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Evolution of Cataclysmic Binaries

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ABSTRACT

Cataclysmic binaries with short orbital periods have low mass secondary components. Their nuclear time scale is too long to be of evolutionary significance. Angular momentum loss from the binary drives the mass transfer between the two components. As long as the characteristic time scale is long compared with the Kelvin-Helmholtz time scale of the mass losing secondary that star remains close to the main sequence, and the binary period decreases with time. If angular momentum loss is due to gravitational radiation then the mass transfer time scale becomes comparable to the Kelvin-Helmholtz time scale when the secondary's mass decreases to 0.12 $M_\odot$, and the binary period is reduced to 80 minutes. Later, the mass losing secondary departs from the main sequence and gradually becomes degenerate. Now the orbital period increases with time. The observed lower limit to the orbital periods of hydrogen rich cataclysmic binaries implies that gravitational radiation is the main driving force for the evolution of those systems.

It is shown that binaries emerging from a common envelope phase of evolution are well detached. They have to lose additional angular momentum to become semi-detached cataclysmic variables.

1. Introduction

The origin and evolution of cataclysmic variables is poorly understood. General information about these systems may be found in recent reviews by Robinson (1976), Warner (1976), and Gallagher and Starrfield (1978). These binaries have a white dwarf primary component surrounded with an accretion disk. The secondary component overflows its Roche lobe, and matter flows from the inner Lagrangian point towards the disk. The secondary may be directly observed if the orbital period is longer than about 4 or 6 hours (SS Cygni, U Geminorum), and it appears to be close to the lower main sequence. The shortest period binaries have unseen secondaries, which are believed to be either lower main sequence, or degenerate dwarfs. In any case, the nuclear evolution cannot be of any importance for the secondary components of the shortest period cataclysmic binaries. The question arises: if the star cannot expand due to nuclear evolution, how can it overflow its Roche lobe? It is natural to suppose, that if the star cannot expand then perhaps the Roche lobe can shrink. This may be acomplished if a binary system loses angular momentum.

Angular momentum may be lost through a number of various processes (Paczyński 1980a):

(a) gravitational radiation,
(b) magnetic winds,
(c) mass outflow through the outer Lagrangian point,
(d) excretion disks,
(e) common envelopes.

Very likely the list may be extended. The processes c, d, and e are not likely to operate in the short period cataclysmic binaries, or at least nothing is known about this subject. Magnetic winds were suggested to be important for the evolution of close binaries (Mestel 1967, Eggleton 1976), but it is not known at this time how to make quantitative estimates. Presumably, this process may be efficient if the secondary is convective, as is the case with the lower main sequence stars. Ultimately, the energy required for the winds comes from nuclear burning. Another possibility is to produce the magnetic winds driven by the disk accretion. No quantitative estimates are available.

The only well understood process of angular momentum loss is the gravitational radiation. This effect was suggested to be important for the evolution of cataclysmic variables by Kraft, Mathews and Greenstein (1962) and by Paczyński (1967). Subsequently, a lot of work has been done on this subject (Vila 1971, Faulkner 1971, Faulkner, Flannery and Warner 1972, Faulkner 1976, Chau and Lauterborn 1977, Chau 1978, Tutukov and Yungelson 1979, Whyte and Eggleton 1980, Taam, Flannery and Faulkner 1980, Ostriker and Żytkow 1980, Joss and Rappaport 1980). It has been suggested that in some systems gravitational radiation is the dominant mode of angular momentum loss (WZ Sge: Kraft *et al.* 1962, Paczyński 1967, Z Cha: Ritter 1979, 1980b, OY Car: Ritter 1980a, AM Her: Young and Schneider 1979). Nevertheless, it is not known how important is the gravitational radiation for the evolution of a typical cataclysmic binary.

It is frequently claimed that cataclysmic variables are the products of a common envelope binary evolution (Ostriker 1973, Paczyński 1976, Eggleton 1976, Ritter 1976, Meyer and Meyer-Hofmeister 1979). Very likely, the immediate progenitors of cataclysmic variables are the short period detached binaries with a degenerate or a subdwarf component, like V 471 Tau (Young and Nelson 1972, Paczyński 1976), UU Sge (Miller, Krzemiński and Priedhorsky 1976, Bond, Liller and Mannery 1978), PG 1413+01 (Green, Richstone and Schmidt 1978), and LB 3459 (Kilkenny, Hilditch and Penfold 1978, Paczyński 1980b). Angular momentum losses will bring these systems from a semidetached to a detached stage and will induce the mass transfer necessary for a development of cataclysmic activity.

There seems to be a cut-off to the binary periods of cataclysmic variables at about 80 minutes. Almost two decades ago Krzemiński (1962) discovered that WZ Sge has a period of 81.5 minutes. Only recently this record has been slightly improved with a discovery that a polar 2A0311−227 has a binary period of 81 minutes (Tapia 1979). It looks like there are no *hydrogen rich* cataclysmic binaries with periods below about 80 minutes, while about a dozen of systems is known with periods between 81 and 100 minutes.

The aim of this paper is to investigate the relationship between the minimum binary period and the evolution of cataclysmic variables. It turns out that the value of minimum period depends on the rate of mass transfer between the two components, and ultimately on the rate of angular momentum loss which drives the binary evolution. Preliminary results were presented at the IAU Colloquium No. 53 in Rochester (Paczyński and Krzemiński 1979). Detailed evolutionary computations will be published elsewhere (Paczyński and Sienkiewicz 1981).

2. Evolutionary considerations

Let us assume that the secondary component of a binary system fills its Roche lobe. In all known cataclysmic variables this is the less massive of the two components. The binary orbits are always circular. For a mass ratio $M_2/M_1 < 0.8$ the Roche lobe radius of the secondary is given with an accuracy of 2% as

$$\frac{R_2}{A} = \frac{2}{3^{4/3}}\left(\frac{M_2}{M_1+M_2}\right)^{1/3} = 0.46224\left(\frac{M_2}{M_1+M_2}\right)^{1/3}, \tag{1}$$

(Paczyński 1971), where A is the separation between the centers of the two components. The Kepler's law

$$A^3 = \left(\frac{P}{2\pi}\right)^2 G(M_1+M_2), \tag{2}$$

may be combined with Eq. (1) to obtain

$$P = \left(\frac{243\pi}{8G}\right)^{1/2} \langle\varrho_2\rangle^{-1/2} = 3.78 \times 10^4 \langle\varrho_2\rangle^{-1/2}, \text{ (c.g.s. units)}, \tag{3}$$

where P is the orbital period, and $\langle\varrho_2\rangle$ is the mean density of the secondary component. The last equation may be written as

$$P^* = 8.85 R_2^{*3/2} M_2^{*-1/2}, \tag{4}$$

where P^* is the orbital period in hours, and R_2^* and M_2^* are the secondary's radius and mass in solar units. Of course, these formulae are well known. It follows, that a short period semidetached binary has a secondary component with a large mean density.

Let us consider hydrogen rich stars with the highest mean density. These are the lower main sequence stars and degenerate dwarfs. Graboske and Grossman (1971) calculated models of 0.2, 0.1, and 0.085$M_\odot$ with

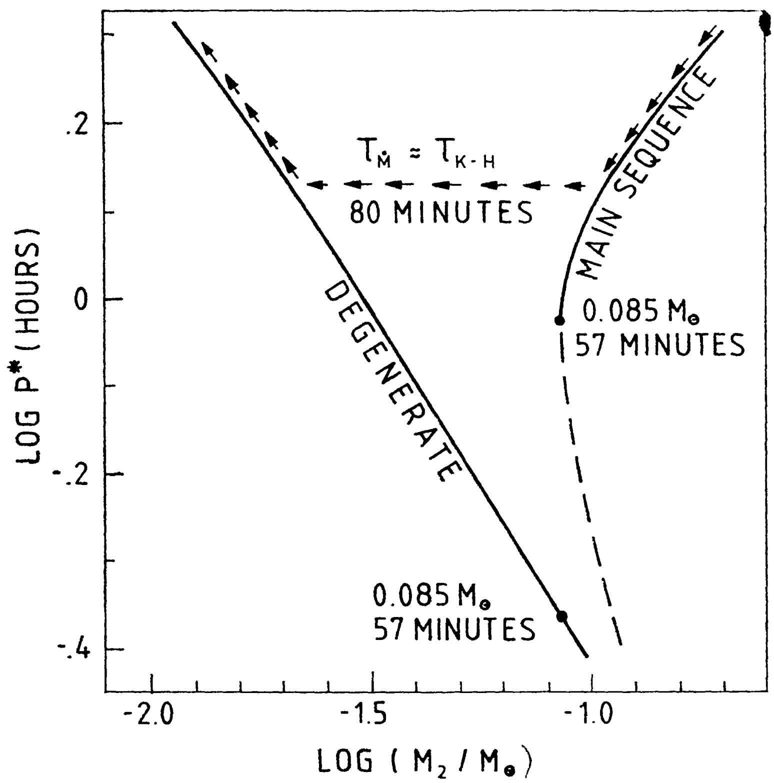

Fig. 1. The relation between the secondary's mass (in solar units) and the binary period (in hours) is shown for the lower main sequence and for degenerate stars with normal chemical composition ($X = 0.68$, $Y = 0.29$, $Z = 0.03$). The minimum mass for hydrogen burning model is 0.085 $M_\odot$. The high density branch of the main sequence is shown schematically with a broken line. Angular momentum losses due to gravitational radiation drive the evolution of a cataclysmic binary along a track indicated with small arrows. Notice, the minimum binary period is about 80 minutes.

a normal chemical composition ($X = 0.68$, $Y = 0.29$, $Z = 0.03$). The $0.085 M_\odot$ model has the lowest mass for which hydrogen burning is possible. Vila (1971) calculated the radii of degenerate dwarfs with various hydrogen content using the equation of state given by Zapolsky and Salpeter (1969). I interpolated the radii of those models for $X = 0.68$. The orbital periods were calculated with Eq. (4) using the models of Graboske and Grossman and Vila. The secondary mass-orbital period relations are shown in Fig. 1. The shortest orbital period for hydrogen burning secondary is 57 minutes ($M_2 = 0.085 M_\odot$). A degenerate dwarf of $0.085 M_\odot$ allows a period as short as 26 minutes. This is 3 times shorter than the shortest observed period. Apparently, binary evolution makes it impossible to get down to the range between 26 and 81 minutes.

We may imagine two ways to form a cataclysmic binary with a very low mass hydrogen rich secondary. In the first scenario the initial mass of the secondary is larger than about $0.1 M_\odot$. Angular momentum loss drives a mass transfer between the two components, and reduces the secondary's mass below $0.1 M_\odot$. In the second scenario the semidetached system is formed with the secondary's mass smaller than $0.1 M_\odot$ from the beginning. For some reason none of these scenarios is capable of producing hydrogen rich cataclysmic binaries with periods between 26 and 81 minutes.

Let us consider the first scenario in some details. The rate of angular momentum loss may be written as

$$\frac{d\ln J}{dt} = a\left(\frac{d\ln J}{dt}\right)_{GR} = -a\frac{32}{5}\left(\frac{2\pi}{P}\right)^{8/3}\frac{G^{5/3}}{c^5}\frac{M_1 M_2}{(M_1+M_2)^{1/3}} = \tag{5}$$
$$= -\frac{a}{7.9\times10^7 \text{ years}}\frac{M_1^* M_2^*}{(M_1^*+M_2^*)^{1/3}}P^{*-8/3}, \qquad a \geqslant 1,$$

where J is the orbital angular momentum, the subscript GR indicates the loss due to gravitational radiation, and a is a dimensionless parameter which must be larger than one, if some additional mechanisms remove angular momentum from the binary. I neglect the spin angular momentum of the two components and the accretion disk.

Let us assume that the secondary fills its Roche lobe all the time. Then Eqs. (1) and (2) may be combined to find a relation

$$J^2 = GA\frac{M_1^2 M_2^2}{M_1+M_2} = \frac{3^{4/3}}{2}GR_2\frac{M_1^2 M_2^{5/3}}{(M_1+M_2)^{2/3}}. \tag{6}$$

If there is no mass loss from the system Eq. (6) may be differentiated to obtain

$$2d\ln J = d\ln M_2\left(\frac{5}{3} - 2\frac{M_2}{M_1} + \beta\right), \tag{7}$$

where

$$\beta \equiv \frac{\mathrm{d}\ln R_2}{\mathrm{d}\ln M_2}, \tag{8}$$

is the slope of the mass-radius relation for the secondary.

Combining Eqs. (5) and (7) we obtain the time scale for the mass transfer induced by the angular momentum losses

$$\tau_{\dot{M}} \equiv \left(-\frac{\mathrm{d}\ln M_2}{\mathrm{d}t}\right)^{-1} = \\ = 7.9\times 10^7 \text{ years } \frac{(M_1^* + M_2^*)^{1/3}}{a M_1^* M_2^*} P^{*8/3}\left(\frac{5}{6} - \frac{M_2^*}{M_1^*} + \frac{\beta}{2}\right). \tag{9}$$

The Kelvin-Helmholtz (i.e. thermal) time scale is defined as

$$\tau_{K-H} \equiv \frac{GM_2^2}{R_2 L_2} = 3\times 10^7 \text{ years} \frac{M_2^{*2}}{R_2^* L_2^*}, \tag{10}$$

where L_2^* is the luminosity of the secondary component, in solar units. As long as the mass transfer time scale, $\tau_{\dot{M}}$, is longer than the Kelvin-Helmholtz time scale, τ_{K-H}, the secondary remains in a thermal equilibrium, *i.e.* it remains on the main sequence. Using the $0.1 M_\odot$ and $0.2 M_\odot$ models of Graboske and Grossman (1971) we find

$$\begin{aligned} \tau_{K-H} &= 1.8\times 10^8 \text{ years } M_2^{*-1.18}, \\ \beta_{MS} &= 0.82, \\ P_{MS}^* &= 6.64 \text{ hours } M_2^{*0.73}, \end{aligned} \tag{11}$$

where P_{MS}^* is the orbital period of a binary with a lower main sequence secondary component filling up its Roche lobe, and β_{MS} is the slope of the mass-radius relation for the lower main sequence stars. Eqs. (9) and (11) may be combined to obtain

$$\begin{aligned} \tau_{\dot{M}} &= 1.53\times 10^{10} \text{ years } \frac{M_2^{*0.95}}{a M_1^*}(M_1^* + M_2^*)^{1/3}\left(1 - 0.804\frac{M_2^*}{M_1^*}\right) \approx \\ &\approx 1.53\times 10^{10} \text{ years } a^{-1} M_1^{*-2/3} M_2^{*0.95}, \\ &\text{for } \tau_{\dot{M}} \gg \tau_{K-H}, \quad M_2^* \ll M_1^*. \end{aligned} \tag{12}$$

As a result of mass transfer the secondary's mass is reduced. This leads to a decrease of the mass transfer time scale (cf. Eq. 12) and to an increase of the Kelvin-Helmholtz time scale (cf. Eq. 11). The two time scales become equal for

$$M_2^* = 0.124 a^{0.47} M_1^{*0.31}, \quad \text{for } \tau_{\dot{M}} = \tau_{K-H}. \tag{13}$$

The lower main sequence stars are fully convective and are closely approximated by the polytropes with an index $n = 1.5$. If the time scale for mass loss from the secondary becomes much shorter than the Kelvin-Helmholtz time scale, the star reacts adiabatically, *i.e.* it expands with the mass radius relation having a slope $\beta_{ad} = -1/3$ (cf. Paczyński 1965, Whyte and Eggleton 1980). Therefore, we have approximate relations

$$\beta = \begin{cases} +0.82, & \text{for } \tau_{\dot{M}} \gg \tau_{K-H}, \\ 0, & \text{for } \tau_{\dot{M}} \approx \tau_{K-H}, \\ -0.33, & \text{for } \tau_{\dot{M}} \ll \tau_{K-H}. \end{cases} \tag{14}$$

Eq. (4) may be differenciated to obtain

$$\frac{d\ln P^*}{d\ln M_2^*} = -\frac{1}{2} + \frac{3}{2}\beta \approx \begin{cases} +0.73, & \text{for } \tau_{\dot{M}} \gg \tau_{K-H}, \\ 0, & \text{for } \tau_{\dot{M}} \approx \tau_{K-H}, \\ -1.00, & \text{for } \tau_{\dot{M}} \ll \tau_{H-K}. \end{cases} \tag{15}$$

Therefore, the minimum orbital period is obtained approximately at the time when the mass transfer proceeds on a Kelvin-Helmholtz time scale. Combining Eqs. (11), (13), and (15) we find the expression for the minimum orbital period

$$P^*_{min} = 1.45 \text{ hours } a^{0.34} M_1^{*0,23}. \tag{16}$$

According to this equation the minimum value is 87 minutes if the primary has a mass of $1M_\odot$, and only gravitational radiation is responsible for angular momentum losses, *i.e.* $a = 1$. This result depends only weakly on the primary's mass: we obtain $P_{min} = 74$ minutes for $M_1 = 0.5M_\odot$. This is remarkably close to the observed value of 81 minutes. According to eq. (13) the secondary has a mass of about $0.12M_\odot$ when it can no longer remain close to the main sequence.

The first evolutionary scenario may now be described in the following way. The precursor of a cataclysmic binary may be a system like V471 Tau, a detached binary with two components of about $0.7M_\odot$ each, one degenerate dwarf and one main sequence dwarf. As a result of angular momentum loss this binary becomes semidetached and mass transfer proceeds from the main sequence dwarf that overflows its Roche lobe, towards the degenerate dwarf component. An accretion disk is formed, and a cataclysmic activity begins. Initially, the mass transfer time scale, which is equal roughly to the angular momentum loss time scale, is longer than the Kelvin-Helmholtz time scale. Therefore, initially the secondary remains in a thermal equilibrium, *i.e.* it stays on the main sequence, with its mass steadily decreasing. The mean density increases while we proceed down the main sequence, and therefore the binary period decreases. This process changes when the Kelvin-Helmholtz time scale becomes so long that the secondary cannot adjust its thermal structure on a relatively short mass transfer time scale. The secondary expands now, while it is still losing mass. Therefore, its mean density decreases, and the orbital period begins to increase. Nuclear burning becomes unimportant, and radiative losses from the surface reduce the heat content of the secondary. This star gradually approaches the state of degenerate dwarf. This evolutionary process is shown schematically in Fig. 1 on the secondary mass--binary period diagram. If gravitational radiation is the only process responsible for angular momentum losses, the minimum orbital period is about 80 minutes, as observed. If additional processes increase the rate of angular momentum loss, *i.e.* if $a > 1$, then the minimum orbital period is much longer. For example, if $a = 8$ then the minimum orbital period is about 3 hours. It is impossible to obtain orbital periods significantly shorter than 80 minutes within this evolutionary scenario.

Let us consider now the second scenario, *i.e.* the progenitor detached binary with a secondary's mass below $0.085 M_\odot$. Such a secondary is not able to burn hydrogen and it contracts to the degenerate state. A binary like this is already known, it is LB 3459. The mass of the secondary is about $0.054 M_\odot$. As a result of angular momentum loss this system will become semidetached with an orbital period of only 40 minutes or so (Paczyński 1980b) much shorter than observed for any hydrogen rich cataclysmic variable. However, because of a small mass of LB 3459, it is impossible, or at least very unlikely, for that binary to have any activity leading to winds or other forms of mass and angular momentum loss. We are left with the gravitational radiation as the only sink of angular momentum, and it turns out that the corresponding time scale is longer than the Hubble time (Paczyński 1980b). Therefore, it seems that the second scenario may produce cataclysmic binaries with periods below 80 minutes, provided we can wait long enough. As no such cataclysmic system is observed it is very likely that our Galaxy is too young for systems like LB 3459 to become semidetached. In other words, for some reason systems like LB 3459 are not produced with much shorter periods, as they would have been able to become semidetached within a Hubble time. This has important implications on the products of binaries that evolve through a common envelope phase. Let us be more quantitative.

Let us consider a detached binary with the initial period P_0^* significantly longer than the period given with Eq. (4). If the secondary has a mass too small for hydrogen burning then the only available sink of angular momentum is due to gravitational radiation, and the time scale for the binary to become semidetached is approximately given with a formula

$$\tau_{GR} = 9.8 \times 10^6 \text{ years} \frac{(M_1^* + M_2^*)^{1/3}}{M_1^* M_2^*} P_0^{*8/3}, \tag{17}$$

(Paczyński 1967). For that time to be less than the age of our Galaxy, *i.e.* about 10^{10} years, the period P_0^* has to satisfy inequality

$$P_0^* < 13^{\rm h}\!.4 \frac{M_1^{*3/8} M_2^{*3/8}}{(M_1^* + M_2^*)^{1/8}} \approx 13^{\rm h}\!.4 M_1^{*1/4} M_2^{*3/8}, \tag{18}$$

for $M_2^* \ll M_1^*$. This restriction does not apply to systems with secondary components burning hydrogen, as in that case energetic winds are possible, and the time scale for angular momentum loss may be strongly reduced.

The four semidetached systems which may become in future cataclysmic have the following orbital periods: $12^{\rm h}\!.5$ (V 471 Tau), $11^{\rm h}\!.2$ (UU Sge), $8^{\rm h}\!.3$ (PG 1413+01), $6^{\rm h}\!.3$ (LB 3459). The masses are known only for V 471 Tau and LB 3459. In both cases the time scales given with Eq. (17) are longer than the age of the Galaxy. V 471 Tau has an active chromosphere, and presumably a strong wind. It may be expected to become cataclysmic on a time much shorter than given with Eq. (17), and to evolve subsequently according to the first scenario. LB 3459 has too small mass to have any winds, and it cannot become cataclysmic within Hubble time. Unfortunately, nothing can be said now about the other two binaries.

All four binaries are very likely the products of common envelope evolution, and they are well detached. I believe this is not a selection effect. If similar binaries with shorter periods existed in a significant number, their discovery would be easier, as the probability of eclipses would be higher. Notice, that all four binaries mentioned above are eclipsing. Also, if binaries like LB 3459 but with much shorter periods could emerge from a common envelope stage, they would have time to become cataclysmic with orbital periods well below 80 minutes within the lifetime of our Galaxy. Such cataclysmic systems are not observed. Therefore, it seems very likely, that all, or most of common envelope systems lose their envelope while the two cores have relatively long periods, contrary to the suggestion made by Meyer and Meyer-Hofmeister (1979). For this reason the second evolutionary scenario is not effective in producing cataclysmic systems with periods below 80 minutes.

3. Discussion

Considerations presented in the previous chapter demonstrated that if the evolution of a cataclysmic binary is driven by the gravitational radiation only then the minimum possible value for the binary period is about 80 minutes, in agreement with available observations. This indicates that gravitational radiation as the dominant process for angular momentum loss for all very short period cataclysmic systems, in particular for the objects of SU UMa and AM Her type. However, some other types of cataclysmics: novae, novae-like, and Z Cam stars have periods always longer than three hours. It is likely that additional processes increase the angular momentum losses from these systems. To make the minimum period as long as 3 hours those processes have to be about one order of magnitude more efficient than gravitational radiation. It may be convenient to ask what rate of mass transfer, $\dot{M}$, is required to have a given value of P_{min}. The mass transfer time scale is defined as

$$\tau_{\dot{M}} \equiv (-M_2/\dot{M}_2). \tag{19}$$

This should be equal to the Kelvin-Helmholtz time scale of the secondary component at the minimum value of the orbital period. Combining Eqs. (11) and (19) we obtain

$$P^*_{min} = 3\overset{h}{.}7\left(\frac{-\dot{M}_2}{10^{-9}M_\odot/\mathrm{yr}}\right)^{0.33}, \tag{20}$$

i.e. the higher the mass transfer rate, the longer the minimum orbital period.

Eq. (20) implies that the mass transfer rate in novae, novae-like and Z Cam stars should be above $5\times10^{-10}M_\odot$/year in order to explain why none of them has a period below 3 hours. The reason for this high rate in not clear but it may be related to novae eruptions themselves. According to eq. (20) the rate of mass transfer must be as low as $5\times10^{-11}M_\odot$/year in a cataclysmic binary with a period of 80 minutes. It is interesting that U Gem itself has a period of $4^{\mathrm{h}}15^{\mathrm{m}}$, and the secondary component has the mean density significantly lower than that proper for a main sequence star (Wade 1979). This implies that the mass transfer proceeds on a Kelvin-Helmholtz time scale and that U Gem is close to its minimum period. The corresponding rate is $1.5\times10^{-9}M_\odot$/year. This may be compared with a similar estimate made by Paczyński and Schwarzenberg-Czerny (1980).

It should be emphasized that the mass transfer rates given with Eq. (20) are averaged over a Kelvin-Helmholtz time scale. It is entirely possible that there may be short term fluctuations in this rate in any particular system on any time scale shorter than 10^8 years, or so, and these fluctuations would not affect the present discussion. However, such fluctuations may very strongly affect the estimates based on the rates observed at present time in any particular binary. I believe, that the estimate based on the observed minimum value of the binary periods of any subclass of cataclysmic variables is of real evolutionary importance.

The discussion of the second evolutionary scenario presented in the previous chapter implies that binaries emerging from a common envelope phase of evolution are well detached and have relatively long orbital periods. In particular this applies to low mass systems like LB3459, and makes it impossible for them to become cataclysmic within the age of our Galaxy.

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