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STELLAR LUMINOSITY FUNCTIONS IN THE *R*, *I*, *J*, AND *K* BANDS OBTAINED BY TRANSFORMATION FROM THE VISUAL BAND

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ABSTRACT

We transform the stellar luminosity function that has been measured in the visual band into the *R*, *I*, *J*, and *K* bands, where it has not yet been obtained directly. The transformation is accomplished by subdividing the total visual function, which includes all stars, into sub-luminosity functions for each luminosity class (supergiants through white dwarfs), applying the known $(V - D)$ color, $D = R, I, J, K$, for each spectral type, and then summing the resultant transformed sub-luminosity functions into a total luminosity function for the band *D*. Simple analytic forms which accurately fit the transformed luminosity functions are given. The possibility of a systematic error resulting from the existence of a very red stellar population not accounted for in the visual band luminosity function is considered. Application of these luminosity functions and sub-luminosity functions to the study of galactic structure, including star count models in the *R*, *I*, *J*, and *K* bands is briefly discussed.

Subject headings: infrared: general — luminosity function — spectrophotometry — stars: stellar statistics

I. INTRODUCTION

For most of this century observational data on stars have been accumulated primarily in the visual and blue bands and their associated bands such as photovisual, photographic, certain photoelectric bands, etc. During the last 20 years or so interest has slowly migrated into bluer bands (such as ultraviolet and X-ray) and redder bands (such as infrared and radio) because of the availability of new detectors and observing opportunities from space. It has also became clear that many of the pending astronomical questions are more likely to be answered with data from these newer bands. Still, the principal data base is in the visual bands, and it is often necessary to transform the available visual data into these other bands in order to obtain a preliminary estimate of what is to be measured, and then once the data in the new bands is on hand, to transform it back to the visual band for comparison in order to determine whether anything new has been learned.

In this paper we transform the stellar luminosity function from the visual band where it has been measured, into the red and infrared *R*, *I*, *J*, and *K* Johnson bands, where it has not yet been obtained directly. The transformation is accomplished by subdividing the total visual function, which includes all stars, into sub-luminosity functions for each luminosity class (supergiants through white dwarfs), and applying the known $V - D$ color, $D = R, I, J, K$, for each spectral type, and then summing the resultant transformed sub-luminosity functions into a total luminosity function for the band *D*. The sub-luminosity functions for each luminosity class and the fraction of stars on and off the main sequence are also examined.

Figure 1 compares the Johnson (1965*b*) *B*, *V*, *R*, *I*, *J*, and *K* band response functions used in this paper (effective wavelengths of 4400 Å, 5500 Å, 7000 Å, 9000 Å,

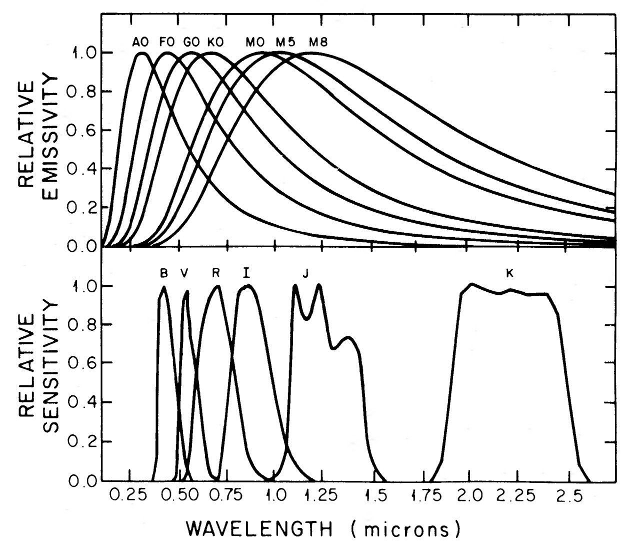

FIG. 1.—Comparison of the Johnson (1965*b*) *B*, *V*, *R*, *I*, *J*, and *K* band response functions with approximate blackbody spectra for main-sequence dwarf stars of spectral type A0 to M8 derived from photometric temperatures listed in Paper III. The response functions and spectra are shown normalized to the same peak sensitivity or emissivity.

1.25 μm, and 2.2 μm, respectively) with approximate blackbody spectra for main sequence (luminosity class V) dwarf stars of spectral type A0–M8 derived from the photometric temperatures listed by Bahcall and Soneira (1981*a*, hereafter Paper III). The response functions and spectra are shown normalized to the same peak sensitivity or emissivity.

We have selected the Johnson bands because they are both widely used and well documented. While the U, B, V color system of Johnson and Morgan (1953) has become the undisputed standard color system in the wavelength interval between 3000 and 6000 Å, there are a large number of competing systems for R and beyond. Of these, the most commonly used, besides the Johnson system, are the Kron R_K and I_K bands (Kron, White, and Gascoigne 1953) and the Becker (1946) R_B band (from his RGU system). The Kron R_K and I_K bands are quite similar to the Johnson bands, but are 200 and 750 Å bluer, respectively. The Becker R_B band has an effective wavelength of 6400 Å, but a full width at half-maximum of only 430 Å (Buser 1978), a factor of 5 narrower than the Johnson R band.

While the spectra of Figure 1 are very broad and there is considerable overlap among the different bands, it is clear that the B and V bands are most sensitive to F and G dwarfs, the R and I bands to K and early M dwarfs, and the J and K bands to late M dwarfs and even redder stars, possibly from a yet to be discovered luminosity class or population of stars (Bahcall and Soneira 1981*b*, hereafter Paper IV). There is an obvious danger in extrapolating data accumulated in one band into another that is far removed from the first in wavelength. The problem is more serious than a reduction in accuracy due to a far reaching extrapolation in wavelength: there is a possibility or even likelihood that unusually red classes of stars will be missed altogether when stars are selected according to their apparent visual magnitude. When the visual data is transformed, such stars, which may contribute significantly in the redder bands at relatively bright absolute magnitudes, will be missing. This problem is discussed further in § II*d*.

The faint end of the disk luminosity function in the visual band is well determined out to only M5 V (corresponding to $M_V \approx +13$ mag) (Paper IV; Bahcall and Soneira 1980*a*, hereafter Paper I; Bahcall and Soneira 1980*b*, hereafter Paper II). The uncertainty in the M dwarfs in the visual band will manifest itself at increasingly brighter absolute magnitudes for the R, I, J, and K bands. These redder bands are therefore particularly well suited for extending our knowledge of the dim end of the luminosity function. The technique for accomplishing this from star counts is discussed extensively in Paper IV.

Discussion in the main text of § II is limited to disk population stars; the spheroid stellar population is treated in § II*f*. Section II*d* examines the available data on star counts for evidence on the existence of a red population of stars which would make a negligible contribution in the visual band but a significant one in the redder R, I, J, and K bands. Section III discusses the application of the luminosity functions and sub-luminosity functions considered in this paper to the study of galactic structure.

II. THE TRANSFORMATION METHOD

In principle, in order to transform the stellar luminosity function from some reference band, which we choose here to be V, into some other band D, it is necessary to know the $V - D$ color for each star in the distance limited sample that is used to calculate the original function in V. This transformation can be considered in terms of a $(V - D, M_V)$ scatter plot for all stars in the sample: summing stars in horizontal strips gives the visual luminosity function, and summing stars along diagonal strips (45° to the horizontal if the axis scales are the same) gives the D band luminosity function.

Such detailed information is not currently available, hence it is necessary to use an indirect mean color method. Sub-luminosity functions, which specify the number of stars belonging to each luminosity class (supergiants through white dwarfs), are determined for each absolute magnitude, M_V, by subdividing the total luminosity function. For the nth sub-luminosity function, stars of absolute visual magnitude, M_V, are transformed to the absolute D band magnitude, M_D, by $M_D = M_V - C_{D_n}(M_V)$, where $C_{D_n}(M_V)$ is the mean $V - D$ color for stars in the nth luminosity class with absolute magnitude M_V. The mean colors are more readily available, and have been tabulated, for example, by Johnson (1966). Figure 2 shows $V - I$ colors as a function of absolute visual magnitude for each of the luminosity classes. Each point in the figure represents the mean color for a given spectral type and luminosity class. The absolute magnitudes plotted are the means for each spectral type and luminosity class. The problem with the mean color method is that it neglects the intrinsic scatter of the colors of stars of a given absolute magnitude and luminosity class. Fortunately, the sequences in the H-R diagram are sufficiently narrow that the use of mean colors should not produce any significant errors.

A complication arises because several of the color functions C_{D_n} are not single-valued functions of M_V. We have therefore further subdivided these luminosity classes into subclasses so that the C_{D_n} are single valued. A total of 11 luminosity subclasses are used: two for each of the supergiant classes (Ia and Ib), two each for the bright giants (II) and giants (III), and one for the subgiants (IV), main sequence (V), and white dwarfs (VII). Figure 2 shows these subdivisons of the $V - I$ color diagram: the filled circles denote the positive-slope branches, and the open circles the negative-slope branches. These subdivisions are identical for all bands discussed here.

After transformation to the D band, each of the sub-luminosity functions must be renormalized to stars per unit magnitude interval because the colors C_{D_n} are not constant functions, i.e., a unit magnitude interval in V does not transform to a unit magnitude interval in D. The transformed sub-luminosity functions are then added together to give the desired total luminosity function in the D band.

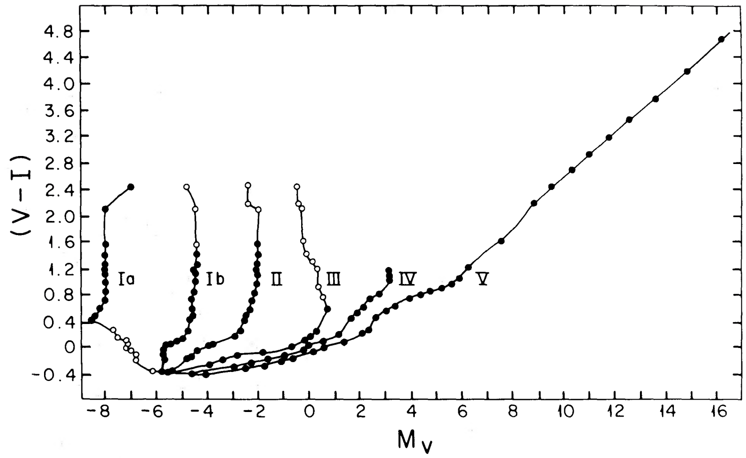

FIG. 2.—Mean $(V-I)$ colors as a function of absolute visual magnitude for each luminosity class. The filled circles denote the positive-slope subclasses, and the open circles denote the negative-slope subclasses defined so as to make the color functions C_{D_n} single valued.

a) *Visual Band Source Data*

The visual band luminosity function we use is an analytic fit to the data of McCuskey (1966), Luyten (1968), and Wielen (1974) given by equation (1) of Paper I (see eq. [3] and Table 2, below), with the bright-end cutoff $M_b = -9$, and the dim-end cutoff $M_d = +\infty$. The sub-luminosity functions, ϕ_n, are estimated by multiplying the total luminosity function, ϕ, by the fraction of stars on and off the main sequence for each absolute magnitude. For $M_V \geq 3.7$ mag, there are only white dwarfs (class VII) and main sequence stars (class V). From Paper I,

$$\begin{aligned} \phi_{\rm V}(M_V) &= \phi(M_V)[1 - f_{\rm VII}(M_V)] \,, \\ \phi_{\rm VII}(M_V) &= \phi(M_V) f_{\rm VII}(M_V) \,, \\ f_{\rm VII}(M_V) &= 0.15 \times 10^{-0.25|M_V - 15|} \,, \\ M_V &\geq 3.7 \,, \end{aligned} \tag{1}$$

with $\phi_n(M_V > 3.7) = 0$ for all other classes. For the bright absolute magnitudes $M_V \lesssim 3.7$ mag, there are main sequence stars and several classes of giants (Ia–IV). From Paper IV, which specifies the total number of giants (all classes) as a function of absolute magnitude, we obtain

$$\begin{aligned} \phi_n(M_V) &= \phi(M_V)[1 - f_V(M_V)]\tau_n(M_V) \,, \\ & n \neq {\rm V, VI, VII} \,; \\ \phi_{\rm V}(M_V) &= \phi(M_V) f_V(M_V) \,; \\ \phi_{\rm VII}(M_V) &= 0 \,; \\ f_V(M_V) &= 0.44 \exp[1.5 \times 10^{-4}(M_V + 8)^{3.5}] \,; \\ \sum_n \tau_n(M_V) &\equiv 1 \,; \quad n \neq {\rm V, VI, VII} \,; \\ M_V &< 3.7 \,, \end{aligned} \tag{2}$$

where n refers to classes Ia through IV, and $\tau_n(M_V)$ is the fraction of *all* giants with absolute magnitude M_V that are in class n. The τ_n functions were estimated from the H-R diagrams of all stars in Volumes **1** and **2** of the Michigan Spectral Catalog within 100 pc of the Sun (Houk and Fesen 1978). (Note that τ_n refers to stars of a given absolute magnitude so that it can also be accurately estimated from apparent magnitude limited surveys.)

Figure 3 shows our estimates of $\tau_n(M_V)$ in such a manner that the height of each region n for a given absolute magnitude, M_V, corresponds to the fraction τ_n at that absolute magnitude. The designations + and − in many of the regions refer to the positive and negative-slope luminosity subclasses defined so as to make the color functions C_{D_n} single valued, as discussed above. Figure 4*a* displays the resultant visual band sub-luminosity functions for each luminosity class (ignoring the + and − distinctions).

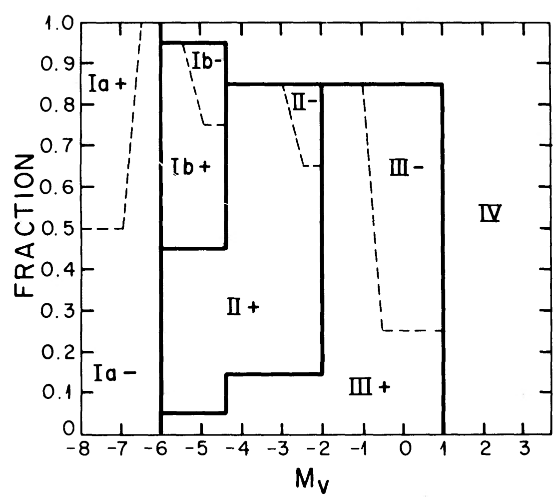

FIG. 3.–Fraction, $\tau_n(M_V)$, of all non-main sequence stars (giants) with absolute magnitude M_V that are in luminosity class n estimated from the H-R diagrams of Houk and Fesen (1978). The designations + and − in many of the regions refer to subclasses defined so as to make the color functions C_{D_n} single valued (see text). The height of each region n for a given absolute magnitude, M_V, corresponds to the fraction τ_n at that absolute magnitude.

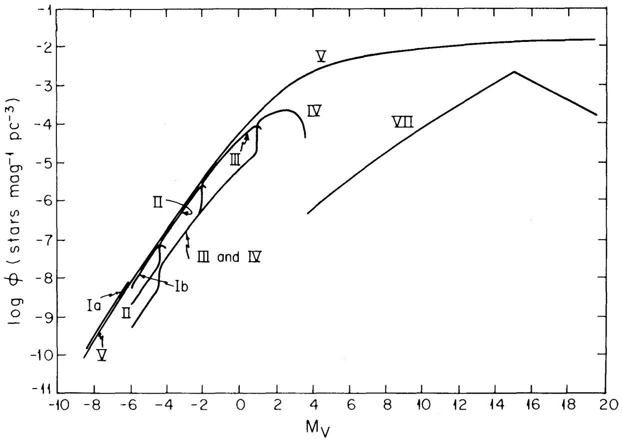

FIG. 4*a*

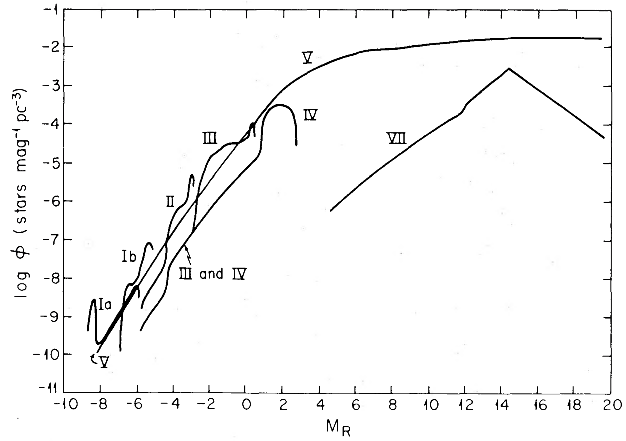

FIG. 4*b*

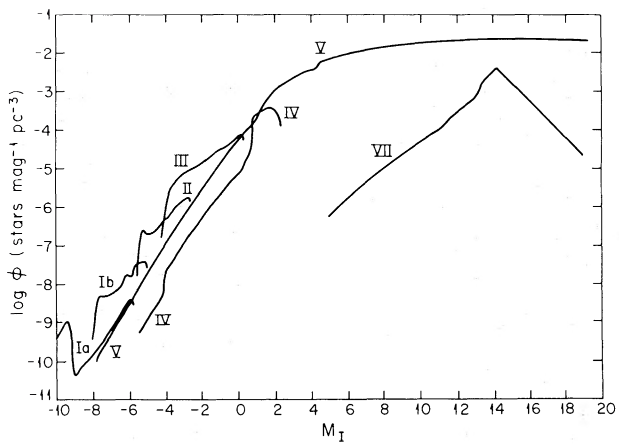

FIG. 4*c*

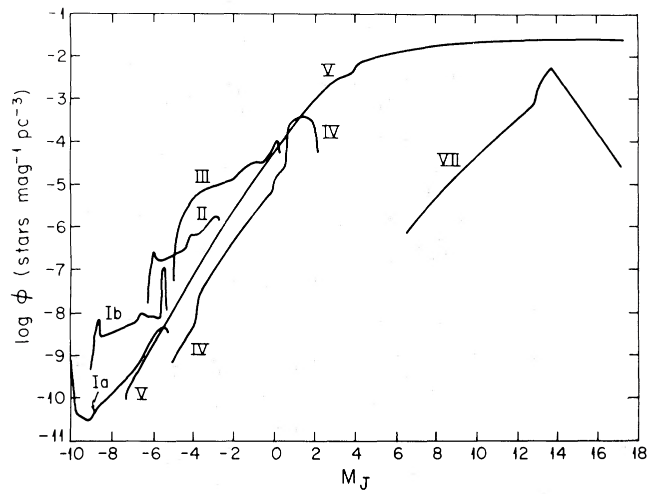

FIG. 4*d*

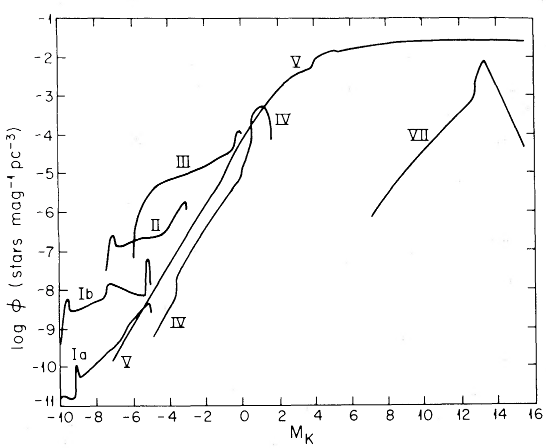

FIG. 4*e*

FIG. 4.—Sub-luminosity functions for each luminosity class in the (*a*) V, (*b*) R, (*c*) I, (*d*) J, and (*e*) K bands. The sum of all the sub-luminosity functions gives the total luminosity function, ϕ.

b) Color Data

The mean $V - D$ colors, $D = R, I, J, K$ as a function of spectral type for luminosity classes Ia, Ib, III (for spectral types G5 and later), and V have been obtained from Johnson (1966). The mean colors for luminosity classes and spectral types not listed by Johnson were interpolated as follows: the mean colors for luminosity class II bright giants were assumed equal to the class I supergiants. Mean colors for class III spectral types earlier than G5 were interpolated from supergiant and main-sequence colors. Mean colors as a function of spectral type for luminosity class IV have been obtained by interpolation of the class III and class V values. The difference in color between each pair of tabulated Johnson values used for interpolation is typically 0.10 mag, and is seldom larger than 0.30 mag. The uncertainties in the interpolated values should be of order one-third of these color differences.

The H-R diagram is needed to translate the spectral types into absolute visual magnitudes. We have used the values from Blaauw (1963) for luminosity classes Ia, Ib, and II. Values for spectral types F5 and earlier in class III are from FitzGerald (1969). Values for class III with spectral types later than F5, for class IV (all spectral types), and for class V spectral types K3 and earlier are from Keenan (1963). Values for class V spectral types K5 and later are from Johnson (1965*a*).

For white dwarf stars (class VII), the mean $V - R$ and $V - I$ colors as a function of absolute visual magnitude, M_V, are directly calculated from the $V - R_K$ and $V - I_K$ colors in the Kron band listed by Greenstein (1976), using the relations given by Greenstein (1976) and Eggen (1971). Similarly, the mean $V - J$ and $V - K$ white dwarf colors are from Mould and Liebert (1978), using the transformations into the Johnson bands given by Frogel *et al.* (1978). Table 1 lists the adopted white dwarf mean colors as a function of absolute visual magnitude. The values for absolute magnitudes greater than $+15$ are linear extrapolations. For the J and K bands, the colors for absolute magnitudes less than $+11$ are also extrapolated. The uncertainties in the transformed luminosity functions resulting from the use of these extrapolated colors is small because the contribution of white dwarfs is sharply peaked near $M_V = +15$, never amounting to more than 15% of the total luminosity function (see Fig. 4).

TABLE 1

ADOPTED WHITE DWARF MEAN COLORS

M_V	$B-V$	$V-R$	$V-I$	$V-J$	$V-K$
8	−0.50	−0.52	−0.79	−1.39	−1.75
9	−0.35	−0.46	−0.72	−1.09	−1.37
10	−0.20	−0.40	−0.65	−0.79	−0.99
11	−0.05	−0.34	−0.58	−0.49	−0.61
12	0.10	−0.20	−0.34	−0.19	−0.23
13	0.25	0.08	−0.10	0.11	0.15
14	0.40	0.36	0.38	0.74	0.81
15	0.76	0.64	0.86	1.37	1.67
16	1.26	0.92	1.34	2.00	2.43
17	1.70	1.20	1.82	2.63	3.19
18	1.81	1.48	2.30	3.26	3.95

c) The Transformed Luminosity Functions

Using the mean color data of § II*b*, the visual band sub–luminosity functions are transformed into sub–luminosity functions in the R, I, J, and K bands. Because the mean colors C_{D_n} are not constant functions, a unit magnitude interval in V does not transform to a unit magnitude interval in D, hence the transformed sub–luminosity functions must be renormalized to stars per unit magnitude interval. (The computation method we use automatically performs the renormalization in the final rebinning of the sub–luminosity functions in the D band because the absolute magnitude interval used in performing the transformation, 0.01 mag, is much smaller than either the final magnitude interval of 0.2 mag that is used in estimating the value of the luminosity function or the color gradient dC/dV.)

Figure 4 displays the resultant sub–luminosity functions for each luminosity class in each band.

The solid curves in Figure 5 show the R, I, J, and K band luminosity functions that result from adding together the transformed sub–luminosity functions. Also, included in the figure are the original V luminosity function and a transformed B band luminosity function (see below). The functions have been multiplied or divided by the indicated factors for convenience in display. As expected, the basic form of the visual luminosity function: a steeply falling bright end and a rather flat dim end is retained by each of the transformed luminosity

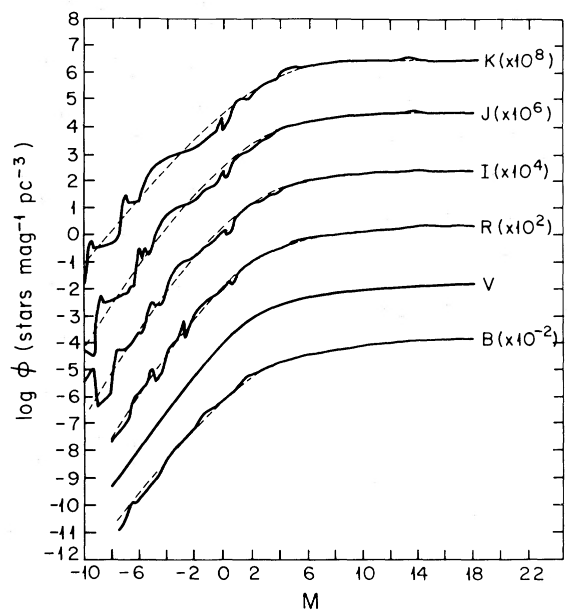

FIG. 5.—Total stellar luminosity functions, ϕ, in the B, V, R, I, J, and K bands. The solid curves give the calculated values obtained by transformation from the V band. The dashed curves are the analytic approximations given by eq. (3) with the parameters listed in Table 2.

functions. The R, I, J, and K luminosity functions have values (stars mag^{-1} pc^{-3}) that are greater than the visual luminosity function by factors of order 1.5, 2, 2.5, and 3, respectively, for absolute magnitudes $M \gtrsim 0$. Of course, when integrated over a specified range of spectral types, the normalizations for all the luminosity functions are identical.

The several prominent peaks and valleys at the bright end, $M \lesssim 0$, of the transformed R, I, J, and K luminosity functions are due to the fact that occasionally $dC_n(M_V)/dM_V \approx 1$ over a small range in M_V. Small variations to either $C_n(M_V)$ or $\tau_n(M_V)$ substantially affect the location and size of these features in the computed luminosity functions. The smaller variations at the bright end are caused by the sharp cutoffs introduced in $\tau_n(M_V)$, Figure 3. Hence, the features are most likely artifacts of the computation (see below).

In order to smooth over these artificial features and allow the luminosity functions to be used in a convenient manner, we have fitted the computed functions to the same analytic form used in Paper I,

$$\phi(M) = \frac{n^* 10^{\beta(M-M^*)}}{[1+10^{-(\alpha-\beta)\delta(M-M^*)}]^{1/\delta}}, \quad M_b \le M \le M_c,$$
$$= \phi(M_c), \qquad M_c \le M \le M_d,$$
$$= 0, \qquad M \le M_b, \quad M \ge M_d. \qquad (3)$$

These parameters for the B, V, R, I, J, and K luminosity functions are listed in Table 2. The fitted functions have been plotted as dashed lines in Figure 5. The quality of the fits is excellent.

We have also transformed the visual luminosity function into the blue (B) band using the $B-V$ color data from Johnson (1966). This allows a comparison with luminosity functions measured directly in the B band (Paper I) and with other B luminosity functions obtained by transformation from the V band. Our transformed B luminosity function is plotted as a solid curve in Figure 5 together with the analytic fit to a luminosity function determined directly in the B band in Paper I (*dashed curve*, using the parameter values shown in Table 2). The agreement is excellent.

d) *Uncertainties in the Transformed Luminosity Functions*

The uncertainties in the transformed luminosity functions cannot be calculated easily because the uncertainties in the source visual band data and the colors are not well known. For $M_V \lesssim -3$, the visual luminosity function is uncertain because of the paucity of objects in this range. The bright-end luminosity function cutoff, M_b, listed in Table 2 corresponds to the brightest absolute magnitude in the visual band for which normalized luminosity function data are available. For the transformations, the analytic form for the visual luminosity function was extrapolated to absolute magnitude -9 in order to eliminate "edge effects" in the computation. The brightest stars known extend to about -8.5 mag visual.

In the magnitude range $M_V \le +4$, the relative proportions, τ_n, of the five kinds of giants (classes Ia–IV) are known only roughly at best. Changes of order 20% in the relative proportions of the giant classes produce fluctuations in the total R and I luminosity functions of order 20% and 50%, respectively. For the J and K luminosity functions, similar variations in the relative proportions of the giant classes produce fluctuations of order 100% or more, and occasionally up to an order of magnitude (1000%). These fluctuations result from the interaction of the steeply falling total (visual) luminosity function (a factor of 10 every 2 magnitudes) with the varying color shifts produced by each giant class. (Shifting even a small fraction of stars from, say, absolute magnitude -1 to absolute magnitude -3 can overwhelm the comparatively small number of stars already at -3.) For the reddest J and K bands, the color shifts are quite large (up to 4 magnitudes) and differ significantly from one giant class to another, so that variations in the relative proportions of the giant classes are greatly amplified by the rapidly decreasing luminosity function.

The faint end of the visual luminosity function is well determined out to only $M_V \approx +13$ (M5 V) (see Papers I and IV). For the transformed functions, this uncertainty begins at brighter absolute magnitudes: $M_R \approx +11$, $M_I \approx +9$, $M_J \approx +8$, and $M_K \approx +7$. The data, however, do suggest that the visual luminosity function is approximately constant for $+11 \lesssim M_V \lesssim +16$, which corresponds to $+9 \lesssim M_R \lesssim +13$, $+8 \lesssim M_I \lesssim +11$, $+7 \lesssim M_J \lesssim +10$, and $+6 \lesssim M_K \lesssim +9$. The dim-end luminosity function cutoff, M_d, listed in Table 2 corresponds to the fainter limit of the above values. For the transformations, the dim-end cutoff of the visual luminosity function was extended to absolute magnitude $+\infty$, again in order to eliminate "edge effects" in the computation.

TABLE 2

LUMINOSITY FUNCTION PARAMETERS

Parameter	B	V	R	I	J	K
n^*	2.55(−3)	4.03(−3)	4.80(−3)	1.75(−2)	2.50(−2)	2.90(−2)
M^*	2.20	1.28	1.40	0.85	0.60	2.0
α	0.60	0.74	0.74	0.81	0.80	0.625
β	0.05	0.04	0.045	0.01	0.01	0.01
$1/\delta$	2.30	3.40	3.0	5.2	5.65	4.0
M_b	$\lesssim -6$	$\lesssim -6$	$\lesssim -6$	$\lesssim -6$	$\lesssim -5$	$\lesssim -5$
M_c	+15	+15	+14	+14	+11	+9
M_d	$\gtrsim +18$	$\gtrsim +16$	$\gtrsim +13$	$\gtrsim +11$	$\gtrsim +10$	$\gtrsim +9$

A potentially large systematic error may arise in the transformed luminosity functions if there exists a new luminosity class or population of stars that has not been accounted for in the source visual luminosity function. If such stars are unusually red then they will most likely be missed when stars are selected according to their apparent visual magnitude (because of their faintness in the band). However, in the longer wavelength bands, such red stars will appear increasingly brighter and could make significant contributions to a long wavelength band luminosity function over absolute magnitude ranges otherwise thought to be complete and accurately measured based on the traditional luminosity classes established in the visual band. Hence there is a possibility that the transformed R, I, J, and K luminosity functions we have calculated in this paper are underestimates; they are, strictly speaking, lower limits to the expected volume densities. Classes of very red stars are known, such as T Tauri stars (Mendoza 1966, 1968) and long period variables (Mendoza 1967), but their number densities are too low to affect the R, I, J, and K luminosity functions in the solar neighborhood (Kuhi 1964; Oort 1958; using the mass estimates of Smak 1966). Paper I found no evidence for such a new class of red stars in the range $20 \leq V \leq 22$ using an argument based on $B - V$ colors. (The test is significant, but not definitive—see below.)

There are several ways to test for incompleteness. One is to measure the luminosity function in a red band directly and compare it with the transformed function. Another method is to compare the observed star counts as a function of apparent magnitude in a red band with the counts predicted by a Galaxy model using a transformed luminosity function. A third method (which we consider here) is to examine the distribution of stars in a color-color plot as a function of apparent magnitude in order to search for red non-main sequence stars.

The source of data we use is the Chiu (1980) proper motion catalog for fields centered on SA 57 ($l^{\rm II} = 65°$, $b^{\rm II} = 86°$) and SA 68 ($l^{\rm II} = 111°$, $b^{\rm II} = -46°$), based on photometry by King (unpublished). (A third field, SA 51, is not considered here because it lies at low galactic latitude, $b^{\rm II} = 21°$.) The catalog has B, V, and R_B magnitudes for all stars brighter than $V = 21.3$ in SA 57 and $V = 20.5$ in SA 68. Each field has an area on 0.09 square degrees. The B and V magnitudes are on the Johnson system, and the R magnitude is approximately on the Becker system (§ I, above). The stars in the catalog were selected for observation based solely upon their apparent visual magnitude; hence if there is a class of unusually red stars, it is likely that most will be missed. However, the catalog extends to sufficiently dim apparent visual magnitudes that if there are a significant number of very red stars (e.g., a $V - R$ color of $+3$ or $+4$), many should appear brighter than $V = 20$. (A quantitative estimate of the limits on such a component established by this method requires detailed model building and is beyond the scope of the present work.)

Figures 6*a* and 6*b* are two-color scatter plots, $B - V$ versus $V - R_B$, for stars in the Chiu fields SA 57 and SA 68, respectively. The two-color plots permit us to separate very red main sequence stars from any stars in a new stellar luminosity class or stellar population which would, by definition, not lie near the main sequence line. The distribution of stars around the main sequence line in each plot is consistent with color measurement errors of order ± 0.15 mag (Chiu 1980). (The small color offset between the main sequence lines in SA 57 and SA 68 appears to be due to obscuration.) We conclude that there

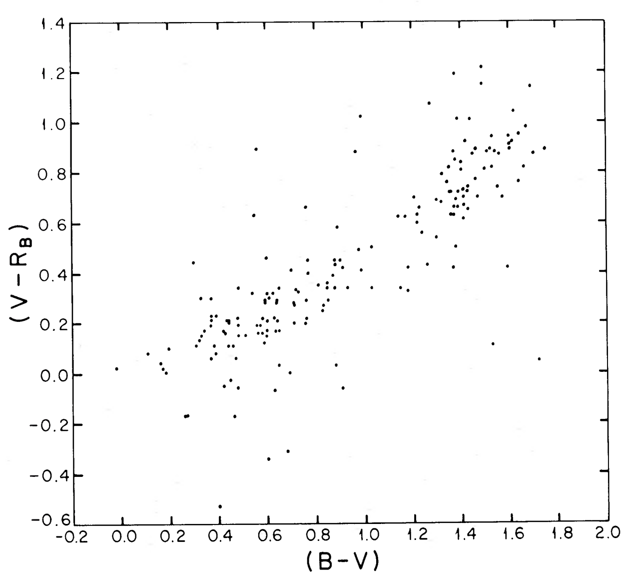

FIG. 6*a*

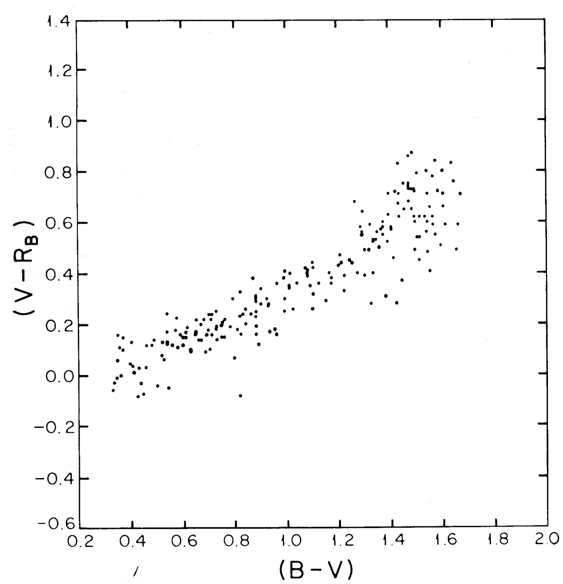

FIG. 6*b*

FIG. 6.—Two-color $B - V$ versus $V - R_B$ plot for stars in the Chiu (1980) catalog in (*a*) SA 57 brighter than $V = 21.3$, and in (*b*) SA 68 brighter than $V = 20.5$. The B and V magnitudes are on the Johnson system, and the R magnitude is on the Becker system.

is no evidence for any new luminosity class or population of stars in the Chiu catalog. A more definitive test would require that the stars selected for color measurement be chosen according to their apparent brightness in the reddest band to be measured.

e) Fraction of Non-Main Sequence Stars

The giants, which are older stars that have evolved off the main sequence, have scale heights in the direction z perpendicular to the galactic plane of 250 pc, significantly larger than for main sequence stars of the same absolute magnitude (Paper I and Paper III). White dwarf stars are presumed to be distributed like the typically older dim main sequence stars.

Hence, in constructing models of the Galaxy it is necessary to know the fraction of stars that are giants as a function of absolute magnitude in the plane of the disk ($z = 0$). This is readily calculated by summing the sub-luminosity functions for all the giant classes (Ia–IV) and dividing by the total luminosity function. We have fitted the resulting function to the same analytic form used in Paper III:

$$f = 1 - C \exp[\alpha(M + \beta)^\gamma], \quad M < M_a,$$
$$= 0, \quad M \geq M_a. \qquad (4)$$

This form accurately fits visual band data as well as the transformed data. These parameters for the B, V, R, I, J, and K luminosity functions are listed in Table 3.

f) The Spheroid Stellar Population

The luminosity function of the high-velocity stellar population that makes up the spheroidal component of the Galaxy is not as accurately known over as broad a range in absolute magnitude as the disk luminosity function. In Paper I the spheroid luminosity function was found to have the same shape as the disk luminosity function in the restricted range of absolute magnitudes over which the spheroid function has been observed, $4 \leq M_V \leq 12$. The ratio of star densities in the disk and spheroid components in the solar neighborhood is 800:1 over the range $4 \leq M_V \leq 12$. Here, as in Paper I, we assume that the shapes of the spheroid and disk luminosity functions in the visual band are the same over the larger range studied for the disk. This assumption is motivated by the fact that the luminosity function for globular clusters is similar to the function derived for the solar neighborhood in the available data range: $-3 \leq M_V \leq 8$ (see Sandage 1954; Oort and van Herk 1959; Simoda and Kimura 1968).

The color transformations from the visual band to the R, I, J, and K bands for spheroid stars are (slightly) different from those for disk stars because the high-velocity spheroid stars are metal-poor. In the standard ($B - V$, M_V) H-R diagram, the main sequence for the high-velocity stars (often referred to as the subdwarf sequence, luminosity class VI) is parallel to the disk main sequence but shifted blueward by $\Delta(B - V)$ of 0.15 mag (Eggen 1976 and Paper I). The color shift for the high-velocity stars in $R - I$ has been determined by Greenstein (1965) and Gliese (1968) to be $\Delta(R - I) = 0.1$ mag. These color shifts can be used to estimate the desired $V - D$ shifts, $D = R$, I, J, and K of high-velocity star colors in comparison with disk star colors for a given absolute V magnitude using the data of Johnson (1966). (The color excess or error that results from using one color to estimate another is greatest in the ultraviolet [typically 0.20 mag for high-velocity stars; see Paper I] and should not significantly affect our calculated color offsets in the infrared.) The estimates for the color offsets are as follows: $\Delta(V - R) = 0.15$ mag, $\Delta(V - I) = 0.25$ mag, $\Delta(V - J) = 0.3$ mag, $\Delta(V - K) = 0.4$ mag. These offsets are all sufficiently small and sufficiently uncertain that it is questionable whether the accuracy of the already poorly known spheroid luminosity function is much improved through their use.

The color-magnitude relation for the high-velocity star giant branch resembles that for an old galactic disk cluster (Roman 1965; Augensen and Buscombe 1978) and should therefore transform into other colors like the disk stars. Since the shape of the spheroid luminosity function in the absolute magnitude range for giants, $M_V \lesssim +4$, is unknown, but assumed to be identical to the disk luminosity function in the visual band as discussed above, we further assume that it is identical in shape to the disk function in the R, I, J, and K bands.

III. DISCUSSION

The stellar luminosity functions in the R, I, J, K bands obtained here by transformation from the V band represent only a first approximation in their determination. In principle, a luminosity function can be transformed from any one band into any other band to any degree of accuracy desired. In practice, the data needed to accomplish this are either unavailable, incomplete, or not in a form that can be conveniently or accurately used.

TABLE 3

NON-MAIN SEQUENCE PARAMETERS

Parameter	B	V	R	I	J	K
C	0.53	0.44	0.31	0.08	6.9(−3)	4.7(−3)
α	2.4(−31)	1.5(−4)	6.5(−4)	0.37	2.0	2.5
β	75	8.0	7.5	4.4	4.0	3.0
γ	16	3.5	3.2	1.0	0.5	0.5
M_a	4.6	3.7	2.9	2.4	2.2	1.6

The mean color method we use to accomplish the transformation is based on the assumption that all stars in the H-R diagram lie in well defined sequences (luminosity classes). Fortunately, the sequences in the *known* region of the H-R diagram are sufficiently narrow that the use of mean colors should not produce any significant errors. The intrinsic widths of the sequences are not generally known and have been neglected in our calculation, thereby introducing a number of artificial features in the transformed functions. The analytic function fits (eq. [3] and Table 2) effectively smooth over not only these artificial features but also real features that are undoubtedly present but which must escape notice in this approach.

The principal source of uncertainty at the bright end of the luminosity function, $M_V \leq +4$, lies with the lack of detailed knowledge of the variation in the relative proportions of the five kinds of giants with absolute magnitude (Fig. 3 and § II). At the dim end, the source visual luminosity function is well determined out to only $M_V \approx +13$ (M5 V). Furthermore, a population of unusually red stars would most likely be overlooked in the source visual band because the stars used in estimating this luminosity function are selected according to their apparent brightness in that band. In the longer wavelength bands, such red stars will appear increasingly brighter and could make significant contributions to a long wavelength band luminosity function over absolute magnitude ranges otherwise thought to be complete.

The sub-luminosity functions shown in Figure 4 for the different luminosity classes in the various bands can be a useful guide in the study of stellar populations and galactic structure, and vice versa. (Similar applications for the total luminosity functions in each of the bands considered here have been described in detail in Papers I–IV.) For example, the relative importance of the main sequence for absolute magnitudes brighter than $M_V \lesssim 0$ (or equivalent) decreases in the redder bands. The cleanest separation of class Ia and Ib supergiants and class II bright giants from one another and from all other classes of stars is best accomplished in the *I*-band. On the other hand, the cleanest separation of class III giants occurs in the *K*-band. Class IV subgiants are the hardest to isolate: they constitute about half of the stars with absolute magnitudes in the range 0.5–1.5 in the *K*-band.

The number of stars in each luminosity class brighter than a given apparent magnitude can also be estimated by applying the sub-luminosity functions individually to the Galaxy models described in Papers I–IV. Figure 7 shows the differential star counts in the *I*-band for luminosity classes I–VII in the direction of the galactic pole. (Scale heights for the class I supergiants are assumed to be 90 pc, identical to that for the very bright and short-lived main sequence O and B stars [Lee 1970]. Scale heights for the class II bright giants are assumed to be 170 pc, equal to the mean of the class I and class III values.) The curves for classes Ia, Ib, and II have been multiplied by the indicated factors for convenience in display.

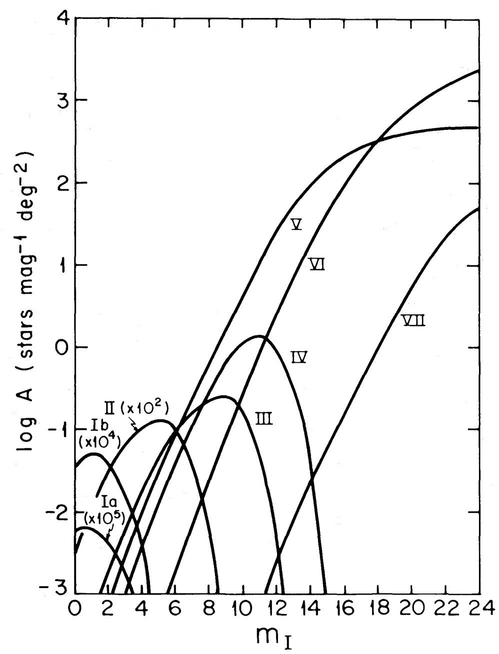

FIG. 7.—Differential star counts (stars per magnitude per square degree) in the *I* band for luminosity classes I–VII in the direction of the galactic pole. The curves for classes Ia, Ib, and II have been multiplied by the indicated factors for convenience in display.

With the recent availability of new photographic emulsions, CCDs, and other red- and infrared-sensitive instruments, there should soon be a wealth of data in the *R*, *I*, *J*, and *K* bands, which we have attempted to explore here (and in Paper IV) by transformation and extrapolation from the traditional visual band. Our calculations will hopefully be useful in preparing observing programs and in interpreting the results.

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