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COMET SHOWERS AND THE STEADY-STATE INFALL OF COMETS FROM THE OORT CLOUD

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ABSTRACT

The appearance of an inner edge to the Oort comet cloud at a semimajor axis of $a = (1\text{–}2) \times 10^4$ AU is an observational artifact. Stellar perturbations are frequent enough and strong enough to assure that a constant fraction of the comets with semimajor axes greater than this are in orbits which bring them within the planetary region. Only infrequent, close stellar encounters are able to repopulate the planet-crossing orbits of comets with smaller semimajor axes. Owing to their relatively short orbital periods which return them frequently to the planetary system, the comets in these more tightly bound orbits will be deflected by Jupiter into drastically different orbits or be destroyed by solar heating before another close stellar passage repopulates their numbers. Comets with semimajor axes less than 2×10^4 AU appear in the inner solar system only in intense bursts or showers which last for a few orbital periods after the close passage of a star to the Sun. This is followed by a much longer span of time during which only comets with $a > 2 \times 10^4$ AU enter the planetary system. The theoretically determined location of the boundary between the semimajor axes of those comets which enter the planetary system only in bursts or showers and those which arrive in a steady stream is very abrupt and falls at the observed inner edge of the Oort cloud. We propose that the comets formed in the outer parts of the collapsing protosun, which had a radius of less than 5×10^3 AU. If this produced a first-generation comet cloud with a radius of 10^3 AU or greater, the coupled dynamical perturbations of passing stars and Jupiter will, of necessity, lead to the formation of a comet cloud similar that of the observed Oort comet cloud. If the comets in the Oort cloud originated in this inner cloud and were later ejected into their present orbits by the perturbations produced by passing stars and Jupiter, the total mass of the inner cloud is about two orders of magnitude greater than that of the Oort cloud. The total number of comets that has entered the planetary system in showers from the inner cloud is more than an order of magnitude greater than the number that has entered from the Oort cloud. The peak intensity of a comet shower produced by a star passing close to the Sun is many orders of magnitude greater than the steady-state comet flux from the Oort cloud. The integrated comet flux from such a shower can be great enough that several comets will actually hit the Earth during the shower. This may show up in the geological record. The dynamical evolution of a strongly interacting trinary star will frequently lead to the orbit of the least massive star being pumped up to the point where a passing star gives it enough angular momentum for it to avoid its binary companion. This is probably responsible for the large number of binary stars with semimajor axes of 0.1 pc $= 2 \times 10^4$ AU found by Bahcall and Soneira.

I. INTRODUCTION

As noted by Oort (1950), those long-period comets which enter the planetary system appear to originate in a Sun-centered spherical shell having an inner semimajor axis of 2×10^4 AU and an outer semimajor axis of 2×10^5 AU. Subsequently, more precise work has confirmed Oort's original summary of the observational evidence which was available to him. The recent work of Marsden, Sekanina, and Everhart (1978) lists 200 long-period comets with well determined semimajor axes. This work shows that the inner edge of the Oort cloud is between 10 000 and 20 000 AU from the Sun. Oort showed that comets in this "Oort Cloud" are strongly perturbed by passing stars. He found that these perturbations deflect a steady stream of Oort-cloud comets into orbits which bring them into the inner solar system. Recently, a number of more detailed computer and analytic studies have been made of the effect of passing stars on the orbits of comets in the Oort cloud (cf. Yabushita 1972; Rickman 1976; Weissman 1977, 1979, 1980b). Detailed studies have also been made of the dispersive effect of Jupiter on the orbits of those long-period comets that visit the planetary realm (cf. Everhart 1969, 1972; Yabushita 1979). Weissman (1977, 1979) and Everhart (1979) combined these two "end games" by using computer simulations to study both the effect of passing stars on the orbits of comets in the Oort cloud

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and the subsequent dynamical interaction of these comets with Jupiter and Saturn if they are deflected into orbits that take them within the inner solar system. Weissman (1980a) summarizes and evaluates various mechanisms which deplete the Oort-cloud comets that enter the inner solar system.

A weak point of previous studies is that they have considered only the comets within the *observed* Oort cloud. In this paper we develop analytic procedures for evaluating the effect of passing stars on comets having any arbitrary semimajor axis. As summarized in the abstract of this paper, we shall show that the abrupt apparent termination of the Oort cloud at a critical inner semimajor axis $a = a_c \sim 2 \times 10^4$ AU is due to observational selection. Comets with $a < a_c$ are observed only for a relatively brief time after a star has come sufficiently close to the Sun to deflect a new supply of them into orbits that penetrate the planetary system. This suggests that most comets may actually have semimajor axes smaller than a_c with the Oort cloud being only the outer halo of a more massive comet cloud. We present the case for the comets having formed in the collapsing protosun, whose radius was considerably smaller than a_c. We show that about one percent of the comets initially present in such an inner comet cloud, where the comets have *a* semimajor axes on the order of 3×10^3 AU, would have been ejected by the perturbations of Jupiter and Saturn into the higher-energy orbits presently occupied by the Oort-cloud comets. Self-consistency then requires that the present-day mass of the inner-comet cloud be about two orders of magnitude greater than that of the Oort cloud.

II. THE LOSS CONE AND THE SMEAR CONE

At distance r from the Sun a fraction F_l of the comets have velocity vectors which place them in orbits that penetrate the planetary system where we can eventually observe them. This bundle of velocity vectors forms a Sun-centered *loss cone* in velocity space. *Unless perturbed again by a passing star,* comets in this cone remain in it until they are scattered into hyperbolic orbits by Jupiter or are destroyed (e.g., by solar heating). Jupiter may scatter a comet within the loss cone into another bound orbit having a radically different semimajor axis, but the comet is within the loss cone associated with this new semimajor axis. This is due to the well-known fact (cf. Yabushita 1979) that while intersections with Jupiter can radically change the orbital energy of a comet, they cannot give a comet which crosses Jupiter's orbit the angular momentum that it needs to escape Jupiter's influence in future pericenter passages. The comet must cross Jupiter's orbit at each subsequent passage until Jupiter ejects it into a hyperbolic orbit or until a passing star gives it the orbital angular momentum required to avoid Jupiter.

The thermalization of the comet orbits due to passing stars will lead to their dynamical evolution to a state in which the comets have an isotropic distribution of velocities at each distance r from the Sun. This state is much more easily reached and maintained at large r, where significant encounters with passing stars are more likely owing to the collision cross sections of the comet orbits increasing with increasing r. If the typical lifetime τ_l of a comet in the loss cone at r is less than the local charateristic time τ_s required for passing stars to refill the loss cone by deflecting new comets into it, there will be a marked deficiency of comets in the loss cone compared to the number in the velocity space around the loss cone. If $\tau_t > \tau_s$, there is no marked deficiency of comets in the loss cone, and the fraction F_l of comets at r which find themselves in the loss cone is just the angular area of the loss cone in units of 4π steradians.

It is convenient to characterize the rate of thermalization of the comet orbits in terms of a *smear cone*. In the loss-cone lifetime τ_l, passing stars will change the velocity of the comets at r by some average amount ΔV. In velocity space this produces a smear cone of angular radius $\theta \sim \Delta V / V_0$, where V_0 is the average speed of the comets at r. If at some distance r from the Sun the angular area of this smear cone exceeds that of the loss cone, there is no net depletion of comets in the loss cone. The fraction of the comets at r in the loss cone is just F_l. On the other hand, if the smear cone angle θ at r is less than that of the loss cone, there will be a marked deficiency of comets at r in orbits that take them into the planetary system.

[This loss cone–smear cone formulation is similar to that considered in finding the rate at which massive black holes consume stars in galactic nuclei (Hills 1975a).]

We will now find the sizes of the loss and smear cones as a function of distance from the Sun.

a) The Size of the Loss Cone

It is more convenient to determine this cone size as a function of the semimajor axis a of the comet orbits rather than as a function of their distance r from the Sun. Observations tell us the semimajor axes of those comets which enter the inner solar system. The use of the semimajor axes also allows us to utilize a very simple but powerful theorem which was first derived for binary stars in statistical equilibrium but which is also applicable to comet-Sun pairs.

If the individual orbits of an ensemble of binary stars (or comets orbiting the Sun) have been thermalized by close stellar encounters, the distribution of orbital eccentricities required by statistical equilibrium is such that the fraction of the binaries with eccentricities between e and unity is just

$$F_e = 1 - e^2 \tag{1}$$

(Jeans 1919; Hills 1975b). This result is independent of semimajor axis. Computer experiments show that the distribution of orbital eccentricities of binary stars in a stellar cluster obeys Eq. (1) after these binaries have made a few close encounters with their neighbors (cf.

Aarseth and Hills 1972).

The closest-approach distance q of a comet to the Sun is related to its semimajor axis a and its orbital eccentricity e by the simple relation $q = a(1-e)$. Using this result and Eq. (1), we see that the fraction of comets with semimajor axis a which pass within a distance q of the Sun is

$$F_q = \frac{2q}{a}\left(1 - \frac{q}{2a}\right). \quad (2)$$

For small values of q, $F_q = 2q/a$. The number of comets per pericenter interval dq is

$$f_q = \frac{dF_q}{dq} = \frac{2}{a}\left(1 - \frac{q}{a}\right). \quad (3)$$

For small values of q, $f_q = 2/a$, which is independent of q. This is in agreement with the well-known fact that the number of long-period comets with a given q which enter the planetary system per unit time is independent of q. This is seen, for example, in the computer simulations of Weissman (1977). It can also be seen in the observational data of Everhart (1967).

We are primarily interested in comets which cross the orbit of Jupiter, which has a semimajor axis of 5 AU, because of its severe effect on comet orbits. Table I shows F_q as a function of comet semimajor axis a for $q = 5$ AU.

b) The Size of the Smear Cone

We shall now consider the effect of a passing star on the orbit of a comet. The star gives both the comet and the Sun impulsive velocities which are perpendicular to the path of the star. With the customary approximation that the trajectory of the star with respect to the Sun is a straight line, the impulse velocity is given by the well-known result,

$$\Delta V_i = 2GM_s/V_s P_i \quad (4)$$

(cf. Ogorodnikov 1965). This approximation is quite good for the weak stellar encounters with which we are primarily concerned. Here M_s is the mass of the perturbing star and V_s is its velocity with respect to the Sun. For the comet, $P_i = P_c$, the closest-approach distance of the star to the comet, and $\Delta V_i = \Delta V_c$, the impulse velocity given the comet. For the sun, $P_i = P_\odot$, the closest-approach distance of the star to the Sun, and $V_i = V_\odot$, the impulse velocity given the Sun. To within the level of approximation used to derive Eq. (4), $\Delta V_\odot$ and ΔV_c are parallel. The star gives the comet an impulse velocity ΔV with respect to the Sun. This is given by

$$\Delta V = |\Delta V_c - \Delta V_\odot| = \frac{2GM_s}{V_s}\left(\left|\frac{P_\odot - P_c}{P_c P_\odot}\right|\right). \quad (5)$$

When averaged over all comets at the same distance r from the Sun, $\langle|P_\odot - P_c|\rangle \sim r$ and $\langle P_c P_\odot\rangle \sim P_\odot^2$. The time-averaged separation of two objects in a Keplerian orbit of semimajor axis a and eccentricity e is

$$\langle r\rangle = a(1 + \tfrac{1}{2}e^2). \quad (6)$$

This was derived from the power series in mean anomaly given by Moulton (1970). An average over mean anomaly is equivalent to a time average over an orbital period. As $e \to 1$, $\langle r\rangle \to 1.5a$. Thus

$$\langle \Delta V\rangle \simeq 3aGM_s/V_s P_\odot^2. \quad (7)$$

Similar values of $\langle\Delta V\rangle$ are derivable from the work of Yabushita (1972), which utilized both analytic approximations and computer simulations of encounters.

The orbital velocity of a comet of semimajor axis a at a distance $r = 1.5a$ from the Sun is easily shown from energy conservation to be

$$V_c = (GM_\odot/3a)^{1/2}. \quad (8)$$

Thus the angular radius of the smear cone produced by the passage of the star is

$$\theta = \frac{\langle\Delta V\rangle}{V_c} \simeq \frac{9M_s}{M_\odot}\left(\frac{a^2(GM_\odot/3a)^{1/2}}{P_\odot^2 V_s}\right) = \frac{9M_s V_c}{M_\odot V_s}\left(\frac{a}{P_\odot}\right)^2. \quad (9)$$

The fraction of velocity space occupied by the smear cone for the case where $\Delta V/V_c \ll 1$ is

$$F_s = \frac{\pi\theta^2}{4\pi} = \frac{\theta^2}{4} = \frac{1}{4}\left(\frac{\langle\Delta V\rangle}{V_c}\right)^2$$

TABLE I. Theoretical properties of the solar comet cloud.

a (AU)	F_q	N	F_L	τ_s (yr)	τ_l (yr)	τ_l/τ_s
1×10^3	1.0×10^{-2}	1	0.010	4.4×10^9	1.3×10^5	3.3×10^{-5}
2×10^3	5.0×10^{-3}	4	0.020	1.1×10^9	3.6×10^5	3.3×10^{-4}
3×10^3	3.3×10^{-3}	9	0.030	4.9×10^8	6.6×10^5	1.3×10^{-3}
5×10^3	2.0×10^{-3}	25	0.049	1.8×10^8	1.4×10^6	7.9×10^{-3}
1×10^4	1.0×10^{-3}	100	0.095	4.4×10^7	4.0×10^6	9.1×10^{-2}
2×10^4	5.0×10^{-4}	400	0.55	1.1×10^7	1.1×10^7	1.0
3×10^4	3.3×10^{-4}	900	0.25	4.9×10^6	2.1×10^7	4.2
5×10^4	2.0×10^{-4}	2500	0.077	1.8×10^6	4.5×10^7	2.5×10^1
1×10^5	1.0×10^{-4}	10000	0.014	4.4×10^5	1.3×10^8	2.9×10^2
2×10^5	5.0×10^{-5}	40000	2.5×10^{-3}	1.1×10^5	3.6×10^8	3.3×10^3

$$= \frac{27}{4}\left(\frac{M_s}{M_\odot}\right)^2\left(\frac{a}{P_\odot}\right)^4\left(\frac{GM_\odot}{aV_s^2}\right). \quad (10)$$

From Eqs. (2) and (10), we see that the ratio of the angular area of the smear cone due to the passage of this single star to the angular area of the loss cone is

$$\frac{F_s}{F_q} = \frac{27}{8}\left(\frac{M_s}{M_\odot}\right)^2\left(\frac{a}{P_\odot}\right)^4\left(\frac{GM_\odot}{qV_s^2}\right) \quad (11)$$

for the case where $q \ll a$. If $M_s = M_\odot$, $a = P_\odot$, and $V_s = 30$ km/s, then $F_s/F_q = 0.7$ when $q = 5$ AU and $F_s/F_q = 3.4$ when $q = 1$ AU.

We see from these calculations that for those comets which come close enough to the Sun to be observed, the size of the smear cone due to a solar-mass star passing near the Sun is comparable to or greater than the loss cone if the closest-approach separation $P_\odot$ of the star and the Sun is equal to or smaller than the semimajor axes of the comet orbits. The result is somewhat sensitive to the mass of the perturbing star and its velocity with respect to the Sun. A few massive stars with low velocities with respect to the sun are much more effective than many low-mass, high-velocity stars. Because F_s/F_q is proportional to $P_\odot^{-4}$, the large number of distant stellar encounters makes no significant contribution to the smear cone. If $P_\odot$ is a little smaller than the semimajor axis a, even low-mass stars can fill the loss cone (although we must be careful that our assumptions do not breakdown at too small values of a). In summary, the passage of an individual star through the comet cloud will fill the loss cone of those comets with semimajor axes greater than or equal to the closest approach of the star to the Sun, but it will make no significant contribution to filling the loss cones of comets with smaller semimajor axes. If the loss cone is filled, the fraction F_q of the comets of semimajor axis a with pericenter distances equal to or less than q is just that given by F_q in Eq. (2).

We emphasize again that after a comet enters the loss cone it remains in it until it is deflected out of it by a passing star, it is ejected into a hyperbolic orbit by Jupiter (or Saturn), or it is destroyed (e.g., by sublimation and breakup owing to repeated passes near the Sun or by hitting the Sun or a planet). Jupiter (or Saturn) cannot give the comet the orbital angular momentum required to escape the loss cone, although it can change greatly the semimajor axis of the comet. The comet must face new perturbations by Jupiter (or Saturn) at each close approach to the Sun until one of the above conditions is met.

III. THE STEADY-STATE (OORT) COMET RESERVOIR

If the median loss-cone lifetime τ_l of comets with semimajor axis a is greater than the mean time τ_s between close stellar encounters having $P_\odot < a$, then the fraction of these comets which find themselves in the loss cone approaches the steady-state value given by Eq. (2). However, if $\tau_l < \tau_s$, which occurs when a is less than some critical semimajor axis a_c, *usually* there will be a marked deficiency of comets of semimajor axis a entering the planetary system. When a star *does* pass within distance $P_\odot = a$ of the Sun, the resulting perturbations will fill the loss cone of these comets, which causes a burst or shower of them to enter the planetary system. This shower starts a fraction of a comet orbital period after the star makes its closest approach to the Sun, and it lasts for the characteristic time τ_l. The fraction of the time during which a shower is in progress is just τ_l/τ_s. As we shall in Sec. IIIa, this ratio rapidly drops below unity when $a < a_c$.

a) The Inner Boundary of the Oort Reservoir

We shall now find the critical semimajor axis a_c which separates the outer, steady-state comets from the inner, shower comets.

The mean time between encounters in which a star passes within a distance $P_\odot = a$ of the Sun is given by the well-known equation of kinetic theory (cf. Ogorodnikov 1965),

$$\tau_s = (\pi a^2 n_s V_s)^{-1}. \quad (12)$$

This equation applies in the limiting case where the closest-approach distance from a passing star to the Sun is sufficiently large that the Sun does not pull the star toward it by any significant amount. This is the case of interest. Here V_s is the average velocity of a passing star with respect to the Sun and n_s is the number of stars per unit volume in the solar neighborhood. Table I shows τ_s as a function of a. We have used $V_s = 30$ km/s and $n_s = 0.1$ pc^{-3} in this evaluation.

The peculiar motion of the Sun will cause a bias in the direction of approach of incoming stars. The direction of approach rotates through 180° in half an orbital period of the Sun around the center of the Galaxy. This interval of $\sim 10^8$ yr is too small to produce much of an asymmetry in the direction of approach of comets from the Oort cloud. The tidal field of the Galaxy, which is responsible for the outer termination of the Oort cloud, may produce such an asymmetry, but we shall not discuss this point in this paper.

The minimum possible lifetime of a comet in the loss cone is just its orbital period. The actual lifetime must be some multiple α of this. Yabushita (1979) finds from his own work and that of Everhart (1976) that the probability of a comet surviving N passages through the planetary system before being ejected into a hyperbolic orbit is about $N^{-1/2}$ for large N. Thus the effective median lifetime τ_l of comets in the loss cone is about four times their orbital period, or

$$\tau_l = \alpha[2\pi a^{3/2}/(GM_\odot)^{1/2}], \quad (13)$$

where $\alpha = 4$. Table I gives τ_l as a function of semimajor axis.

From Eqs. (12) and (13) we find that the critical semimajor axis, $a = a_c$, for which $\tau_s = \tau_l$ is

$$a_c = [(GM_\odot)^{1/2}/2\pi^2 \alpha n_s V_s]^{2/7} = 2.0\times 10^4 \text{ AU}. \quad (14)$$

This value is well defined because of the error compression which results from the 2/7 power factor. Even a factor-of-2 change in $\alpha n_s V_s$ would change a_c by only 20%. A factor-of-10 change in this parameter would change a_c by less than a factor of 2. Thus, the large uncertainty in the effective value of n_s to use in this equation produces a much smaller uncertainty in a_c. The boundary at $a = a_c$ is quite sharp. We can see this by examining the ratio τ_l/τ_s in Table I. When a drops to $\frac{1}{2}a_c$, τ_s is already more than an order of magnitude longer than the loss-cone lifetime. Comet showers containing comets with $a = \frac{1}{2}a_c$ are present only 9% of the time. This is in marked contrast to the situation for comets with $a \geqslant a_c$. The number of these comets entering the planetary system per unit time is in a steady state and is at the maximum rate allowed by the size of the loss cone and the orbital period of the comets.

We see that the theoretically derived a_c *is*, in fact, just at the *observed* inner semimajor axis of the comets in the Oort cloud. This can hardly be an accident. It very strongly suggests that the observed inner edge of the Oort cloud is due to observational selection. *The observed comet cloud may be only the outer halo of a much more massive comet cloud whose center of mass is well inside the observed inner boundary of the Oort cloud.* The possibility of this being the case was already realized by Oort (1950) in his classical paper.

b) The Formation of the Comets

From the standpoint of the theory of star formation, the existence of a hypothetical inner comet cloud is much more reasonable than the observed existence of the Oort-cloud comets. The difficulty of forming comets in the low-density environment of the Oort cloud is well known. It is more reasonable to suppose that the comets formed in the outer parts of the collapsing protosun.

The maximum possible (Jeans) radius of the protosun when it separated from its neighboring gas fragments in a collapsing interstellar cloud was

$$R_{\max} = 0.41 GM_\odot/\mathscr{R}T, \quad (15)$$

where the gas constant $\mathscr{R} = 3.36\times 10^7$ erg g^{-1} K and the gas temperature $T = 20$ K (Larson and Starrfield 1971). This gives

$$R_{\max} = 5.4\times 10^3 \text{ AU} = 0.026 \text{ pc}$$

for the maximum initial radius of the protosun. (We note that this radius implies a minimum local gas density of $1.3\times 10^4\ M_\odot/\text{pc}^3$ at the time when the Sun separated gravitationally from its neighboring protostars.) This suggests that about 3×10^3 AU is a reasonable value to assume for the typical semimajor axis of the comets in the inner cloud.

The early theoretical work of Larson and others (cf. Larson and Starrfield 1971) indicated that the core of the protosun collapsed first to form a central object which was initially much more luminous than the present-day Sun. The remaining material in the protosun then fell into this core over a protracted period of time. Several possible mechanisms for producing the comets in this infalling material suggest themselves. I find that the radiation pressure due to the luminous core is enough to drive together the dust in a protocomet in less than the free-fall time from a distance of $r = 3\times 10^3$ AU. This is due to the self-shielding of the dust in a protocomet causing a net inward radiation pressure which drives together the dust grains (snow) and leaves behind the gas. The details of this mechanism will be presented in a later paper.

The discussion of comet dynamics which follows is independent of the details of how the comets actually formed. We show that if an inner comet cloud exists, stellar perturbations will of necessity have led to the formation of an outer comet cloud with dimensions identical to that of the observed Oort cloud. The observed Oort cloud is the required final product of any inner comet cloud with a radius of the order of 10^3 AU or greater. Unfortunately, because of this fact, the distribution of comet orbits in the Oort cloud tells us little about the distribution of the comet orbits in the inner cloud.

c) The Number of Comets in the Oort Cloud

If the orbits of the comets in the Oort cloud are completely thermalized, the fraction of the comets with semimajor axis a having a pericenter distance equal to or less than q is $F_q = 2q/a$. The rates at which these comets enter a sphere of radius q centered on the Sun is given by

$$N'_c = \frac{N_c F_q}{P} = \frac{N_c}{\pi}\left(\frac{q(GM_\odot)^{1/2}}{a^{5/2}}\right). \quad (16)$$

Here N_c is the number of comets of semimajor axis a and orbital period P. If N'_c is 5 comets per year for $q = 2$ AU, then $N_c = 7.1\times 10^{10}$ comets for $a = a_c = 2.0\times 10^4$ AU and $N_c = 4.0\times 10^{12}$ comets for $a = 10^5$ AU.

We can improve our determination of N_c by allowing for a range of comet semimajor axes within the Oort comet halo. We shall assume that the number of comets in semimajor-axis interval da is given by the power-law distribution

$$dN_c = N_0[(a/a_0)^{-n}]da/a_0. \quad (17)$$

(We can expect a steep decrease in the number of comets as a increases. Any such rapidly decreasing, well behaved function should be approximated fairly well by a power law over the relatively small interval between the inner and outer edge of the comet cloud.) Here N_0, a_0, and n are constants. The total number of comets with semimajor axes between $a = a_{\min}$ and $a = a_{\max}$ is

$$N_c = \frac{N_0}{n-1}\left[1 - \left(\frac{a_{\min}}{a_{\max}}\right)^{n-1}\right]\left[\left(\frac{a_0}{a_{\min}}\right)^{n-1}\right]. \quad (18)$$

By replacing N_c in Eq. (16) with the expression for dN_c given by Eq. (17) and integrating, we find that for comets with semimajor axes in the range $a = a_{\min}$ to $a_{\max}$,

$$N'_c = \left(\frac{2q(GM_\odot)^{1/2}}{(2n+3)\pi}\right)\left(\frac{N_0 a_0^{n-1}}{a_{\min}^{n+3/2}}\right)\left[1-\left(\frac{a_{\min}}{a_{\max}}\right)^{n+3/2}\right]. \quad (19)$$

Using the last two equations to eliminate the scaling parameter $N_0 a_0^{n-1}$, we find that

$$N'_c = \left(\frac{2(n-1)N_c}{(2n+3)\pi}\right)\left(\frac{q(GM_\odot)^{1/2}}{a_{\min}^{5/2}}\right)H. \quad (20)$$

Here

$$H = \left[1-\left(\frac{a_{\min}}{a_{\max}}\right)^{n+3/2}\right] \Big/ \left[1-\left(\frac{a_{\min}}{a_{\max}}\right)^{n-1}\right]. \quad (21)$$

The best value of the power-law index n to use in these equations is unknown, but it is likely to be at least 2. If $n = 2$ then the number of comets per interval of orbital binding energy is independent of the binding energy E_b ($E_b \alpha a^{-1}$, and $dE_b \alpha a^{-2}\, da$). If $n < 2$, the number of comets per interval of binding energy would increase as the binding energy decreases. If the comets in the Oort cloud have diffused out to their present orbits from orbits with much larger values of E_b, we expect the number of comets per interval of binding energy to decrease as the binding energy decreases. This requires that $n > 2$. If $n \gtrsim 2$ and $a_{\max} \gg a_{\min}$, we note that $H \to 1$. We shall assume that this limit holds in the discussion that follows.

In evaluating Eq. (20), we let $q = 2$AU, $N'_c = 5\ \mathrm{yr}^{-1}$, and $a_{\min} = 2.0 \times 10^4$ AU. We find that the number of comets in the Oort halo is $N_c = 2.5 \times 10^{11}$ if $n = 2$ and $N_c = 1.9 \times 10^{11}$ if $n = 2.5$. A similar result would be obtained for $n = 3$. The derived number of comets in the Oort cloud is not a sensitive function of the rate of decrease in the number of comets with increasing semimajor as long as the rate of decrease is at least as fast as a decreasing power law with an index $n = 2$ or greater. If the average comet has a radius of 1 km and a density of 1 g/cm^3, which requires an individual comet mass $m_c = 4.2 \times 10^{15}$ g, the total mass of the Oort cloud is $M_c = N_c m_c = 1.1 \times 10^{27}$ g $= 0.18 M_\oplus$ for $n = 2$ and $0.13 M_\oplus$ for $n = 2.5$. In Sec. IV b of this paper we shall use the procedure developed in this section to estimate the number of comets with $a < a_c$.

d) Loss of Comets from the Oort Halo

We might suppose that because the Oort-cloud comets are bound very loosely to the Sun they would be easily ejected into hyperbolic orbits by passing stars. This is not the case. Computer simulations and analytic analysis show that only about 10% of all comets in the classical Oort cloud have been ejected into hyperbolic orbits by the impulses given them by passing stars (cf. Yabushita 1972; Hills 1975b; Weissman 1980b). The physics responsible for this result is not difficult to see. Let us consider the case where a passing star comes close enough to a comet for its maximum gravitational pull on the comet to be comparable to that of the Sun. In this case the star gives the comet an impulse velocity which is much less than the orbital velocity of the comet because the star passes through the Oort cloud in a time which is short compared to the orbital period of the comet. It takes the Sun a sizable fraction of the comet's orbital period to produce a change in the (vector) velocity of the comet which is comparable in magnitude to its initial orbital velocity. The time during which a passing star is in the vicinity of the comet is very short compared to the orbital period of the comet. Only if the closest-approach separation between the star and the comet is much less than the semimajor axis of the orbit will the impulse velocity be great enough to eject the comet into a hyperbolic orbit. Comets with semimajor axes smaller than those of the Oort-cloud comets are even less likely to be pulled away from the Sun by passing stars. This is due to the smaller cross sections such orbits present to passing stars.

There exists another comet-loss mechanism which can be more effective than direct ejection by stars passing close to the comets. Computer simulations (Hills 1975b) show that passing stars are much more effective in changing the orbital angular momentum (velocity direction) of the stellar components of a binary than in changing their orbital energy (amplitude of the velocity). (This is consistent with the well-known fact that it is easier to change the perihelion distance of a comet than to change its semimajor axis.) It is this redistribution of angular momentum which refills the loss cone of the comets of a given semimajor axis and sends them plunging into the planetary system. Most of the comets which enter the planetary system from the Oort cloud eventually are ejected into hyperbolic orbits by Jupiter (or Saturn) or are destroyed. Only a small fraction return to the Oort cloud by being deflected out of the loss cone by a passing star after they have spent one (usually) or more (if the new $a < a_c$) orbital periods within the loss cone. (This point will be discussed in more detail later in this paper.) The rate of depletion of Oort-cloud comets by this mechanisms is given by

$$\frac{dN_c}{dt} = -\frac{N_c F_q}{P}. \quad (22)$$

Here P is the orbital period, and $F_q \simeq 2q/a$ is the fraction of the comets of semimajor axis a in the loss cone at any given time. Separating the variables and integrating, we find that the number of comets which remain in the cloud after time t is

$$N_c = (N_c)_0 \exp\left(-\frac{tF_q}{P}\right) = (N_c)_0 \exp\left[\left(-\frac{q(GM_\odot)^{1/2}}{\pi a^{5/2}}\right)t\right], \quad (23)$$

where $(N_c)_0$ is the original number of the comets in the cloud. Table I shows the fraction $F_L = [1 - N_c/(N_c)_0]$ of comets of semimajor axis $a > a_c$ which have been ejected into hyperbolic orbits by this mechanism for the case where $q = 5$ AU and $t = 4.5 \times 10^9$ yr.

We note from Table I that F_L rapidly drops from $F_L = 0.55$ for $a = 2.0 \times 10^4$ AU to $F_L = 0.014$ for $a = 10^5$ AU. This rapid decrease in F_L with increasing a is due to the fact that the size of the loss cone decreases rapidly with increasing a (the leak in the comet reservoir becomes smaller), and the orbital periods (hence lifetimes) of the comets in the loss cone increase (the flow velocity of the comets through the leak decreases). In Sec. IV a, we shall show that for comets with $a < a_c$, F_L decreases with decreasing a. The comets that have been the most seriously depleted by being scattered into hyperbolic orbits by Jupiter are those with $a = a_c$.

Because most comets in the Oort reservoir are not expected to have semimajor axes much greater than $a = a_c$, we estimate that for the comets in the reservoir as a whole, $F_L \simeq 0.25$, or about half the value of F_L at the inner boundary. Adding this comet loss to the loss due to comets being scattered directly into hyperbolic orbits by passing stars, we estimate that about 35% of the comets initially in the Oort reservoir have been ejected into hyperbolic orbits.

IV. THE INNER COMET CLOUD AND COMET SHOWERS

Most comets initially in orbits with semimajor axes $a < 2 \times 10^4$ AU enter the planetary system in intense, but brief showers. A shower associated with the comets of a given a begins a fraction of an orbital period after the perturbing star makes its closest approach ($P_\odot \leqslant a$) to the Sun. The characteristic duration of the shower is only a few orbital periods, since the median change (increase or decrease) in the orbital binding energy of a comet at each plunge through the planetary system is an order of magnitude greater than the binding energy of a comet with $a = a_c$; i.e., the semimajor axes of these loosely bound comets change drastically at each pass through the planetary system (cf. Everhart 1968 and Everhart and Raghavan 1970).

a) The Depletion of the Inner-Cloud Comets

The comets with $a < a_c$, the shower comets, do not remain in this semimajor interval for long after a passing star sends them into the loss cone. This is due to their frequent excursions into the planetary system which results from their relatively short orbital periods. These frequent visits into the inner solar system assure that either they are quickly destroyed by solar heating or the semimajor axes of their orbits are greatly changed by Jupiter and Saturn. The latter effect will result in their quickly being ejected either into hyperbolic orbits or into the range of semimajor axes occupied by the Oort-cloud comets ($a > a_c$). If the semimajor axis of an inner-cloud comet is pumped up to $a > a_c$, a passing star will usually knock it out of the loss cone before it can reenter the inner solar system. The smear cone is so much larger than the loss cone for the Oort-cloud comets that the chance of an individual comet being knocked back into the loss cone on its next trip to perihelion is quite small. A few comets may be deflected into short-period orbits by Jupiter and Saturn, but their residence time in these orbits is very short even when compared to the typical orbital period of a comet in the inner cloud. These short-period comets are quickly ejected into long-period or hyperbolic orbits. Nearly all shower comets are gone by the time another star comes close enough to the Sun to repopulate their numbers. Thus the integrated fraction of inner-cloud comets which has been lost is just the fraction which has entered the loss cone since the formation of the inner cloud.

We assume that $F_q = 2q/a$ is the fraction of comets of semimajor axis a which enters the loss cone when a star comes close enough to the Sun to produce a smear cone for these comets which is comparable to or larger than their loss cone (i.e., $P_\odot \leqslant a$). This assumption holds if the orbital eccentricities are completely thermalized prior to the encounter (except for the near absence of comets within the loss cone).

The thermalization of the orbital eccentricities certainly has occurred within the Oort cloud ($a > a_c$). It may not have occurred for the comets with $a < a_c$ because fewer stars have come close enough to the Sun to affect their more tightly bound orbits. However, the initial relaxation of these comet orbits would have been helped by the high stellar density in the Sun's neighborhood at its birth if it formed as one of the stellar fragments of a collapsing interstellar gas cloud. Blaauw (1964) notes that the σ Per cluster, an infantile open cluster, has a density of 600 $M_\odot/\mathrm{pc}^3$ and that the Trapezium cluster has a density of 10^2–10^3 times that of a normal (dynamically relaxed) open cluster. The density in these two infantile clusters is 10^3–10^4 times greater than the present-day density in the solar neighborhood. Theoretical work suggests that even higher densities may have existed at the time of formation of stars in such clusters (Hills 1980). We found in Sec. III b that the density of the collapsing gas cloud in which the Sun was born had to exceed 1.3×10^4 $M_\odot/\mathrm{pc}^3$ at the time that the Sun separated gravitationally from its neighboring protostars. It is evident that the very high stellar densities present in the infantile clusters would have helped to dynamically relax the comet orbits.

If the distribution of eccentricities of the comet orbits with $a \ll a_c$ has never been fully thermalized, it will reflect the conditions present when the comets formed. If the comets formed in the outer parts of the collapsing protosun after it became gravitationally detached from neighboring fragments, their initial orbits would be expected to have a higher average eccentricity than is the case for thermalized orbits. In this case the true value of F_q, the fraction of the comets in the loss cone after the passage of a star, is expected to be greater than that for thermalized orbits for which $F_q = 2q/a$. As we assume in the work which follows that thermalization has occurred, our results provide a lower limit to the total fraction of comets with $a < a_c$ which have entered the loss cone.

Using Eq. (12) and assuming that the age of the solar system is 4.5×10^9 yr, we can determine N, the total number of stars that have passed within distance $P_\odot = a$ of the Sun over this time interval. This is given in Table I. As before, we use $n_s = 0.1$ pc^{-3} and $V_s = 30$ km/s. After N encounters in which a fraction F_q of the remaining comets is lost per encounter, the final integrated fraction of comets lost is (for $a < a_c$)

$$F_L = 1 - (1 - F_q)^N. \quad (24)$$

Table I shows F_L as a function of semimajor axis a. We note that for $a < a_c$, F_L increases monotonically with increasing semimajor axis to $a = a_c$. Ths is due to the fact that the decrease in the size of the loss cone with increasing semimajor axis a is more than compensated by the increased frequency of those stellar encounters that are capable of refilling the loss cone. For $a > a_c$ the required stellar encounters are frequent enough that the loss cone is always filled. We saw in Sec. III d that this causes F_L to decrease with increasing a. Thus, F_L reaches its maximum when $a = a_c$.

b) The Number of Comets in the Inner Cloud

We shall use three different methods to estimate the number of comets in the inner cloud and its mass: First, we shall assume that the same power-law distribution of semimajor axes holds in the inner cloud as in the Oort halo so we can extrapolate it into the inner cloud. Next, we find the number of inner-cloud comets required by self-consistency if the Oort-halo comets originated in the inner cloud and were later elevated up to their present-day orbits by the coupled perturbations of passing stars and Jupiter. Finally, we shall make use of the fact that the most loosely bound long-period comets appear, at most, to be only weakly hyperbolic (when it is assumed that only the combined mass of the Sun and planets is available to gravitationally bind them to the solar system), in order to place an upper limit on the integrated *mass* of the inner comet cloud.

1) Extrapolation from the Oort Halo

We shall assume that the distribution of comet semimajor axes given by Eq. (17) holds both in the inner cloud and in the Oort halo. In this case, we see from Eq. (18) that for $a_{\max} \gg a_{\min}$ the ratio of the number of comets in the entire comet cloud to that in the Oort halo is

$$N_T/N_0 = (a_c/a_i)^{n-1}. \quad (25)$$

Here $a_c = 2 \times 10^4$ AU is the inner semimajor axis of the Oort halo and a_i is the inner semimajor axis of the entire comet cloud. If $a_i = 10^3$ AU, then $N_T/N_0 = 20$ for $n = 2$ and 89 for $n = 2.5$. Using our previous estimates for the mass of the Oort cloud, we find that the mass of the total cloud is $3.5M_\oplus$ for $n = 2$ and $12M_\oplus$ for $n = 2.5$. The number of comets in the total cloud is estimated to be 5.0×10^{12} for $n = 2$ and 1.7×10^{13} for $n = 2.5$. (These calculations are quite crude and uncertain because of their weak physical basis. It is, for example, entirely possible that the distribution of comet semimajor axes is bimodal with one concentration of comets in the inner cloud and another in the Oort cloud with relatively few comets between these two clouds. But we feel that these estimates probably bracket the true solution.)

2) Number Required to Produce the Oort Halo

The great difficulty of forming comets in the low-density environment of the Oort halo is well known. In Sec. III b we proposed that the comets formed in the outer parts of the collapsing protosun which had a radius of less than 5×10^3 AU. Using the observed number of comets in the Oort halo and assuming that they first formed in this inner cloud and then later were ejected into their present-day orbits, we can estimate the number of comets in the inner cloud.

Computer simulations show that stellar encounters tend, on the average, to increase the semimajor axis of a loosely bound binary such as a Sun-comet pair (cf. Hills 1975b). However, this mechanism is not nearly as effective as the one whereby the passing star reduces the orbital angular momentum of the comet so it passes within the loss cone and is subsequently scattered into a higher-energy orbit by Jupiter.

A comet caught in the loss cone remains in it and suffers scattering by Jupiter at each passage until it is destroyed, ejected into a hyperbolic orbit, or ejected into the Oort halo, through which stars pass frequently enough to deflect it out of the loss cone before it returns to the inner solar system. The destruction of the comet is much less probable than the two other loss mechanisms (cf. Weissman 1980a).

For a comet caught in the loss cone, the median change in the orbital binding energy per orbital period due to scattering by Jupiter is about an order of magnitude greater than the orbital binding energy of a comet with a semimajor axis $a = a_c$ or of about the same order as the binding energy of the comets in the inner cloud (cf. Everhart 1968 and Everhart and Raghavan 1970). We estimate that in the random walk to higher energies far fewer than half the comets in the loss cone eventually end up in the Oort halo, with the bulk being ejected into hyperbolic orbits. Assuming that 25% of these comets make it into the Oort halo and combining this figure with our previously derived result that about 35% of the comets originally in the Oort cloud have been lost, we estimate that about 16% of the comets scattered into the loss cone of the inner cloud have ended up as the present-day, Oort-cloud comets. If $a = 3\times10^3$ AU is the median semimajor axis of the comets in the inner cloud, we find from Table I that about 3% of these comets have been lost by being scattered into the loss cone. Thus, we estimate that the ratio of the mass of the inner cloud to that of the Oort cloud is

$$M_T/M_0 \sim (0.03\times0.16)^{-1} \sim 2\times10^2.$$

We expect at least two orders of magnitude more comets in the inner cloud than in the Oort halo. This estimate requires that there be a few times 10^{13} comets in the inner cloud and that the mass of the inner cloud be a few tens of earth masses.

3) Maximum Dynamical Mass

A sufficiently massive inner comet cloud would increase the incoming velocity of a loosely bound comet from the Oort cloud to the point where it would appear to be in a hyperbolic orbit as it passed through the planetary system. Some long-period comets do, in fact, appear to be in slightly hyperbolic orbits even after allowance is made for planetary perturbations. However, there is strong evidence that the apparently hyperbolic nature of these orbits is due to observational errors and to nongravitational forces [a rocket effect resulting from ice sublimation near the sun (Marsden and Sekanina 1973)].

A simple, but adequate gravitational model of the comet cloud is one where the inner cloud forms a thin shell of mass M_T and radius R_T. Comets with $2a \gtrsim R_T$ will feel a mass $M_\odot + M_T$ when they are outside this shell and a mass $M_\odot$ when they are inside it. Comets with $a \gg R_T$ will appear hyperbolic when observed near the Sun because of the increased velocity which results from their being accelerated by this comet shell.

It is the custom of observers to parametrize the energy of a hyperbolic orbit in terms of an effective semimajor axis a_H. The most strongly hyperbolic comets have $a_H = -10^4$ AU (Marsden and Sekanina 1973). This indicates an excess kinetic energy at infinity equal to the binding energy of a comet with a semimajor axis $a = 10^4$ AU. If this effect is due to the added gravitational acceleration provided by the comet cloud and the actual semimajor axis of the comet is $a \gg R_T$, then

$$\frac{M_T}{M_\odot} \sim \frac{1}{2}\left|\frac{R_T}{a_H}\right|. \tag{26}$$

With $a_H = -10^4$ AU, we find that

$$M_T = 5\times10^{-2}\ M_\odot = 1.7\times10^4\, M_\oplus$$

for $R_T = 10^3$ AU and

$$M_T = 5\times10^{-3}\ M_\odot = 1.7\times10^3\, M_\oplus$$

for $R_T = 10^2$ AU. Because the true a_H is likely to be much larger than 10^4 AU when allowance is made for various observational errors, it is evident that these mass estimates must provide a firm and very generous upper limit to the mass of the inner comet cloud.

4) Dynamical Limits on any Stellar Companions of the Sun

We note that the added gravitational attraction due to any group of planets, black holes, or other massive objects outside the known planetary system but with $a < a_c$ would mimic that due to the inner comet cloud and would cause the very long-period comets to appear hyperbolic. In this case the hyperbolic orbits would show a marked anisotropy if this mass is concentrated in one or two objects. From the work of Sec. IV b 3, a firm upper limit to the total mass of any such objects is $M = 5\times10^{-3}\ M_\odot$ if their distance from the Sun is $r = 10^2$ AU, $M = 5\times10^{-2} M_\odot$ if $r = 10^3$ AU, and $M = 0.5 M_\odot$ if $r = 10^4$ AU.

The fact that the inner edge of the Oort cloud is observed to be at $a = (1\text{–}2)\times10^4$ AU which is near the theoretical inner semimajor axis for steady-state comets, indicates that the Sun has no companion more massive than $\sim 0.1 M_\odot$ with a semimajor axis less than 2×10^4 AU. A single star of this mass would tend to refill the loss cones of comets with $a < a_c$. This does not preclude there being many less massive companions with a total mass greater than $0.1 M_\odot$, as long as they do not violate the integrated mass limit given in the previous paragraph.

c) Comet Showers

We will now discuss the comets which enter the planetary system from the inner comet cloud ($a < a_c$). These comets are seen only in brief but intense bursts or showers which appear shortly after the passage of a star through the inner cloud. We shall first determine the total number of these comets which have entered the planetary system since its formation. We shall then determine the maximum intensity of a comet shower produced by the close passage of a star to the Sun.

1) Total Number of Shower Comets

In Sec. IV b 2, we estimated the total number of inner-cloud comets which have been deflected into the loss cone which funnels them into the planetary system to be $0.16^{-1} = 6$ times the present-day number of comets in the Oort halo, or about 10^{12} comets. This is based on the assumption that the comets in the Oort halo originated in the inner cloud and were subsequently ejected into their present-day orbits. We found in Sec. III d that about 25% of the Oort comets have reentered the planetary system. These figures indicate that the total number of comets which have entered the planetary system directly from the inner cloud is about $6/0.25 = 24$ times the number that have come from the Oort halo.

2) Shower Intensities

As we see from the data in Table I, the duration of a comet shower is much less than the time between showers. Thus the peak intensity of such showers, the number of comets entering the planetary system per unit time, is much greater than the steady-state flux of comets from the Oort halo. As an example, a close stellar encounter in which $P_\odot = 3 \times 10^3$ AU happens about every 5×10^8 yr; i.e., it has happened about nine times since the solar system formed. The characteristic duration of a shower associated with comets having $a = 3 \times 10^3$ AU is about four orbital periods or 7×10^5 yr. Thus the duration of such a shower is only about 10^{-3} of the mean time between showers. From Eq. (19) we see that for $a_{\max} \gg a_{\min}$ the number of comets entering the planetary system per unit time scales as

$$a_{\min}^{-(3/2)-n} = P_\odot^{-(3/2)-n},$$

where $a_{\min}$ is the minimum semimajor axis of those comets for which the loss cone is filled. If $N_c' = 5$ comets/yr for the present-day, steady-state flux of comets from the Oort halo when $q = 2$ AU and $a_{\min} = a_c = 2.0 \times 10^4$ AU, then for $a_{\min} = 3 \times 10^3$ AU and $q = 2$ AU, we find that $N_c' = 3.8 \times 10^3$ comets/yr for $n = 2.0$ and $N_c' = 9.9 \times 10^3$ comets/yr for $n = 2.5$. A rate of $\sim 10^4$ comets/yr is one comet/hr. Such a high comet flux would be a major nuisance to astronomical observers engaged in research on low-light objects!

Evidence for ancient comet showers should be looked for in the geological record of the Earth. Increases in cosmic dust concentrations during showers are expected to show up in ocean and lake sediments. One should also look for evidence of entire comets hitting the Earth. Under present-day circumstances, Weissman (1980a) estimates that a comet hits the earth about every 10^8 yr, while Everhart (1969) estimates that a comet brighter than magnitude 13.9 at 1 AU hits the Earth about every 5×10^6 yr. During the intense shower resulting from a star passing within a distance $P_\odot = 3 \times 10^3$ AU of the Sun about 7×10^3 comets/yr would pass within $q = 2$ AU of the Sun compared to 5 comets/yr from the Oort cloud. During the shower a comet hits the Earth about every 7×10^4 yr if we use Weissman's figure and every 3.6×10^3 yr if we use Everhart's figure. Over a shower lifetime of 7×10^5 yr we find that about ten comets would hit the Earth if we scaled up from Weissman's figure, whereas 200 would hit the Earth if we scaled up from Everhart's figure. The environmental stress resulting from such a intense, repeated bombardment of the Earth may be expected to sufficiently affect the distribution of species on the Earth to show up in the fossil record. It is tempting to join the "me too" parade and suggest that this bombardment could have led to the extinction of the dinosaurs. Alvarez, Alvarez, Asaro, and Michel (1980) have recently presented geological and fossil evidence for a meteoritic impact being responsible for this extinction.

V. COMET CLOUDS AND DISTANT STELLAR COMPANIONS ABOUT OTHER STARS

From the picture developed in this paper, comet clouds similar to the Oort cloud can be expected to be present about other stars having a massive planet like Jupiter and about binary stars. The companions are needed to pump up the orbital energy of the comets so that a significant fraction of them will be forced up from their point of orgin in the outer parts of the protostar to the stable halo where passing stars can give them the angular momentum needed to prevent their return to the inner part of the system, where they would be perturbed by the massive companion.

A similar phenomenon should occur if three or more stars initially form in strongly interacting, bound orbits. The two more massive stars would be expected to remain as a bound binary. They will pump energy into the lower-mass companion until it is sent into a hyperbolic orbit or until it is sent into an orbit with a sufficiently large semimajor axis that a passing star can knock it out of the loss cone and into a stable, long-period orbit. This would form a stable trinary star (at least until a second passing star knocks the outer stellar component back down into the loss cone, where it is again strongly perturbed by the closer binary.)

The range of semimajor axes of the comets or stellar companions in these stable, outer orbits has to be very similar to that of the Oort-cloud comets of the Sun. The inner radius would be about 2×10^4 AU and the outer radius would be about 2×10^5 AU. That the inner boundary must be at about 2×10^4 AU is evident from Eq. (14). The only property of the central star which appears in this equation is its mass M. This is a very weak dependence: $a_c \propto M^{1/7}$. The other parameters in the equation are weak functions of the stellar environment in which the star finds itself. These factors are similar for most stars in the solar neighborhood. The outer boundary at 2×10^5 AU would also be similar for all stars since it is determined by the tidal field of the Galaxy and has a weak dependence on the mass of the star: $R_{\max} \propto M^{1/3}$.

As in the Oort Comet Cloud of the Sun, we expect most of the objects in these outer stable orbits to clump near the inner boundary at 2×10^4 AU. We would predict the existence of a large number of binary stars with separations on the order of 2×10^4 AU, in which at least one of the binary components is in turn a much closer binary. There is some evidence for the existence of these objects. The nearest star, α Cen, is of this type. It is composed of a relatively close binary made up of solar-mass components and a distant, low-mass companion, Proxima Cen, at a projected distance of 2° or about 1×10^4 AU. Recent work by Bahcall and Soneira (1981) shows that about 15% of the stars that they looked at near the North Galactic Pole have stellar companions at projected distances of 0.1 pc $= 2 \times 10^4$ AU. It would be interesting to know whether either component of these pairs is a "close" ("normal" visual as well as spectroscopic) binary. These distant pairs could serve as probes of the "Oort-cloud" phenomenon around other stars.

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