— 1981, paper presented at the *Second Cambridge Workshop on Cool Stars, Stellar Systems, and the Sun*, 21 October 1981 (and to appear in the symposium volume).
Layzer, D., Rosner, R., and Doyle, H. T. 1979, *Ap. J.* 229, 1126.
Leahy, D. A., and Valenkin, A. 1981, *Ap. J.* 248, 13.
Mestel, L., and Roxburgh, I. W. 1962, *Ap. J.* 136, 615.
Roxburgh, I. W. 1966, *M.N.R.A.S.* 132, 201.
Schlüter, A. 1950, *Zs. Naturforschung* 5a, 72.