

Analysis of lunar occultations – II. Personal equation

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Summary. From an analysis of 42 000 timings of occultations of stars by the Moon in the years 1943 to 1979 it is found that the observers' personal equation is dependent on the phase of the Moon and the magnitude of the star. Expressions for this dependence are derived for two different methods of timing.

1 Introduction

Timings of the occultations of stars by the Moon provide useful data on the orbit of the Moon and the stellar reference frame (see Morrison 1979, hereafter referred to as Paper I). A worldwide distribution of observers makes these timings which have been collected and collated by HM Nautical Almanac Office, Royal Greenwich Observatory, since 1943. By far the greatest proportion of the observations is made visually and the times are recorded using a hand-operated device, such as a stop-watch which is usually started at the instant of occultation and stopped a few minutes later on the minute-marker of a broadcast time-signal. About 20 per cent of the visual observations are made by the eye-and-ear method, whereby the observer estimates the instant of occultation relative to the audible second-markers of a time-signal. In both these methods a precision of 0.1 s can be achieved, but in the former, especially, some allowance has to be made for the observer's personal equation. About 30 per cent of the observers estimate their personal equation and apply a correction to the time recorded on the clock. Where observers using a stop-watch or similar device do not apply a correction, -0.3 s is applied to the time of disappearances and -0.5 s to reappearances in the reduction of the observations.

It is difficult to be sure that these estimates for the personal equation are correct. Simulated experiments in laboratories do not reproduce all the circumstances that occur in practice, such as the varying brightness of the Moon with phase and the contrast between the magnitude of the star and the sky-background. The most satisfactory way of determining the systematic errors in visual observations is to compare a large number of them with the times predicted from a lunar ephemeris from which systematic errors have been removed as far as possible. The separation of some of these systematic errors from errors in timing visual observations has proved to be troublesome in the past, because, to a large extent, systematic errors in timing occultations can be compensated by adjusting the Moon's ephemeris position

in mean longitude and changing the value of the mean semi-diameter. Mathematically, this can be expressed as

$$\frac{\partial \sigma}{\partial t} \delta t \approx \frac{\partial \sigma}{\partial \lambda} \delta \lambda + \delta s, \quad (1)$$

where σ is the computed angular separation between the star and the limb of the Moon at the observed time of occultation, δt is the delay in timing, and $\delta \lambda$ and δs are corrections to the longitude and semi-diameter.

Fortunately, in recent years several thousand occultations have been timed to a precision of 0.01 s or better using photoelectric equipment and, of course, any systematic errors in these times are negligible compared with the personal equation. In this paper we use these photoelectric observations as a control on the longitude and semi-diameter of the Moon and thus distinguish the personal equation in the visual observations. We find that all visual observations are subject to a personal equation which varies with the phase of the Moon and the magnitude of the star. The corrections found in Paper I to the personal equations, the semi-diameter and the constant of parallactic inequality are revised as a result of this investigation.

2 Observations and reduction

The observations used in this analysis were taken from the catalogues of lunar occultations published by Morrison (1978) for the years 1943 to 1971 and Morrison & Appleby (1981a) for 1972 to 1979. The methods of selecting, reducing and weighting observations are described in detail in Paper I. Only the following types of observation were retained in the present analysis: occultations at the dark limb with reliable profile corrections; timings marked good or very good; and stars with positions in Robertson's (1940) catalogue. Thus about 42 000 of the best visual observations and 2200 photoelectric observations were retained for analysis.

The topocentric angular separation between the star and the centre of the Moon calculated from the ephemeris designated $j = 2$ (IAU 1968) was computed for the time of each observation. The lunar semi-diameter and limb-profile height taken from Watts' (1963) charts were subtracted from the separation to give the residual separation $\Delta\sigma$. The systematic and periodic corrections to the lunar ephemeris, the stellar frame of reference and the shape of the datum of Watts' charts found by Morrison (Paper I) were subtracted from $\Delta\sigma$. However, the corrections found by Morrison for the semi-diameter (or, equivalently, the radius of Watts' datum), the constant of parallactic inequality (α_1) and the personal equations were not applied, because these unknowns are correlated, and are the subject of the present investigation. Therefore, the value of the semi-diameter used in the reduction of the observations is that in the ephemeris $j = 2$, which is 932.58 arcsec at a mean distance corresponding to an equatorial horizontal parallax of 3422.608 arcsec and an equatorial radius of 6 378 160 m for the Earth. In Paper III (Morrison & Appleby 1981b) we find errors in Watts' datum varying with libration, but as these enter randomly in the treatment of the personal equation we have ignored them here.

The visual observations were divided into three main classes according to the method of timing and application of personal equation:

- (a) stop-watch or similar device with no estimate of the personal equation reported or applied by the observer;
- (b) stop-watch or similar device with estimate of the personal equation reported by the observer; or

Table 1. Corrections for personal equation applied in reduction of observations.

Method	Phenomenon	
	Disappearance	Reappearance
(a) Stop-watch, NAO estimate	$-0^s.30$	$-0^s.50$
(b) Stop-watch, observer estimate	$-0^s.30$	$-0^s.35$
(c) Eye-and-ear, NAO estimate	$-0^s.00$	$-0^s.20$

(c) eye-and-ear, where it is assumed that no correction for the personal equation has been applied by the observer.

The values of the personal equations for these three classes, which were applied in reducing the observations, are given in Table 1, where NAO stands for HM Nautical Almanac Office. The values for class (b) are the means of all the estimates reported by the observers: the individual estimate was actually applied in reducing each observation. Observations in which the observers applied a correction for the personal equation, but did not report its value to us, have not been used in this analysis.

Before embarking on the analysis of the visual observations, the photoelectric timings of 2175 occultations were analysed to determine corrections to the adopted mean longitude and semi-diameter of the Moon. These quantities are correlated with the observers' personal equations, but by using only photoelectric observations this is avoided. The corrections were found to be:

Mean longitude -0.013 ± 0.017 arcsec,

Semi-diameter $+0.044 \pm 0.015$ arcsec.

The correction to mean longitude was ignored and $+0.04$ arcsec was adopted for the correction to the provisional value of 932.58 arcsec for the semi-diameter. All the residuals ($\Delta\sigma$) were adjusted for this revised value of the semi-diameter. We now regard these residuals as being free of any significant systematic errors that might be confused with the personal equation.

3 Analysis and results

3.1 PERSONAL EQUATION AS A FUNCTION OF THE MOON'S ELONGATION

The visual observations in the classes (a), (b) and (c) defined in Section 2 were subdivided into 10° ranges of elongation (D) of the Moon from the Sun. Within each class the distribution of the observations with elongation follows that shown in Paper I (Fig. 2), where disappearances at the leading dark limb occur up to elongation 180° (i.e. full Moon), and reappearances at the following dark limb occur thereafter. The numbers of observations in the classes are in the proportion 5 : 3 : 2, respectively.

The weighted mean residual ($\overline{\Delta\sigma}$) and its standard error were calculated for each 10° range of elongation within each class. These were converted to corrections in time by forming $\overline{\Delta\sigma}/(\partial\sigma/\partial t)$, where the partial derivative $\partial\sigma/\partial t$ was the average value in each 10° range; this average value was found to be very close to 0.35 arcsec s^{-1} in all the ranges of elongation. These corrections in time were added to the values of the assumed personal equation applied in the reduction (Table 1) to give the total correction for the personal equation involved in timing occultations. The results for the three classes are plotted in Fig. 1.

There is clearly a systematic difference between the eye-and-ear method and the methods which require some physical response, such as starting a stop-watch; but they do share the

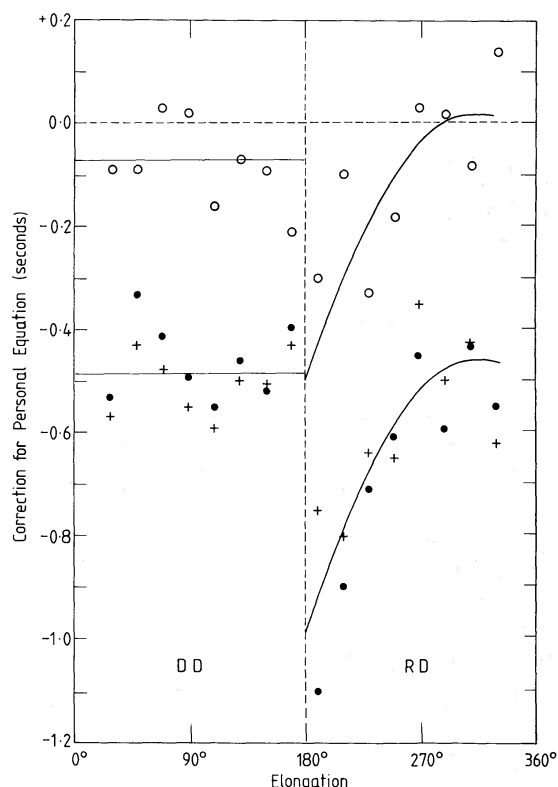


Figure 1. Correction for personal equation as a function of elongation. Dots represent timings made with a stop-watch, or similar device, where the observer made no estimate of the personal equation (class a in text). Crosses represent timings made with a stop-watch or similar device, where the observer made and reported an estimate of personal equation (class b in text). Open circles represent timings made using the eye-and-ear method. The equations of the best-fitting lines and parabolae are given in Table 2.

same trend with elongation. The increase just after full Moon is evident in the three classes. Assuming that they have the same functional dependence on elongation, the results for the three classes were combined by removing the mean differences between them. The combined results for disappearances showed no particular dependence on elongation, so constant values were adopted for the stop-watch method (i.e. classes a and b combined) and the eye-and-ear method. A parabola was fitted to the combined results for the reappearances and the resultant parabolae for the two methods are shown in Fig. 1.

In the disappearances it seems that the personal equation of observers who estimate and apply a correction (class b) is greater systematically by about 0.1 s than those who do not (class a). This is a surprising result. It may also apply to reappearances, but there are too few observations, and hence too great a scatter in the points in Fig. 1, to detect the effect. We conclude that observers who do not estimate their personal equation tend to round down their times to the earlier tenth of a second, whereas observers who estimate their personal equation do not do this.

The standard deviations of the points about the functions shown in Fig. 1 are ± 0.08 s for the disappearances and ± 0.11 s for the reappearances. The points are also subject to systematic errors arising from the errors in the adopted values of the mean longitude and semi-diameter. Using equation (1) with the standard errors ± 0.017 arcsec in longitude and ± 0.015 arcsec in semi-diameter found in Section 2, we find the resultant systematic error in the personal equations to be ± 0.06 s, where the average values of $\partial\sigma/\partial t$ and $\partial\sigma/\partial\lambda$ are 0.35 and 0.87, respectively.

In order to check the consistency of our solution for the personal equations as a function of elongation, their effects were removed from the residuals $\Delta\sigma$ and the visual observations of all types were combined in a solution to determine corrections to the semi-diameter, mean longitude and constant of parallactic inequality. The corrections were found to be:

Semi-diameter	$+0.042 \pm 0.005$ arcsec,
Mean longitude	-0.002 ± 0.003 arcsec,
Parallactic inequality	-0.088 ± 0.041 arcsec.

As we should expect, the corrections to the semi-diameter and mean longitude are in agreement with those from the photoelectric observations because the personal equations were derived with respect to those values. The correction to the constant of parallactic inequality (α_1) is greatly reduced from the value of $+0.46 \pm 0.03$ arcsec obtained in Paper I, and this therefore demonstrates that that spurious correction was mainly the result of the variation of the personal equation with elongation: it is recalled that the principal term in the partial derivative $\partial\lambda/\partial\alpha_1$ is $-0.24 \sin D$.

3.2 PERSONAL EQUATION AS A FUNCTION OF STELLAR MAGNITUDE

It was found that about 80 per cent of the occultations are of stars of visual magnitude between 5.0 and 8.0. This distribution is largely dictated by the stars in Robertson's zodiacal catalogue which is nearly complete to magnitude 7.5. Thus the solutions for the personal equations as a function of elongation have effectively been determined for stars in the visual magnitude range 5.0–8.0.

In order to investigate the dependence of personal equation on stellar magnitude, the effect of the personal equations in Table 2 was subtracted from the residuals $\Delta\sigma$ and the three classes of visual observations were combined. The means and their standard errors were calculated in 1-mag steps from 3.0 to 10.0 and in a 2-mag step from 1.0 to 3.0 because there were so few observations in this range. The results are plotted in Fig. 2. Separate solutions for disappearances and reappearances gave similar results to those shown in Fig. 2. Solutions were also made for the three classes separately, but the comparatively few observations of stars brighter than 4.0 in classes (b) and (c) led to unreliable results in the range 1.0–4.0 mag. The results for stars fainter than 4.0 mag showed no significant variation with class of observation, so it was decided to combine the classes on the assumption that there would be no variation for bright stars either.

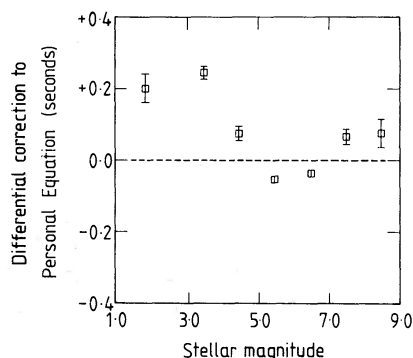


Figure 2. Differential correction to the personal equation as a function of stellar magnitude. The error bars represent standard errors of the means.

Table 2. Corrections for personal equation deduced from observations.

Method	Phenomenon	
	Disappearance	Reappearance
(a) Stop-watch	$-0^s.48$	$-0^s.99 + 0^s.79 ((D - 180)/100) - 0^s.29 ((D - 180)/100)^2$
(b) Eye-and-ear	$-0^s.07$	$-0^s.50 + 0^s.79 ((D - 180)/100) - 0^s.29 ((D - 180)/100)^2$

For stars brighter than 4.0 mag, +0.20 s should be applied to the correction for personal equation.

The results in Fig. 2 show that the personal equation for timings of stars brighter than 4.0 mag is about 0.2 s less than that for stars fainter than 4.0 mag. This result is included in Table 2 with the corrections for personal equation as a function of elongation (D).

From an analysis of 137 duplicate timings made by a single observer using both a switch operated manually and photoelectric apparatus, Sinzi & Suzuki (1967) found their mean correction to the personal equation (presumably for disappearances) to be -0.5 s for stars fainter than 4.0 mag. This is in agreement with our result. There were too few observations of stars brighter than 4.0 mag, however, to obtain a reliable mean for that range of magnitude.

If the brightness of occulted stars were correlated with the elongation of the Moon, this would invalidate our method of analysis whereby we treat them independently. There is a tendency for observers to select brighter stars around new and full Moon, indeed the predictions encourage this; but even at these phases the greater number of observations of stars fainter than 4.0 mag dominate the solution. We therefore regard our method of analysis as valid. If observers were to restrict themselves, just after full Moon, to observing the reappearances of bright stars, the large increase in their personal equation would be rectified somewhat. The results for the eye-and-ear method imply that for disappearances, and reappearances at elongations greater than 225° , the correction for personal equation for bright stars is positive; i.e. the observers over-compensate for their delay in timing. If this is not the case, we are probably wrong in our assumption that the behaviour of the personal equation as a function of magnitude is independent of the method of timing.

4 Conclusion

The personal equation in timing lunar occultations is shown to be a function of the elongation of the Moon (for reappearances) and the magnitude of the star. The dependence on elongation is found to be approximately parabolic for reappearances, with the greatest correction for the personal equation occurring just after full Moon. There is little or no dependence on elongation for disappearances. We attribute the behaviour of the personal equation for reappearances to the difficulty of locating the star close to the illuminated limb just after full Moon.

The personal equation for observers using the eye-and-ear method is about 0.4 s less systematically than that of those using some hand-operated switch linked to a clock, such as a stop-watch. The expressions for the corrections for personal equation are given in Table 2. They apply to observations of stars fainter than 4.0 mag and are subject to a systematic uncertainty of ± 0.06 s. The personal equation for timings of stars brighter than 4.0 mag, made by whatever visual method, is about 0.2 s less than that for stars fainter than 4.0 mag.

The results obtained in Paper I for the value of the lunar semi-diameter implicit in Watts' charts and the constant of parallactic inequality are shown to have been distorted as a result of the behaviour of the personal equations uncovered by this analysis. The revised value of

the semi-diameter is 932.62 ± 0.02 arcsec. The correction of -0.09 ± 0.04 arcsec to the constant of parallax inequality is regarded as spurious and is probably an indication that our model for the variation of personal equation with elongation does not completely satisfy the observations.

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Note added in proof

Recently, Hammerton & Stretch (1981, *J. Br. astr. Ass.*, **91**, 245) have reported the results of an analysis of timings made by the eye-and-ear method of simulated occultations using a star-like point of light at the end of a blackened tube. They find that on the average the observers timed the events early by about 0.2 s. This supports our conclusion that using the eye-and-ear method, the correction for personal equation for occultations of bright stars is positive. This implies that their simulations reproduce the circumstances of an occultation of a bright star, where the contrast between the star and sky-background is greatest.

Analysis of lunar occultations – III. Systematic corrections to Watts' limb-profiles for the Moon

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Summary. Timings of 66 000 occultations of stars by the Moon are analysed for systematic corrections to the limb-profile heights taken from Watts' charts of the marginal zone of the Moon. The radius, shape and location of the centre of the datum implicit in the charts are found to vary with libration and these variations produce systematic errors in the limb-profile heights which attain 0.4 arcsec in some position angles.

1 Introduction

In 1956, C. B. Watts completed a photographic survey of the marginal zone of the Moon and he later published a set of charts (Watts 1963) giving limb-profile heights with respect to an adopted smooth reference surface (or datum). These are particularly useful in reducing observations of occultations of stars which occur at different position angles and librations; for this purpose the reference surface should be a sphere with its centre at the centre of mass of the Moon, otherwise systematic distortions will be introduced into the residuals and these may be mistakenly interpreted as, for example, errors in the ephemeris of the Moon.

Analyses of occultations of stars by Van Flandern (1970) and Morrison (1979), (hereafter referred to as Paper I) have revealed that the average cross-section of Watts' datum is slightly elliptical rather than circular, and that the implicit centre of the datum is displaced from the point to which the current lunar ephemeris refers. Recently, whilst analysing grazing occultations, we have also found that the radius of the datum appears to vary with libration. Since grazing observations are restricted to the polar regions of the Moon, it is not possible using these observations alone to decide whether the variation in radius occurs around the whole limb. We therefore decided to extend the analysis of total occultations in the years 1943 to 1974 reported in Paper I in order to investigate whether there were any systematic changes in the datum varying with libration.

The main results of this investigation are that the heights of the limb-profiles at different librations in Watts' charts are referred to different datums which are elliptical rather than circular and are displaced from one another. Corrections are given to the heights of the profiles such that the revised heights refer to one spherical datum, the centre of which is located at the centre of mass.

2 Observations and reduction

About 20 000 observations of total occultations made in the years 1975 to 1979 were added to the 62 000 observations in the years 1943 to 1974 which had been analysed in Paper I. For the reasons given in that previous work, only occultations of stars with positions in the *Catalog of 3539 Zodiacal Stars for the Equinox 1950.0* (Robertson 1940) were retained for analysis. Occultations at the bright limb and at position angles and librations lying outside the scope of Watts' charts were rejected.

The distributions of the remaining 66 000 observations with libration in longitude (l) and latitude (b), and position angle (Q) from the projection of the north polar axis of the Moon are displayed in Figs 1 and 2. Apart from the librations near $l = +7^\circ.5$, $b = -6^\circ.5$ the observations are quite uniformly distributed, but there are comparatively few in the range $180^\circ - 360^\circ$ of position angle (appearances).

The method of reduction of the observations is set out in detail in Paper I. The residual geocentric separation ($\Delta\sigma$) between the star and the limb of the Moon was calculated for the observed time of occultation using the catalogue position of the star, the Moon's position and semi-diameter taken from the ephemeris $j=2$ (IAU 1968) and the profile heights of the Moon (scaled to the true distance) taken from Watts' charts.

For the purposes of the reduction, it was assumed that Watts' limb-profile heights are referred to one spherical datum, the centre of which is coincident with the ephemeris

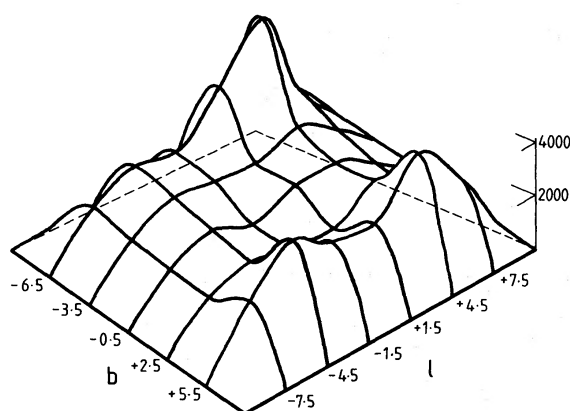


Figure 1. Distribution of observations with libration in longitude (l) and latitude (b).

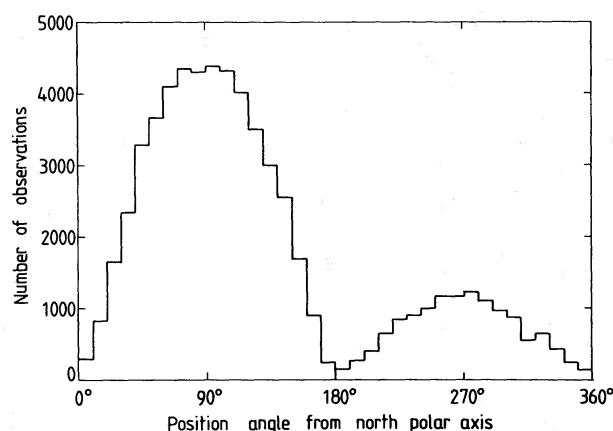


Figure 2. Distribution of observations with position angle from the projection on the geocentric sphere of the north polar axis.

position. The assumed radius of the datum, 1737.97 km, is equivalent to a semi-diameter of 932.58 arcsec at the mean distance of the Moon corresponding to a value of 3422.608 arcsec for the equatorial horizontal parallax (IAU 1966). The limb-profile height for position angle Q was found from Watts' charts by entering for the argument $Q + 0^\circ.25$ in accordance with the systematic error in the argument found by Morrison (1970).

The residual $\Delta\sigma$ was adjusted for corrections to the lunar ephemeris $j=2$ contained in Paper I in order to refer the ephemeris to the FK4 frame of reference. Adjustments were also made for corrections to the elements in $j=2$ found in that paper. Systematic corrections found in that paper to the limb-profile heights in Watts' charts were not applied because they are the subject of the present investigation. The values of the personal equations determined by Morrison & Appleby (1981, Paper II) were used in computing $\Delta\sigma$ in preference to those contained in Paper I, because the latter solutions for the personal equations were distorted as a result of their high correlation with the correction to the mean radius of Watts' datum. This problem was circumvented by us by adopting a correction of +0.04 arcsec to the heights of the limb-profiles in Watts' charts which we had determined from an analysis of photoelectric timings of occultations. A model for the behaviour of the personal equation, which is dependent on the phase of the Moon and magnitude of the star, was then derived by us using this revised radius. Therefore, the application of our model for the personal equation in the reduction of the observations should lead to a correction of +0.04 arcsec when averaged over all librations.

The correction to the constant of parallactic inequality α_1 (Paper I) was not applied in this analysis because we (Paper II) have shown that this is a spurious correction arising from the variation of the personal equation with the phase of the Moon. Its effect has been absorbed in the revised model for the behaviour of the personal equation which has been applied in this analysis.

3 Analysis and results

The residuals $\Delta\sigma$, reduced as described in the previous section, were analysed for systematic corrections to the limb-profile heights in Watts' charts varying with libration. Watts' charts cover the librations -9° to $+8^\circ$ in longitude (l) and -8° to $+8^\circ$ in latitude (b). In order that there should be no fewer than about 1000 observations in any subdivision of these ranges of libration, it was decided to divide l into six ranges of 3° and b into four ranges of 3° and one of 4° , the centres of the 30 resultant frames being at the intersections of the contours shown in Fig. 1. It was found that ~ 1000 observations were required in each range of libration to obtain reliable solutions for the systematic corrections.

The systematic corrections in each range of libration were considered to be harmonic series as far as the second order in the position angle. Thus, the following equation of condition in five unknowns, $\delta r_0, \delta r_1, R_1, \delta r_2, R_2$ was formulated:

$$\frac{\pi_0}{\pi} \Delta\sigma = \delta r_0 + \delta r_1 \cos(Q - R_1) + \delta r_2 \cos 2(Q - R_2),$$

where Q is the position angle of the occultation measured through east on the geocentric celestial sphere from the projection of the Moon's north polar axis, and the factor π_0/π (the ratio of the mean to the true parallax) reduces $\Delta\sigma$ to mean distance.

The unknown, δr_0 , is the constant correction to the limb-profile heights in Watts' charts which is required when a provisional value of 932.58 arcsec is used for the semi-diameter at mean distance. The term in Q may be expressed as the sum of two components: $\delta x \sin Q$, where x is measured positive in the direction of increasing ecliptic longitude (i.e. east on the

geocentric sphere), and $\delta y \cos Q$, where y is measured positive in the direction of the north lunar pole. The quantities $-\delta x$, $-\delta y$ are the amounts by which the centre of the datum in Watts' charts should be moved in order that the resultant profile should agree with the 'observed' one when the charts are used in conjunction with the lunar ephemeris $j=2$. The term in $2Q$ in the equation of condition allows for an approximately elliptical correction to the limb-profile. This is equivalent to stating that the datum departs from circularity by the amount $-\delta r_2 \cos 2(Q - R_2)$.

The observations were weighted as in Paper I, (Section 4.1) and the unknowns were determined by the method of least-squares for each frame at the points of intersection shown in Fig. 1. The solutions for the position angle R_2 did not differ significantly from 146° which was obtained by combining all the observations. Therefore, R_2 was fixed at the value 146° and the least-squares analysis was repeated to determine the other unknowns. The average standard errors of the solutions in each frame were:

$$\delta r_0 \pm 0.01 \text{ arcsec},$$

$$\delta x, \delta y \pm 0.02 \text{ arcsec},$$

$$\delta r_2 \pm 0.02 \text{ arcsec}.$$

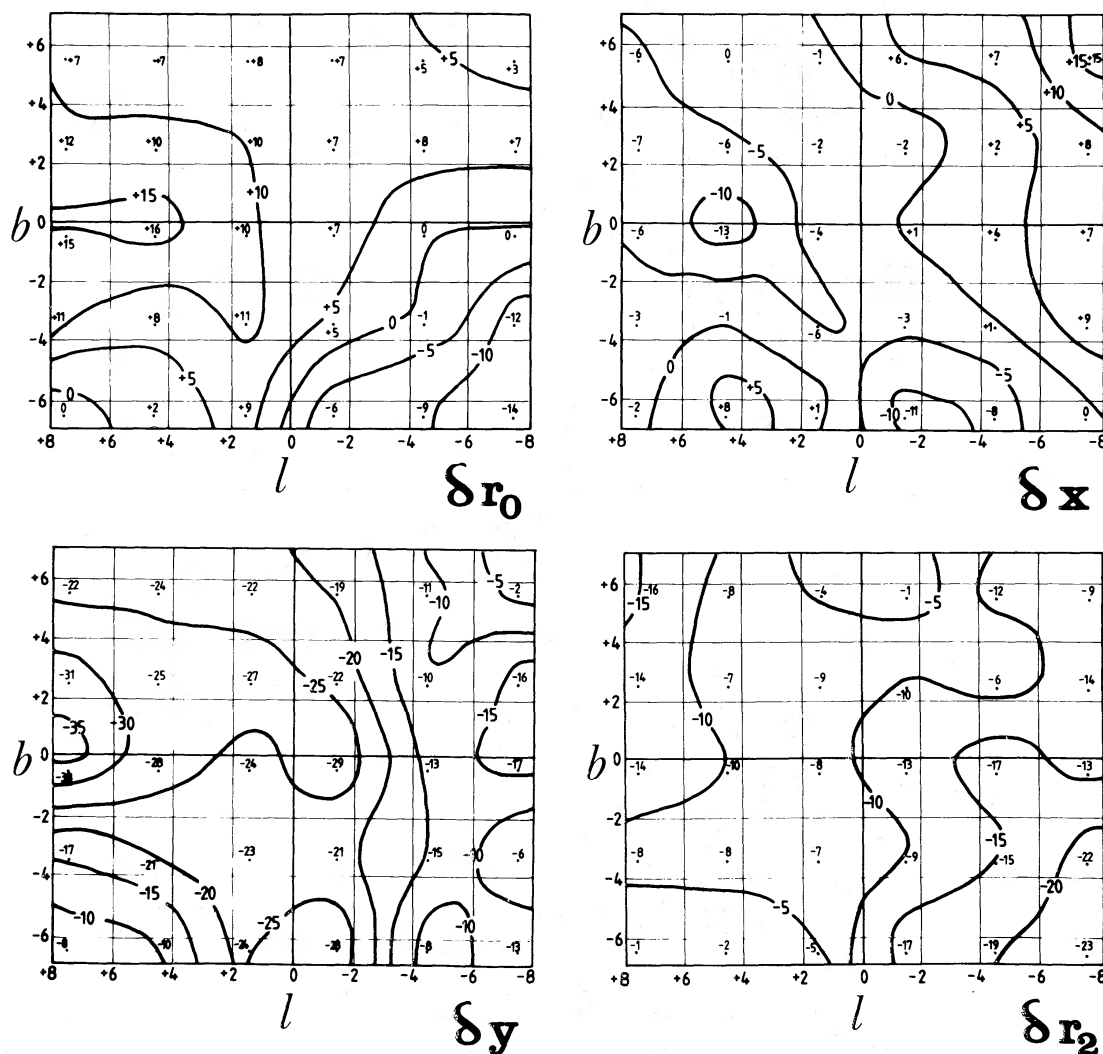


Figure 3. Systematic corrections to the limb-profile heights in Watts' charts varying with libration in longitude (l) and latitude (b). Contours (in units of 0.01 arcsec) have been drawn through the grid of solution points.

The results at the points of intersection in Fig. 1 are given in Fig. 3 rounded to the nearest 0.01 arcsec. It is seen that, in general, these results vary systematically with libration and we therefore considered it appropriate to represent the results by contours in units of 0.01 arcsec and at an interval of 0.05 arcsec. The contours were constructed using the GINO routine CONTR.

4 Discussion

4.1 GENERAL

This analysis shows that when Watts' charts are used in conjunction with the lunar ephemeris $j=2$, the heights of the limb-profiles should be amended by

$$\delta r_0 + \delta x \sin Q + \delta y \cos Q + \delta r_2 \cos 2(Q - 146^\circ),$$

where the values of δr_0 , δx , δy and δr_2 should be taken from Fig. 3 according to the libration. The expression above with the opposite sign gives the correction to reduce the cross-section of the datum of Watts' charts to a circle of semi-diameter 932.58 arcsec whose centre is such that, when it is superimposed on the position of the Moon given by the ephemeris $j=2$ (as amended in Section 2), the lunar profile is obtained with respect to the FK4 frame of reference.

4.2 RADIUS CORRECTION

The corrections to the heights of the limb-profile given in Fig. 3 (δr_0) have a mean value over all librations of +0.04 arcsec as a result of the adoption of the model for the behaviour of the personal equations reported in Paper II. In Paper I Morrison had found a correction of +0.15 arcsec, but this result is now known to be in error because, as we have shown, he underestimated the size of the personal equation in some methods of timing and did not allow for the possibility that the personal equation might vary with the elongation of the Moon from the Sun.

4.3 LONGITUDE AND LATITUDE COMPONENTS δx AND δy

The variation of the longitude component of the correction to the limb-profile heights in Watts' charts is shown in Fig. 3 (δx).

A source of systematic error in δx arises from the possibility that part of this correction varies linearly with libration in longitude as a result of the possible displacement in the direction towards the Earth of the centre of figure from the centre of mass.

If this were the case, then it may be represented by

$$\delta x = \frac{m}{6^\circ} l + k,$$

where m is the change in 6° (say) of libration in longitude and k is a constant. Hence,

$$\delta x \approx m \sin M + k,$$

where M is the Moon's mean anomaly. Also, $\delta \lambda \approx 2\delta e \sin M + \delta L$, where λ is the true longitude, e is the orbital eccentricity and L is the mean longitude. Since δx and $\delta \lambda$ are indistinguishable in the occultation residuals, $m \approx 2\delta e$, $k \approx \delta L$.

Thus, any contribution k will have been absorbed in the correction δL , and any contribution m , in the correction to the orbital eccentricity, which were derived in Paper I and

removed from the residuals $\Delta\sigma$ in Section 2. Hence, our results in Fig. 3 should be regarded as those parts of δx which exhibit sufficient variation with libration in latitude to enable them to be separated from a correction to the orbital eccentricity. If the centre of figure in Watts' charts were 2 km (say) nearer the Earth than the centre of mass, the value m would be about $+0.1$ arcsec, and thus the spurious contribution to δe in Paper I would have been $+0.05$ arcsec.

It is our understanding that Watts intended that his charts should be used in conjunction with the lunar ephemeris to obtain the profile of the Moon with respect to the stellar reference frame. By its development, (see Paper I) the ephemeris $j=2$ in longitude refers to the centre of the mean profile which we shall regard as the centre of figure. Watts intended the position of the centre of his datum in longitude to be consistent with this centre of figure. If this were not the case, the offset of his datum in longitude would have been absorbed in the correction to the mean longitude deduced in Paper I which was removed from $\Delta\sigma$ in this analysis. The displacement between the centre of figure and the centre of mass can be deduced from the correction to the elongation of the Moon (D) found by Morrison in Paper I. He found $\delta D = -0.72 \pm 0.08$ arcsec and interpreted it as arising from a correction of the opposite sign to the longitude of the Sun in Newcomb's Tables. However, as the correction to D is derived from dynamical considerations, it may also be interpreted as a measure of the displacement between the centre of figure used in the calculated longitude of the Moon and the centre of mass. If we accept that the longitude of the Sun in Newcomb's tables does not require a correction (Fricke 1980), we can conclude that the centre of figure leads the centre of mass in longitude by 0.72 ± 0.08 arcsec. Mulholland (1977) has determined the displacement between the centres of figure and mass from an analysis of occultations of stars combined with precise measurements of the Moon's distance derived from laser ranging. He found that the centre of figure leads the centre of mass by 2.5 ± 0.5 km (i.e. 1.3 ± 0.3 arcsec). Using laser altimetry data from *Apollo 15*, *16* and *17* missions, Sjogren & Wollenhaupt (1976) find that the centre of figure leads the centre of mass by about 1 km (i.e. 0.5 arcsec), regardless of whether the reference points are taken on the maria or the highlands. Bills & Ferrari (1977), however, obtain a result which is about half that of Sjogren & Wollenhaupt from a combination of laser altimetry data with orbital and Earth-based photogrammetry.

All the results considered here are in agreement that the centre of figure leads the centre of mass in the direction of increasing longitude, but they differ in size. The results presumably do not all refer to the same centre of figure. Mulholland's result should be amended to allow for the offset in right ascension of the FK4 equinox from the dynamical equinox. The centre of figure to which occultations refer must be some mixture of the profiles of the maria and highlands, depending on the particular libration. We shall assume that the result obtained by Sjogren & Wollenhaupt applies to Watts' datum, but, bearing in mind our result of 0.72 arcsec from occultations, we regard their displacement of 0.5 arcsec as being uncertain by about ± 0.2 arcsec in the present context. Thus, if Watts' charts are used in conjunction with a lunar ephemeris which refers to the centre of *mass* in orbital longitude, we recommend that a correction of $+0.50 \sin Q$ arcsec be applied to the limb-profile heights in addition to the correction $\delta x \sin Q$ which is dependent on libration.

The variation of the latitude component of the correction to the limb-profile heights in Watts' charts is shown in Fig. 3 (δy). By an argument analogous to that in the discussion of the systematic errors in δx , the results for δy should be regarded as comprising those components which exhibit sufficient variation with libration in longitude to enable them to be separated from a correction to the inclination of the Moon's orbit or the possible displacement between the centre of figure and centre of mass in the direction towards the Earth.

The average correction over all librations is -0.18 ± 0.01 arcsec, which is in good agreement with the result -0.19 ± 0.01 arcsec reported in Paper I. As explained in that paper, the lunar ephemeris $j=2$ gives the position of the centre of mass in latitude, and the centre implicit in Watts' charts is superimposed on this centre in the reduction of occultations. The quantity $-\delta y$ is therefore the amount that the centre of Watts' datum should be moved in latitude in order that it should be coincident with the centre of mass. In the construction of his charts Watts moved the centre of his original datum 0.3 arcsec along position angle 333° : our results for δy show that on the average he should have moved it a further 0.18 arcsec north in latitude.

4.4 ELLIPTICITY CORRECTION δr_2

Fig. 3 (δr_2) shows the variation with libration of the spurious ellipticity in the datum of Watts' charts. This can be removed from the limb-profile heights by adding the correction $\delta r_2 \cos 2(Q - 146^\circ)$. Watts' suspected that his original reference datum might be elliptical and he applied the following corrective term to the profile heights in the construction of his charts to remove the ellipticity: $+0.15 \text{ arcsec} \cos 2(Q - 153^\circ)$. The average value of the coefficient over all librations is about -0.09 arcsec, so it would appear that Watts should have applied $+0.06$ arcsec, rather than $+0.15$ arcsec.

4.5 COMPARISON OF THE LIMB-PROFILES OF WATTS WITH WEIMER

Meyer (1976) has compared the limb-profile heights taken from Watts' charts with the atlas of Weimer (1952). He found systematic differences in the heights varying with position angle which attain ~ 1 arcsec in some librations. In Fig. 4 we show the difference, in the sense Weimer minus Watts, for libration $l=+2^\circ.6$, $b=-5^\circ.1$ derived by Meyer. For comparison we plot the result of our harmonic series, in the sense occultations minus Watts, for the solution point $l=+1^\circ.5$, $b=-6^\circ.5$. The difference $\sim 1^\circ$ between the librations in the comparisons can not account for the considerable differences in the curves.

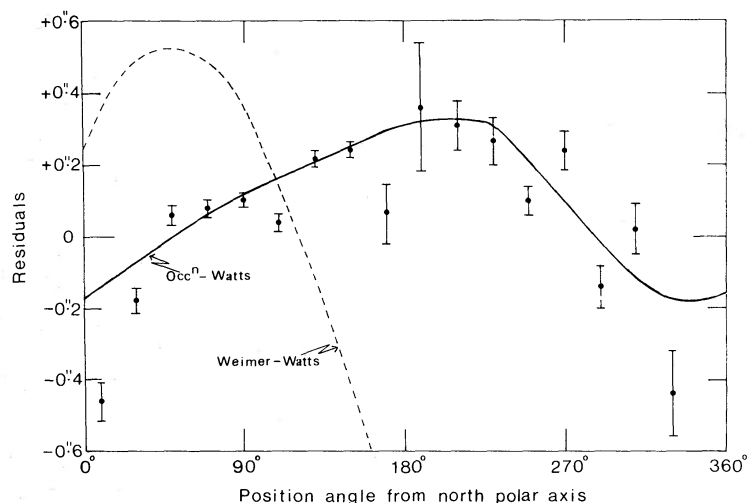


Figure 4. Systematic corrections to the limb-profile heights in Watts' charts deduced from occultations (solid line) and by comparison with Weimer's atlas (dashed line) for a libration near $l=+2^\circ$, $b=-6^\circ$. The solid line was evaluated from the harmonic expression given in Section 4.1, and the error bars show the means and standard errors of the differences between occultations and Watts for 20° ranges of position angle.

In order to test the goodness of fit of our harmonic series which was truncated at the second order, we formed the means of the residuals $\Delta\sigma$ in ranges of 20° in position angle for each of the 30 libration frames shown in Fig. 1. The means and their standard errors are plotted in Fig. 4 for the frame centred on $l=+1^\circ.5$, $b=-6^\circ.5$. The general agreement between the means and our harmonic series for this frame is typical of the others. We conclude that there must be greater systematic errors in Weimer's profiles than in Watts'.

4.6 SYSTEMATIC ERRORS IN THE POLAR REGIONS

The stimulus for this investigation was that the radius of Watts' datum deduced from grazing occultations in the region of the lunar north pole appeared to vary linearly with libration in latitude. The evaluation of the harmonic series in Section 4.1 for a position angle of zero and a range of libration in latitude produces a linear change in the same direction, but of quarter the magnitude. In Fig. 4 we note that our harmonic series is not a good fit to the means near the north lunar pole and from Fig. 2 it is seen that there are comparatively few observations within $\pm 10^\circ$ of the lunar poles. We conclude that our global solution may not be valid in the polar regions, particularly in the north.

5 Conclusions

The systematic corrections to the limb-profile heights in Watts' charts, which follow from the requirements that the reference datum should be a sphere with its centre at the centre of mass, are given by

$$\delta r_0 + (\delta x + 0.50 \text{ arcsec}) \sin Q + \delta y \cos Q + \delta r_2 \cos 2(Q - 146^\circ),$$

where the values of δr_0 , δx , δy and δr_2 are taken from Fig. 3 according to the libration, and Q is the position angle measured through east from the projection on the geocentric sphere of the north polar axis. If Watts' charts are used in conjunction with the lunar ephemeris $j=2$, which refers to the centre of figure in longitude, the constant 0.50 arcsec in the coefficient of $\sin Q$ should not be applied. These systematic corrections may not be valid close to the poles of the Moon.

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