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RECOMMENDATIONS FOR CALIBRATION OF MILLIMETER-WAVELENGTH SPECTRAL LINE DATA

MARC L. KUTNER
Physics Department, Rensselaer Polytechnic Institute
AND
B. L. ULICH
Multiple Mirror Telescope Observatory
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ABSTRACT

We examine the methods currently used to calibrate millimeter-wavelength photometric data and make recommendations on procedures to be followed. Since confusing references to the quantity $T_A{}^*$ have appeared in the literature, we introduce a series of formal definitions of quantities related to intensity measurements. A new quantity called $T_R{}^*$ is introduced which is the source antenna temperature corrected for atmospheric, ohmic, and all spillover losses. Physically, $T_R{}^*$ corresponds to the source brightness distribution convolved with the diffraction and error beam patterns of the telescope. We also reexamine the relative merits of the so-called "direct" and the "chopper-wheel" methods of correcting for atmospheric absorption and conclude that the chopper-wheel technique is generally preferable.

Subject headings: instruments — interstellar: molecules — radio sources: lines

I. INTRODUCTION

With the growing variety of astrophysical problems that are now being studied by millimeter-wavelength observations of spectral lines, there is an increasing demand for reliable relative and absolute intensity calibration. Such calibration is made difficult by a number of factors:

1. At millimeter wavelengths, the Earth's atmosphere is only partially transparent. Moreover, this transparency can change on a relatively short time scale as the water content of the atmosphere over the observatory changes.

2. Since there is generally no amplification of the incoming signal before it is mixed with the local oscillator signal, millimeter receivers are sensitive to both signal and image sidebands. Because local calibration standards are usually broad-band sources such as hot and cold blackbody loads, the calibration signals enter both sidebands, while the spectral line enters only the signal sideband. The relative response of the system to the two sidebands is often unknown and uncontrolled. Recently, image sideband rejection filters with low insertion loss have been made using quasi-optical techniques and have been quite effective in Cassegrain systems with large effective focal lengths (e.g., Wannier *et al.* 1976).

3. With the current tendency to observe at higher frequencies, telescopes are often operating in regimes where the error pattern caused by small scale reflector surface errors plays an important (but difficult to evaluate) role in the coupling between the source and the antenna.

In principle, the intensity of a spectral line can be determined by the "direct" method in which a source antenna temperature T_A is measured (antenna temperature is simply the detected power divided by the product of the predetection bandwidth and Boltzmann's constant). The atmospheric zenith opacity τ_a is then determined (usually from a measurement of the equivalent sky brightness temperature as a function of airmass A). An exponential correction factor ($e^{\tau_a A}$) is then applied to obtain a corrected antenna temperature $T_A{}'$. This technique requires that τ_a be determined accurately.

The more commonly used alternative is an indirect method first described by Penzias and Burrus (1973). An ambient temperature "chopper" or "vane" absorber is alternated with cold sky as the calibration signal. Thus, as the sky becomes more opaque, the apparent sky brightness temperature gets larger, and the calibration signal is reduced. Under ideal circumstances this method corrects exactly for atmospheric opacity and rear spillover losses (see the Appendix). However, Davis and Vanden Bout (1973) have pointed out that since much of the atmospheric opacity is due to O_2, which is distributed with a scale height of about 8 km, the effective mean temperature of the atmosphere T_M is significantly lower than the ambient temperature T_{AMB}. They also pointed out that the atmospheric opacity can be very different in the two sidebands (especially near the CO $J = 1 \rightarrow 0$ transition at 115 GHz). More recent improvements in the correction for atmospheric opacity are discussed below. The chopper method automatically corrects for antenna ohmic losses and for the spillover that looks like an ohmic loss terminated at ambient temperature. However, there is usually some part of the antenna pattern that looks at the sky but not at the source, and an additional correction is required for this forward spillover. This need has resulted in some confusion over what corrections are included in the commonly reported quantity T_A^*, introduced by Phillips, Jefferts, and Wannier (1973) and defined in detail for prime focus telescopes by Ulich and Haas (1976, hereafter referred to as UH). In this paper, we set down a proposed set of definitions for the various observational quantities that removes these ambiguities. We then look at how this affects data already in the literature, and, finally, we reexamine the methods of calibration.

II. COUPLING OF ANTENNA TO SOURCE

Following UH, we assume that the emission from a radio source at any point can be characterized by a blackbody brightness temperature T_B or by the effective source radiation temperature $J(\nu, T_B)$, where

$$J(\nu, T) \equiv (h\nu/k)/[\exp(h\nu/kT) - 1] , \quad (1)$$

ν is the frequency, T is the temperature, h is Planck's constant, and k is Boltzmann's constant.

For example, a spectral line source with a constant excitation temperature T_E, a background brightness temperature T_{BG}, and a source optical depth $\tau(\nu)$ will have an effective radiation temperature given by

$$J(\nu, T_B) = \{1 - \exp[-\tau(\nu)]\}[J(\nu, T_E) - J(\nu, T_{BG})] . \quad (2)$$

In the literature the quantity $J(\nu, T_B)$ is often referred to as simply the radiation temperature T_R for the source, and we will use that terminology. Variations in source brightness temperature with direction angle on the sky Ψ are accounted for in the normalized source brightness distribution $B_n(\Psi)$, which takes on values between zero and unity.

The observed source antenna temperature in the direction Ω on the sky will then be

$$T_A = T_R \eta_r \left[\frac{\iint_{\Omega_s} P_n(\Psi - \Omega) B_n(\Psi) d\Psi}{\iint_{4\pi} P_n(\Omega) d\Omega}\right] e^{-\tau_a A} , \quad (3)$$

where P_n is the normalized antenna power pattern $[P_n(0) = 1]$, Ω_s is the solid angle subtended by the source, τ_a is the atmospheric optical depth at the zenith, A is the airmass at which the observation is made, and η_r is the radiation efficiency of the telescope. η_r accounts for the fraction of the incoming power lost to ohmic (resistive) heating of the antenna and is formally defined in terms of the maximum antenna gain G as

$$\eta_r \equiv (G/4\pi) \iint_{4\pi} P_n(\Omega) d\Omega . \quad (4)$$

Because of the convolution integral in equation (3), the conversion from T_A to T_R requires a detailed knowledge of the source structure and of the antenna power pattern. It is therefore desirable to report a more directly observable quantity that does not depend on a convolution process. For this purpose we divide the antenna pattern into two zones, one involving the normal diffraction pattern (including the error pattern) and the other involving spillover (both forward and rearward) as well as scattering from the feed support legs and other structures. There is some ambiguity in drawing the dividing line between these two regions, since some of the spillover and scattering can be in the forward hemisphere and close to the main lobe (especially in Cassegrain systems with spillover past the secondary mirror). However, spillover and scattering falling into the central diffraction pattern must be treated as part of that pattern. Therefore, if we say that all of the power within a solid angle Ω_d is part of the diffraction pattern, then everything outside Ω_d is considered spillover and scattering. In practice, the exact choice of Ω_d should not cause serious problems, as long as the procedures are followed consistently.

We can then define the efficiency η_c with which the antenna couples to the source as

$$\eta_c \equiv \iint_{\Omega_s} P_n(\Psi - \Omega) B_n(\Psi)\, d\Psi \Big/ \iint_{\Omega_d} P_n(\Omega) d\Omega . \quad (5)$$

For most telescopes, Ω_d will encompass a region within a few degrees of the telescope axis. Therefore, there may be sources (giant molecular clouds observed in CO, for example) which are larger than Ω_d and actually extend into the spillover region. If such sources are of roughly uniform brightness then it is possible to have $\eta_c > 1$. In such circumstances, the coupling of the spillover to the source is still a problem which must be taken into account, whether nor not our particular definition of η_c is used.

Using our definition of η_c from equation (5), we define the corrected source intensity in radiation temperature units as

$$T_R^* \equiv \eta_c T_R . \quad (6)$$

For the following reasons we believe that T_R^* is often the most useful quantity to report:

1. It is as close as one can get to source intensity without inserting particular knowledge about source structure. That is, T_R^* is corrected for everything (ohmic loss, atmospheric loss, spillover, and scattering) except the actual coupling of the antenna diffraction pattern to the source.
2. For many observations η_c is not very different from unity, so T_R^* gives a reasonable estimate of T_R which is adequate for the purposes of many observers.
3. With the exception of quantities that amount to filling factors (which can at least be estimated on a relative basis for different telescopes), T_R^* is a relatively telescope-independent quantity for a number of objects that might be used as standard sources.
4. With a growing emphasis on source modeling for many problems, the usual procedure would be to develop a source model, to use radiative transfer calculations to predict the emergent intensity as a function of position on the source, and then to do the convolution of this intensity with the antenna pattern. This is because the convolution of a particular source model with an antenna pattern can be done rather easily, but a deconvolution of the observations is much more difficult. Thus, it seems

that it may be most convenient to compare theory and observations in terms of $T_R{}^*$ rather than of T_A or of T_R.

It should also be noted that our definition of $T_R{}^*$ conforms to the definition that some authors have been assuming for $T_A{}^*$. The relationship between our definitions and previous usage will be discussed in § III.

Using equations (3), (5), and (6), we find that $T_R{}^*$ is related to the source antenna temperature $T_A{}'$ corrected for atmospheric attenuation $[T_A{}' \equiv T_A \exp(\tau_a A)]$ by

$$T_A{}' = T_R{}^* \eta_r \left[\iint_{\Omega_d} P_n(\Omega) d\Omega \Big/ \iint_{4\pi} P_n(\Omega) d\Omega \right] . \quad (7)$$

The quantity in brackets is the spillover and scattering efficiency of the telescope. That is, it is the fraction of the radiated power that is not lost in spillover and scattering outside the diffraction zone. As shown in the Appendix, the spillover and scattering that fall on the forward hemisphere enter into the chopper calibration scheme differently than the spillover and scattering that fall on the rearward hemisphere. We therefore write the quantity in brackets as the product of two efficiencies:

$$\eta_{\text{fss}} \equiv \iint_{\Omega_d} P_n(\Omega) d\Omega \Big/ \iint_{2\pi} P_n(\Omega) d\Omega , \quad (9)$$

and

$$\eta_{\text{rss}} \equiv \iint_{2\pi} P_n(\Omega) d\Omega \Big/ \iint_{4\pi} P_n(\Omega) d\Omega , \quad (9)$$

where the double integral over 2π denotes an integral over the forward hemisphere. In terms of these quantities

$$T_A{}' = T_R{}^* \eta_r \eta_{\text{fss}} \eta_{\text{rss}} . \quad (10)$$

We can also define an extended source efficiency $\eta_s \equiv \eta_r \eta_{\text{fss}} \eta_{\text{rss}}$ such that

$$T_A{}' \equiv T_R{}^* \eta_s . \quad (11)$$

In principle, while the product $\eta_{\text{fss}} \eta_{\text{rss}}$ remains constant, the individual quantities may change with the elevation angle of the telescope. That is, some of the spillover may switch from looking at the sky to looking at the ground, or from looking at the ground to looking at the sky. This is probably worst for elevations very near the horizon when some of the rear spillover, just past the edge of the dish, now looks at the sky, and when some of the spillover past the secondary in Cassegrain systems goes from the sky to the ground. However, as a practical matter, for most positions at which observations will be made, this does not pose a significant problem. To the extent that it may interfere with very accurate absolute calibration, the elevations at which these effects become significant, and the amount by which they affect the calibration, can be determined by measurements made over a variety of elevations under very clear and stable atmospheric conditions.

III. COMPARISON OF $T_R{}^*$ AND $T_A{}^*$

As shown in the Appendix, the chopper calibration method automatically corrects for losses that appear to be terminated at the ambient temperature, namely ohmic losses and rearward spillover and scattering. It is therefore convenient to combine these effects into a single efficiency, η_l, originally introduced by UH. In terms of the quantities presented here, it is defined as

$$\eta_l \equiv \eta_r \eta_{\text{rss}} . \quad (12)$$

When the antenna is pointed at the sky, the observed antenna temperature is then

$$T_A{}^{\text{SKY}} = \eta_l J(\nu, T_{\text{SKY}}) + (1 - \eta_l) J(\nu, T_{\text{SPILL}}) , \quad (13)$$

where T_{SKY} is the brightness temperature of the sky and T_{SPILL} is the temperature at which the spillover is terminated and is usually equal to the ambient temperature T_{AMB}. From this we see that if the antenna temperature is measured as a function of airmass A (as in an antenna tipping procedure) and extrapolated to $A = 0$, η_l can be determined. For many purposes, the calibration scale can be considered to be essentially independent of η_l, but, as shown in the Appendix, when all effects are included a very weak dependence on n_l is introduced. Also, if the atmospheric opacity τ_a is to be directly determined from tipping curves, then η_l must be known.

The antenna temperature scale, corrected for atmospheric attenuation, resistive losses, and rearward spillover and scattering, is then

$$T_A{}^* \equiv T_A{}'/\eta_l = [T_A \exp(\tau_a A)]/\eta_l . \quad (14)$$

$T_R{}^*$ is related to $T_A{}^*$ by the forward spillover and scattering efficiency such that

$$T_R{}^* = T_A{}^*/\eta_{\text{fss}} . \quad (15)$$

In introducing the symbol $T_A{}^*$, Phillips, Jefferts, and Wannier (1973) defined it as "antenna temperature corrected for all telescope and atmospheric losses" with no further details. In the definition of $T_A{}^*$ of UH, it was implicitly assumed that $n_{\text{fss}} = 1$, in which case these two temperature scales would be identical. This assumption was made because the discussion in UH deals only with prime focus antennas which have primarily rearward spillover. However, it now appears that some prime focus antennas can have a significant amount of forward spillover and scattering. In addition, when the extension was made to Cassegrain systems, some observers used this definition of $T_A{}^*$. That is, $T_A{}^*$ was taken to be the antenna temperature corrected for atmospheric attenuation, ohmic losses, and rearward spillover. As such, it is the natural quantity that falls out of chopper calibration. However, other observers assumed another definition of $T_A{}^*$ based on its relationship to the source. They took $T_A{}^*$ to be corrected for everything except the actual coupling of the source of the antenna diffraction pattern (which is our definition of $T_R{}^*$). As a result, numbers have been reported as $T_A{}^*$ which may have different physical meanings. The exact relationship for several telescopes is given in Table 1.

TABLE 1

THREE MILLIMETER TELESCOPE EFFICIENCIES

Telescope	MWO 4.9 m	NRAO 11 m	NRAO 11 m	NRAO 11 m
Focus	Prime	Prime	Cassegrain	Cassegrain
Feed type	Pyramidal horn	Conical horn	Gustincic horn/lens[a]	Ulrich horn/lens[a]
η_r	1.00	0.99	0.99	0.99
η_{rss}	0.93	0.68	0.79	0.91
η_l	0.93	0.67	0.78	0.90
η_{fss}	0.86	0.99	0.74	0.79
η_s	0.80	0.66	0.58	0.71
η_{Sun}	0.77	0.59	0.52	0.63
T_R^*/T_A^* (reported)	1.17 ± 0.05	1.09 ± 0.09	1.02 ± 0.09	1.27 ± 0.09

NOTE.—For observations at other frequencies, assuming appropriate scaling of the feeds, all quoted efficiencies should be approximately the same, except for η_{Sun}, which will get lower at higher frequencies. At higher frequencies, more power generally goes from the main beam to the error pattern, but η_{fss} includes both, so it does not change.

[a] Ulich 1980.

In general, for the NRAO 11 m telescope, prime focus observations that were reported as T_A^* are T_A^* as defined here, and Cassegrain observations reported as T_A^* are actually T_R^* as defined here. For example, if Cassegrain observations were interpreted as T_A^* according to Ulich and Haas, (1976), then an attempt to correct for forward spillover (a second time) would result in a 35% overestimate in line strengths. For the most part, observations done on the Aerospace Corporation 5 m telescope, on the University of Texas Millimeter Wave Observatory (MWO) 5 m telescope, and on the Columbia University 1 m telescope that were reported as T_A^* are T_A^* as defined here, and not T_R^*. Results from the Bell Telephone Laboratories 7 m telescope, reported as T_A^*, are generally T_R^* (with $\eta_{fss} = 0.9$) as defined here when some statement about correction for "beam efficiency" is made (R. A. Linke 1981, private communication).

IV. RECOMMENDATIONS FOR CALIBRATION PROCEDURE

Given that the chopper technique naturally corrects for only (the rearward) part of the spillover, and that a knowledge of η_{fss} is required to convert T_A^* to T_R^*, one might ask whether it now becomes preferable to abandon the chopper technique and return to the direct method. We examine this question with regard to two criteria: (1) correction for antenna coupling, and (2) correction for atmospheric attenuation. In making the comparison, we will assume that the goal is the best value of T_R^*, since, in either method, one must still contend with the convolution of the source structure and the antenna pattern.

a) *Correction for Antenna Coupling*

In the chopper technique, T_A^* is well determined since η_l can be determined rather precisely, so the only correction to get to T_R^* is η_{fss}. In the direct method, T_A' must be corrected by $\eta_s = n_l \eta_{fss}$. The question is thus whether η_s is better known than η_{fss}. In practice, η_s can be found by observing appropriate sources whose intensity and structure are well known. Ideally, these sources should be of uniform brightness and should exactly fill Ω_d, but this is rarely possible, so some correction is needed for the part of the diffraction pattern (within Ω_d) that does not couple to the reference source. η_{fss} cannot be directly determined in practice. However, η_l can be directly determined, and η_{fss} is then found as the ratio η_s/η_l. Since there is also some uncertainty in η_l, η_{fss} will have a larger uncertainty than η_s. The difference in uncertainties depends on the relative uncertainties in η_s and η_l. In summary, the way in which the quantities are determined makes η_{fss} slightly more uncertain than η_s, but under most circumstances, the advantage in the direct approach should amount to at most a few percent.

b) *Correction for Atmospheric Attenuation*

As shown by Kutner (1978), for any given uncertainty in the atmospheric opacity, the chopper method gives significantly smaller errors in the overall calibration than the direct method. The actual numerical advantage of the chopper method depends on the atmospheric opacity. The advantage is greater when the atmosphere is more opaque. Under the range of circumstances in which observations might be made at wavelengths between 2 and 4 mm, the accuracy advantage of the chopper method might range from 5% to 25%. At even shorter wavelengths, as the atmospheric opacity increases sharply, the advantage becomes even greater.

This advantage arises from the fact that there are offsetting effects in the chopper technique. Under idealized circumstances, these offsetting effects exactly cancel (see the Appendix). Under real circumstances, the advantage persists only if care is taken to correct for the fact that the effects do not completely offset each other. However, this is not a serious problem, since it generally involves the application of an easily calculated elevation-dependent scale factor, such as that given by UH (and reproduced in the Appendix), along with an appropriate atmospheric model. Alternatively, the cooled, variable-temperature chopper proposed and tested by Ulich (1980) provides a "hardware" solution. In either case, the simple models that have already been published appear to be adequate to bring the calibration uncertainty down to less than the 5% level in the 2 to 4 mm wavelength range.

A serious problem in calibration can be short term changes in the atmospheric opacity (essentially changes in the water vapor opacity). The cooled chopper system is the least sensitive to such changes, except in situations where no image rejection is used and the water vapor opacity is very different in the two sidebands. If an uncooled chopper is used, and especially if the direct method is used, the usual technique of doing an antenna tipping every few hours may not be satisfactory because of rapid changes in the water vapor opacity and of differences in the opacity from one part of the sky to another. For this reason, a system that puts two different temperature loads in front of the receiver as part of the normal calibration process is desirable in that it allows frequent and independent monitoring of the atmospheric attenuation, as is done on the BTL 7 m telescope (Goldsmith 1977). Given the three step calibration (two loads and the sky), one is then free to apply the calibration scale either in the direct way or in the chopper-prescribed manner. The same arguments for using the chopper method, mentioned above, still apply. In summary, it appears that the chopper technique offers more reliable correction for atmospheric attenuation, and this appears to more than offset the slight advantage that the direct approach has in the correction for spillover losses.

c) *Standard Sources*

Finally, we make a few comments about the use of standard sources as an aid in calibration. For a source to be an appropriate standard, the source brightness distribution must be known. Two simple extremes are a point source and a source that uniformly fills the diffraction pattern. (It should be noted that one of the most popular standard sources, Ori A, falls in neither of these categories.) Standard sources may be used in two basic ways: (1) If all of the free parameters that enter into the calibration are known, then the standard sources can be used as a check of the calibration scale. (2) If some quantities are unknown, then standard sources can be used to determine them. However, it is important to identify the quantities (or groups of quantities) being determined. All parameters do not enter directly into the equivalent chopper calibration temperature as scale factors. Adjusting the scale of $T_A{}^*$ or of $T_R{}^*$ to match observations with other telescopes is generally incorrect since both scales depend on antenna parameters. Thus $T_A{}^*$ and $T_R{}^*$ will in general vary from one telescope to another. Only T_R is truly independent of the telescope characteristics.

V. SUMMARY

We have examined the existing work on millimeter spectral line calibration. To avoid the confusion that has grown out of multiple usage of the quantity $T_A{}^*$, we define the quantity $T_R{}^*$ which is the source antenna temperature corrected for all atmospheric, ohmic, and spillover losses. This corresponds to the source intensity convolved with the antenna diffraction and error patterns. The quantity $T_A{}^*$ is actually $T_R{}^*$ without the correction for forward spillover. For work already published, data quoted as $T_A{}^*$ are generally consistent with the original definition. An exception is Cassegrain focus data taken on the NRAO 11 m telescope, for which previously reported values of $T_A{}^*$ are really $T_R{}^*$.

We have reexamined the relative merits of direct and chopper-wheel calibration schemes, and we conclude that the chopper method provides the better overall absolute calibration. However, in order to realize this potential, one must use more than just the simplest model for atmospheric emission. One improvement is provided by the two-layer atmospheric model of Kutner (1978). The cooled chopper wheel suggested by Ulich (1980) also provides adequate correction for second-order errors through hardware improvements. If the cooled chopper is not used, a calibration procedure involving both the sky and two known temperature loads is preferable.

Much of this paper is an outgrowth of a workshop on Millimeter Wavelength Calibration, hosted by the NRAO[1] in May 1980. We would like to thank the participants for their many significant suggestions. We would also like to thank the participants in the U.R.S.I. Symposium on Millimeter Wavelength Technology, in Grenoble, France, in August 1980 for their helpful criticisms. M. L. K. was partially supported by National Science Foundation grant AST 79-23584.

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APPENDIX

CHOPPER WHEEL CALIBRATION SCALE FACTOR

In this section, we briefly look at the considerations appropriate to establishing a scale factor for chopper wheel calibrations. These have been discussed more extensively by Davis and Vanden Bout (1973), Ulich and Haas (1976), Kutner (1978), and Ulich (1980).

a) *Simplified Example*

To see the basics in establishing a scale factor, we first go through a simplified case in which we assume the following: (1) the receiver is single sideband; (2) the chopper and spillover are at the ambient temperature T_{AMB}, (3) the cosmic background radiation is negligibly small; (4) $h\nu \ll kT$ for all temperatures involved; and (5) the sky brightness temperature can be written in terms of a mean effective temperature of the sky T_M as

$$T_{\rm SKY} = T_M[1 - \exp(-\tau_a A)] . \qquad (A1)$$

If V is the voltage response of the usual square-law detector and g is the factor for converting equivalent input temperatures into output voltages, then when looking at a spectral line with antenna temperature T_A the output voltage is

$$V_L = gT_A = g\eta_l T_A^* \exp(-\tau_a A) . \qquad (A2)$$

The chopper calibration signal is the difference between the ambient temperature absorber and the antenna temperature of the sky

$$T_{\rm CAL} = T_{\rm AMB} - T_A^{\rm SKY} , \qquad (A3)$$

so

$$V_{\rm CAL} \equiv g(T_{\rm AMB} - T_A^{\rm SKY}) . \qquad (A4)$$

Using equation (13), this becomes

$$V_{\rm CAL} = g\eta_l(T_{\rm AMB} - T_{\rm SKY}) . \qquad (A5)$$

We define the quantity T_C such that

$$T_A^* = T_C(V_L/V_{\rm CAL}) , \qquad (A6)$$

in which case

$$T_C \equiv (V_{\rm CAL}/V_L)T_A^* \qquad (A7)$$

$$= (T_{\rm CAL}/\eta_l) \exp(\tau_a A) \qquad (A8)$$

$$= T_{\rm AMB} + (T_{\rm AMB} - T_M)[\exp(\tau_a A) - 1] . \qquad (A9)$$

An initially attractive feature of the chopper method is that, for this simplest case, if the atmosphere is at the ambient ground temperature, one has the simple result that $T_C = T_{\rm AMB}$, independent of airmass or atmospheric conditions. In addition, it can be seen that T_C is independent of η_l, which means that T_A^*, as defined in equations (14) and (A6), is the quantity that comes out of the chopper calibration with the least amount of additional information. However, if we want a scale factor that produces T_R^*, we must define a corrected calibration temperature T_C^* so that by analogy with equation (A7),

$$T_C^* = (V_{\rm CAL}/V_L)T_R^* , \qquad (A10)$$

which, from equation (15), gives

$$T_C^* = T_C/\eta_{\rm fss} = (T_{\rm CAL}/\eta_s) \exp(\tau_a A) . \qquad (A11)$$

b) General Case

The more general case, with signal and image sidebands at frequencies ν_s and ν_i, with relative receiver power gains G_s and G_i (normalized such that $G_s + G_i = 1$), and with zenith atmospheric opacities τ_s and τ_i, is worked out by Ulich and Haas (1976). On the assumption that $J(\nu_s, T) = J(\nu_i, T)$ for all T including $T_{\rm BG}$ (the brightness temperature of the background radiation), one has

$$\begin{aligned} T_C = {} & (1 + G_i/G_s)[J(\nu_s, T_M) - J(\nu_s, T_{\rm BG})] \\ & + (1 + G_i/G_s)[J(\nu_s, T_{\rm SPILL}) - J(\nu_s, T_M)]e^{\tau_s A} \\ & + (G_i/G_s)[J(\nu_s, T_M) - J(\nu_s, T_{\rm BG})]\{\exp[(\tau_s - \tau_i)A] - 1\} \\ & + [(1 + G_i/G_s)/\eta_l][J(\nu_s, T_{\rm CHOP}) - J(\nu_s, T_{\rm SPILL})]e^{\tau_s A} , \end{aligned} \qquad (A12)$$

where $T_{\rm CHOP}$ is the temperature of the chopper and $T_{\rm SPILL}$ is the temperature at which the rear spillover is terminated. It should be noted that when the chopper is not at the same temperature as the spillover, T_C will depend weakly on η_l.

Ulich (1980) has pointed out that one can use the second and the last terms in equation (A12) to advantage by controlling the temperature of the chopper such that

$$J(\nu_s, T_{\rm CHOP}) = J(\nu_s, T_{\rm SPILL}) + \eta_l[J(\nu_s, T_M) - J(\nu_s, T_{\rm SPILL})] , \qquad (A13)$$

in which case

$$T_C = [J(\nu_s, T_M) - J(\nu_s, T_{\rm BG})]\{1 + (G_i/G_s) \exp[(\tau_s - \tau_i)A]\} . \qquad (A14)$$

From equation (A13) we see that a dependence on n_l is still there, but, again, it is a weak one. From equation (A14) we see that T_C is simply a constant if the system is single sideband ($G_i/G_s = 0$) or at frequencies where the atmospheric opacity is the same in the two sidebands ($\tau_s = \tau_i$). In any case, T_C is not strongly dependent on the atmospheric opacity.

For the situations in which a cooled chopper is not used, an appropriate atmospheric model should be used in evaluating equation (A12). Kutner (1978) has suggested a two-layer model in which the oxygen and water contributions to the opacity are treated separately. The oxygen contribution is very stable over time, while the water contribution is taken to be variable. The effective temperature for the water is given by

$$T_w = T_{\rm AMB} - \Delta \,, \tag{A15}$$

where $\Delta \approx 10$ K, and the effective temperature of the oxygen is given by

$$T_{\rm O} = (0.90 + 0.02\tau_{\rm O} A)T_{\rm AMB} \,, \tag{A16}$$

where $\tau_{\rm O}$ is the oxygen zenith opacity (similarly, τ_w is the water opacity). The mean sky brightness temperature is then given by

$$J(\nu, T_M) = J(\nu, T_{\rm O})\{1 - \exp[-(\tau_{\rm O} + \tau_w)A]\} + [J(\nu, T_w) - J(\nu, T_{\rm O})][1 - \exp(-\tau_w A)] \,. \tag{A17}$$

If this model is used, then tipping curves may be analyzed on the assumption that $\tau_{\rm O}$ is known at the frequency of interest, and τ_w is then found. If a self-consistent model is used in the analysis of the tipping curves and in the calculation of T_C, the sensitivity to uncertainties in τ_w is reduced over the direct method, especially for total sky opacities in the direction of observation greater than 0.2. It should also be noted that if a cooled chopper is used, then the two-layer model can also be used to give a more accurate value of T_M (which is almost independent of air mass).

c) *Comparison of Calibration Scales*

The intensities observed with telescopes of equal apertures and of equal reflector surface precisions may be directly compared for consistency. In general, however, molecular cloud structure will result in different observed intensities with different telescopes, and comparisons of peak intensities will not guarantee consistent calibration schemes. This difficulty with the coupling of the antenna pattern to the source brightness distribution may be partly overcome by completely mapping the source. Since the total flux density from a source is independent of the telescope power pattern, comparison of integrated intensities may be used to compare the thermal calibration scales of different telescopes. The source flux density S observed in the direction Ω is given by the convolution of the source brightness distribution $B(\Psi)$ with the normalized antenna power pattern so that

$$S(\Omega) = \iint_{4\pi} P_n(\Psi - \Omega)B(\Psi)d\Psi = (2kT_R/\lambda^2) \iint_{4\pi} P_n(\Psi - \Omega)B_n(\Psi)d\Psi \,. \tag{A18}$$

The flux density is also related to the antenna temperature by

$$S(\Omega) = 2kT_A{}'(\Omega)/A_E = 2k\eta_s T_R{}^*(\Omega)/A_E \,, \tag{A19}$$

where A_E is the effective collecting area of the antenna ($A_E = \lambda^2 G/4\pi$, where λ is the wavelength). The total flux density of the source S_T is found by integrating the source brightness distribution over the source solid angle and is given by

$$S_T = \iint_{\Omega_s} B(\Psi)d\Psi = (2kT_R/\lambda^2) \iint_{\Omega_s} B_n(\Psi)d\Psi \,. \tag{A20}$$

Combining equations (4), (A18), and (A20), one can show that the integral of flux density is related to the total flux density by

$$\iint_{4\pi} S(\Omega)d\Omega = (\eta_R \lambda^2/A_E)S_T \,, \tag{A21}$$

or by substitution from equation (A19) we have

$$S_T = (2k/\eta_r \lambda^2) \iint_{4\pi} T_A{}'(\Omega)d\Omega \,. \tag{A22}$$

Similarly, from equation (10), we have

$$S_T = (2k\eta_{\rm fss}\eta_{\rm rss}/\lambda^2) \iint_{4\pi} T_R{}^*(\Omega)d\Omega \,, \tag{A23}$$

which is independent of the antenna directional pattern. Comparisons of total source flux densities calculated from equation (A23) may thus be used to check the consistency of the thermal calibration scales of different antennas, but agreement does not necessarily indicate that the spillover factors $\eta_{fss}\eta_{rss}$ are accurately known. These may be checked only by comparison of peak $T_R{}^*$ values rather than of integrated $T_R{}^*$ values.

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MARC L. KUTNER: Department of Physics, Rensselaer Polytechnic Institute, Troy, NY 12181

BOBBY L. ULICH: Multiple Mirror Telescope Observatory, University of Arizona, Tucson, AZ 85721