THE ASTROPHYSICAL JOURNAL, 248:L27–L30, 1981 August 15

GRAVITATIONAL RADIATION AND THE EVOLUTION OF CATACLYSMIC BINARIES

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Received 1980 October 27; accepted 1981 March 27

ABSTRACT

Model evolutionary computations were done for a low-mass secondary component filling its Roche lobe. Mass transfer from the secondary onto a primary component was driven by angular momentum loss caused by gravitational radiation. The secondary was initially a lower-main-sequence star, and it gradually became degenerate as a result of mass loss. The orbital period initially decreased to a minimum value close to 80 minutes and then increased. The observed hydrogen-rich cataclysmic binaries have a short-period cutoff at 81 minutes. This indicates that gravitational radiation is the main driving force for the evolution of such short-period systems.

Subject headings: gravitation — stars: binaries — stars: evolution — stars: variables

I. INTRODUCTION

Qualitative discussions of the evolution of a short-period cataclysmic binary indicate that the observed cutoff of orbital periods at 81 minutes is caused by gravitational radiation being the dominant driving force for that evolution (Paczyński 1981). According to that estimate, the minimum period expected theoretically is 81 ± 6 minutes. This remarkable coincidence may be a result of the crudeness of the theoretical estimate, but it may also be real. If real, the coincidence would be of considerable importance. It could be used as indirect evidence for the existence of gravitational radiation and for the correctness of commonly used formulae for gravitational energy losses (Landau and Lifshitz 1951). This evidence would be less clear than that provided by the binary radio pulsar (Taylor and McCulloch 1980), but it would be entirely independent. The aim of this *Letter* is to present more precise model computations that agree quantitatively with the previous estimate.

II. OUTLINE OF THE PROBLEM

We consider a semidetached binary with a low-mass ($< 0.4\ M_\odot$) secondary component filling up its Roche lobe. We assume that there is no mass loss from the binary and that all matter lost from the secondary is accreted onto the primary. We do not follow the evolution of the primary component, but it may be any compact star: a degenerate dwarf, a neutron star, or a black hole. The low mass of the secondary makes the nuclear time scale so long that it cannot be of any evolutionary significance. The binary has to lose angular momentum in order to maintain mass transfer; this causes it to evolve. A lot of work has been published on this subject (cf. Whyte and Eggleton 1980; Paczyński 1981 for references), as this is the generally accepted model for a cataclysmic variable.

It is well known that lower-main-sequence stars are fully convective, i.e., they are adiabatic in space except for a thin layer close to the surface (see, e.g., Grossman, Hays, and Graboske 1974). We assume that the secondary component of our binary is adiabatic in space, even if it departs from the main sequence. Of course, the specific entropy of our models may vary with time, and, therefore, the surface luminosity does not have to be equal to the nuclear luminosity. The difference is commonly called a "gravitational luminosity," and it may be calculated as

$$L_g = L - L_n = -\frac{dS}{dt} T_M, \quad (1)$$

where

$$T_M \equiv \int_0^M T dM_r, \quad (2)$$

T is the temperature, S is the specific entropy, L is the surface luminosity, L_n is the nuclear luminosity, and M is the total mass of the star. We also have

$$L_n = \int_0^M \varepsilon_n dM_r, \quad (3)$$

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where ϵ_n is the nuclear energy generation rate per unit mass.

Entropy may be written as a general function of density and temperature in the form:

$$S = S(\rho_c, T_c), \tag{4}$$

where ρ_c and T_c are the density and temperature in the stellar center. The equations of stellar structure may be integrated from the center outward by assuming hydrostatic and adiabatic equilibrium. Throughout this *Letter*, we adopt the same chemical composition in all models, with

$$X = 0.70, \quad Y = 0.27, \quad Z = 0.03. \tag{5}$$

By integrating the stellar structure equations all the way to the surface, we can obtain:

$$M = M(\rho_c, T_c),$$

$$R = R(\rho_c, T_c),$$

$$L_n = L_n(\rho_c, T_c),$$

$$T_M = T_M(\rho_c, T_c), \tag{6}$$

where R is the stellar radius.

There is a thin, nonadiabatic layer close to the stellar surface. This layer has been neglected in calculating equations (6) because its thickness is much smaller than R. The surface luminosity may be calculated taking this layer into account. The structure of the layer depends on the effective temperature, T_e, and the surface gravity, g. In particular, the value of the entropy in the adiabatic interior is a function of T_e and g:

$$S = S(T_e, g), \tag{7}$$

where

$$T_e = \left(\frac{L}{4\pi R^2 \sigma}\right)^{1/4}, \quad g = \frac{GM}{R^2}. \tag{8}$$

Combining equations (4), (6), (7), and (8), we may obtain

$$L = L(\rho_c, T_c). \tag{9}$$

Thus, $(L - L_n)/T_M$, i.e., dS/dt in equation (1), is also a function of central density and temperature.

We have assumed that the star filled the Roche lobe and that it was the less massive secondary component of a binary. The orbital angular momentum, J, the masses of the two components, M_1 and M_2, and the radius of the secondary, R_2, are related by the equation:

$$J^2 = \frac{3^{4/3}}{2} G R_2 \frac{M_1^2 M_2^{5/3}}{(M_1 + M_2)^{2/3}} \tag{10}$$

(cf. eq. [6] of Paczyński 1981). Let us suppose there is angular momentum loss but no mass loss from the binary. Let the secondary fill its Roche lobe all the time, and let the mass be transferred from the secondary to the primary, i.e.,

$$\frac{dM_1}{dt} = -\frac{dM_2}{dt} > 0. \tag{11}$$

Equation (10) may be differentiated to obtain

$$\frac{d\ln R_2}{dt} + \left(\frac{5}{3} - 2\frac{M_2}{M_1}\right)\frac{d\ln M_2}{dt} = 2\frac{d\ln J}{dt}. \tag{12}$$

Equations (1) and (12) thus determine the evolution of the secondary component, if the rate of angular momentum loss from the binary is specified. It is convenient to write the two equations in the form:

$$S_\rho \frac{d\ln \rho_c}{dt} + S_T \frac{d\ln T_c}{dt} = \frac{L_n - L}{T_M}, \tag{13a}$$

$$\left[R_\rho + \left(\frac{5}{3} - 2\frac{M_2}{M_1}\right)M_\rho\right]\frac{d\ln \rho_c}{dt} + \left[R_T + \left(\frac{5}{3} - 2\frac{M_2}{M_1}\right)M_T\right]\frac{d\ln T_c}{dt} = 2\frac{d\ln J}{dt}, \tag{13b}$$

where

$$S_\rho \equiv \left(\frac{\partial S}{\partial \ln \rho_c}\right)_{T_c}, \quad R_\rho \equiv \left(\frac{\partial \ln R_2}{\partial \ln \rho_c}\right)_{T_c},$$

$$M_\rho \equiv \left(\frac{\partial \ln M_2}{\partial \ln \rho_c}\right)_{T_c},$$

$$S_T \equiv \left(\frac{\partial S}{\partial \ln T_c}\right)_{\rho_c}, \quad R_T \equiv \left(\frac{\partial \ln R_2}{\partial \ln T_c}\right)_{\rho_c},$$

$$M_T \equiv \left(\frac{\partial \ln M_2}{\partial \ln T_c}\right)_{\rho_c}. \tag{14}$$

Equations (13a, b) may be numerically integrated provided that we know the thermodynamic conditions and the nuclear burning rate in the stellar interior, as well as the opacities and the mixing length in the nonadiabatic layer close to the stellar surface. Of course, we also have to specify the rate of angular momentum loss, dJ/dt. When the variations of central temperature and density with time are known, we may use equations (6), (9), and (8) to find the other stellar parameters: M_2, R_2, L_2, T_{e2}, dM_2/dt, and so on. We may also find the binary orbital period with the equation:

$$P = \frac{9\pi}{(2G)^{1/2}} \frac{R_2^{3/2}}{M_2^{1/2}} \qquad (15)$$

(cf. eq. [3] of Paczyński 1981).

III. PHYSICS OF THE STELLAR INTERIOR

Nuclear Reactions.—Only the *p-p* reaction was included, and for $T > 9 \times 10^6$ K, we used the formula given by Hofmeister, Kippenhahn, and Weigert (1964), which takes into account the three chains: PPI, PPII, and PPIII. For $T < 7 \times 10^6$ K, we used only the first two reactions according to formulae given by Fowler, Caughlan, and Zimmerman (1975). A smooth interpolation was made between the two formulae for intermediate temperatures. Screening effects were taken into account following DeWitt, Graboske, and Cooper (1973) and Graboske *et al.* (1973).

Thermodynamics.—All of the thermodynamic functions were calculated from tables given by Fontaine, Graboske, and Van Horn (1977). When necessary, an interpolation was made between those tables and the zero-temperature equation of state given by Zapolsky and Salpeter (1969). The details of the interpolation scheme will be published elsewhere (Sienkiewicz 1981). In the low temperature region, molecular hydrogen was taken into account following Paczyński (1969).

Opacities.—Opacities given by Cox and Stewart (1970) were used with molecular H_2O accounted for following Paczyński (1969). If extrapolation was necessary, then the value of the opacity at the table boundary was adopted.

Mixing Length.—Mixing length in the nonadiabatic layer close to the stellar surface was taken to be equal to one pressure scale height. The temperature gradient was calculated with the code described by Paczyński (1969).

All of the quantities of interest (M, R, L, L_n, T_M, etc.) were tabulated as a function of the central density and temperature for the chemical composition given by equation (5). These tables will be published elsewhere (Sienkiewicz 1981).

IV. RESULTS

The tables described in the last section make it easy to find the location of the main-sequence models and the degenerate models. On the main sequence, we require $L = L_n$, i.e., the surface luminosity is equal to the nuclear luminosity, and the model is in thermal equilibrium. Such models form two branches that are sep-

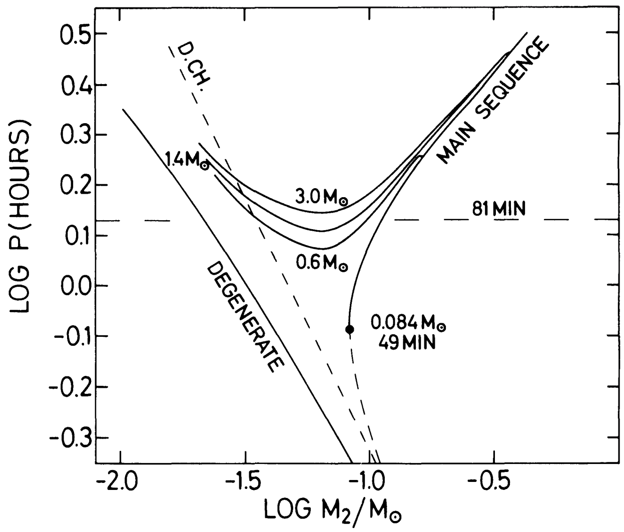

FIG. 1.—The relation between the secondary's mass (in solar units) and the binary period (in hours) is shown with thick lines for lower-main-sequence stars and for degenerate stars with normal chemical composition ($X = 0.70$, $Y = 0.27$, $Z = 0.03$). The high-density branch of the main sequence and the Chandrasekhar degenerate sequence are shown with broken lines. Evolutionary tracks are shown for three binaries with a total mass of 0.6 $M_\odot$, 1.4 $M_\odot$, and 3 $M_\odot$. Notice the minimum binary period along the evolutionary tracks is close to the observed cutoff at 81 minutes. Gravitational radiation is the only sink of angular momentum.

arated by a model with the minimum mass for which hydrogen burning is possible, i.e., 0.084 $M_\odot$. (cf. Paczyński 1973 for a discussion). Our normal main-sequence branch agrees well with that calculated by Grossman, Hays, and Graboske (1974). Models on the high-density branch are unstable and are of no astrophysical importance. On the degenerate branch, we require $T_c \approx 0$, and we obtain the mass-radius relation which is very close to that calculated by Vila (1971); however, it differs significantly from the Chandrasekhar relation (cf. Fig. 1).

Equations (13a, b) were numerically integrated to follow the evolution of a low-mass star filling its Roche lobe. All of the quantities defined with equations (14), as well as $(L_n - L)/T_M$, are available in tables as functions of the central density and temperature. The loss of angular momentum because of gravitational radiation is given as

$$\frac{d \ln J}{dt} = -\frac{32}{5} \left(\frac{2\pi}{P} \right)^{8/3} \frac{G^{5/3}}{c^5} \frac{M_1 M_2}{(M_1 + M_2)^{1/3}} \qquad (16)$$

(cf. Paczyński 1967, 1981), with the orbital period given by equation (15). A large number of evolutionary tracks were calculated for many values of a total mass of a binary, $M_1 + M_2$, and the initial mass of the secondary,

M_{10}. The tracks with various M_{10} rapidly converged to a common evolutionary track that depended only on the total mass. Some tracks are shown on the secondary-mass/orbital-period diagram in Figure 1. The evolution begins with the secondary close to the main sequence. Initially, the secondary component evolves down the main sequence, and the orbital period decreases because the mass transfer time scale is longer than the Kelvin-Helmholtz time scale. When the secondary's mass is reduced to about 0.12–0.15 $M_{\odot}$, the two time scales become comparable and the secondary departs from the main sequence and evolves toward a degenerate condition. The binary period passes through a minimum which depends on the total mass of a binary. For a larger total mass, the gravitational radiation is stronger and the minimum period is longer.

We evolved model binaries with total masses as large as 3 $M_{\odot}$, as they may be applicable to some X-ray sources, like those observed in globular clusters and in the galactic bulge (Ostriker and Żytkow 1981; Joss and Rappaport 1981).

V. DISCUSSION

Model calculations described in this paper confirm the estimate made by Paczyński (1981). The observed minimum value of orbital periods of hydrogen-rich cataclysmic binaries (81 minutes) shows that gravitational radiation is the main driving force for the evolution of these short-period systems. There is the potential here to make a test of general relativity since the models predict the relationship between the minimum period and the total mass of a cataclysmic binary. This relation should be studied in more detail theoretically (its dependence on mixing length and chemical composition), and observationally (mass determination of many short-period systems).

We are very grateful to Dr. G. Fontaine who kindly provided a magnetic tape with tables of thermodynamic functions. One of us (R. S.) would like to thank the staff of the Max-Planck-Institut für Physik und Astrophysik, especially Dr. R. Ward for suggesting some improvements in the writing, and Mrs. Ana Hrala who helped with the preparation of the opacity tables. Preliminary results were obtained with the PDP 11/45 computer donated to the N. Copernicus Astronomical Center by the National Academy of Sciences. This project was partly supported by the NSF grant AST 79-22074.

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