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Analysis of lunar occultations – III. Systematic corrections to Watts' limb-profiles for the Moon

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Summary. Timings of 66 000 occultations of stars by the Moon are analysed for systematic corrections to the limb-profile heights taken from Watts' charts of the marginal zone of the Moon. The radius, shape and location of the centre of the datum implicit in the charts are found to vary with libration and these variations produce systematic errors in the limb-profile heights which attain 0.4 arcsec in some position angles.

1 Introduction

In 1956, C. B. Watts completed a photographic survey of the marginal zone of the Moon and he later published a set of charts (Watts 1963) giving limb-profile heights with respect to an adopted smooth reference surface (or datum). These are particularly useful in reducing observations of occultations of stars which occur at different position angles and librations; for this purpose the reference surface should be a sphere with its centre at the centre of mass of the Moon, otherwise systematic distortions will be introduced into the residuals and these may be mistakenly interpreted as, for example, errors in the ephemeris of the Moon.

Analyses of occultations of stars by Van Flandern (1970) and Morrison (1979), (hereafter referred to as Paper I) have revealed that the average cross-section of Watts' datum is slightly elliptical rather than circular, and that the implicit centre of the datum is displaced from the point to which the current lunar ephemeris refers. Recently, whilst analysing grazing occultations, we have also found that the radius of the datum appears to vary with libration. Since grazing observations are restricted to the polar regions of the Moon, it is not possible using these observations alone to decide whether the variation in radius occurs around the whole limb. We therefore decided to extend the analysis of total occultations in the years 1943 to 1974 reported in Paper I in order to investigate whether there were any systematic changes in the datum varying with libration.

The main results of this investigation are that the heights of the limb-profiles at different librations in Watts' charts are referred to different datums which are elliptical rather than circular and are displaced from one another. Corrections are given to the heights of the profiles such that the revised heights refer to one spherical datum, the centre of which is located at the centre of mass.

2 Observations and reduction

About 20 000 observations of total occultations made in the years 1975 to 1979 were added to the 62 000 observations in the years 1943 to 1974 which had been analysed in Paper I. For the reasons given in that previous work, only occultations of stars with positions in the *Catalog of 3539 Zodiacal Stars for the Equinox 1950.0* (Robertson 1940) were retained for analysis. Occultations at the bright limb and at position angles and librations lying outside the scope of Watts' charts were rejected.

The distributions of the remaining 66 000 observations with libration in longitude (l) and latitude (b), and position angle (Q) from the projection of the north polar axis of the Moon are displayed in Figs 1 and 2. Apart from the librations near $l = +7°.5$, $b = -6°.5$ the observations are quite uniformly distributed, but there are comparatively few in the range 180°–360° of position angle (appearances).

The method of reduction of the observations is set out in detail in Paper I. The residual geocentric separation ($\Delta\sigma$) between the star and the limb of the Moon was calculated for the observed time of occultation using the catalogue position of the star, the Moon's position and semi-diameter taken from the ephemeris $j = 2$ (IAU 1968) and the profile heights of the Moon (scaled to the true distance) taken from Watts' charts.

For the purposes of the reduction, it was assumed that Watts' limb-profile heights are referred to one spherical datum, the centre of which is coincident with the ephemeris

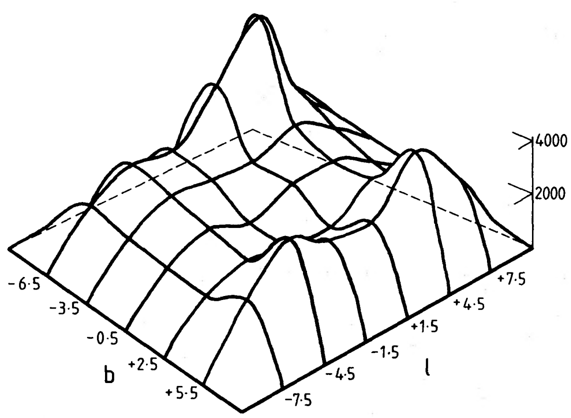

Figure 1. Distribution of observations with libration in longitude (l) and latitude (b).

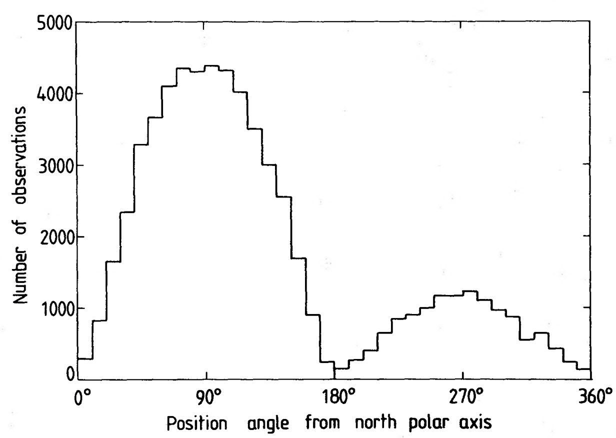

Figure 2. Distribution of observations with position angle from the projection on the geocentric sphere of the north polar axis.

position. The assumed radius of the datum, 1737.97 km, is equivalent to a semi-diameter of 932.58 arcsec at the mean distance of the Moon corresponding to a value of 3422.608 arcsec for the equatorial horizontal parallax (IAU 1966). The limb-profile height for position angle Q was found from Watts' charts by entering for the argument $Q + 0°.25$ in accordance with the systematic error in the argument found by Morrison (1970).

The residual $\Delta\sigma$ was adjusted for corrections to the lunar ephemeris $j = 2$ contained in Paper I in order to refer the ephemeris to the FK4 frame of reference. Adjustments were also made for corrections to the elements in $j = 2$ found in that paper. Systematic corrections found in that paper to the limb-profile heights in Watts' charts were not applied because they are the subject of the present investigation. The values of the personal equations determined by Morrison & Appleby (1981, Paper II) were used in computing $\Delta\sigma$ in preference to those contained in Paper I, because the latter solutions for the personal equations were distorted as a result of their high correlation with the correction to the mean radius of Watts' datum. This problem was circumvented by us by adopting a correction of +0.04 arcsec to the heights of the limb-profiles in Watts' charts which we had determined from an analysis of photoelectric timings of occultations. A model for the behaviour of the personal equation, which is dependent on the phase of the Moon and magnitude of the star, was then derived by us using this revised radius. Therefore, the application of our model for the personal equation in the reduction of the observations should lead to a correction of + 0.04 arcsec when averaged over all librations.

The correction to the constant of parallactic inequality α_1 (Paper I) was not applied in this analysis because we (Paper II) have shown that this is a spurious correction arising from the variation of the personal equation with the phase of the Moon. Its effect has been absorbed in the revised model for the behaviour of the personal equation which has been applied in this analysis.

3 Analysis and results

The residuals $\Delta\sigma$, reduced as described in the previous section, were analysed for systematic corrections to the limb-profile heights in Watts' charts varying with libration. Watts' charts cover the librations $-9°$ to $+8°$ in longitude (l) and $-8°$ to $+8°$ in latitude (b). In order that there should be no fewer than about 1000 observations in any subdivision of these ranges of libration, it was decided to divide l into six ranges of 3° and b into four ranges of 3° and one of 4°, the centres of the 30 resultant frames being at the intersections of the contours shown in Fig. 1. It was found that ~ 1000 observations were required in each range of libration to obtain reliable solutions for the systematic corrections.

The systematic corrections in each range of libration were considered to be harmonic series as far as the second order in the position angle. Thus, the following equation of condition in five unknowns, δr_0, δr_1, R_1, δr_2, R_2 was formulated:

$$\frac{\pi_0}{\pi}\Delta\sigma = \delta r_0 + \delta r_1 \cos(Q - R_1) + \delta r_2 \cos 2(Q - R_2),$$

where Q is the position angle of the occultation measured through east on the geocentric celestial sphere from the projection of the Moon's north polar axis, and the factor π_0/π (the ratio of the mean to the true parallax) reduces $\Delta\sigma$ to mean distance.

The unknown, δr_0, is the constant correction to the limb-profile heights in Watts' charts which is required when a provisional value of 932.58 arcsec is used for the semi-diameter at mean distance. The term in Q may be expressed as the sum of two components: $\delta x \sin Q$, where x is measured positive in the direction of increasing ecliptic longitude (i.e. east on the geocentric sphere), and $\delta y \cos Q$, where y is measured positive in the direction of the north lunar pole. The quantities $-\delta x$, $-\delta y$ are the amounts by which the centre of the datum in Watts' charts should be moved in order that the resultant profile should agree with the 'observed' one when the charts are used in conjunction with the lunar ephemeris $j = 2$. The term in $2Q$ in the equation of condition allows for an approximately elliptical correction to the limb-profile. This is equivalent to stating that the datum departs from circularity by the amount $-\delta r_2 \cos 2\,(Q - R_2)$.

The observations were weighted as in Paper I, (Section 4.1) and the unknowns were determined by the method of least-squares for each frame at the points of intersection shown in Fig. 1. The solutions for the position angle R_2 did not differ significantly from 146° which was obtained by combining all the observations. Therefore, R_2 was fixed at the value 146° and the least-squares analysis was repeated to determine the other unknowns. The average standard errors of the solutions in each frame were:

$\delta r_0 \pm 0.01$ arcsec,

$\delta x, \delta y \pm 0.02$ arcsec,

$\delta r_2 \pm 0.02$ arcsec.

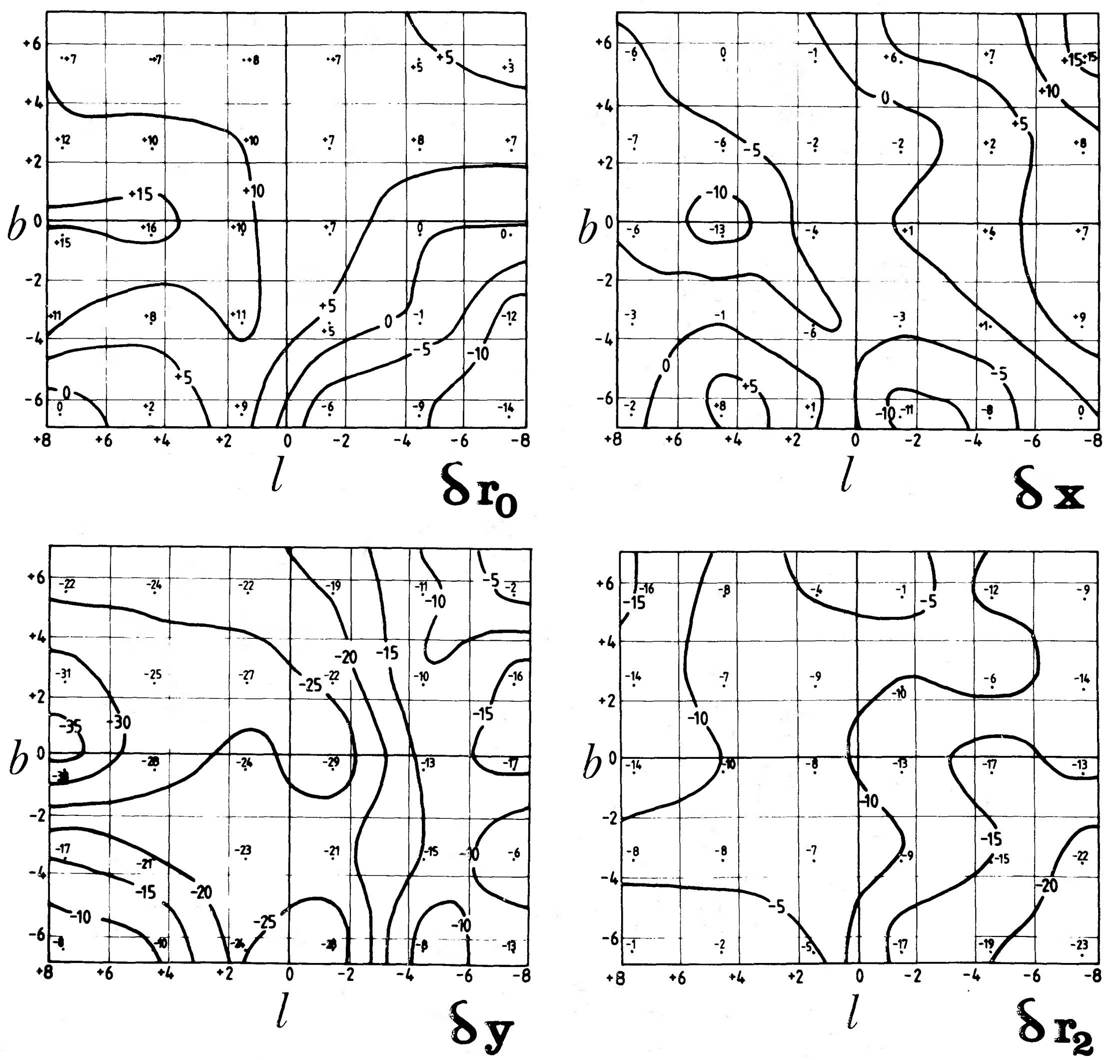

Figure 3. Systematic corrections to the limb-profile heights in Watts' charts varying with libration in longitude (l) and latitude (b). Contours (in units of 0.01 arcsec) have been drawn through the grid of solution points.

The results at the points of intersection in Fig. 1 are given in Fig. 3 rounded to the nearest 0.01 arcsec. It is seen that, in general, these results vary systematically with libration and we therefore considered it appropriate to represent the results by contours in units of 0.01 arcsec and at an interval of 0.05 arcsec. The contours were constructed using the GINO routine CONTR.

4 Discussion

4.1 GENERAL

This analysis shows that when Watts' charts are used in conjunction with the lunar ephemeris $j=2$, the heights of the limb-profiles should be amended by

$$\delta r_0 + \delta x \sin Q + \delta y \cos Q + \delta r_2 \cos 2\,(Q - 146^\circ),$$

where the values of δr_0, δx, δy and δr_2 should be taken from Fig. 3 according to the libration. The expression above with the opposite sign gives the correction to reduce the cross-section of the datum of Watts' charts to a circle of semi-diameter 932.58 arcsec whose centre is such that, when it is superimposed on the position of the Moon given by the ephemeris $j=2$ (as amended in Section 2), the lunar profile is obtained with respect to the FK4 frame of reference.

4.2 RADIUS CORRECTION

The corrections to the heights of the limb-profile given in Fig. 3 (δr_0) have a mean value over all librations of +0.04 arcsec as a result of the adoption of the model for the behaviour of the personal equations reported in Paper II. In Paper I Morrison had found a correction of +0.15 arcsec, but this result is now known to be in error because, as we have shown, he underestimated the size of the personal equation in some methods of timing and did not allow for the possibility that the personal equation might vary with the elongation of the Moon from the Sun.

4.3 LONGITUDE AND LATITUDE COMPONENTS δx AND δy

The variation of the longitude component of the correction to the limb-profile heights in Watts' charts is shown in Fig. 3 (δx).

A source of systematic error in δx arises from the possibility that part of this correction varies linearly with libration in longitude as a result of the possible displacement in the direction towards the Earth of the centre of figure from the centre of mass.

If this were the case, then it may be represented by

$$\delta x = \frac{m}{6^\circ}\ l + \mathrm{k},$$

where m is the change in 6° (say) of libration in longitude and k is a constant. Hence,

$$\delta x \simeq m \sin M + k,$$

where M is the Moon's mean anomaly. Also, $\delta\lambda \simeq 2\delta e\ \sin M + \delta L$, where λ is the true longitude, e is the orbital eccentricity and L is the mean longitude. Since δx and $\delta\lambda$ are indistinguishable in the occultation residuals, $m \simeq 2\delta e$, $k \simeq \delta L$.

Thus, any contribution k will have been absorbed in the correction δL, and any contribution m, in the correction to the orbital eccentricity, which were derived in Paper I and removed from the residuals $\Delta\sigma$ in Section 2. Hence, our results in Fig. 3 should be regarded as those parts of δx which exhibit sufficient variation with libration in latitude to enable them to be separated from a correction to the orbital eccentricity. If the centre of figure in Watts' charts were 2 km (say) nearer the Earth than the centre of mass, the value m would be about +0.1 arcsec, and thus the spurious contribution to δe in Paper I would have been +0.05 arcsec.

It is our understanding that Watts intended that his charts should be used in conjunction with the lunar ephemeris to obtain the profile of the Moon with respect to the stellar reference frame. By its development, (see Paper I) the ephemeris $j=2$ in longitude refers to the centre of the mean profile which we shall regard as the centre of figure. Watts intended the position of the centre of his datum in longitude to be consistent with this centre of figure. If this were not the case, the offset of his datum in longitude would have been absorbed in the correction to the mean longitude deduced in Paper I which was removed from $\Delta\sigma$ in this analysis. The displacement between the centre of figure and the centre of mass can be deduced from the correction to the elongation of the Moon (D) found by Morrison in Paper I. He found $\delta D = -0.72 \pm 0.08$ arcsec and interpreted it as arising from a correction of the opposite sign to the longitude of the Sun in Newcomb's Tables. However, as the correction to D is derived from dynamical considerations, it may also be interpreted as a measure of the displacement between the centre of figure used in the calculated longitude of the Moon and the centre of mass. If we accept that the longitude of the Sun in Newcomb's tables does not require a correction (Fricke 1980), we can conclude that the centre of figure of leads the centre of mass in longitude by 0.72 ± 0.08 arcsec. Mulholland (1977) has determined the displacement between the centres of figure and mass from an analysis of occultations of stars combined with precise measurements of the Moon's distance derived from laser ranging. He found that the centre of figure leads the centre of mass by 2.5 ± 0.5 km (i.e. 1.3 ± 0.3 arcsec). Using laser altimetry data from *Apollo 15*, *16* and *17* missions, Sjogren & Wollenhaupt (1976) find that the centre of figure leads the centre of mass by about 1 km (i.e. 0.5 arcsec), regardless of whether the reference points are taken on the maria or the highlands. Bills & Ferrari (1977), however, obtain a result which is about half that of Sjogren & Wollenhaupt from a combination of laser altimetry data with orbital and Earth-based photogrammetry.

All the results considered here are in agreement that the centre of figure leads the centre of mass in the direction of increasing longitude, but they differ in size. The results presumably do not all refer to the same centre of figure. Mulholland's result should be amended to allow for the offset in right ascension of the FK4 equinox from the dynamical equinox. The centre of figure to which occultations refer must be some mixture of the profiles of the maria and highlands, depending on the particular libration. We shall assume that the result obtained by Sjogren & Wollenhaupt applies to Watts' datum, but, bearing in mind our result of 0.72 arcsec from occultations, we regard their displacement of 0.5 arcsec as being uncertain by about ± 0.2 arcsec in the present context. Thus, if Watts' charts are used in conjunction with a lunar ephemeris which refers to the centre of *mass* in orbital longitude, we recommend that a correction of +0.50 arcsec sin Q be applied to the limb-profile heights in addition to the correction δx sin Q which is dependent on libration.

The variation of the latitude component of the correction to the limb-profile heights in Watts' charts is shown in Fig. 3 (δy). By an argument analogous to that in the discussion of the systematic errors in δx, the results for δy should be regarded as comprising those components which exhibit sufficient variation with libration in longitude to enable them to be separated from a correction to the inclination of the Moon's orbit or the possible displacement between the centre of figure and centre of mass in the direction towards the Earth.

The average correction over all librations is -0.18 ± 0.01 arcsec, which is in good agreement with the result -0.19 ± 0.01 arcsec reported in Paper I. As explained in that paper, the lunar ephemeris $j = 2$ gives the position of the centre of mass in latitude, and the centre implicit in Watts' charts is superimposed on this centre in the reduction of occultations. The quantity $-\delta y$ is therefore the amount that the centre of Watts' datum should be moved in latitude in order that it should be coincident with the centre of mass. In the construction of his charts Watts moved the centre of his original datum 0.3 arcsec along position angle 333°: our results for δy show that on the average he should have moved it a further 0.18 arcsec north in latitude.

4.4 ELLIPTICITY CORRECTION δr_2

Fig. 3 (δr_2) shows the variation with libration of the spurious ellipticity in the datum of Watts' charts. This can be removed from the limb-profile heights by adding the correction $\delta r_2 \cos 2 (Q - 146°)$. Watts' suspected that his original reference datum might be elliptical and he applied the following corrective term to the profile heights in the construction of his charts to remove the ellipticity: $+0.15$ arcsec $\cos 2 (Q - 153°)$. The average value of the coefficient over all librations is about -0.09 arcsec, so it would appear that Watts should have applied $+0.06$ arcsec, rather than $+0.15$ arcsec.

4.5 COMPARISON OF THE LIMB-PROFILES OF WATTS WITH WEIMER

Meyer (1976) has compared the limb-profile heights taken from Watts' charts with the atlas of Weimer (1952). He found systematic differences in the heights varying with position angle which attain ~ 1 arcsec in some librations. In Fig. 4 we show the difference, in the sense Weimer minus Watts, for libration $l = +2°.6, b = -5°.1$ derived by Meyer. For comparison we plot the result of our harmonic series, in the sense occultations minus Watts, for the solution point $l = +1°.5, b = -6°.5$. The difference ~ 1° between the librations in the comparisons can not account for the considerable differences in the curves.

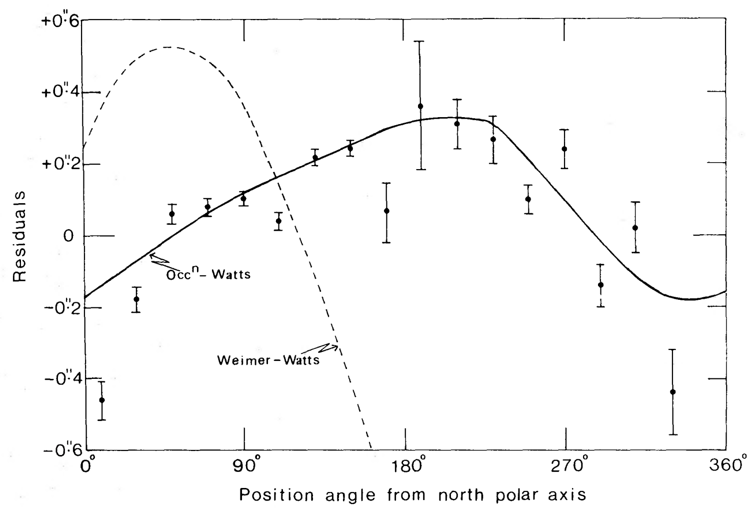

Figure 4. Systematic corrections to the limb-profile heights in Watts' charts deduced from occultations (solid line) and by comparison with Weimer's atlas (dashed line) for a libration near $l = +2°, b = -6°$. The solid line was evaluated from the harmonic expression given in Section 4.1, and the error bars show the means and standard errors of the differences between occultations and Watts for 20° ranges of position angle.

In order to test the goodness of fit of our harmonic series which was truncated at the second order, we formed the means of the residuals $\Delta\sigma$ in ranges of 20° in position angle for each of the 30 libration frames shown in Fig. 1. The means and their standard errors are plotted in Fig. 4 for the frame centred on $l=+1°.5$, $b=-6°.5$. The general agreement between the means and our harmonic series for this frame is typical of the others. We conclude that there must be greater systematic errors in Weimer's profiles than in Watts'.

4.6 SYSTEMATIC ERRORS IN THE POLAR REGIONS

The stimulus for this investigation was that the radius of Watts' datum deduced from grazing occultations in the region of the lunar north pole appeared to vary linearly with libration in latitude. The evaluation of the harmonic series in Section 4.1 for a position angle of zero and a range of libration in latitude produces a linear change in the same direction, but of quarter the magnitude. In Fig. 4 we note that our harmonic series is not a good fit to the means near the north lunar pole and from Fig. 2 it is seen that there are comparatively few observations within ±10° of the lunar poles. We conclude that our global solution may not be valid in the polar regions, particularly in the north.

5 Conclusions

The systematic corrections to the limb-profile heights in Watts' charts, which follow from the requirements that the reference datum should be a sphere with its centre at the centre of mass, are given by

$$\delta r_0 + (\delta x + 0.50\,\text{arcsec}) \sin Q + \delta y \cos Q + \delta r_2 \cos 2\,(Q - 146°),$$

where the values of δr_0, δx, δy and δr_2 are taken from Fig. 3 according to the libration, and Q is the position angle measured through east from the projection on the geocentric sphere of the north polar axis. If Watts' charts are used in conjunction with the lunar ephemeris $j=2$, which refers to the centre of figure in longitude, the constant 0.50 arcsec in the coefficient of sin Q should not be applied. These systematic corrections may not be valid close to the poles of the Moon.

References

Bills, B. G. & Ferrari, A. J., 1977. *Icarus*, **31**, 244.
Fricke, W., 1980. *Mitt, Astr. Gesell.*, **48**, 29.
IAU, 1966. Report on Astronomical Constants, *Trans. int. Astr. Un.*, **12B**, 593.
IAU, 1968. Epoch of Ephemeris Time, *Trans. int. Astr. Un.*, **13B**, 48.
Meyer, C., 1976. *Moon*, **16**, 27.
Morrison, L. V., 1970. *Mon. Not. R. astr. Soc.*, **149**, 81.
Morrison, L. V., 1979. *Mon. Not. R. astr. Soc.*, **187**, 41 (Paper I).
Morrison, L. V. & Appleby, G. M., 1981. *Mon. Not. R. astr. Soc.*, **196**, 1005 (Paper II).
Mulholland, J. D., 1977. *Astr. J.*, **82**, 306.
Robertson, J., 1940. *Astr. Pap. Washington*, **X**, Part II.
Sjogren, W. L. & Wollenhaupt, W. R., 1976. *Moon*, **15**, 143.
Van Flandern, T. C., 1970. *Astr. J.*, **75**, 744.
Watts, C. B., 1963. *Astr. Pap. Washington*, **XVII.**
Weimer, Th., 1952. *Atlas de Profils Lunaire*, Observatoire de Paris.