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LARGE-SCALE FLUCTUATIONS IN THE MICROWAVE BACKGROUND AND THE SMALL-SCALE CLUSTERING OF GALAXIES[1]

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ABSTRACT

Two groups have discovered large-scale irregularities in the microwave background in excess of the dipole part that might be due to our peculiar motion (Fabbri *et al.*; Boughn, Cheng, and Wilkinson). More observations are needed before we can decide whether this is a local effect, perhaps emission from dust clouds, or a true irregularity in the microwave background. My purpose here is to point out that if the effect is extragalactic there is a ready interpretation. The irregular distribution of mass in clusters of galaxies causes large-scale gradients in the gravitational potential that in turn perturb the microwave background. If the mass autocorrelation function $\xi(r)$ is negligibly small on large scales, the fluctuations in brightness on angular scale θ vary as $\langle(T_1 - T_2)^2\rangle^{1/2} \propto \theta^{1/2}$. If $\xi(r)$ agrees with the galaxy two-point correlation function observed at $r \lesssim 30h^{-1}$ Mpc, the brightness fluctuations produced by this effect are comparable to the recent observations.

Subject headings: cosmic background radiation — cosmology — galaxies: clusters of

I. INTRODUCTION

The $\delta T \propto \theta^{1/2}$ relation for large-scale irregularities in the microwave background due to the relatively small-scale clumping of matter in clusters of galaxies assumes the mass autocorrelation function $\xi(r)$ is negligibly small at large separations. If so, the fluctuation in mass found in a large, randomly placed sphere of radius r is $\delta M \propto r^{3/2}$, so the typical gravitational potential difference at separation r is $\delta\phi \propto \delta M/r \propto r^{1/2}$. This causes a like fluctuation $\delta T/T \sim \delta\phi$ in the brightness of the radiation arriving from points at separation r, so $\delta T/T \propto \theta^{1/2}$.

The following discussion uses the convenient relation of Sachs and Wolfe (1967; eq. [1] below). It assumes the density parameter Ω_0 is close to unity or else $\theta \lesssim \Omega_0(1 - \Omega_0)^{-1/2}$. The computation of the expected size of the quadrupole moment of the brightness distribution thus applies only if $\Omega_0 \sim 1$, but it does give a reasonable indication of the expected size of the effect. The interesting point is that if $\xi(r)$ agrees with the observed galaxy two-point correlation function then the computed $\delta T/T$ is comparable to the indications of large-scale fluctuations recently obtained by Fabbri *et al.* (1980*a*, *b*) and Boughn, Cheng, and Wilkinson (1981).

II. LARGE-SCALE ROUGHNESS OF THE RADIATION BACKGROUND

The Sachs-Wolfe (1967) relation slightly generalized to allow for an open or closed cosmological model is (Peebles 1980, §§ 11 and 93):

$$\frac{T_1 - T_2}{T} = \frac{2}{15}\frac{\Omega_0^{-1} - 1}{c^2 D(\Omega_0)} G\langle\rho\rangle \int \frac{d^3r}{r}\,[\delta(r_2 - r) - \delta(r_1 - r)]\,. \tag{1}$$

To this must be added the dipole anisotropy due to our peculiar motion, any initial fluctuations (departure from adiabatic perturbations), and fluctuations caused by the clumpy matter distribution along the line of sight. Hubble's constant is $H_0 = 100h$ km s^{-1} Mpc^{-1}, $\Lambda = 0$, and $\Omega_0 = 2q_0$ is the density parameter. The function D is the growing mode of the mass density perturbation $\delta(r, t) = \delta\rho/\rho$ normalized to the present value:

$$D_0 = 1 + \frac{3}{x} + \frac{3(1 + x)^{1/2}}{x^{3/2}} \ln\,[(1 + x)^{1/2} - x^{1/2}]\,, \qquad x = \Omega_0^{-1} - 1\,. \tag{2}$$

If $\Omega_0 \sim 1$, $D_0 \sim 2x/5$. In equation (1), $\delta(r)$ is the present value of $\delta\rho/\rho$, r is proper separation, and r_1 and r_2 are the present positions of origin of the radiation detected along the lines of sight 1 and 2. The fractional difference of radiation temperatures along the two lines of sight is $(T_1 - T_2)/T$. The relation neglects curvature of the t = constant surfaces, so in an open model we require

$$|r_1 - r_2| < a_0|R| = cH_0^{-1}(1 - \Omega_0)^{-1/2}\,. \tag{3}$$

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The dimensionless mass autocorrelation function is

$$\xi(r) = \langle \delta(r + s)\delta(s) \rangle . \tag{4}$$

I assume $\xi(r) = 0$ at separations greater than the limit

$$r > r_{mx} \ll |r_1 - r_2| = 2a_0 x \sin \theta/2 , \quad x = 2/(H_0 \Omega_0 a_0) , \tag{5}$$

where x is the coordinate distance the radiation has traveled. Then equation (1) yields

$$\langle (T_1 - T_2)^2/T^2 \rangle = K \sin \theta/2 , \quad K = \frac{4}{25} \frac{(1 - \Omega_0)^2}{D(\Omega_0)^2 \Omega_0} \left(\frac{H_0}{c}\right)^3 J_3 ,$$

$$J_3 = \int_0^\infty s^2 ds \xi(s) = 960 \pm 125 h^{-3} \text{ Mpc}^3 . \tag{6}$$

J_3 is based on the galaxy correlation function (Clutton-Brock and Peebles 1981). It assumes $r_{mx} \sim 30h^{-1}$ Mpc. The standard deviation of J_3 reflects only the uncertainty in translation from the angular function, not that due to ξ at large r. The factor K is approximately

$$K \approx 3.6 \times 10^{-8} \Omega_0^{-0.4} . \tag{7}$$

When $\theta \ll 1$ rad, equation (6) with θ in degrees is

$$\langle (T_1 - T_2)^2/T^2 \rangle^{1/2} = 1.8 \times 10^{-5} \Omega_0^{-0.2} (\theta^\circ)^{1/2} . \tag{8}$$

The spherical harmonic expansion of the distribution is

$$T(\theta, \phi) = T \, \Sigma a_l^m Y_l^m(\theta, \phi) . \tag{9}$$

By following § 46 of Peebles (1980), one finds

$$\langle |a_l^m|^2 \rangle = a_l^2 = \pi K I_l , \quad I_l = -\int_0^\pi d \cos\theta \sin\theta/2 P_l(\cos\theta) . \tag{10}$$

If $m \neq 0$, the real and imaginary parts of a_l^m satisfy

$$\langle (\text{Re } a_l^m)^2 \rangle = \langle (\text{Im } a_l^m)^2 \rangle = a_l^2/2 . \tag{11}$$

The integral is

$$I_1 = 4/15 , \quad I_2 = 4/105 , \quad I_3 = 4/315 , \tag{12}$$

and, for $l \gg 1$, is

$$I_l = (2l^3)^{-1} . \tag{13}$$

III. EFFECTS OF SCATTERING AND DECOUPLING

If the free electron density is $n_e \propto (1 + z)^3$, the depth for scattering into the line of sight since redshift z is

$$\tau = \int_0^z \sigma n_e c(dt) ,$$

$$= \frac{2\sigma n_e(0) c}{3\Omega_0^2 H_0} [2 - 3\Omega_0 + (1 + \Omega_0 z)^{1/2}(\Omega_0 z + 3\Omega_0 - 2)] , \tag{14}$$

where 0 means present value. Table 1 lists the redshift z_1 at which $\tau = 1$ assuming $h = 0.75$, neutral helium abundance

TABLE 1

SMOOTHING BY INTERGALACTIC PLASMA

Density Parameter	Ionization = 1		Ionization = 0.1	
	z_1	θ_1[a]	z_1	θ_1[a]
1.0	10.6	11.9	52	4.6
0.3	17.6	7.0	80	2.4
0.1	28.0	4.3	119	1.37
0.03	47.5	2.4	188	0.75

[a] Degrees.

$Y = 0.25$, and two values of the fractional ionization of the hydrogen. The angle that subtends $c/H(z_1)$ at epoch z_1 is (e.g., Peebles 1980, Appendix):

$$\theta_1 = \frac{0.5\Omega_0^2(1 + z_1)(1 + \Omega_0 z_1)^{-1/2}}{[(\Omega_0 - 2)(1 + \Omega_0 z_1)^{1/2} + 2 - \Omega_0 + \Omega_0 z_1]} . \tag{15}$$

This is a rough measure of the angle at which the $\theta^{1/2}$ relation would be truncated by scattering by a homogeneous plasma. Since ionization $x \sim 1$ at $z \sim 10$ would be hard to maintain (the time for cooling by radiation drag is less than the expansion time; Peebles 1971, § VII*a*), a reasonable bound is $\theta_1 \lesssim 5°$.

Irregularities on scales less than the coupled matter-radiation Jeans length depend on the details of decoupling of matter and radiation. This Jeans length at decoupling is subtended by

$$\theta_x \sim 20\Omega_0^{1/2}(1 + 30\Omega_0 h^2)^{-1/2} \text{ arcmin} . \tag{16}$$

The computations of decoupling made by Peebles and Yu (1970), Doroshkevich, Zel'dovich, and Sunyaev (1978), and Silk and Wilson (1980) all take account of gravity and so include the effect of large-scale potential fluctuations; and Wilson and Silk (1980) computed the quadrupole moment a_2. But since none of these computations were adjusted to fit the empirical value of J_3, none can be compared to the results presented here.

IV. COMPARISON WITH OBSERVATIONS

Fabbri *et al.* (1980*b*) find at $\theta = 6°$:

$$\langle (T_1 - T_2)^2/T^2 \rangle^{1/2} = (3.0 \pm 0.7) \times 10^{-5} . \tag{17}$$

As Fabbri *et al.* (1980*b*) point out, this could be caused by local sources. However, it is remarkably close to equation (8),

$$\langle (T_1 - T_2)^2/T^2 \rangle^{1/2} = 4 \times 10^{-5}\Omega_0^{-0.2} . \tag{18}$$

Upper limits on $\delta T/T$ at $\theta < 6°$ are listed by Partridge (1980); none conflicts with equation (8).

Fabbri *et al.* (1980*a*, *b*) and Boughn, Cheng, and Wilkinson (1981) find evidence of a nonzero quadrupole moment in the brightness distribution. I discuss the results of Boughn *et al.* because they measure more components. Their results are expressed in the Berkeley convention (Smoot and Lubin 1979) where Q_2 through Q_5 are the real and imaginary parts of Sa_{21} and Sa_{22}, with $S = \pm(2.7 \text{ K})(15/8\pi)^{1/2}$. (They could not measure a_{20}.) Since the Q_i estimates are not strongly correlated, we can set $Q_i = Q_i(0) + m_i$, where $Q_i(0)$ is drawn from a population with zero mean and variance Q^2, and m_i is the measuring error; write

$$\chi^2 = \sum_2^5 Q_i^2/(Q^2 + m_i)^2 , \tag{19}$$

and then seek the range of Q that yields the expected range of values of χ^2. The result at the 10 and 90% probabilities for χ^2 is

$$0.20 < Q < 0.65 \text{ mK} . \tag{20}$$

(Though it is not apparent from this expression, the formal probability for $Q = 0$ is negligible.) We have from equations (11) and (20):

$$1.3 \times 10^{-4} < a_2 < 4.4 \times 10^{-4} , \tag{21}$$

which can be compared to the prediction of the model (eqs. [7], [10], and [12]):

$$a_2 = 6.5 \times 10^{-5}\Omega_0^{-0.2} . \tag{22}$$

In view of the uncertainties, the discrepancy may not be serious, and there is the interesting coincidence that theory and observation yield comparable values of a_2.

Equations (10) and (12) yield dipole moment

$$D^2 \equiv T_x^2 + T_y^2 + T_z^2 = 0.6 \text{ K } (2700 \text{ mK})^2 , \tag{23}$$

where the T^α are the amplitudes of the 24 h anisotropy. If our peculiar motion were negligible, we would expect from equations (7) and (23):

$$D = 0.4\Omega_0^{-0.2} \text{ mK} . \tag{24}$$

The observed value (Smoot and Lubin 1979; Boughn, Cheng, and Wilkinson 1981) is

$$D = 3.3 \pm 0.4 \text{ mK} . \tag{25}$$

If $\delta T \propto \theta^{1/2}$, we cannot fit this D and the 6° fluctuations; peculiar motion is indicated. However, if as discussed in the next section (§ V) $\delta T/T$ increases more rapidly than $\theta^{1/2}$ because of large-scale clustering, we could arrange the clustering spectrum to fit the present observations at 6°, a_2, and a_1, as suggested by Fabbri *et al.* (1980*b*).

V. IMPLICATIONS

If further measurements show the fluctuations are independent of frequency and vary as $\theta^{1/2}$, it will be strong evidence that the effect is in the microwave background and the result of uncorrelated fluctuations in the mass distribution. Scattering by an intergalactic plasma may truncate the $\theta^{1/2}$ relation at $\theta_1 \lesssim 10°$ (Table 1). A detection of this effect would be of great interest as a measure of the amount of diffuse ionized matter at redshifts $z \gtrsim 10$.

Other contributions to $\delta T/T$ would be expected to add in quadrature to the effect of potential fluctuations, so we have in any case an upper bound on J_3 (eq. [6]). Many authors have considered the possibility of large-scale clustering (de Vaucouleurs 1970; Grishchuck and Zel'dovich 1978; Oort 1980). If $\xi \sim 0.1$ to $r \sim 100h^{-1}$ Mpc, it would increase $\delta T/T$ by a factor of 6, which would conflict with the measurement of Fabbri *et al.* (1980*a*, *b*) at $\theta = 6°$. Thus it appears that if structures existed at $r \sim 100h^{-1}$ Mpc, they would have to be rare enough to make $\xi \lesssim 0.003$, or else their gravitational effects would have to be balanced by suitably arranged holes around them.

Arrangement of holes can make $\xi < 0$ at large r and so considerably reduce J_3 and $\delta T/T$. In Kantowski's (1969) "Swiss cheese" model the mass excess in each cluster is taken from a surrounding hole, so $J_3 = 0$. Rees and Sciama (1968) and Dyer (1976) used this model and so were led to discuss the smaller effect on $\delta T/T$ of the time-varying potentials in clusters. No evidence has been found of anticorrelation in the galaxy distribution (Peebles 1974, § III). Improved measures of ξ at large r would be helpful here.

The power spectrum of the large-scale mass distribution often is supposed to have a power-law form,

$$\langle |\rho_{\mathbf{k}}|^2 \rangle \propto k^{\nu} . \tag{26}$$

If $-3 < \nu < 0$, J_3 diverges at large r as (Peebles 1980, § 42):

$$J_3(r) = \int_0^r \xi r^2 dr \propto r^{|\nu|} . \tag{27}$$

This would change equation (8) to

$$\langle (T_1 - T_2)^2/T^2 \rangle^{1/2} \propto \theta^{\epsilon} , \quad \epsilon = (1 + |\nu|)/2 . \tag{28}$$

Unless $|\nu|$ is small, $\delta T/T$ at 6° would be unacceptably large. If $0 < \nu < 1$, then, at large r, $\xi < 0$, $J_3 \propto r^{-\nu}$, and $\epsilon = (1 - \nu)/2$; so $\delta T/T$ grows only slowly with increasing θ. As noted by Wilson and Silk (1980), adopting $\nu > 0$ appreciably decreases the expected quadrupole moment a_2. If $\nu = 1$, the potential diverges only as log r, and so one can suppose the primeval power spectrum k^{ν} extends to all interesting lengths (Peebles and Yu 1970; Harrison 1970; Zel'dovich 1972). However, as Press and Vishniac (1980) point out, this limits the rms density fluctuation at decoupling to $\delta\rho/\rho \sim 10^{-4}$, which seems much too small to make galaxies. Of course, one can always assume the $\nu = 1$ power spectrum applies only at $r \gtrsim 30h^{-1}$ Mpc. If $\nu > 1$, then $\epsilon = 0$, but $\delta\phi$ diverges at small r, and so there would have to be a break in the power law.

VI. CONCLUSIONS

Recent indications of irregularities in the microwave background are comparable to what is expected under a not unreasonable model for the large-scale structure of the universe. It appears that we are within reach of a convincing measure of the structure or else a considerable narrowing of the options. The computation presented here is preliminary because it neglects curvature of the t = constant surfaces. I hope to present the results of more detailed computations in due course.

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