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LARGE-SCALE BACKGROUND TEMPERATURE AND MASS FLUCTUATIONS DUE TO SCALE-INVARIANT PRIMEVAL PERTURBATIONS

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ABSTRACT

The large-scale anisotropy of the microwave background and the large-scale fluctuations in the mass distribution are discussed under the assumptions that the universe is dominated by very massive, weakly interacting particles and that the primeval density fluctuations were adiabatic with the scale-invariant spectrum $P \propto$ wavenumber. This model yields a characteristic mass comparable to that of a large galaxy independent of the particle mass, m_x, if $m_x \gtrsim 1$ keV. The expected background temperature fluctuations are well below present observational limits.

Subject headings: cosmic background radiation — cosmology — galaxies: formation

I. INTRODUCTION

It is useful to consider which scenarios for the nature and evolution of the mass distribution in the universe can fit the observations without undue contrivance. We may hope that as the observations improve the list of candidates will narrow, and that this process may in time help guide us to a fundamental theory of the origin of structure in the universe. The picture discussed here is motivated by the argument that, if the initial conditions for conventional classical cosmology were set at some exceedingly high redshift (perhaps the Planck time or the grand unified theory epoch), and if the initial conditions did not involve exceedingly large or small numbers, then the cosmological density parameter ought to be $\Omega = 1$, and, assuming adiabatic density perturbations, the power spectrum P ought to be proportional to the wavenumber k (e.g., Hawking 1982). Density fluctuations on scales greater than the horizon are defined as in Peebles (1980, § 91, hereafter LSS). If $P \propto k$, perturbations to the geometry diverge only as $\log k$ so the cutoffs can be at very large and small k and the spectrum can be truly scale invariant (Harrison 1970; Peebles and Yu 1970, § VI*a*; Zel'dovich 1972). Density fluctuations appearing on the horizon have a fixed value, $\delta M/M \sim 10^{-4}$, and, as this number is not greatly different from unity, we might imagine it is fixed by fundamental physics.

Observational constraints include the measurements of background temperature fluctuations, $\delta T/T \lesssim 1 \times 10^{-4}$ (Boughn, Cheng, and Wilkinson 1981; Melchiorri *et al.* 1981) and the estimate of the coherence length of the galaxy distribution. They tell us $P \propto k$ is unacceptable in the usual cosmology with baryons, electrons, radiation, and massless neutrinos. The observations of $\delta T/T$ imply $\varepsilon \lesssim 1 \times 10^{-4}$. The wanted growth factor $\sim 10^4$ to make nonlinear density fluctuations form by the present epoch is obtained if $\Omega \sim 1$ (Silk and Wilson 1981) because density fluctuations on scales greater than the matter-radiation Jeans length, λ_x, grow before decoupling. However, λ_x is large so it makes the mass autocorrelation function unacceptably broad (Silk and Wilson 1981; Peebles 1981*a*). Press and Vishniac (1980) emphasized that there is no hope for the development of appreciable density fluctuations on scales smaller than λ_x.

The problem is relieved if the universe is dominated by massive, weakly interacting particles because density fluctuations on small scales can grow before decoupling. This effect has been widely discussed in the case that the weakly interacting particle mass, m_x, is some tens of electron volts (e.g., Doroshkevich *et al.* 1981 and references therein). If $\Omega \sim 1$, the mass coherence length is broad but perhaps not unacceptable (Peebles 1982*a*). The case $m_x \sim 1$ keV is discussed by Bond, Szalay, and Turner (1982) and Blumenthal, Pagels, and Primack (1982). I discuss here a particularly simple and perhaps important limiting case, $m_x \gtrsim 1$ keV. The main results are the spectrum of mass fluctuations, which seems quite reasonable for the production of galaxies and clusters of galaxies, the statistical character of the background temperature fluctuations, and the expected size of the mass density anticorrelation at large separations.

II. CALCULATION

I assume zero cosmological constant and $\Omega = 1$, the mass being mainly in weakly interacting particles, mass m_x. Following Davis *et al.* (1981), I take the particle distribution in phase space at $v \ll c$ in the absence of perturbations to be

$$\mathcal{N} = (\exp m_x vc/kT_x \pm 1)^{-1}, \quad (1)$$

meaning the particles thermally decoupled from the radiation when the particles were relativistic. The parameter T_x is adjusted to make $\Omega = 1$. Hubble's constant and the present radiation temperature are written as

$$H = 100\,h\ \mathrm{km\ s^{-1}\ Mpc^{-1}}, \quad T = 2.7\,\tau\ \mathrm{K}. \quad (2)$$

a) *Free Streaming*

The smoothing of the mass distribution by the motions of the free particles has been widely discussed (e.g., Wasserman 1981). For either sign in equation (1), one finds that, to good accuracy, the rms peculiar velocity is

$$v = (12)^{1/2} \pi^{2/3} \hbar \rho_c^{1/3} m_x^{-4/3} (1 + z), \quad (3)$$

where $v \ll c$, ρ_c is the Einstein–de Sitter density, and z is the redshift. The rms coordinate displacement is

$$r = \int_0^{t_0} dt\, v(t)(1 + z), \quad (4)$$

expressed in units of proper displacement at the present epoch. The dominant part of this integral comes between the epochs z_1, where v first drops appreciably below c, and z_{eq}, where $\rho_r \sim \rho_x$ (the universe becomes matter dominated). Taking account only of this interval, one gets (for $h = \tau = 1$)

$$r \sim (0.8 + 0.3 \log m_x) m_x^{-4/3}\ \mathrm{Mpc}, \quad (5)$$

where m_x is expressed in units of kilo-electron volts. Thus, if $m_x \gtrsim 1$ keV, thermal motions are unimportant on the characteristic scale, $r \sim 3\ h^{-1}$ Mpc, of large galaxies. This is seen also in Figure 2 of Peebles (1982*a*), Figure 1 of Bond, Szalay, and Turner (1982), and Figure 1 of Blumenthal, Pagels, and Primack (1982).

b) *Spectrum of Mass Distribution*

When thermal motions are negligible, the free particles can be described as a zero pressure ideal fluid. At high redshift, the density fluctuations are supposed to be adiabatic with spectrum $P \propto k$. The part that appears within the horizon while $\rho_r > \rho_x$ stops growing until $\rho_r \sim \rho_x$ (because the radiation distribution is oscillating as an acoustic wave). This tilts the mass spectrum to $P \propto k^{-3}$ at large k (LSS § 92D). At small k, the spectrum keeps its primeval shape, $P \propto k$. The results of a numerical integration of the shape of the spectrum between these limiting cases (Peebles 1982*a*) are approximated by the formulae

$$P = Ak(1 + \alpha k + \beta k^2)^{-2},$$

$$\alpha = 6(\tau/h)^2\ \mathrm{Mpc}, \quad \beta = 2.65(\tau/h)^4\ \mathrm{Mpc}^2. \quad (6)$$

Here and below, k is expressed in units of radians per megaparsec at the present epoch.

c) *Large-Scale Background Temperature Fluctuations*

If the primeval density fluctuations are adiabatic, the angular distribution of the background temperature is (LSS eq. [93.35])

$$T(\theta, \phi)/T_b - 1 = -\tfrac{1}{2} H^2 \sum k^{-2} \delta_{\boldsymbol{k}} \exp i\boldsymbol{k} \cdot \boldsymbol{x}. \quad (7)$$

This equation assumes that the density fluctuations on scales of interest appear on the horizon when $\rho_r < \rho_x$ and that the universe subsequently has behaved like the Einstein–de Sitter model. The term $\propto Y_1^m$ due to our peculiar motion is not included. The vector $\boldsymbol{x}$ with length $2cH^{-1}$ points in the direction of observation. In the linear perturbation approximation, the present mass density is

$$\rho = \rho_b\left(1 + \sum \delta_{\boldsymbol{k}} \exp i\boldsymbol{k} \cdot \boldsymbol{r}\right). \quad (8)$$

The expansion of the background temperature in spherical harmonics is

$$T(\theta, \phi) = T_b\left(1 + \sum a_l^m Y_l^m\right),$$

$$a_l^m = -2\pi i^l H^2 \sum_{\boldsymbol{k}} k^{-2} \delta_{\boldsymbol{k}} j_l(kx) Y_l^m(\Omega_k). \quad (9)$$

With the normalization

$$\sum_{\boldsymbol{k}} \to \int k^2\, dk\, d\Omega_k / 4\pi, \quad (10)$$

we get

$$a_l^2 \equiv \langle |a_l^m|^2 \rangle = \pi H^4 \int_0^\infty dk\, k^{-2} |\delta_k|^2 j_l(kx)^2. \quad (11)$$

At large angular scales, $|\delta_k|^2 \propto k$, and the integral is

$$\int_0^\infty \frac{dy}{y} j_l(y)^2 = [2l(l+1)]^{-1}. \quad (12)$$

It is convenient to express the normalization in terms of the quadrupole moment a_2. Then (eq. [6]),

$$A = 12(a_2)^2 / (\pi H^4), \quad (13)$$

and the angular power spectrum of the background is

$$(a_l)^2 = 6(a_2)^2/[l(l+1)]. \tag{14}$$

The s-wave part a_0 diverges, corresponding to the logarithmic divergence in the primeval spectrum $P \propto k$. The p-wave does not include the effect of our peculiar motion.

The autocorrelation function of the background temperature after the p-wave has been eliminated is (LSS § 46)

$$\begin{aligned} w(\theta_{12}) &= \langle T_1 T_2 \rangle / \langle T \rangle^2 - 1 \\ &= \sum_{l>1} (a_l)^2 (2l+1) P_l(\cos\theta_{12})/4\pi. \end{aligned} \tag{15}$$

At $\theta_{12} \ll 1$ radian, this with equation (14) gives

$$w(\theta) = (3/\pi) a_2^2 \log(\Theta/\theta), \tag{16}$$

where Θ is on the order of the size of the field within which $w(\theta)$ is measured.

d) *Mass Fluctuations*

A convenient measure is the rms fluctuation in the mass found within a randomly placed sphere of radius R. We have from equation (8)

$$\delta M/M = 3 \sum \delta_k (\sin kR - kR\cos kR)/(kR)^3. \tag{17}$$

With equations (6) and (10), this gives the rms value

$$\frac{\delta M}{M} = \left(\frac{108}{\pi}\right)^{1/2} a_2 \left(\frac{c}{H}\right)^2 \left(\frac{h}{\tau}\right)^4 I(Rh^2/\tau^2),$$

$$[I(R)]^2 = \int_0^\infty \frac{k^3\,dk}{(1+6k+2.65\,k^2)^2} \times \frac{(\sin kR - kR\cos kR)^2}{(kR)^6}. \tag{18}$$

Figure 1 shows $\delta M/M$ for $h = \tau = 1$ and the normalization

$$\frac{\delta M}{M}(R = 8\ \mathrm{Mpc}) = 1, \tag{19}$$

which agrees with the rms fluctuation, $\delta N/N$, in the counts of bright galaxies at $R = 8\ h^{-1}$ Mpc (LSS § 59; Davis and Peebles 1983). At $R \lesssim 0.1$ Mpc, $\delta M/M$ varies only slowly, as $|\log R|^{1/2}$, fluctuations with fixed variance per octave of R having been stored when $\rho_r \gtrsim \rho_x$. Shortward of the scale r fixed by m_x (eq. [5]), the power spectrum is truncated by thermal motions so $\delta M/M$ is independent of R. At large R, $\delta M/M \propto R^{-2}$, which is the primeval spectrum. At $1 \lesssim R \lesssim 30$ Mpc, $\delta M/M$ varies roughly as $R^{-1.25}$.

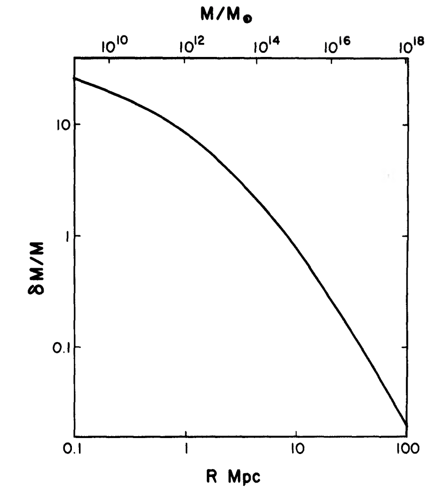

FIG. 1.—The rms fluctuation in the mass found within a sphere of radius R. The top scale is the mean mass within the sphere. The vertical normalization is adjusted to make $\delta M/M$ agree with the observed fluctuation, $\delta N/N$, in the count of bright galaxies at $R = 8\ h^{-1}$ Mpc. This curve is based on the assumptions that the universe is dominated by very massive, weakly interacting particles and that the initial density fluctuations were adiabatic with power spectrum $P \propto k$.

Figure 1 can be compared to the curve $n = 1$ in Figure 2 of Bond, Szalay, and Turner (1982). The short-wavelength cutoff in the latter curve results from the assumption $m_x \sim 1$ keV. The Bond *et al.* curve peaks at $M \sim 10^{12}\ M_\odot$ as does the Jeans mass found by Blumenthal, Pagels, and Primack (1982), both of which agree with the position of the break in Figure 1. This is consistent with the fact that $m_x = 1$ keV is roughly equivalent to the limit of very large m_x.

By equations (18) and (19), the expected quadrupole moment of the microwave background is

$$a_2 = 3.5 \times 10^{-6}. \tag{20}$$

For the case of baryon-dominated matter, Silk and Wilson (1981) found $a_2 = 4 \times 10^{-5}$. The larger value is the result of the much broader mass coherence length, which increases the integral $J_3 = \int r^2\,dr\,\xi$ (Peebles 1981*b*).

The Boughn, Cheng, and Wilkinson (1981) measurements imply $a_2 \sim 3 \times 10^{-4}$, but the more recent measurements of Lubin (1982) and Fixsen (1982) suggest the extragalactic anisotropy may be appreciably less than that and hence perhaps not inconsistent with equation (20).

The expected temperature anisotropy at intermediate angular scales is given by equation (16). The rms fluctuation in T smoothed over $\theta = 10°$ in a sample of size $\Theta = 100°$ is

$$\delta T/T = w^{1/2} \sim 5 \times 10^{-6}. \tag{21}$$

The mass autocorrelation function is

$$\xi(r) = \langle\rho(r)\rho(0)\rangle/\langle\rho\rangle^2 - 1 = \int_0^\infty k^2\, dk\, P \sin kr/kr. \tag{22}$$

At large r, this is dominated by the primeval spectrum, $P = Ak$, so that (LSS § 42)

$$\xi = -(24/\pi)a_2^2(c/Hr)^4 = -(9.4\ \mathrm{Mpc}/r)^4. \tag{23}$$

III. DISCUSSION

Figure 1 is based on the primeval spectrum $P \propto k$ that has some theoretically attractive and perhaps important properties. It is normalized so $\delta M/M$ averaged over a sphere of radius 8 Mpc agrees with the rms fluctuation, $\delta N/N$, in the counts of bright galaxies. This radius is small enough that $\delta N/N$ is fairly well known yet large enough that the time evolution of $\delta M/M$ in the absence of nongravitational forces is accurately given by the linear perturbation calculation (Peebles and Groth 1976). If galaxy formation were aided by the Ostriker-Cowie (1981) process, the primeval amplitude could be lower. However, $\delta M/M$ is almost flat at $R \lesssim 0.1$ Mpc, so if we wanted the first generation of objects to form at redshift $z \gtrsim 10$, we could not decrease a_2 by a factor of more than about 3 unless we were willing to go to exceedingly small values of R.

This scenario yields a characteristic length on the order of 1 Mpc, which certainly is observationally interesting. It is fixed by the horizon size when $\rho_r \sim \rho_x$, independent of the free particle mass m_x if $m_x \gtrsim 1$ keV. At R between 3 and 30 Mpc, $\delta M/M$ scales roughly as $M^{-1/2}$ so the similarity argument (LSS §§ 26 and 73) suggests this could develop into the observed mass clustering hierarchy (Davis and Peebles 1983).

All of this discussion has dealt with the second moment of the mass distribution. It remains to be seen whether the rather rapidly decreasing large-scale fluctuations, $\delta M/M \propto R^{-2}$, could produce large-scale features such as the cluster-cluster coherence length, $r_c \sim 25\ h^{-1}$ Mpc (where $\xi_{cc} = 1$; Hauser and Peebles 1973; Bahcall and Soneira 1982).

In the spirit of the scenario, we expect $\Omega = 1$. The relative velocity data at $r \lesssim 1\ \mathrm{h}^{-1}$ Mpc in the Center for Astrophysics (CfA) redshift sample yield $\Omega \sim 0.3$, but there is an indication of a more broadly distributed mass component, so $\Omega = 1$ certainly is not improbable. The p-wave part of the background would be almost entirely due to our peculiar velocity, v. The expected rms velocity is ~ 1000 km s^{-1} if $\Omega \sim 1$ (LSS § 74), which is larger than the observed background p-wave but not to be ruled out: as we are not in a particularly strongly clustered spot, we might expect our velocity is no greater than the median, and as the degree of clustering is highly variable, we might expect the distribution of v is broad and hence that the median is well below the rms value.

Another interesting possibility is that the mass autocorrelation function, $\xi(r)$, vanishes at $r \gtrsim 20\ h^{-1}$ Mpc. If so, the mass fluctuations at smaller r, where $\xi > 0$, produce long-range gravitational potential fluctuations that lead to a value of a_2 that agrees with the present observations (Peebles 1981*b*, 1982*b*). This is an attractive coincidence. Of course, if a_2 were substantially lowered, this picture would be much less attractive. By going from a flat spectrum to $P \propto k$ in the present scenario, we have introduced anticorrelated mass fluctuations, which strongly reduces a_2 (Peebles 1981*b*, § V; Silk and Wilson 1981). If this model is correct, a measurement of the background temperature autocorrelation function, $w(\theta)$, at large θ (eqs. [16] and [21]) will require a considerable advance in the observations. The computed mass anticorrelation amounts to $\xi(20\ \mathrm{Mpc}) \sim -0.05$. As this number is based on the linear approximation, it is not highly accurate, but certainly $\xi(r)$ must be negative at large r to reduce a_2. There is a tentative indication of anticorrelation at $hr \sim 30$ Mpc in the CfA redshift catalog (Davis and Peebles 1983), but a firm test awaits deeper samples.

If the extragalactic part of a_2 were found to be appreciable, say $a_2 \gtrsim 3 \times 10^{-5}$, several interpretations could be considered. It is doubtful that the discrepancy with equation (20) could be due to an underestimate of ξ (by a factor ~ 100 at $r \sim 5\ h^{-1}$ Mpc) for that would conflict with the relative velocity data (Davis and Peebles 1983). The anisotropy could be inserted by sources along the line of sight. If the source density were proportional to the mass density, the angular spectrum in the present scenario would be $a_l \propto l^{1/2}$ (compared to $a_l \propto l^{-1}$ for the intrinsic part in eq. [14]). Hogan (1982) has pointed out that one could assume that the luminosity per unit mass averaged over the mass coherence length $\sim 5\ h^{-1}$ Mpc is a random variable with broad dispersion, so that the background temperature fluctuations approximate white noise, $a_l \sim$ constant. In these two cases, the rms fluctuation in the background temperature smoothed over the angle θ scales as $\delta T/T \sim$

$\theta^{-3/2}$ and θ^{-1} respectively. Thus, if a_2 can be detected, it ought to be possible to check the spectrum. If these two models can be ruled out, likely interpretations of a large a_2 will be that the growth of clustering at $r \lesssim 20$ h^{-1} Mpc was hindered, perhaps by thermal motions ($m_x \ll 1$ keV), or else that the primeval spectrum was not adiabatic with $P \propto k$.

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