THE ASTRONOMICAL JOURNAL VOLUME 87, NUMBER 7 JULY 1982

RESONANCES IN SATURN'S RINGS

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Received 5 March 1982

ABSTRACT

We calculate the locations and strengths of the major resonances of Saturn's known moons with particles orbiting within Saturn's rings. The forcing at an l:$(m-1)$ resonance depends on the moon's eccentricity to the $|l-m|$ power. As most of Saturn's inner moons are in very nearly circular orbits, only their strongest resonances, with $l=m$ and $l=m+1$, are calculated. For Mimas, which has a somewhat larger e, resonances with $l=m+2$ are also computed. All of the resonances of these forms which are located between Saturn's cloud tops and $r=2.267R_{♄}$, except some of those due to tiny 1980 S28, are tabulated.

I. INTRODUCTION

Resonant forcing of ring particles by external moons, particularly Mimas, has long been invoked to explain the structure in Saturn's rings which is observable from Earth, e.g., Cassini's division and overall ring boundaries (Franklin and Colombo 1970). The much more detailed structure of the rings viewed by the Voyager spacecrafts has yet to be fully explained; however some features correlate very well with predicted resonant phenomena (Cuzzi, Lissauer, and Shu 1981; Smith *et al.*, 1982; Lane *et al.* 1982; Holberg *et al.* 1982). Additionally, the recent discoveries of several satellites orbiting between Mimas and the rings, and thus having many strong resonances located within the ring system, suggest that resonant effects may be widespread.

Owing to the close spacing of resonances, especially in the A ring, precise locations, both theoretical and observational, must be obtained before specific features can be related to individual resonances. The strengths of resonances are important in determining these correspondences, as a consistent picture can only be developed if resonances of comparable "strength" have similar observed effects in a given region of the rings.

II. RESONANCE LOCATIONS

The basic theory underlying most of the calculations of this paper can be found in Goldreich and Tremaine (1980). In what follows, we use their notation. In order to analyze the resonant effects of an outer satellite on Saturn's rings, the satellite's potential is Fourier expanded as

$$\phi^s(r,\theta,t) = \sum_{l=-\infty}^{\infty} \sum_{m=0}^{\infty} \phi^s_{l,m}(r) \times \exp(i\{m\theta - [m\Omega_s + (l-m)\kappa_s]t\}), \quad (1)$$

where Ω_s and κ_s are the satellite's mean motion and epicyclic frequency, respectively, $\Omega_s - \kappa_s$ being the satellite's apse precession rate. For small satellite eccentricity e, the largest term in $\phi^s_{l,m}$ is proportional to $e^{|l-m|}$. As all of Saturn's satellites have low eccentricity, resonances with $|l-m|>2$ are quite weak and will not concern us here.

The pattern speed of the (l,m) potential component, i.e., the rotation rate of the frame in which this component of the potential is constant, is

$$\Omega^p_{l,m} = \Omega_s + \frac{l-m}{m}\kappa_s. \quad (2)$$

An external satellite exerts significant torque at inner Lindblad resonances (ILR), where (Lin and Shu 1966)

$$\Omega(r) - \kappa(r)/m = \Omega^p_{l,m} \quad (3)$$

or, equivalently, $m[\Omega(r) - \Omega^p_{l,m}] = \kappa(r)$. That is, the frequency with which the particle "experiences" the forcing is a simple fraction of its "natural," or epicyclic, frequency. The deposit of this torque causes induced ring-particle eccentricities which may clear narrow gaps, excite spiral density waves, or both, at ILR.

To determine these resonant radii, we adopt the following procedure. As the satellite period is its best determined observational parameter, we use the measured value of $\Omega = \Omega_s$ to define a radius, $r = r_s$, where centrifugal balance holds:

$$\Omega^2 = \frac{GM_{♄}}{r^3}\left[1 + \frac{3}{2}J_2\left(\frac{R_{♄}}{r}\right)^2 - \frac{15}{8}J_4\left(\frac{R_{♄}}{r}\right)^4 + \frac{35}{16}J_6\left(\frac{R_{♄}}{r}\right)^6\right], \quad (4)$$

where the J_n's are Saturn's gravitational harmonic coefficients and the subscript ♄ refers to Saturn. The epicyclic frequency κ_s is then calculated at $r = r_s$ from

$$\kappa^2 = \frac{1}{r^3}\frac{d}{dr}(r^4\Omega^2) = \frac{GM_♄}{r^3}\left[1 - \frac{3}{2}J_2\left(\frac{R_♄}{r}\right)^2 + \frac{45}{8}J_4\left(\frac{R_♄}{r}\right)^4 - \frac{175}{16}J_6\left(\frac{R_♄}{r}\right)^6\right], \quad (5)$$

with fractional errors of the order $J_8(R_♄/r)^8$ and $(e^2 + \sin^2 i)J_2(R_♄/r)^2$, i being the inclination of the satellite's orbit to Saturn's equator. To the same order, $r_s = a_s$, the satellite's semimajor axis. Equation (3) is then solved iteratively, using Eqs. (4) and (5) for Ω and κ, to give the location, r_L, of the ILR. In (5), we have ignored any forced precession of the lines of apsides due to e-type resonant locking of one satellite to another. This may cause a slight shift in location for $l = m + 1$ resonances of 1980 S26, if it is resonantly locked to Mimas. Note that as $\kappa \approx \Omega$, the ratio of the moon's orbital period to that of a particle at ILR is roughly $l/(m-1)$ for the $m \neq 1$ resonances (the $m = 1$ "apsidal" resonances exist only because of the slight difference of κ from Ω owing to Saturn's oblateness); thus, e.g., the $l = 2$, $m = 2$ resonance of Mimas is frequently referred to as Mimas' 2:1 resonance.

Iteration was done to five-digit accuracy, thus the errors in our ILR locations are dominated by uncertainties in satellite parameters and in Saturn's mass and gravitational moments, which are greater than the omitted contribution of the rings to the gravitational field. Errors should be 10 km or less except for the apsidal ($m = 1$) resonances, which could be off by 40 km owing to the larger effects of uncertainties in measured values of Saturn's gravitational harmonics.

III. RESONANCE STRENGTHS

Calculations of resonance strengths are not as precise as those of their locations. Uncertainties in the measured masses and eccentricities of the forcing satellites may lead to errors of nearly a factor of 2 for some of the resonances with the small, newly discovered moons. For simplicity, we thus approximated the Fourier components of the satellite's potentials to the lowest nonvanishing order in eccentricity. In addition, for nonapsidal resonances we used the Keplerian approximation of $\Omega = \kappa$ for the calculations of strengths. These approximations produce errors on the order of a few percent.

With these simplifications, the relevant Fourier components in the satellite's potential become (Brouwer and Clemence 1961, ch. 15)

$$\phi^s_{1,1} = -\frac{GM_s}{a_s}\left[b^m_{1/2}(\beta) - \beta\right], \quad (6a)$$

$$\phi^s_{m,m} = -\frac{GM_s}{a_s} b^m_{1/2}(\beta), \quad m > 1, \quad (6b)$$

$$\phi^s_{m+1,m} = -\frac{GM_s e}{a_s} \times\left(\frac{1}{2} + m + \frac{\beta}{2}\frac{d}{d\beta}\right) b^m_{1/2}(\beta), \quad m > 1, \quad (6c)$$

$$\phi^s_{m+2,m} = -\frac{GM_s e^2}{a}\frac{1}{8}\left[(4m^2 + 9m + 4) + (4m + 5)\beta\frac{d}{d\beta} + \left(\beta\frac{d}{d\beta}\right)^2\right] b^m_{1/2}(\beta), \quad m > 1, \quad (6d)$$

where M_s is the satellite's mass, a_s its semimajor axis, $\beta \equiv r/a_s$, and $b^m_{1/2}(\beta)$ is the Laplace coefficient

$$b^m_{1/2}(\beta) \equiv \frac{2}{\pi}\int_0^\pi \frac{\cos m\theta \, d\theta}{(1 - 2\beta\cos\theta + \beta^2)^{1/2}}. \quad (7)$$

Two types of resonant effects have been suggested as causes of structure within Saturn's rings: a "natural" collisionally induced gap and the excitation of density waves. Although all of the measures of resonance strengths we tabulate in this paper are related to the production of density waves in the rings, the classical idea of creating a collisionally caused gap at resonance is worth some discussion.

Particles orbiting near ILR will be perturbed into slightly eccentric orbits by the resonant moon's gravity. The steady-state amplitude of the perturbation can be calculated using epicyclic theory (e.g., Goldreich and Tremaine 1982) and is inversely proportional to the particle's distance from resonance. There is also a phase reversal of lines of apsides across resonance, which causes particles inside resonance to be at apoapse at the same phase at which particles outside resonance are at periapse. If it is assumed that particles are cleared from these regions of intersecting streamlines, then the azimuthally averaged full width of the gap created is

$$w_{1,1} = 2\left(\frac{2F_{1,1}}{21J_2\Omega^2_L}\right)^{1/2}, \quad (8a)$$

$$w_{l,m} = 2\left(\frac{F_{l,m}}{3(m-1)\Omega^2_L}\right)^{1/2}, \quad l,m > 1, \quad (8b)$$

where

$$F_{l,m} = \left| r\frac{d\phi^s_{l,m}}{\partial r} + \frac{2\Omega^s_{l,m}}{\Omega - \Omega^p_{l,m}}\right|_{r_L}. \quad (9)$$

This theory gives the correct order-of-magnitude width for the Kirkwood gaps in the asteroid belt, but predicts dozens of resonantly causes gaps of average width between 5 and 50 km in Saturn's rings, most in regions where no clear gaps have been found (Lane *et al.* 1982). To resolve this apparent contradiction, we note that the time scale to attain these "steady-state" eccentricities is on the order of $[\Omega(r_L) - \Omega(r)]^{-1}$, which is a very large number of orbits for $r \approx r_L$. In particle disks of moderate optical depth, collisions will quickly result that are initially gentle and orbits will be randomized or merely averaged well before the high eccentricities that would be required to clear gaps are developed. Therefore, this classical mechanism would only produce gaps in disks with optical depth comparable to or less than

TABLE I. Saturn parameters used.

Gravitational mass[a] : $GM_♄$ $= (3.792\,914 \pm 0.000\,240) \times 10^{16}$ cm^3 s^{-2}	
Equatorial radius[b] : $R_♄ = 60\,330$ km	
Gravitational moments[c] :	$J_2 = (16\,299.1 \pm 18) \times 10^{-6}$
	$J_4 = (-916.7 \pm 38) \times 10^{-6}$
	$J_6 = 81.3 \times 10^{-6}$ (assumed)

[a] Null *et al.* (1981); satellite masses have been subtracted out.
[b] *Voyager* result; e.g., Smith *et al.* (1981).
[c] Null *et al.* (1981); corrected to $R_♄ = 60\,330$ km.

TABLE II. Satellite parameters used.

Satellite	Period (s)	e	$M_s/M_♄$
1980 S28	51 993.4[a]	0.0020[b]	0.177×10^{-10}[c]
1980 S27	52 962.8[b]	0.0030[b]	0.124×10^{-8}[c]
1980 S26	54 304.5[b]	0.0040[b]	0.766×10^{-9}[c]
1980 S3	59 989.6[b]	0.0090[b]	0.186×10^{-8}[c]
1980 S1	60 019.0[b]	0.0070[b]	0.778×10^{-8}[c]
Mimas	81 425.0[d]	0.0201[e]	0.659×10^{-7}[e]
Enceladus	118 387.0[d]	0.0044[e]	0.148×10^{-6}[e]
Titan	1 377 684.0[e]	0.0289[e]	0.237×10^{-3}[e]
Hyperian	1 838 592.0[d]	0.1042[e]	0.259×10^{-7}[e]
Iapetus	6 854 112.0[d]	0.0283[e]	0.331×10^{-5}[f]

[a] Synnott, S., private communication (1982).
[b] Synnott, S. *et al.* (1981). Periods as of 1 October 1980.
[c] Calculated from observed sizes (Smith *et al.* 1982) and density $= 1.2$ g cm^{-3} assumed.
[d] Duxbury, T. C., and Lieske, J., private communication (1981); obtained as part of reanalysis of all classical and modern observations preparatory to the *Voyager* Saturn encounters.
[e] Morrison *et al.* (1977).
[f] Smith *et al.* (1982).

the fractional gap width obtained from (8a); a condition satisfied in the asteroid belt but not in Saturn's rings.

A complete derivation of the torque $T^L_{l,m}$ exerted on a uniform fluid disk by a satellite at one of its inner Lindblad resonances is given by Goldreich and Tremaine (1979). We use only their result:

$$T^L_{l,m} = -m\pi^2 \left[\sigma\left(\frac{rdD}{dr}\right)^{-1}\left(\frac{rd\phi^s_{l,m}}{dr} + \frac{2\Omega\phi^s_{l,m}}{\Omega - \Omega^p_{l,m}}\right)^2\right]_{r_L}, \tag{10}$$

where σ is the surface mass density of the disk and $D \equiv \kappa^2 - m^2(\Omega - \Omega^p_{l,m})^2$. In addition to the torque, we compute two other parameters developed by Goldreich and Tremaine (1978) that are important in characterizing the transport of angular momentum by density waves excited by Saturn's moons.

In the absense of damping, the amplitude of long density waves will grow linearly near ILR (Shu 1970). Thus a good measure of the strength of a resonance is the fractional distance from resonance, X_{NL} (distance divided by resonant radius r_L), at which the wave becomes nonlinear; that is, where the perturbation in surface density equals the base surface density. This distance depends on both the local surface density of the rings, and the particular resonance involved, as follows:

$$X_{NL} = \frac{2\pi^2}{dD/dr}\left(-\frac{m\sigma}{r_L T^L_{l,m}}\right)^{1/2}(G\sigma)^{3/2}. \tag{11}$$

One of the possible effects of density waves is a gap just outside their excitation point (Goldreich and Tremaine 1978). Although the workings of this process in Saturn's rings is quite complex and controversial (Cuzzi, Lissauer, and Shu 1981), one important parameter is easily calculated. If we assume that all of the torque exerted at the resonance over the age of the solar system, $t_{ss} = 1.5 \times 10^{17}$ s, was used to move material from outside resonance to just inside it, and no material diffused into the gap, we get a maximum theoretical gap width

$$X_{\max} = \left(-\frac{2T^L_{l,m}t_{ss}}{\pi\sigma\Omega_L r_L^4}\right)^{1/2}. \tag{12}$$

Equations (11) and (12) assume X_{NL} and $X_{\max}$ are small compared to unity.

IV. RESULTS

We have used the most recent and reliable values for satellite and planet masses and for satellite orbital parameters (Tables I and II). Our results are presented in Table III. We tabulate resonance locations r_L and strengths $T^L_{l,m}$, X_{NL}, and $X_{\max}$ as obtained from Eqs. (3)

TABLE III. Resonance locations and strengths.

$l/(m-1)$	$r\,(R_♄)$	$T^L_{l,m}/\sigma$ (cm^4/s^2)	$X_{NL}\sigma_{100}^{-3/2}$	$X_{\max}$
		1980 S28		
2/ 1	1.452 87	-1.69×10^{12}	2.03×10^{-1}	3.39×10^{-4}
3/ 2	1.748 35	-6.26×10^{12}	1.24×10^{-1}	5.17×10^{-4}
4/ 3	1.888 16	-1.37×10^{13}	8.43×10^{-2}	6.96×10^{-4}
5/ 4	1.969 82	-2.41×10^{13}	6.19×10^{-2}	8.75×10^{-4}
6/ 5	2.023 41	-3.74×10^{13}	4.79×10^{-2}	1.05×10^{-3}
7/ 6	2.061 28	-5.36×10^{13}	3.84×10^{-2}	1.23×10^{-3}
8/ 7	2.089 47	-7.27×10^{13}	3.17×10^{-2}	1.41×10^{-3}
9/ 8	2.111 27	-9.47×10^{13}	2.67×10^{-2}	1.59×10^{-3}
10/ 9	2.128 64	-1.20×10^{14}	2.29×10^{-2}	1.77×10^{-3}
20/ 19	2.205 97	-5.33×10^{14}	8.25×10^{-3}	3.57×10^{-3}
30/ 29	2.231 47	-1.25×10^{15}	4.51×10^{-3}	5.38×10^{-3}
40/ 39	2.244 16	-2.26×10^{15}	2.93×10^{-3}	7.20×10^{-3}
50/ 49	2.251 76	-3.59×10^{15}	2.10×10^{-3}	9.03×10^{-3}

TABLE III. (continued)

$l/(m-1)$	$r\,(R_{\hbar})$	$T^{L}_{l,m}/\sigma$ (cm^4/s^2)	$X_{NL}\sigma_{100}^{-3/2}$	$X_{\max}$
60/ 59	2.256 82	-5.22×10^{15}	1.59×10^{-3}	1.09×10^{-2}
70/ 69	2.260 43	-7.17×10^{15}	1.26×10^{-3}	1.27×10^{-2}
80/ 79	2.263 14	-9.44×10^{15}	1.03×10^{-3}	1.46×10^{-2}
90/ 89	2.265 24	-1.20×10^{16}	8.63×10^{-4}	1.64×10^{-2}
100/ 99	2.266 93	-1.49×10^{16}	7.36×10^{-4}	1.83×10^{-2}
110/109	2.268 30	-1.82×10^{16}	6.37×10^{-4}	2.01×10^{-2}
120/119	2.269 45	-2.17×10^{16}	5.58×10^{-4}	2.20×10^{-2}
3/ 1	1.119 98	-1.10×10^{7}	3.17×10^{1}	1.19×10^{-6}
4/ 2	1.448 20	-8.75×10^{7}	1.70×10^{1}	2.45×10^{-6}
5/ 3	1.630 46	-3.34×10^{8}	1.02×10^{1}	4.12×10^{-6}
6/ 4	1.746 88	-9.06×10^{8}	6.62×10	6.23×10^{-6}
7/ 5	1.827 83	-2.01×10^{9}	4.57×10	8.77×10^{-6}
8/ 6	1.887 43	-3.90×10^{9}	3.30×10	1.17×10^{-5}
9/ 7	1.933 17	-6.89×10^{9}	2.48×10	1.51×10^{-5}
10/ 8	1.969 39	-1.13×10^{10}	1.91×10	1.90×10^{-5}
20/ 18	2.128 54	-2.52×10^{11}	3.44×10^{-1}	8.12×10^{-5}
40/ 38	2.205 95	-4.73×10^{12}	6.11×10^{-2}	3.37×10^{-4}
60/ 58	2.231 46	-2.54×10^{13}	2.21×10^{-2}	7.69×10^{-4}
80/ 78	2.244 16	-8.29×10^{13}	1.08×10^{-2}	1.38×10^{-3}
100/ 98	2.251 76	-2.07×10^{14}	6.14×10^{-3}	2.17×10^{-3}
120/118	2.256 82	-4.35×10^{14}	3.88×10^{-3}	3.14×10^{-3}
140/138	2.260 43	-8.17×10^{14}	2.64×10^{-3}	4.29×10^{-3}
160/158	2.263 14	-1.41×10^{15}	1.88×10^{-3}	5.62×10^{-3}
180/178	2.265 24	-2.28×10^{15}	1.40×10^{-3}	7.14×10^{-3}
200/198	2.266 93	-3.50×10^{15}	1.07×10^{-3}	8.84×10^{-3}
220/218	2.268 30	-5.15×10^{15}	8.44×10^{-4}	1.07×10^{-2}
240/238	2.269 45	-7.35×10^{15}	6.77×10^{-4}	1.28×10^{-2}
		1980 S27		
2/ 1	1.470 43	-8.39×10^{15}	3.00×10^{-3}	2.35×10^{-2}
3/ 2	1.769 78	-3.11×10^{16}	1.83×10^{-3}	3.59×10^{-2}
4/ 3	1.911 38	-6.81×10^{16}	1.25×10^{-3}	4.83×10^{-2}
5/ 4	1.994 07	-1.20×10^{17}	9.17×10^{-4}	6.07×10^{-2}
6/ 5	2.048 33	-1.86×10^{17}	7.09×10^{-4}	7.31×10^{-2}
7/ 6	2.086 68	-2.66×10^{17}	5.69×10^{-4}	8.55×10^{-2}
8/ 7	2.115 23	-3.61×10^{17}	4.70×10^{-4}	9.79×10^{-2}
9/ 8	2.137 31	-4.70×10^{17}	3.96×10^{-4}	1.10×10^{-1}
10/ 9	2.154 90	-5.95×10^{17}	3.40×10^{-4}	1.23×10^{-1}
11/ 10	2.169 23	-7.33×10^{17}	2.95×10^{-4}	1.35×10^{-1}
12/ 11	2.181 15	-8.87×10^{17}	2.60×10^{-4}	1.48×10^{-1}
13/ 12	2.191 20	-1.05×10^{18}	2.31×10^{-4}	1.60×10^{-1}
14/ 13	2.199 81	-1.24×10^{18}	2.07×10^{-4}	1.73×10^{-1}
15/ 14	2.207 25	-1.44×10^{18}	1.87×10^{-4}	1.85×10^{-1}
16/ 15	2.213 75	-1.65×10^{18}	1.70×10^{-4}	1.98×10^{-1}
17/ 16	2.219 48	-1.87×10^{18}	1.56×10^{-4}	2.10×10^{-1}
18/ 17	2.224 57	-2.12×10^{18}	1.43×10^{-4}	2.23×10^{-1}
19/ 18	2.229 12	-2.37×10^{18}	1.32×10^{-4}	2.35×10^{-1}
20/ 19	2.233 21	-2.65×10^{18}	1.22×10^{-4}	2.48×10^{-1}
21/ 20	2.236 90	-2.93×10^{18}	1.14×10^{-4}	2.60×10^{-1}
22/ 21	2.240 26	-3.23×10^{18}	1.06×10^{-4}	2.73×10^{-1}
23/ 22	2.243 32	-3.55×10^{18}	9.93×10^{-5}	2.85×10^{-1}
24/ 23	2.246 13	-3.88×10^{18}	9.32×10^{-5}	2.98×10^{-1}
25/ 24	2.248 71	-4.23×10^{18}	8.77×10^{-5}	3.11×10^{-1}
26/ 25	2.251 09	-4.59×10^{18}	8.27×10^{-5}	3.23×10^{-1}
27/ 26	2.253 30	-4.97×10^{18}	7.82×10^{-5}	3.36×10^{-1}
28/ 27	2.255 34	-5.36×10^{18}	7.40×10^{-5}	3.48×10^{-1}
29/ 28	2.257 24	-5.76×10^{18}	7.03×10^{-5}	3.61×10^{-1}
30/ 29	2.259 02	-6.19×10^{18}	6.68×10^{-5}	3.74×10^{-1}
31/ 30	2.260 68	-6.62×10^{18}	6.36×10^{-5}	3.86×10^{-1}
32/ 31	2.262 24	-7.08×10^{18}	6.06×10^{-5}	3.99×10^{-1}
33/ 32	2.263 70	-7.54×10^{18}	5.79×10^{-5}	4.11×10^{-1}
34/ 33	2.265 07	-8.02×10^{18}	5.54×10^{-5}	4.24×10^{-1}
35/ 34	2.266 37	-8.52×10^{18}	5.30×10^{-5}	4.37×10^{-1}
36/ 35	2.267 59	-9.03×10^{18}	5.08×10^{-5}	4.49×10^{-1}
37/ 36	2.268 75	-9.56×10^{18}	4.88×10^{-5}	4.62×10^{-1}
38/ 37	2.269 85	-1.01×10^{19}	4.69×10^{-5}	4.75×10^{-1}
3/ 1	1.133 22	-1.23×10^{11}	3.13×10^{-1}	1.24×10^{-4}
4/ 2	1.465 82	-9.77×10^{11}	1.68×10^{-1}	2.54×10^{-4}
5/ 3	1.650 42	-3.73×10^{12}	1.01×10^{-1}	4.29×10^{-4}
6/ 4	1.768 32	-1.01×10^{13}	6.54×10^{-2}	6.48×10^{-4}
7/ 5	1.850 30	-2.24×10^{13}	4.51×10^{-2}	9.13×10^{-4}
8/ 6	1.910 66	-4.36×10^{13}	3.26×10^{-2}	1.22×10^{-3}

TABLE III. (continued)

$l/(m-1)$	$r\,(R_{\mathrm{h}})$	$T^{L}_{l,m}/\sigma$ (cm^4/s^2)	$X_{NL}\sigma_{100}^{-3/2}$	$X_{\max}$
9/ 7	1.956 97	-7.69×10^{13}	2.45×10^{-2}	1.58×10^{-3}
10/ 8	1.993 65	-1.27×10^{14}	1.89×10^{-2}	1.97×10^{-3}
11/ 9	2.023 41	-1.97×10^{14}	1.50×10^{-2}	2.42×10^{-3}
12/ 10	2.048 05	-2.93×10^{14}	1.21×10^{-2}	2.91×10^{-3}
13/ 11	2.068 79	-4.22×10^{14}	9.90×10^{-3}	3.44×10^{-3}
14/ 12	2.086 48	-5.88×10^{14}	8.25×10^{-3}	4.02×10^{-3}
15/ 13	2.101 76	-7.99×10^{14}	6.95×10^{-3}	4.65×10^{-3}
16/ 14	2.115 08	-1.06×10^{15}	5.92×10^{-3}	5.32×10^{-3}
17/ 15	2.126 80	-1.39×10^{15}	5.10×10^{-3}	6.03×10^{-3}
18/ 16	2.137 19	-1.78×10^{15}	4.42×10^{-3}	6.79×10^{-3}
19/ 17	2.146 47	-2.25×10^{15}	3.87×10^{-3}	7.60×10^{-3}
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21/ 19	2.162 33	-3.47×10^{15}	3.01×10^{-3}	9.34×10^{-3}
22/ 20	2.169 16	-4.24×10^{15}	2.68×10^{-3}	1.03×10^{-2}
23/ 21	2.175 38	-5.13×10^{15}	2.40×10^{-3}	1.13×10^{-2}
24/ 22	2.181 08	-6.15×10^{15}	2.16×10^{-3}	1.23×10^{-2}
25/ 23	2.186 32	-7.32×10^{15}	1.95×10^{-3}	1.34×10^{-2}
26/ 24	2.191 15	-8.65×10^{15}	1.77×10^{-3}	1.45×10^{-2}
27/ 25	2.195 62	-1.01×10^{16}	1.61×10^{-3}	1.57×10^{-2}
28/ 26	2.199 76	-1.18×10^{16}	1.47×10^{-3}	1.69×10^{-2}
29/ 27	2.203 61	-1.37×10^{16}	1.35×10^{-3}	1.82×10^{-2}
30/ 28	2.207 21	-1.58×10^{16}	1.24×10^{-3}	1.95×10^{-2}
31/ 29	2.210 57	-1.82×10^{16}	1.14×10^{-3}	2.08×10^{-2}
32/ 30	2.213 72	-2.08×10^{16}	1.05×10^{-3}	2.22×10^{-2}
33/ 31	2.216 67	-2.37×10^{16}	9.76×10^{-4}	2.37×10^{-2}
34/ 32	2.219 45	-2.68×10^{16}	9.06×10^{-4}	2.51×10^{-2}
35/ 33	2.222 07	-3.03×10^{16}	8.43×10^{-4}	2.67×10^{-2}
36/ 34	2.224 54	-3.41×10^{16}	7.86×10^{-4}	2.83×10^{-2}
37/ 35	2.226 88	-3.82×10^{16}	7.34×10^{-4}	2.99×10^{-2}
38/ 36	2.229 09	-4.27×10^{16}	6.86×10^{-4}	3.16×10^{-2}
39/ 37	2.231 19	-4.76×10^{16}	6.43×10^{-4}	3.33×10^{-2}
40/ 38	2.233 18	-5.29×10^{16}	6.04×10^{-4}	3.50×10^{-2}
41/ 39	2.235 08	-5.86×10^{16}	5.67×10^{-4}	3.68×10^{-2}
42/ 40	2.236 88	-6.48×10^{16}	5.34×10^{-4}	3.87×10^{-2}
43/ 41	2.238 60	-7.15×10^{16}	5.04×10^{-4}	4.06×10^{-2}
44/ 42	2.240 24	-7.86×10^{16}	4.76×10^{-4}	4.26×10^{-2}
45/ 43	2.241 81	-8.63×10^{16}	4.50×10^{-4}	4.45×10^{-2}
46/ 44	2.243 31	-9.46×10^{16}	4.25×10^{-4}	4.66×10^{-2}
47/ 45	2.244 74	-1.03×10^{17}	4.03×10^{-4}	4.87×10^{-2}
48/ 46	2.246 12	-1.13×10^{17}	3.82×10^{-4}	5.08×10^{-2}
49/ 47	2.247 43	-1.23×10^{17}	3.63×10^{-4}	5.30×10^{-2}
50/ 48	2.248 70	-1.34×10^{17}	3.45×10^{-4}	5.52×10^{-2}
51/ 49	2.249 91	-1.45×10^{17}	3.29×10^{-4}	5.75×10^{-2}
52/ 50	2.251 08	-1.57×10^{17}	3.13×10^{-4}	5.98×10^{-2}
53/ 51	2.252 20	-1.70×10^{17}	2.98×10^{-4}	6.22×10^{-2}
54/ 52	2.253 28	-1.84×10^{17}	2.85×10^{-4}	6.46×10^{-2}
55/ 53	2.254 33	-1.98×10^{17}	2.72×10^{-4}	6.70×10^{-2}
56/ 54	2.255 33	-2.13×10^{17}	2.60×10^{-4}	6.95×10^{-2}
57/ 55	2.256 30	-2.30×10^{17}	2.49×10^{-4}	7.21×10^{-2}
58/ 56	2.257 23	-2.47×10^{17}	2.38×10^{-4}	7.47×10^{-2}
59/ 57	2.258 14	-2.65×10^{17}	2.28×10^{-4}	7.73×10^{-2}
60/ 58	2.259 01	-2.84×10^{17}	2.19×10^{-4}	8.00×10^{-2}
61/ 59	2.259 86	-3.04×10^{17}	2.10×10^{-4}	8.27×10^{-2}
62/ 60	2.260 67	-3.25×10^{17}	2.01×10^{-4}	8.55×10^{-2}
63/ 61	2.261 46	-3.47×10^{17}	1.93×10^{-4}	8.83×10^{-2}
64/ 62	2.262 23	-3.70×10^{17}	1.86×10^{-4}	9.12×10^{-2}
65/ 63	2.262 97	-3.94×10^{17}	1.79×10^{-4}	9.41×10^{-2}
66/ 64	2.263 69	-4.20×10^{17}	1.72×10^{-4}	9.71×10^{-2}
67/ 65	2.264 39	-4.47×10^{17}	1.66×10^{-4}	1.00×10^{-1}
68/ 66	2.265 07	-4.75×10^{17}	1.60×10^{-4}	1.03×10^{-1}
69/ 67	2.265 72	-5.04×10^{17}	1.54×10^{-4}	1.06×10^{-1}
70/ 68	2.266 36	-5.35×10^{17}	1.49×10^{-4}	1.09×10^{-1}
71/ 69	2.266 98	-5.67×10^{17}	1.43×10^{-4}	1.13×10^{-1}
72/ 70	2.267 59	-6.01×10^{17}	1.38×10^{-4}	1.16×10^{-1}
73/ 71	2.268 17	-6.36×10^{17}	1.34×10^{-4}	1.19×10^{-1}
74/ 72	2.268 75	-6.72×10^{17}	1.29×10^{-4}	1.22×10^{-1}
75/ 73	2.269 30	-7.10×10^{17}	1.25×10^{-4}	1.26×10^{-1}
76/ 74	2.269 84	-7.50×10^{17}	1.21×10^{-4}	1.29×10^{-1}

TABLE III. (continued)

$l/(m-1)$	$r\,(R_{♄})$	$T^L_{l,m}/\sigma$ (cm^4/s^2)	$X_{NL}\sigma_{100}^{-3/2}$	$X_{\max}$
		1980 S26		
2/ 1	1.494 56	-3.24×10^{15}	5.11×10^{-3}	1.43×10^{-2}
3/ 2	1.799 22	-1.20×10^{16}	3.12×10^{-3}	2.19×10^{-2}
4/ 3	1.943 27	-2.64×10^{16}	2.13×10^{-3}	2.94×10^{-2}
5/ 4	2.027 40	-4.64×10^{16}	1.56×10^{-3}	3.70×10^{-2}
6/ 5	2.082 59	-7.19×10^{16}	1.21×10^{-3}	4.46×10^{-2}
7/ 6	2.121 60	-1.03×10^{17}	9.69×10^{-4}	5.22×10^{-2}
8/ 7	2.150 64	-1.40×10^{17}	7.99×10^{-4}	5.97×10^{-2}
9/ 8	2.173 09	-1.82×10^{17}	6.74×10^{-4}	6.73×10^{-2}
10/ 9	2.190 98	-2.30×10^{17}	5.78×10^{-4}	7.49×10^{-2}
11/ 10	2.205 56	-2.84×10^{17}	5.03×10^{-4}	8.25×10^{-2}
12/ 11	2.217 68	-3.44×10^{17}	4.43×10^{-4}	9.01×10^{-2}
13/ 12	2.227 91	-4.09×10^{17}	3.94×10^{-4}	9.77×10^{-2}
14/ 13	2.236 66	-4.80×10^{17}	3.53×10^{-4}	1.05×10^{-1}
15/ 14	2.244 23	-5.56×10^{17}	3.19×10^{-4}	1.13×10^{-1}
16/ 15	2.250 84	-6.39×10^{17}	2.90×10^{-4}	1.21×10^{-1}
17/ 16	2.256 67	-7.27×10^{17}	2.65×10^{-4}	1.28×10^{-1}
18/ 17	2.261 84	-8.21×10^{17}	2.43×10^{-4}	1.36×10^{-1}
19/ 18	2.266 47	-9.20×10^{17}	2.25×10^{-4}	1.43×10^{-1}
3/ 1	1,151 43	-8.41×10^{10}	4.00×10^{-1}	1.01×10^{-4}
4/ 2	1.490 03	-6.72×10^{11}	2.15×10^{-1}	2.07×10^{-4}
5/ 3	1.677 85	-2.57×10^{12}	1.29×10^{-1}	3.49×10^{-4}
6/ 4	1.797 79	-6.97×10^{12}	8.35×10^{-2}	5.27×10^{-4}
7/ 5	1.881 18	-1.55×10^{13}	5.76×10^{-2}	7.42×10^{-4}
8/ 6	1.942 57	-3.00×10^{13}	4.17×10^{-2}	9.93×10^{-4}
9/ 7	1.989 68	-5.30×10^{13}	3.13×10^{-2}	1.28×10^{-3}
10/ 8	2.026 98	-8.72×10^{13}	2.41×10^{-2}	1.61×10^{-3}
11/ 9	2.057 25	-1.36×10^{14}	1.91×10^{-2}	1.97×10^{-3}
12/ 10	2.082 31	-2.02×10^{14}	1.54×10^{-2}	2.36×10^{-3}
13/ 11	2.103 40	-2.90×10^{14}	1.26×10^{-2}	2.80×10^{-3}
14/ 12	2.121 40	-4.05×10^{14}	1.05×10^{-2}	3.27×10^{-3}
15/ 13	2.136 94	-5.51×10^{14}	8.88×10^{-3}	3.78×10^{-3}
16/ 14	2.150 49	-7.32×10^{14}	7.56×10^{-3}	4.32×10^{-3}
17/ 15	2.162 41	-9.56×10^{14}	6.51×10^{-3}	4.90×10^{-3}
18/ 16	2.172 98	-1.23×10^{15}	5.65×10^{-3}	5.52×10^{-3}
19/ 17	2.182 41	-1.55×10^{15}	4.94×10^{-3}	6.18×10^{-3}
20/ 18	2.190 89	-1.94×10^{15}	4.35×10^{-3}	6.87×10^{-3}
21/ 19	2.198 54	-2.39×10^{15}	3.85×10^{-3}	7.60×10^{-3}
22/ 20	2.205 49	-2.92×10^{15}	3.43×10^{-3}	8.36×10^{-3}
23/ 21	2.211 82	-3.53×10^{15}	3.07×10^{-3}	9.17×10^{-3}
24/ 22	2.217 62	-4.24×10^{15}	2.76×10^{-3}	1.00×10^{-2}
25/ 23	2.222 94	-5.04×10^{15}	2.49×10^{-3}	1.09×10^{-2}
26/ 24	2.227 85	-5.96×10^{15}	2.26×10^{-3}	1.18×10^{-2}
27/ 25	2.232 40	-6.99×10^{15}	2.06×10^{-3}	1.27×10^{-2}
28/ 26	2.236 61	-8.16×10^{15}	1.88×10^{-3}	1.37×10^{-2}
29/ 27	2.240 53	-9.46×10^{15}	1.72×10^{-3}	1.48×10^{-2}
30/ 28	2.244 19	-1.09×10^{16}	1.58×10^{-3}	1.58×10^{-2}
31/ 29	2.247 60	-1.25×10^{16}	1.46×10^{-3}	1.69×10^{-2}
32/ 30	2.250 81	-1.43×10^{16}	1.35×10^{-3}	1.81×10^{-2}
33/ 31	2.253 81	-1.63×10^{16}	1.25×10^{-3}	1.92×10^{-2}
34/ 32	2.256 64	-1.85×10^{16}	1.16×10^{-3}	2.04×10^{-2}
35/ 33	2.259 30	-2.09×10^{16}	1.08×10^{-3}	2.17×10^{-2}
36/ 34	2.261 82	-2.35×10^{16}	1.00×10^{-3}	2.30×10^{-2}
37/ 35	2.264 19	-2.63×10^{16}	9.37×10^{-4}	2.43×10^{-2}
38/ 36	2.266 44	-2.94×10^{16}	8.76×10^{-4}	2.57×10^{-2}
39/ 37	2.268 58	-3.28×10^{16}	8.21×10^{-4}	2.71×10^{-2}
		1980 S3		
2/ 1	1.594 82	-2.02×10^{16}	2.57×10^{-3}	3.29×10^{-2}
3/ 2	1.921 44	-7.54×10^{16}	1.57×10^{-3}	5.05×10^{-2}
4/ 3	2.075 68	-1.66×10^{17}	1.07×10^{-3}	6.79×10^{-2}
5/ 4	2.165 71	-2.91×10^{17}	7.86×10^{-4}	8.54×10^{-2}
6/ 5	2.224 77	-4.51×10^{17}	6.08×10^{-4}	1.03×10^{-1}
7/ 6	2.266 50	-6.47×10^{17}	4.88×10^{-4}	1.20×10^{-1}
3/ 1	1.227 16	-2.63×10^{12}	8.96×10^{-2}	5.20×10^{-4}
4/ 2	1.590 59	-2.12×10^{13}	4.82×10^{-2}	1.07×10^{-3}
5/ 3	1.791 73	-8.12×10^{13}	2.88×10^{-2}	1.81×10^{-3}
6/ 4	1.920 10	-2.21×10^{14}	1.87×10^{-2}	2.73×10^{-3}
7/ 5	2.009 33	-4.90×10^{14}	1.29×10^{-2}	3.85×10^{-3}
8/ 6	2.075 02	-9.52×10^{14}	9.33×10^{-3}	5.15×10^{-3}
9/ 7	2.125 41	-1.68×10^{15}	7.00×10^{-3}	6.64×10^{-3}
10/ 8	2.165 32	-2.77×10^{15}	5.40×10^{-3}	8.33×10^{-3}

TABLE III. (continued)

$l/(m-1)$	$r\,(R_♄)$	$T^L_{l,m}/\sigma$ (cm^4/s^2)	$X_{NL}\sigma_{100}^{-3/2}$	X_{max}
11/ 9	2.197 70	-4.31×10^{15}	4.28×10^{-3}	1.02×10^{-2}
12/ 10	2.224 51	-6.42×10^{15}	3.45×10^{-3}	1.23×10^{-2}
13/ 11	2.247 07	-9.22×10^{15}	2.83×10^{-3}	1.45×10^{-2}
14/ 12	2.266 32	-1.29×10^{16}	2.36×10^{-3}	1.70×10^{-2}
		1980 S1		
2/ 1	1.595 33	-3.54×10^{17}	6.16×10^{-4}	1.38×10^{-1}
3/ 2	1.922 06	-1.32×10^{18}	3.76×10^{-4}	2.11×10^{-1}
4/ 3	2.076 35	-2.90×10^{18}	2.56×10^{-4}	2.84×10^{-1}
5/ 4	2.166 41	-5.09×10^{18}	1.88×10^{-4}	3.57×10^{-1}
6/ 5	2.225 49	-7.90×10^{18}	1.45×10^{-4}	4.30×10^{-1}
7/ 6	2.267 24	-1.13×10^{19}	1.17×10^{-4}	5.03×10^{-1}
3/ 1	1.227 55	-2.78×10^{13}	2.76×10^{-2}	1.69×10^{-3}
4/ 2	1.591 10	-2.25×10^{14}	1.48×10^{-2}	3.48×10^{-3}
5/ 3	1.792 31	-8.60×10^{14}	8.87×10^{-3}	5.88×10^{-3}
6/ 4	1.920 73	-2.34×10^{15}	5.75×10^{-3}	8.89×10^{-3}
7/ 5	2.009 98	-5.19×10^{15}	3.97×10^{-3}	1.25×10^{-2}
8/ 6	2.075 69	-1.01×10^{16}	2.87×10^{-3}	1.68×10^{-2}
9/ 7	2.126 11	-1.78×10^{16}	2.15×10^{-3}	2.16×10^{-2}
10/ 8	2.166 02	-2.93×10^{16}	1.66×10^{-3}	2.71×10^{-2}
11/ 9	2.198 41	-4.56×10^{16}	1.32×10^{-3}	3.32×10^{-2}
12/ 10	2.225 23	-6.79×10^{16}	1.06×10^{-3}	3.99×10^{-2}
13/ 11	2.247 80	-9.76×10^{16}	8.71×10^{-4}	4.72×10^{-2}
14/ 12	2.267 05	-1.36×10^{17}	7.25×10^{-4}	5.52×10^{-2}
		Mimas		
2/ 1	1.948 47	-3.03×10^{19}	1.34×10^{-4}	9.94×10^{-1}
3/ 1	1.495 03	-1.92×10^{16}	2.10×10^{-3}	3.48×10^{-2}
4/ 2	1.945 05	-1.59×10^{17}	1.13×10^{-3}	7.22×10^{-2}
5/ 3	2.192 86	-6.12×10^{17}	6.75×10^{-4}	1.22×10^{-1}
4/ 1	1.241 30	-1.15×10^{13}	4.45×10^{-2}	1.08×10^{-3}
5/ 2	1.679 25	-1.51×10^{14}	2.18×10^{-2}	2.67×10^{-3}
6/ 3	1.943 90	-8.58×10^{14}	1.18×10^{-2}	5.31×10^{-3}
7/ 4	2.122 76	-3.23×10^{15}	6.95×10^{-3}	9.23×10^{-3}
8/ 5	2.252 22	-9.51×10^{15}	4.37×10^{-3}	1.47×10^{-2}
		Enceladus		
3/ 1	1.909 58	-5.74×10^{15}	9.10×10^{-3}	1.40×10^{-2}
		Titan		
1/ 0	1.289 50	-1.39×10^{18}	2.79×10^{-3}	3.56×10^{-1}
		Hyperion		
1/ 0	1.794 98	-8.56×10^{9}	5.56×10^{1}	2.53×10^{-5}
		Iapetus		
1/ 0	2.008 67	-7.17×10^{12}	1.49×10^{1}	4.66×10^{-4}

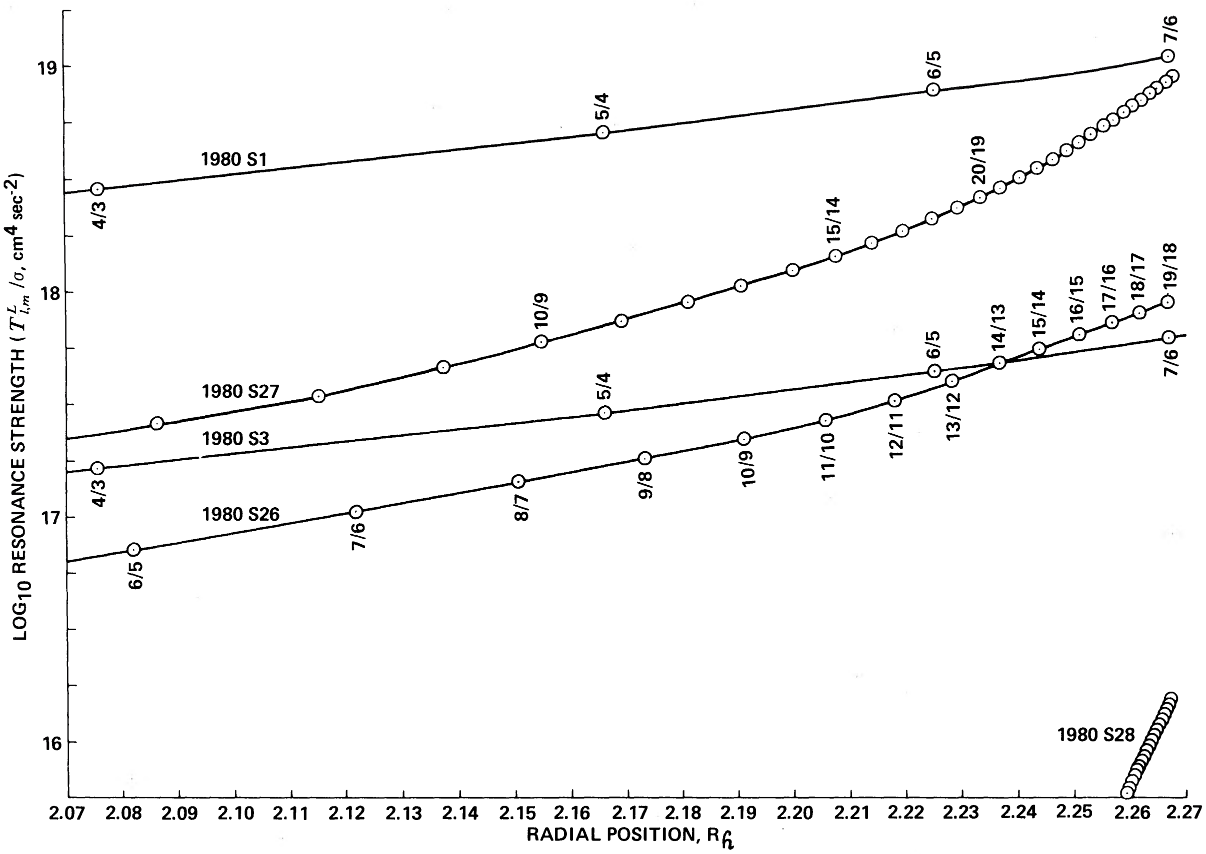

FIG. 1. Locations [Eqs. (2), (3)] and strengths (Eq. 10) of major Lindblad resonances in Saturn's A ring are plotted for Saturn's five closest "ring moons," 1980 S1, 1980 S3, 1980 S26, 1980 S27, and 1980 S28. The closer moons have more closely spaced resonances with strength increasing outward more rapidly.

and (10)–(12); in Table III, $\sigma_{100} = \sigma/100$ g cm^{-2}. We also display some of our results in graphical form in Fig. 1, which shows the locations and strengths of resonances in Saturn's A ring due to the "ring moons." Clearly, the dominant features will be due to the inner "shepherd" 1980 S27 and the larger co-orbital 1980 S1, with the smaller co-orbital 1980 S3 having resonances only ~30–45 km interior to (see Table III) and about an order of magnitude weaker than those of 1980 S1. The outer shepherd 1980 S26 has fewer and weaker resonances than the inner shepherd. It may be seen that the effects of tiny 1980 S28 will be far smaller than the effects of other "ring moons."

We gratefully acknowledge helpful conversations with Frank Shu. Steve Synnott, Jay Holberg, Tom Duxbury, and Jay Lieske provided useful data in advance of publication. We thank Keith Fischer for his valuable assistance with programming the algorithms. Scott Tremaine's comments were also helpful. This work was supported in part by NASA Grant No. NGT-05-003-800.

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