Despite the simplicity of the chosen model, these data agree well, both in amplitude and in phase of the seasonal pressure oscillations. The region of 0° < L_S < 130°, where the theory predicts the appearance of secondary pressure maxima in the atmosphere of the northern hemisphere due to the intense melting of the south polar cap at this time, is very interesting. Unfortunately, observational data for this period are absent.

The results presented in Fig. 5 confirm the assumption made in Ref. 9 concerning the different landing altitudes of the descent craft. If the craft landed at the same altitudes the VL-1 and VL-2 pressure graphs would have intersected at the points L_S = 190° and 10° according to the theoretical calculations of the present report, since the amplitude of the pressure oscillations is larger at higher latitudes than at lower latitudes. Nevertheless, the pressures at these times differ by 0.7 mbar between the VL-1 and VL-2 stations, which corresponds to an altitude difference of 0.8-1 km.

It should be noted that in these calculations one observes not only agreement of the amplitude and phase of the pressure oscillations with the Viking measurements but also agreement of the absolute value of the pressure, which oscillates between 7 and 9 mbar. This result is not trivial, since the initial pressure in the model was taken as 10 mbar, and the data presented in Fig. 5 are the result of stabilization of a transitional process in the model under consideration.

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Computer modeling of the evolution of plane rings of gravitating particles moving around the sun

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The evolution of plane rings of gravitating particles (bodies or material points) moving around the sun is investigated by computer modeling. The algorithm used is investigated. The parameters of the rings of gravitating bodies, which combine during any collisions, correspond to the supply zones of the real planets. The densities of the bodies are close to the present densities of the planets. An analysis of the results obtained shows that the number of planets formed in the supply zone of planets of the terrestrial group would be larger than the actual number of planets if one assumes that all the bodies moved in the same plane and ignores the influence of the drag force of the gas on the motion of the planetesimals and certain other factors. The number of planets formed decreases (possibly to the actual number of planets) if a three-dimensional rather than a plane model is considered.

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1. INTRODUCTION

In the present report the evolution of plane rings of gravitating particles moving around a massive central body (the sun) is investigated by computer modeling.

The results of an investigation of the evolution of the orbits of two gravitationally interacting planetesimals, material points, obtained by the method of spheres of action (or spheres of a larger radius) and by numerical integration of the equations of motion are compared in Sec. 2 of the present report. An algorithm for modeling the evolution of a ring of bodies is proposed on the basis of this comparison and the results of Refs. 1-3. In Sec. 3 the results of the computer modeling of the evolution of the orbits of two, three, four, and several hundred gravitating material points are applied to an investigation of Éneev's hypothesis[4] about the possible character of the evolution of Pluto's orbit and to an investigation of the mutual gravitational influence of the asteroids. The results of Sec. 3 are presented in more detail in Refs. 5 and 6. The evolution of plane rings of gravitating bodies which combine during collisions is considered in Secs. 4 and 5. The parameters of the investigated rings correspond to the supply zones of the real planets. The densities of the bodies are close to the present densities of the planets. The main results of Secs. 4 and 5 are presented in Ref. 3. The evolution of plane rings of gravitating bodies whose parameters do not correspond to the

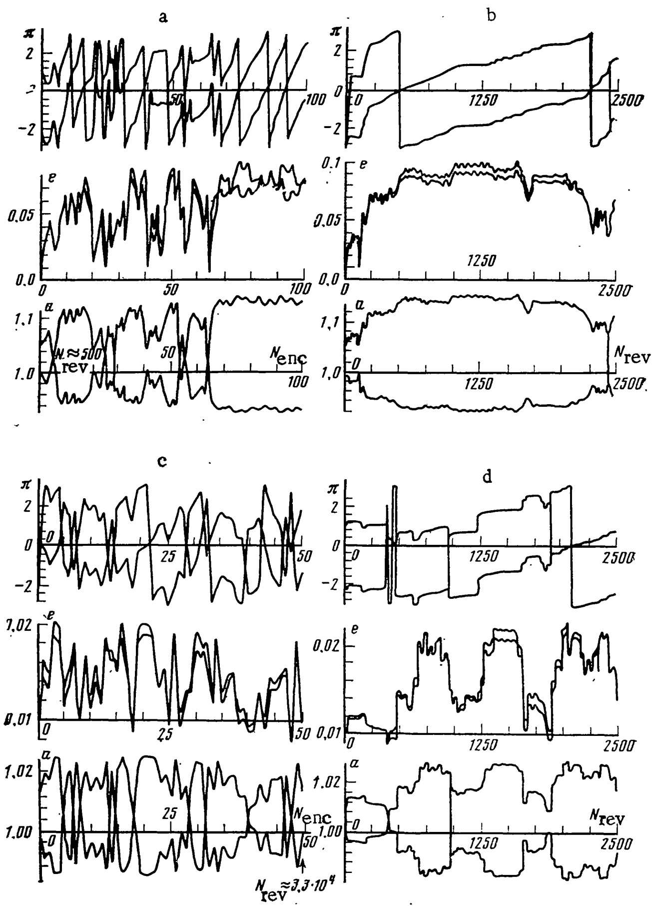

FIG. 1. Dependence of the orbital elements of two gravitationally interacting particles, material points, moving around the sun on time (on the number of revolutions N_{rev} of the first particle around the sun or on the number of encounters N_{enc} of the particles to a distance R_s). Initial data: $a_1^0 = 1$, $m_1 = m_2$, $e_1^0 = e_2^0 = e_0$; a and b) $m_1 = 10^{-5}$, $a_2^0 = 1.05$, $e_0 = 0$; c and d) $m_1 = 10^{-7}$, $a_2^0 = 1.01$, $e_0 = 0.01$.

supply zones of the real planets was investigated in Ref. 7. The evolution of the axial rotations of the bodies during their combinings was allowed for in that report. Some comments on the possible evolution of three-dimensional rings are given in Sec. 6. In the present report principal attention is paid to the investigation of the evolution of rings corresponding to the supply zone of planets of the terrestrial group. It is proposed to present the results of an investigation of the evolution of plane rings corresponding to the supply zone of the giant planets and estimates of the evolution of three-dimensional rings in more detail in a separate publication.

Numerical computer modeling of the evolution of rings of bodies (or of tenuous gas–dust clumps) moving around the sun and combining during collisions has also been performed in Refs. 8-16. Among these reports the mutual gravitational influence of the bodies was allowed for in Refs. 13-16. The algorithm used by us to allow for the gravitational interactions of the bodies differs from the algorithms used in these reports. The initial data of the investigated rings also differ. The initial stage of formation of the planets was considered in Ref. 13, while the formation of planets in a binary star system was investigated in Ref. 14. In Ref. 15 the mass of the ring exceeded the mass of Jupiter, while the initial eccentricities were large. Numerical investigations of the final stages of accumulation of planets of the terrestrial group were made in Ref. 16. In that report the initial orbital eccentricities were chosen randomly from 0 to e_{max}, where e_{max} equalled 0.05, 0.1, and 0.15. The number of bodies formed in the three-dimensional model was close to the actual number of planets, while in the plane model it was close to 10. In contrast to Refs. 15 and 16, we considered close to 10. In contrast to Refs. 15 and 16, we considered other (including zero) initial eccentricities and a large initial number of bodies and allowed for the axial rotations of the bodies. The other initial parameters of the rings also differed.

2. MODELING OF THE MUTUAL GRAVITATIONAL INFLUENCE OF PARTICLES MOVING AROUND THE SUN BY THE METHOD OF SPHERES

The results of numerical integration of the equations of motion of the plane three-body problem show[1-3] that the evolution of the orbits of two gravitationally interacting material points can be divided arbitrarily into three types. In one of them, which we call the second, the limits of variation of the semimajor axes of the orbits are considerably larger than in the other two types. In the first and third types of evolution the variations of the orbital elements have a periodic character, while the minimum distance between particles cannot be very small. In the case of initially circular orbits evolution of the second type occurs (with the condition that the initial angle between the directions toward the particles with the apex at the sun is not small) if the absolute difference between the initial values of the semimajor axes lies in the range from $0.3m^{1/4}$ to $rm^{1/4}$ (where r is the distance of the bodies from the sum and m is the mass of the larger of the encountering bodies in solar masses). For real planets and planetesimals the value of $rm^{1/4}$ is several times larger than the radius of the sphere of action $rm^{2/5}$. Therefore, in addition to the spheres of action, in investigating the evolution of rings of particles by the method of spheres we also used spheres of a larger radius R_s ($R_s \leq rm^{1/4}$). A sphere of radius $R_s = rm^{1/4}$ was evidently considered for the first time by Dole.[8] Remember that in an investigation by the method of spheres it is assumed that outside the sphere the particles move around the sun in unperturbed Keplerian orbits. When particles approach each other to a distance less than the radius of the sphere the relative motion of the particles is analyzed within the framework of the two-body problem under the assumption that their center of mass moves around the central body in a Keplerian orbit at this time. The computer realization of an algorithm for the method of spheres of action is described in Ref. 5. In this algorithm, used in the present report, the pairs of particles approaching to a distance equal to the radius R_s of the sphere and the positions of the particles at the time of encounter are chosen by a certain procedure using a pseudorandom numbers generator. The algorithm under consideration differs from Öpik's algorithm.[17] Öpik took the probability of the encounter

TABLE I

$em_1^{-1/3}$	0	10	50
$N_{(e<i)}$	100%	20%	1%

of a body with a planet as equal to the ratio of the time of motion of the body in the planet's sphere of action to the sidereal period of revolution of the planet. But we consider the probability of entry into the sphere. Öpik's algorithm is valid in the case of a nonzero inclination of the plane of the body's orbit to the plane of the planet's orbit. We are investigating the plane problem. We used a matrix of the probabilities of the encounter of pairs of particles to choose the numbers of the encountering particles. The number of particles in the rings under consideration reached thousands, and hence the number of matrix elements stored in the computer memory reached $5 \cdot 10^5$ (the matrix is symmetric).

It is known[18] that in modeling the mutual gravitational influence of particles by the method of spheres of action the model gives a good approximation for close encounters if $\varepsilon \geq 4$ while $m_2 \leq m_1 \leq 10^{-5}$ (where ε is the eccentricity of the hyperbolic orbit along which one particle moves relative to the other inside the sphere of action; m_1 and m_2 are the masses of the particles in $M_\odot$). Let us consider the results of modeling by the method of spheres of action of the gravitational interaction of two identical particles ($m_1 = m_2$) for which the eccentricities of the heliocentric orbits are $e_1 = e_2 = e$ before their approach to a distance equal to the radius of the sphere of action. In this case the fraction $N_{(\varepsilon < 4)}$ of orbits inside the sphere of action with eccentricities ε less than four is rather large in the case of small values of $em_1^{-1/3}$ (see Table I). Therefore, in the case of small values of $em_1^{-1/3}$ the motion being modeled can differ rather strongly from the real motion in an investigation of the mutual gravitational influence of particles by the method of spheres of action. When spheres larger than the sphere of action are used the energy integral is conserved less well and the model motion inside the sphere differs more strongly from the real motion. As our investigations show, however, through the appropriate choice of the radius of the sphere being used one can allow for the limits and the main tendencies of variations in the elements of the heliocentric orbits of gravitationally interacting particles in modeling by the method of spheres.[6] It should be mentioned that the evolution of the orbits of two particles moving around a massive central body (the sun) can be modeled by the method of spheres only in the case of evolution of the second type. In this case, if $em^{-1/3} > C_{cr}$ ($C_{cr} \approx 2$, m is the mass of the larger particle in $M_\odot$, and e is the larger eccentricity of the heliocentric orbits of the two encountering particles) then one must use the method of spheres of action, while if $em^{-1/3} \leq C_{cr}$ one must use the method of spheres of a larger radius (such as $R_s = rm^{1/4}$). These conclusions follow from a comparative analysis of about 20 pairs of graphs of the time dependence of the orbital elements of two gravitationally interacting particles, planetesimals, obtained by numerical integration of the equations of motion and by the method of spheres.[6] Two pairs of such graphs are presented in Fig. 1. The mutual gravitational influence of the particles is modeled by the method of spheres of radius R_s ($R_s = 2.3rm^{1/3}$ for Fig. 1a, and $R_s = rm^{2/5}$ for Fig. 1c) and by numerical integration of the equations of motion (Fig. 1b, d). The method of action reflects the real particle interaction ever better with an increase in the orbital eccentricities. Gravitational interactions of the second type make the main contribution to the evolution of a ring of particles, although in the N-body problem there are no pure gravitational interactions of two particles or planetesimals.

Let us dwell in more detail on the method which we used to model the evolution of plane rings of gravitating bodies which combine during collisions. The results of the investigation of the evolution of such rings are discussed in Secs. 4 and 5 of the present report. In the majority of the rings considered the initial orbits were circular or almost circular. In this case we usually analyzed the evolution of two rings (primary and secondary) which differ only in the initial eccentricities. First the evolution of the so-called primary ring of bodies, the initial orbits of which are circular, was analyzed by the method of spheres larger than the sphere of action ($R_s = rm^{1/4}$). Then the evolution of the secondary ring was analyzed by the method of spheres of action. The initial eccentricities e_0^{sec} of orbits in the secondary ring did not exceed the values of the average orbital eccentricity which the bodies acquired in the initial stages of evolution of the primary ring, but at the same time they were not very small. In the majority of variants $e_0^{sec} = 0.02$. If the maximum value e_{av}^{max} of the average orbital eccentricity of bodies of the primary ring was greater than e_{av}^{max} of bodies of the secondary ring, then it was assumed that the method of spheres larger than the sphere of action better reflects the real evolution. In this case the primary ring was taken as the main ring and the results of modeling the evolution of the secondary ring were not used further. Conversely, if e_{av}^{max} of orbits of bodies of the secondary ring was larger than e_{av}^{max} of orbits of bodies of the primary ring, then the secondary ring was considered as the main ring. Both the primary and the secondary rings were modeled to determine which ring is the main one, as well as to estimate the values of e_0^{sec} for the secondary ring. But if e_0^{sec} and the main ring were known in advance, then only the evolution of the main ring was investigated. In the variants analyzed the primary ring was the main ring only in the case of initially circular orbits for rings corresponding to the zone of planets of the terrestrial group and described in Sec. 4. The secondary ring was the main ring in the case of large initial eccentricities e_0 (in this case $e_0^{sec} = e_0$), as well as in the case of rings corresponding to the zone of the giant planets (see Sec. 5) and rings of arbitrary bodies of very high density (see Sec. 6). The distance between the orbits of the bodies formed as a result of the evolution of the main ring are larger than the radii R_s of the corresponding spheres, and within the framework of the algorithm being used such bodies do not interact. In reality, however, these bodies can interact gravitationally with each other. In particular, the lines of apsides of these bodies can evolve. Therefore, the evolution of such bodies was subsequently modeled with an algorithm differing from the main algorithm. In the main algorithm the longitudes π_i of the pericenters of the orbits were taken as constant between encounters up to distances not exceeding R_s. In the algorithm used to model the final stage of evolution $R_s = rm^{1/4}$ and the values of π_i were chosen so that encounters of bodies were possible so long as they could occur at any values of π_i.

In the course of the modeling we could also choose the sphere of action as the sphere if $em^{-1/3}$ was larger

TABLE II

Parameters of initial ring	MVE	M	V	E
a_{min} in AU	0.36	0.36	0.533	0.8
a_{max} in AU	1.2	0.533	0.8	1.2
M_Σ in $M_\oplus$	1.871	0.221	0.508	1

than some constant C_{cr} ($C_{cr} \approx 1$–3) or a sphere of larger radius ($R_s = rm^{1/4}$) in the opposite case.

3. EVOLUTION OF THE ORBITS OF GRAVITATIONALLY INTERACTING MATERIAL POINTS

The evolution of the orbits of two, three, and four gravitationally interacting particles, material points, the masses of which were close to the mass of Pluto while their initial orbits had small eccentricities, were modeled by the method of spheres of action. The results of the modeling show[6] that the maximum values of the orbital eccentricities of two gravitationally interacting particles are small, while for three or four particles they can exceed 0.2 in the course of evolution. These results testify in favor of Éneev's hypothesis[4] that Pluto, the semimajor axis of whose orbit was initially about 50 AU, could alter its orbit, owing to the gravitational influence of bodies lying at about the same distance from the sun, so that close encounters with Neptune became possible, which brought Pluto to its present orbit.

The mutual gravitational influence of particles or material points forming a plane ring leads to the spreading out of the ring and to an increase in the average orbital eccentricity, which can grow from zero to values of more than 0.15 (the average orbital eccentricity of the asteroids). The masses M_Σ of the rings considered by us[5] were considerably larger than the present mass of the asteroid belt ($M_\Sigma \approx 10^{-5}\ M_\odot$–$10^{-3}\ M_\odot$), but the results obtained probably give a qualitative picture of the evolution of rings of smaller mass. In order for the average orbital eccentricity of the asteroids to increase from zero to 0.15 during the time of existence of the solar system, the mass of the ring must be greater than the mass of the asteroid belt according to our approximate estimates.[5] Nor can mutual driving of the orbits of the asteroids explain the large orbital inclination of Pallas (34°.7), one of the largest asteroids. Therefore, the statement that the large eccentricities and inclinations of the orbits of the asteroids arose in the process of formation of the solar system planets and the bodies from the supply zone of the giant planets had a great influence on the formation of the asteroid belt[19] seems correct to us.

4. EVOLUTION OF PLANE RINGS OR BODIES CORRESPONDING TO THE SUPPLY ZONE OF PLANETS OF THE TERRESTRIAL GROUP

We will assume that bodies combine during any collisions, regardless of the collision velocity. In Secs. 4 and it is assumed that the bodies move in the same plane, so that the results obtained in these sections can only be extended to the case of very small orbital inclinations of the bodies.

Let us consider the evolution of plane rings corresponding to the supply zone of planets of the terrestrial group (except for Mars). At the initial time the masses m_i^0 and eccentricities e_i^0 of the orbits of the bodies are taken as equal to each other ($e_i^0 = e_0$ and $m_i^0 = M_\Sigma/N$, where M_Σ is the mass of the ring and N is the initial number of bodies in the ring, $N \leq 1000$), the semimajor axes a_i^0 of the the orbits are determined from the equation $a_i^0 = \sqrt{a_{min}^2 + (a_{max}^2 - a_{min}^2)\, i/N}$ $(i = 1, \ldots, N)$, while the directions toward the pericenters of the orbits are chosen randomly in the range from $-\pi$ to π. If not specially stated, it is assumed that we are talking about the case of initially circular or almost circular orbits. The initial data of these "MVE," "M," "V," and "E" rings are given in Table II. We mainly considered the case of $\rho = 2$ g/cm^3 and $\rho = 5.52$ g/cm^3. The character of the evolution was about the same for these two different densities. The evolution of rings with the same values of M_Σ, a_{min}, a_{max}, and e_0 was modeled several times with different pseudorandom numbers and different values of N. Some results of the modeling of the evolution of the "MVE" and "E" rings are presented in Figs. 2 and 3.

The numerical computer modeling shows that the average orbital eccentricity e_{av} of bodies in the "MVE" ring first grows rather rapidly to 0.01 and then somewhat slower to 0.02, and it does not exceed 0.03 in the course of evolution (see Fig. 3a). At the end of the evolution e_{av} declines somewhat. Throwing of bodies into hyperbolic orbits does not occur. More than 10 bodies are formed as a result of the evolution of the "MVE" ring (see Fig. 2a).

In the case of large initial eccentricities, e_{av} mainly declines in the course of evolution (see Fig. 3b, c). With an increase in the initial eccentricities the number of bodies formed as a result of the evolution of the "MVE" ring decreases and equals three for $e_0 \approx 0.3$–0.35 (see Fig. 2c). The origin of such large orbital eccentricities of the planetesimals in the supply zone of planets of the terrestrial group is hard to explain if the orbital inclinations are assumed to be very small. For $e_0 = 0.2$ seven to nine bodies are formed as a result of the evolution of the "MVE" ring (see Fig. 2e, b), while one body is formed, for the most part, as a result of the evolution of each of the isolated "M," "V," and "E" rings. This difference in the number of bodies formed is caused by the fact that for $e_0 = 0.2$ the massive bodies are formed mainly in the region of $[a_{min}(1 + e_0), a_{max}(1 - e_0)]$.

Some of the bodies which form have direct axial rotation and some have reverse rotation. The relatively small values of the periods of axial rotation of the bodies (see Fig. 2) are probably due to the small ratios of the masses of the colliding bodies, which is connected with the small number of initial bodies.

In the real solar system the initial number of planetesimals, and hence the ratio of the masses of the planetary embryos and planetesimals, were considerably larger than in our model. According to our estimates, the orbital eccentricities of the main mass of fine bodies of a ring initially consisting of a large number of bodies should not differ strongly from the orbital eccentricities in the variant which we considered. For a large initial number of bodies the orbital eccentricities of the massive bodies will most

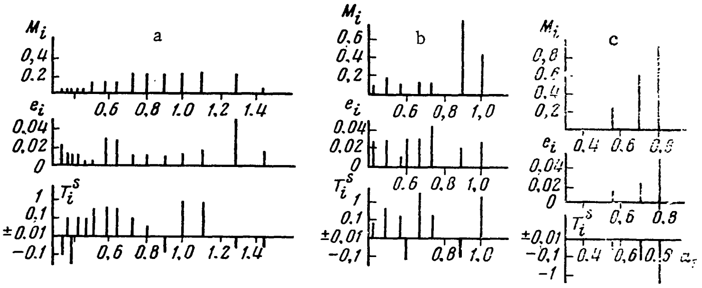

FIG. 2. Some characteristics of the bodies formed as a result of the evolution of plane "MVE" rings of gravitating bodies which combine during collisions. M_i (in $M_\odot$) masses; T_i^s (in days) periods of axial rotation of bodies which form; a_i and e_i) semimajor axes and eccentricities of their orbits: a) $e_0 = 0$, N = 500, $\rho = 5.56$ g/cm³; b) $e_0 = 0.2$, N = 1000, $\rho = 2$ g/cm³; c) $e_0 = 0.35$, N = 250, $\rho =$ g/cm³.

likely be smaller than for a small initial number. The number of "planets" formed is determined mainly by the eccentricities of the majority of the accumulating bodies in the course of evolution, and probably it does not decrease markedly with a strong increase in the initial number of bodies and with allowance for fragmentation of the bodies.

These investigations show that if the combining and gravitational interaction of solid bodies had played the dominant role in the accumulation of planets of the terrestrial group, the orbital inclinations had been small, and the influence of bodies from the supply zone of the giant planets had been insignificant, then the number of planets formed would have been larger than the actual number. The number of "planets" formed changes if one considers a three-dimensional model and also, possibly, if one allows for the resistance of the gas.[16,20]

5. EVOLUTION OF PLANE RINGS OF BODIES CORRESPONDING TO THE SUPPLY ZONE OF THE GIANT PLANETS

Safronov proposed a model in which the initial mass of solid matter in the supply zone of Uranus and Neptune was hundreds of times greater than the earth's mass, and he assumed that a large part of this mass was ejected from the solar system in the course of evolution.[19] In our modeling of the evolution of plane rings corresponding to the supply zone of the giant planets and consisting initially of several hundred bodies no more than half the initial mass of the ring was thrown into hyperbolic orbits for $M_\Sigma = 500\ M_\oplus$ and no more than a third for $M_\Sigma = 200\ M_\oplus$. The mass of ejected matter did not increase with an increase in the initial number N of bodies ($N \leq 750$). The densities of the bodies in these rings were close to the present densities of the planets. If one assumes that the orbital inclinations of the planetesimals were very small during the accumulation of the planets, then the results obtained testify in favor of Levin's conclusion[21] that the chaotic velocities and the mass of solid matter in the supply zone of the giant planets were less than those adopted by Safronov. According to our estimates, if the mass of solid matter in the supply zone of Uranus and Neptune equals the sum of the masses of these planets, Uranus and Neptune could be formed in a time not exceeding the time of existence of the solar system if the orbital inclinations were very small in the course of evolution. The estimates obtained for the time of evolution are less than Safronov's estimates.[19] This is connected with the fact that we are considering a plane rather than a three-dimensional model. The chaotic velocities of the bodies in the course of evolution of a plane ring are considerably smaller than in Safronov's model. We note that allowance for fragmentation of the bodies can lead to a change in the mass of ejected bodies, but this change will hardly be very significant. If the mass of the ring corresponding to the supply zone of the giant planets equals the mass of solid matter in the present planets (the estimate of the mass of the ring is taken from Ref. 12), the number of "planets" formed is close to the number of corresponding real planets.

6. ON THE POSSIBLE EVOLUTION OF THREE-DIMENSIONAL RINGS

The results of Secs. 4 and 5 were obtained on the basis of computer investigations of the evolution of plane rings of gravitating bodies which combine during collisions. The densities of the bodies in these rings are close to the present densities of the planets. The evolution of three-dimensional rings can differ quite strongly from the evolution of plane rings.[16] In the three-dimensional case the number of encounters leading to one combining is considerably larger (probably by one to two orders of magnitude) than in the plane case. The effect of "three-dimensionality" is strongly manifested, most likely, even with orbital inclinations $i \sim m^{2/5}$ (where m is the mass of the larger of two encountering bodies in $M_\odot$), i.e., with inclinations not exceeding the orbital inclinations of the present planets. Therefore, despite the fact that the average change in orbital eccentricity in one encounter is smaller in the three-dimensional case than in the plane case, because of the larger number of encounters of bodies the average orbital eccentricities of bodies in a three-dimensional ring can reach larger values than in the plane case, according to our approximate estimates. Approximate estimates of the evolution of three-dimensional rings were made on the basis of the results of modeling the evolution of plane rings of bodies of very high density (several orders of magnitude higher than the earth's density) by the method of spheres. The idea of the estimates consists in choosing that density of the bodies for which the changes in orbital eccentricities between successive combinings of bodies in the plane model being investigated will be about the same as in the three-dimensional model when the density of the bodies is close to the present density of the planets. It is proposed to give

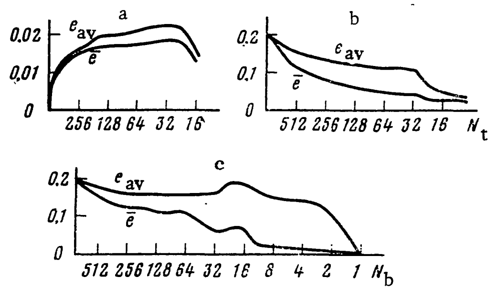

FIG. 3. Dependence of $e_{av} = \left(\sum_{i=1}^{i=N_b} e_i \right) : N_t$ and $\bar{e} = \left(\sum_{i=1}^{i=N_b} m_i e_i \right) : \left(\sum_{i=1}^{i=N_b} m_i \right)$ on the number N_b of bodies in the ring at the current time: a) "MVE," $e_0 = 0$, $N = 500$, $\rho = 5.56\ g/cm^3$; b) "MVE," $e_0 = 0.2$, $N = 1000$, $\rho = 2\ g/cm^3$; c) "E," $e_0 = 0.2$, $N = 750$, $\rho = 2\ g/cm^3$.

estimates of the corresponding values of the density and the results of modeling the evolution of plane rings of bodies of very high density in a separate publication. The small initial number of bodies ($N \leq 500$) and the simplicity of the model under consideration prevent making rigorous estimates of the parameters of the process of accumulation of the planets. However, the results obtained in the modeling can probably extend to the case of a large initial number of bodies if one assumes that in the course of the evolution of rings consisting of a large number of bodies, the masses of the largest bodies do not differ very strongly from each other for a considerable part of the time, i.e., if one does not consider models in which a massive embryo scoops up the fine bodies in the supply zone of each planet while the mass of the body second in size is many orders of magnitude less than the mass of the embryo. The results obtained show that the ejection from the zone of the giant planets of a considerable mass of solid matter, which can be several times greater than the mass of solid matter in the present planets, is possible for certain mutual inclinations of the orbits of the planetesimals. If one admits the possibility of combinings at high velocities of encounter of solid bodies whose masses differ from each other by not many orders of magnitude, then for certain orbital inclinations of the planetesimals in the course of accumulation of the solid bodies one can obtain the required number of planets in the zone of planets of the terrestrial group. We note that to explain the formation of the inclinations of the rotation axes of the planets within the framework of the accumulation of solid bodies it is necessary that the masses of the largest bodies falling onto the planetary embryos be two to three orders of magnitude less than the masses of the respective planets (Ref. 19). The scheme of the falling of only small bodies onto the planetary embryos cannot also explain the large orbital eccentricities of Mercury and Mars or provide the formation of the required number of planets in the zone of the terrestrial group.[1,3] At the same time, a weak point of the model which assumes the collisions of massive bodies of close masses is the assumption of the combinings of these bodies in high-velocity collisions. The question of the high-velocity collisions of massive bodies has not yet been fully clarified. It follows from the data of Ref. 22 that in the collision of an embryo with a diameter of more than 1000 km with a body whose mass is an order of magnitude less than the embryo's mass the catastrophic destruction of the embryo does not occur if the collision vellcity is less than 5 km/sec.

If one assumes much ejection of solid matter from the supply zone of the giant planets, then massive bodies from this zone moving along highly elongated elliptical orbits could probably influence the orbital elements and evolution of the almost-formed planets of the terrestrial group. It is possible that the relatively large (compared with those of the giant planets) inclinations (to the Laplace plane) of the orbits of planets of the terrestrial group and the large eccentricities of the orbits of Mercury and Mars are due to the influence of these bodies. Under the assumption that the massive embryo of Jupiter was formed before the formation of planets of the terrestrial group ended, and there was much solid matter in its supply zone at this time, bodies from Jupiter's zone moving along highly elongated elliptical orbits (possibly with fairly large inclinations) before the transfer to hyperbolic orbits could increase the eccentricities and inclinations of the orbits of planetesimals in the zone of planets of the terrestrial group.

7. CONCLUSION

An approximate method of modeling the evolution of a plane ring of gravitating particles based on the method of spheres of action is investigated in the report.

The results of the investigation of the evolution of plane rings of gravitating bodies which combine during collisions show that if the orbital inclinations of the planetesimals were small and the influence of the drag force of the gas on the motion of the planetesimals could be neglected, then the number of planets formed in the supply zone of planets of the terrestrial group would be greater than the actual number of the corresponding planets. In the case of very small orbital inclinations of the planetesimals, the initial mass of solid matter in the zone of the giant planets would have been close to the mass of solid matter in the present planets.

The formation of the planets of the terrestrial group and the ejection of a large mass of solid matter from the zone of the giant planets are possible in principle in the case of the accumulation of solid bodies when the orbital inclinations of the planetesimals are not very small.

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