THE EMISSION MECHANISMS FOR SOLAR RADIO BURSTS

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Abstract. Emission mechanisms for meter-λ solar radio bursts are reviewed with emphasis on fundamental plasma emission.

The 'standard' version of fundamental plasma emission is due to scattering of Langmuir waves into transverse waves by thermal ions. It may be treated semi-quantitatively by analogy with Thomson scattering provided induced scattering is unimportant. A physical interpretation of induced scattering is given and used to derive the transfer equation in a semi-quantitative way. Solutions of the transfer equation are presented and it is emphasized that 'standard' fundamental emission with brightness temperatures $\gg 10^9$ K can be explained only under seemingly exceptional circumstances.

Two alternative fundamental emission mechanisms are discussed: coalescence of Langmuir waves with low-frequency waves and direct conversion due to a density inhomogeneity. It is pointed out for the first time that the coalescence process (actually a related decay process) can lead to amplified transverse waves. The coalescence process saturates when the effective temperature T^t of the transverse waves reaches the effective temperature T^l of the Langmuir waves. This saturation occurs provided the energy density in the low-frequency waves exceeds a specific value which is about 10^{-9} of the thermal energy density for emission from the corona at ≈ 100 MHz. It is suggested that direct emission has been dismissed as a possible alternative without adequate justification.

Second harmonic plasma emission is discussed and compared with fundamental plasma emission. It also saturates at $T^t \approx T^l$, and this saturation should occur in the corona roughly for $T^l \gtrsim 10^{15}$ K. If fundamental plasma emission is attributed to coalescence with low-frequency waves, then for $T^l \gtrsim 10^{15}$ K the brightness temperatures at the two harmonics should be equal and equal to T^l. This offers a natural explanation for the approximate equality of the two brightness temperature often found in type II and type III bursts.

Analytic treatments of gyro-synchrotron emission are reviewed. The application of the mechanism to moving type IV bursts is discussed in view of bursts with $\gtrsim 10^{10}$ K at 43 MHz.

Contents

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1. Introduction

There are only two accepted mechanisms for nonthermal radio emission from the Sun: plasma emission and gyromagnetic emission. In plasma emission energy in Langmuir waves is converted into energy in escaping radiation either at the fundamental or the second harmonic of the plasma frequency. The theory of plasma emission is complicated due to the variety of competing and complementary processes, many of which are nonlinear. This complexity is exacerbated by the confusing variety of names given to or associated with these processes. Gyromagnetic emission is familiar in two limits: for nonrelativistic electrons *in vacuo* the emission occurs predominantly at the fundamental of the gyrofrequency and is called cyclotron emission; for highly relativistic electrons it occurs over a broad band at very high harmonics and is called synchrotron emission. In the solar corona the gyromagnetic emissions occur at intermediate harmonics and is called gyro-synchrotron emission. Low rather than high harmonics dominate because the electrons are only mildly relativistic ($\approx$100 keV), and the lowest harmonics, including the fundamental, either cannot escape because they are below the plasma frequency or are suppressed through the Razin effect.

In this paper I concentrate on fundamental plasma emission. Some introductory remarks are made in Section 2 where the 'standard' version of fundamental plasma emission is defined. In Sections 3 to 5 some physical arguments are used to derive results which allow one to understand the theory of the 'standard' version and to use it. (The results quoted are justified by quantitative treatments in Appendices.) My objective is to formulate the theory of fundamental plasma emission in such a way as to make it as useful to astronomers as are the theories of synchrotron emission, bremsstrahlung and inverse Compton emiesion. Alternatives to the 'standard' version for fundamental plasma emission are discussed in Sections 6 and 7.

Second harmonic emission and gyro-synchrotron emission are discussed in Sections 8 and 9 respectively.

2. Fundamental Plasma Emission

The strongest evidence for fundamental plasma emission comes from harmonic structure, first reported in type II and type III bursts by Wild *et al.* (1953, 1954). The existence of a fundamental component in type III bursts was questioned by Daigne and Møller-Pedersen (1974), Mercier and Rosenberg (1974), Daigne (1975a, b), and Smerd (1976) argued that the then existing data strongly favoured the existence of a fundamental component in some type III bursts. More recent data on the polarization of type III bursts (Suzuki and Sheridan, 1977; Dulk and Suzuki, 1979) has helped clarify the situation: most type III bursts either show harmonic structure and are polarized, the fundamental more strongly than the second harmonic, or are structureless, showing neither harmonic structure nor substantial polarization. The reason for this is not known. Structureless bursts are at the second harmonic and lie near the limb, implying that the fundamental is more directive than the second harmonic. It is also not known why the fundamental is absent at high frequencies and at low frequencies even for those type III bursts which show clear harmonic structure at decametric wavelengths. The existence of a fundamental component in type II bursts has not been questioned. It is also widely believed that type I emission (which shows no harmonic structure) is fundamental plasma emission; the main evidence for this is the strong o-mode polarization. The polarization of fundamental plasma emission is predicted to be very high in the sense of the o-mode (Kai, 1970; Melrose and Sy, 1972), cf. Figure 1, whereas gyromagnetic emission is expected to favour the x-mode and second harmonic plasma emission can have either sense of polarization (Melrose *et al.*, 1978).

A detailed quantitative theory of fundamental plasma emission was given by Ginzburg and Zheleznyakov (1958, 1959). They considered two alternative mechanisms and favoured the one they called 'Rayleigh scattering'. This mechanism

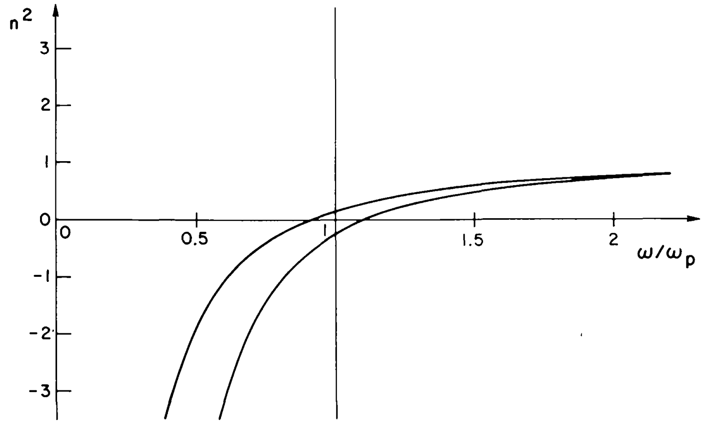

Fig. 1 (a). The equations of the refractive index curves are plotted for $\theta = 0$ and $\Omega_e/\omega_p = 0.2$. The upper curve is the z-mode for $\omega < \omega_p$ and the o-mode for $\omega > \omega_p$. The coupling point is where the two curves join at $\omega = \omega_p$.

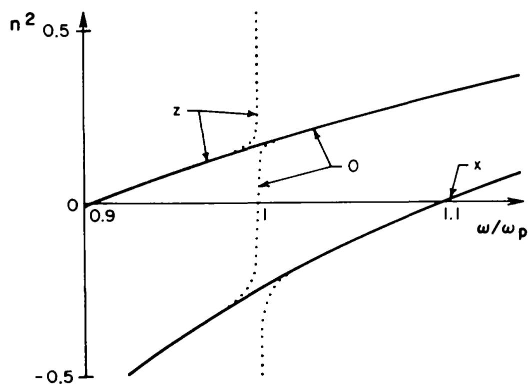

Fig. 1 (b). The region near $\omega = \omega_p$ is illustrated on a different scale for $\theta = 0$ (solid line) and for $\theta = 5°$ (dotted line).

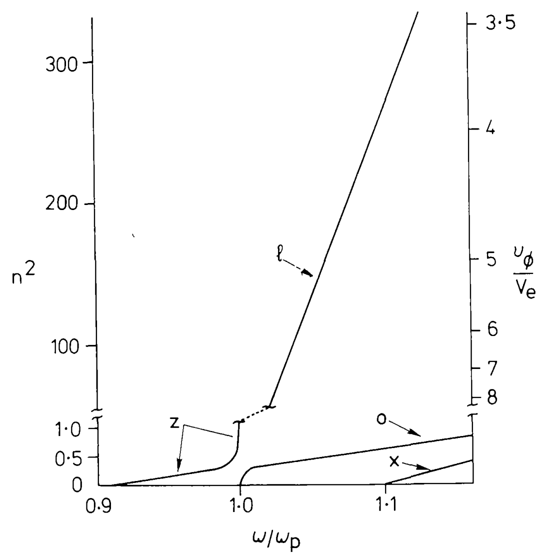

Fig. 1 (c). The same frequency region as in (b) is illustrated on a larger scale with the refractive index curves for Langmuir (l) waves included for $T_e = 1.5 \times 10^6$ K. One has $n^2 = c^2/v_\phi^2$ for $n^2 \gg 1$, and when plotted as a function of v_ϕ/V_e (the scale to the right) the curve is independent of T_e. The l-curve is to the left of the x-curve for $v_\phi \gtrsim 4 V_e$, for the numbers chosen here, and then scattering by ions can produce only o-mode emission.

has also been called 'nonlinear scattering' (Tsytovich, 1966; Kaplan and Tsytovich, 1973) and 'enhanced bremsstrahlung' (Tidman and Dupree, 1965). The scattering is attributed to the charge inhomogeneity in the electron gas associated with the shielding field around a thermal ion. It is the 'standard' version for fundamental plasma emission. Here the mechanism will be referred to either as the 'standard' version or as 'scattering by ions'. The other mechanism considered by Ginzburg and Zheleznyakov (1958, 1959) is the 'direct conversion' mechanism discussed in Section 9. Ginzburg and Zheleznyakov also developed a theory for second harmonic plasma emission which they called 'combination scattering'. The idea (although not their analysis of it) is now accepted: second harmonic plasma emission is attributed to coalescence of two Langmuir waves into a transverse at the sum of the two frequencies. Such coalescence can also occur between a Langmuir wave and certain low-frequency waves; the resulting transverse wave is then at the fundamental plasma frequencies (this is because $\omega - \omega_p \ll \omega_p$). This coalescence with low-frequency waves to produce fundamental plasma emission is discussed below in Section 6.

In Section 3 it is argued that scattering by thermal ions may be treated in a way closely analogous to Thomson scattering.

It was first pointed out by Tsytovich (1966) that the scattering by ions, like any scattering process, can be 'induced' by the presence of scattered waves. Induced scattering causes one set of waves to grow exponentially at the expense of the other. A physical interpretation of induced scattering is presented in Section 4, and the transfer equation for the 'standard' version of fundamental plasma emission is written down on the basis of semi-quantitative arguments. The transverse waves are generated with small refractive index, and such waves experience large refractive effects. Some special solutions of the transfer equation for scattering by ions are presented in Section 5 to illustrate the effects of refraction.

It is important to consider alternatives to the 'standard' version of fundamental plasma emission for at least two reasons. First, there is a problem in accounting quantitatively for fundamental type III emission, e.g. Melrose (1977) and Section 5.6 below, and this can be overcome only by invoking a more efficient conversion of Langmuir waves into transverse waves than is implied by the simplest models based on the 'standard' version. Second, type I emission is qualitatively different from fundamental type II and type III emission and it may be that this is due to a qualitative difference in the emission mechanism. More generally, the suggestion that all fundamental plasma emission can be attributed to scattering by ions is difficult to support convincingly, and one needs to keep an open mind on possible alternatives. Two alternatives are discussed in Sections 6 and 7.

3. The Analogy with Thomson Scattering

3.1. Thomson Scattering

In the simplest case, Thomson scattering is the conversion of one transverse wave into another by an electron *in vacuo*. In Thomson scattering the energy density W in the scattered waves increases in unit length $\mathrm{d}l$ as

$$\frac{\mathrm{d}W}{\mathrm{d}l} = \sigma_{\mathrm{T}} n_e W' \,, \tag{1}$$

where $\sigma_{\mathrm{T}} = (8\pi/3)\, e^4/m_e^2 c^4$ is the Thomson cross-section, n_e is the number density of the scattering electrons and W' is the energy density in the unscattered waves. In a frame in which an individual scattering electron is at rest, the frequency ω of the scattered wave is equal to the frequency ω' of the unscattered wave, and in a frame in which the electron moves with velocity $\mathbf{v}$ this equality transforms into

$$\omega - \mathbf{k}\cdot\mathbf{v} = \omega' - \mathbf{k}'\cdot\mathbf{v} \,, \tag{2}$$

where $\mathbf{k}$ and $\mathbf{k}'$ are the wave vectors of the scattered and unscattered waves respectively. In scattering of monochromatic radiation by thermal electrons, the bandwidth of the scattered radiation is the spread in $\omega - \omega' = (\mathbf{k} - \mathbf{k}') \cdot \mathbf{v}$ and it is of order

$$\Delta\omega = |\mathbf{k} - \mathbf{k}'|\, V_e \,, \tag{3}$$

where V_e is the thermal speed of the electrons.

3.2. The Effect of Shielding

Cooperative effects in a plasma cause the electric field due to any individual charge to be shielded out over about a Debye length λ_{D}. Specifically the electrostatic potential for a charge q at rest at the origin ($\mathbf{r} = 0$) is

$$\phi(\mathbf{r}) = \frac{q}{r}\, e^{-r/\lambda_{\mathrm{D}}} \,. \tag{4}$$

The shielding may be attributed to a localized inhomogeneity in the electron gas giving a net charge $-q$ in a sphere of radius $\approx \lambda_{\mathrm{D}}$.

For wavelengths less than a Debye length, which corresponds to frequencies (for transverse waves)

$$\omega > \omega_p \left(\frac{c}{V_e}\right) , \tag{5}$$

the plasma looks like a collection of individual charges. Thomson scattering then occurs more or less as *in vacuo*. However, for wavelengths greater than a Debye length the plasma looks like a collection of shielded charges. As far as scattering is concerned, a shielded electron is quite ineffective. This is because the individual charge $-e$ and the shielding field of charge $+e$ look like a negative and a positive electron moving together and scattering in phase. Thus the net scattering by electrons is small due to the destructive interference. A thermal ion with charge $Z_i e$ gives entirely negligible Thomson scattering (the cross-section is proportional to (charge)^4/(mass)^2); however its shielding field looks like a particle with charge $Z_i e$ and mass $Z_i m_e$ and has a cross-section $Z_i^2 \sigma_T$ for Thomson-like scattering. Hence for waves of wavelength $\gg \lambda_D$, i.e. when inequality (5) is reversed, one expects the scattering to be due to the shielding field around individual ions. Furthermore, the scattering should still be describable by (1) but with n_e replaced according to

$$n_e \to \sum_i Z_i^2 n_i \,, \tag{6}$$

where the sum is over all ionic species. The frequency spread will now be characteristic of the ions, i.e. (3) is replaced by

$$\Delta\omega = |\mathbf{k} - \mathbf{k}'| V_i \,, \tag{7}$$

where V_i is the thermal speed of the ions.

A further minor effect is associated with the fact that only part of the shielding is due to the electrons, with the remainder being due to ions. The ionic part of the shielding does not contribute to the scattering. The effective Debye length λ_D is related to the Debye lengths of the electrons (λ_{De}) and of the ions (λ_{Di}) by

$$\lambda_D^{-2} = \lambda_{De}^2 + \sum_i \lambda_{Di}^{-2} \tag{8}$$

with $\lambda_{De}^2 \propto T_e/n_e$ and $\lambda_{Di}^2 \propto T_i/Z_i^2 n_i$ and where the sum is over all ionic species. The effective scattering charge is not the total shielding charge $-Ze$ but rather the part $-Ze\ \lambda_D/\lambda_{De}$ due to the electrons. Hence in addition to the change (6) the cross-section is modified by a factor $\lambda_D^4/\lambda_{De}^4$. For simplicity this factor will be replaced by $\frac{1}{4}$ (appropriate to an ionized hydrogen gas) in the following formulae. However, it is relevant to remark that in the detailed theory the factor (9) does not appear when the intrinsic bandwidth $\Delta\omega$ is much greater than $k'V_i$; in the process involving coalescence with low-frequency waves the intrinsic bandwidth satisfies this inequality and the factor $\lambda_D^4/\lambda_{De}^4$ does not appear (Section 6 below). Thus we have for scattering by ions for coalescence with low frequency waves

$$\frac{\mathrm{d}W}{\mathrm{d}l} = \sigma_T \Big(\sum_i Z_i^2 n_i \Big) W' \begin{cases} \frac{1}{4} & \text{for} \quad \Delta\omega \leqslant k'V_i, \\ 1 & \text{for} \quad \Delta\omega \geqslant k'V_i. \end{cases} \tag{9}$$

3.3. Scattering $l \to t$

Thomson scattering by a charge at rest depends only on the frequency of the waves; the wavenumbers of the scattered and unscattered waves are irrelevant. Consequently (1) as modified in accord with (6) should apply to transverse waves even when the refractive index

$$n = \left(1 - \frac{\omega_p^2}{\omega^2} \right)^{1/2} \tag{10}$$

is much less than unity. Furthermore the same equation should also apply to the scattering of Langmuir waves (l) with transverse waves (t). For this scattering one has

$$\frac{\mathrm{d}W^t}{\mathrm{d}l}=\frac{1}{4}\sigma_{\mathrm{T}}\Big(\sum_i Z_i^2 n_i\Big)W^l. \tag{11}$$

The bandwidth is

$$\Delta\omega=|\mathbf{k}-\mathbf{k}'|\,V_i\approx k'V_i\,, \tag{12}$$

where in practice the wave number k' of the Langmuir wave is always much larger than that of the fundamental transverse waves.

The wavenumber and refractive index of the transverse waves at generation can be estimated from the approximate equality between the frequency of the transverse waves and of the Langmuir waves, i.e. from $\omega^t(\mathbf{k})=\omega^l(\mathbf{k}')$, with

$$\omega^t(\mathbf{k})=\{\omega_p^2+k^2c^2\}^{1/2} \tag{13a}$$

$$\omega^l(\mathbf{k}')\approx\omega_p+3k'^2V_e^2/2\omega_p\,, \tag{13b}$$

which give

$$k\approx\sqrt{3}\,k'\frac{V_e}{c},\qquad n=\frac{kc}{\omega}\approx\sqrt{3}\,\frac{V_e}{v_\Phi}, \tag{14}$$

where $v_\Phi=\omega_p/k'$ is the phase speed of the Langmuir waves.

3.4. Propagation effects

It is important to take account of the fact that in a refractive medium W^t is not constant along a ray path even in the absence of emission and absorption. The specific intensity $I=W^t v_g/\Delta\omega\Delta\Omega$, where $\Delta\Omega$ is the solid angle filled by the radiation, is also not constant, but I/n^2 is.

4. Amplified Fundamental Emission

4.1. Induced scattering

Induced scattering occurs for Thomson scattering, for scattering by ions and for any scattering process. Amplification of the fundamental occurs due to induced scattering $l\to t$. This was recognized by Tsytovich (1966), Kaplan and Tsytovich (1967), and Smith (1970). Induced scattering $l\to l$ is a more familiar process in plasma physics where it is often called nonlinear Landau damping. In laser physics induced scattering by molecules is referred to as stimulated Raman scattering; 'induced' and 'stimulated' are interchangeable in this connection.

The important features of induced scattering can be understood by regarding it as related to ordinary ('spontaneous') scattering in the same way as absorption is related to emission. One regards a scattering process as an emission or absorption by the scatterer of the beat fluctuation $(\omega-\omega',\mathbf{k}-\mathbf{k}')$ between the scattered and unscattered waves. This idea is developed below. The only subtle point is that the absorption of the beats needs to be treated differently for $\omega-\omega'>0$ and $\omega-\omega'<0$.

4.2. Semi-classical formalism

It is convenient to introduce a semi-classical formalism to discuss emission, absorption and scattering. The waves are regarded as a collection of wave quanta (photons, plasmons etc.) with energy $\hbar\omega$, momentum $\hbar\mathbf{k}$ and occupation number $N(\mathbf{k})$. First let us define the effective temperatures $T(\mathbf{k})$ of the transverse and Langmuir waves in terms of their energy densities:

$$W^t=\int\frac{d^3\mathbf{k}}{(2\pi)^3}\kappa T^t(\mathbf{k}),\qquad W^l=\int\frac{d^3\mathbf{k}'}{(2\pi)^3}\kappa T^l(\mathbf{k}'),\tag{15a, b}$$

where κ is Boltzmann's constant. The occupation numbers are then given by

$$N^t(\mathbf{k})=\frac{\kappa T^t(\mathbf{k})}{\hbar\omega^t(\mathbf{k})},\qquad N^l(\mathbf{k}')=\frac{\kappa T^l(\mathbf{k}')}{\hbar\omega^l(\mathbf{k}')},\tag{16a, b}$$

where $\hbar$ is Planck's constant divided by 2π. An important advantage of the semi-classical formalism is that it allows one to discuss a single scattering event of one wave quantum into another. In each such scattering event one occupation number decreases by unity and the other increases by unity.

Now consider the fluctuations (f) with $\omega-\omega'$, $\mathbf{k}-\mathbf{k}'$. Let us define $T^f_\pm(\mathbf{k}-\mathbf{k}')$ and $N^f_\pm(\mathbf{k}-\mathbf{k}')$ with the + sign referring to $\omega-\omega'>0$ and the − sign to $\omega-\omega'<0$. As in (16a, b) we have

$$T^f_\pm(\mathbf{k}-\mathbf{k}')=\hbar|\omega-\omega'|N^f_\pm(\mathbf{k}-\mathbf{k}').\tag{16c}$$

Given $N^l(\mathbf{k}')\neq 0$ and $N^t(\mathbf{k})=0$ initially we may set $N^f_\pm(\mathbf{k}-\mathbf{k}')=0$ initially. If the Langmuir waves were monochromatic we could then conclude that for $\omega>\omega'$ we have $N^f_+(\mathbf{k}-\mathbf{k}')=N^t(\mathbf{k})$ due to the fact that both N^f_+ and N^t increase by unity for each scattering $l\to t$ and decrease by unity for scattering $t\to l$. For $\omega<\omega'$ we can set $N^f_-(\mathbf{k}-\mathbf{k}')=N^l(\mathbf{k}')$ initially and then because each scattering $l\to t$ decreases both N^l and N^f_- by unity, the equality holds thereafter.

In a radiation process in a thermal plasma, e.g. bremsstrahlung, emission tends to cause $T^t(\mathbf{k})$ and $N^t(\mathbf{k})$ to increase, and the associated absorption process tends to cause $T^t(\mathbf{k})$ and $N^t(\mathbf{k})$ to decrease. In the optically-thick limit $T^t(\mathbf{k})$ is equal to the plasma temperature T. Thus, if $T^t(\mathbf{k})$ is initially small then spontaneous emission tends to increase $T^t(\mathbf{k})$ until absorption becomes important and limits $T^t(\mathbf{k})$ to less than T. On the other hand, if $T^t(\mathbf{k})$ is initially large then absorption tends to reduce it to T.

4.3. The transfer equation for a scattering process

For the fluctuations with $\omega>\omega'$ the frequency $\omega-\omega'$ is positive and the analogy between induced scattering and absorption is simple. Each scattering $l\to t$ increases both N^t and N_+^f by unity and when $\hbar(\omega-\omega')\,N_+^f$ approaches κT_i absorption of the fluctuations becomes important and limits N_+^f and hence N^t to less than $\kappa T_i/\hbar(\omega-\omega')$. Hence for $\omega<\omega'$, T^t is limited to

$$T^t(\mathbf{k}) \lesssim \frac{\omega}{\omega-\omega'}T_i\,. \tag{17}$$

A transfer equation which describes this limiting effect is

$$\frac{\mathrm{d}T^t(\mathbf{k})}{\mathrm{d}t}=\alpha(\mathbf{k})\left\{1-\frac{\omega-\omega'}{\omega}\frac{T^t(\mathbf{k})}{T_i}\right\}, \tag{18}$$

where $\alpha(\mathbf{k})$ is an emission coefficient. The final term in (18) describes induced scattering.

For $\omega<\omega'$, that is when the scattering generates a transverse wave with frequency less than that of the Langmuir wave, (18) still applies and the induced scattering causes $T^t(\mathbf{k})$ to increase exponentially. To interpret this amplification in terms of the fluctuations N_-^f, one must regard N_-^f as equal to N^l initially, as suggested above, and interpret each scattering as an absorption of these initial fluctuations. The absorption then tends to reduce N_-^f to $\kappa T_i/\hbar(\omega'-\omega)$, and hence $N^t=N^l-N_-^f$ tends to increase exponentially.

Thus induced scattering can be interpreted in terms of absorption of the beats between the two waves provided that one formulates the absorption so that the beat frequency is always positive. Specifically, induced scattering between two distributions $N'(\mathbf{k}')$ and $N(\mathbf{k})$, with frequencies ω' and ω, corresponds to absorption of fluctuations with $N_+^f=N$ for $\omega>\omega$ and with $N_-^f=N'$ for $\omega'>\omega$. Energy is always transferred from higher frequency waves to lower frequency waves, with the energy difference $\hbar|\omega-\omega'|$ being absorbed by the scattering particles. This net transfer from higher to lower frequencies is a characteristic feature of induced scattering processes.

4.4. Broad Langmuir Spectra

For semi-quantitative purposes one can set $\omega-\omega'\approx k'V_i=\omega_p V_i/v_\phi$ in (18) and hence deduce that induced scattering becomes important for

$$T^t \gtrsim \frac{v_\phi}{V_i}T_i \tag{19}$$

for monochromatic Langmuir waves, i.e. for waves with a given k'. For $v_\phi\approx c/3$ and $T_i\approx 10^6$ K (19) becomes $T^t\gtrsim 10^9$ K. For a broad spectrum of Langmuir waves (19) is replaced by (Melrose, 1979b, p. 216)

$$T^t \gtrsim \frac{3m_i}{m_e}\frac{T_e}{g(k_0)} \tag{20}$$

with

$$g(k')=\frac{\mathrm{d}\ln\{k'^3T^l(k')\}}{\mathrm{d}\ln k'} \tag{21}$$

and

$$k_0 = [2(\omega - \omega_p)\omega_p / 3V_e^2]^{1/2}. \tag{22}$$

4.5. The emission coefficient

The emission coefficient $\alpha(\mathbf{k})$ in (18) may be estimated semiquantitatively from (18) by evaluating $\mathrm{d}W^t/\mathrm{d}l$ ignoring the induced scattering term:

$$\frac{\mathrm{d}W^t}{\mathrm{d}l} = \frac{1}{v_g} \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \frac{\kappa\, \mathrm{d}T^t(\mathbf{k})}{\mathrm{d}t} \approx \frac{4\pi}{v_g} \frac{k^2 \Delta k}{(2\pi)^3} \kappa\alpha(\mathbf{k}), \tag{23}$$

where $v_g = cn$ is the group speed and Δk is the range of wavenumbers of the transverse waves. Assuming

$$\Delta k = \frac{\Delta\omega}{\partial\omega/\partial k} \approx \frac{k' V_i}{cn} = \frac{\omega_p}{c}\left(\frac{V_i}{\sqrt{3}\, V_e}\right), \tag{24}$$

where (12) and (14) have been used, and using (14) to estimate k and n, (23) reduces to

$$\frac{\mathrm{d}W^t}{\mathrm{d}l} = 4\pi \frac{\omega_p^2 V_i}{c^4 v_\phi} \frac{\kappa\alpha(\mathbf{k})}{(2\pi)^3}. \tag{25}$$

Comparison of (11) and (25) gives

$$\kappa\alpha(\mathbf{k}) = \frac{\pi}{12} \frac{\omega_p W^l v_\phi}{n_e V_i}, \tag{26}$$

which differs by a factor of order unity from the results of a more detailed calculation. (The small quantitative difference from the more detailed calculation can be attributed mainly to the estimate of the bandwidth in (12) and in (24).)

4.6. The effective length for amplification

Finally, we can estimate the length l_0 required before T^t builds up to the value where induced scattering becomes important. Induced scattering become important for

$$W^t \approx \frac{4\pi}{3} \frac{k^2 \Delta k}{(2\pi)^3} \frac{v_\phi}{V_i} T_i \approx \frac{1}{2\sqrt{3}\, \pi^2} \frac{\omega_p^3 V_e}{c^3 v_\phi} T_i. \tag{27}$$

According to (11) it would take a distance

$$l_0 \approx \frac{\sqrt{3}}{\pi}\left(\frac{n_i \kappa T_i}{W^l}\right) \frac{V_e}{v_\phi} \frac{c}{\omega_p} \tag{28}$$

for W^t to build up to the value (25).

The distance l_0 is important in any semi-quantitative theory. Induced scattering is significantt only if the distance, l say, available for build-up of the waves is greater than l_0. There are two obvious limits on l. One is associated with the gradient in the plasma density, over the characteristic distance L_N say. Then, if $\Delta\omega$ is the intrinsic bandwidth of the emission, l must be less than $(\Delta\omega/\omega_p)L_N/2$. The other is when the Langmuir waves are distributed in an inhomogeneous way, i.e. in clumps; l must then be less than or equal to the thickness of the clump. In the next section the transfer equation is solved for several simple models which illustrate these points.

5. Solutions of the Transfer Equation

To explain brightness temperatures $\gtrsim 10^9$ K for fundamental plasma emission in terms of scattering $l \to t$ by thermal ions one must appeal to induced scattering. However, escaping radiation encounters an absorbing region along any prospective escape path (e.g. Smith, 1970; Smith and Riddle, 1975; Melrose, 1977) and it requires exceptional circumstances to obtain brightness temperature significantly in excess of 10^9 K. This is illustrated by the following examples of solutions of the transfer equation (Melrose, 1969b, pp. 225–229).

5.1. The effective optical depth

It is convenient to introduce an effective optical depth τ for induced scattering. From a semi-quantitative viewpoint τ may be defined by

$$\tau := l/l_0\,, \tag{29}$$

where l_0 is given by (28) and where l is the distance available for generation of fundamental transverse radiation at fixed ω. Changes in the plasma density, over a characteristic distance L_N say ($L_N := n_e|\text{grad}\, n_e|^{-1}$), cause the plasma frequency to change over a characteristic distance $L_N/2$. The distance over which emission at fixed ω can be generated is restricted by the narrow bandwidth to $l \le \bar{l}$ with

$$\bar{l} \approx \frac{\Delta\omega}{\omega_p}\frac{L_N}{2} \approx \frac{V_i}{v_\phi}\frac{L_N}{2}\,, \tag{30}$$

where (12) has been used.

5.2. The slab and linear models

In terms of τ and the temperature $T_0(\approx v_\phi T_i/V_i)$ at which induced scattering becomes important, the transfer Equation (18) may be written in the form

$$\frac{\mathrm{d}T^t}{\mathrm{d}\tau} = T_0\left\{1 - \frac{\omega-\omega'}{\Delta\omega}T^t\right\}. \tag{31}$$

The simplest solution of (31) is for a *slab model* in which the source is assumed to be a uniform slab of thickness $\bar{l}$. The solution is

$$T^t = T_0\begin{cases} 1 - \exp\left[-\dfrac{\omega-\omega'}{\Delta\omega}\bar{\tau}\right] & \text{for}\quad \omega > \omega' \\[2ex] \exp\left[\dfrac{\omega'-\omega}{\Delta\omega}\bar{\tau}\right] - 1 & \text{for}\quad \omega < \omega'\,, \end{cases} \tag{32}$$

with $\bar{\tau}$ given by (29) with $l = \bar{l}$. A more realistic model for a source in the solar corona is a *linear model* in which ω_p, and hence ω', decreases linearly along the ray path. For such a model one has

$$T^t = 2T_0\phi(\sqrt{\bar{\tau}}/2) \tag{33}$$

where

$$\phi(y) := 2y\,e^{-y^2}\int_0^y dt\,e^{t^2} \tag{34}$$

is a form of the plasma dispersion function. It follows from (33) and (34) that the brightness temperature of the fundamental cannot exceed $2T_0$, i.e. several times 10^9 K, for a seemingly reliable source model. Brightness temperatures orders of magnitude in excess of 10^9 K have been observed in type III bursts, e.g. Dulk and Suzuki (1979) found 10^{11} to 10^{12} K in some fundamental type III bursts.

5.3. The sawtooth model

Another model considered by Melrose (1979b, p. 228) is a sawtooth model in which the decrease in density is not uniform but is linear on average with linear rises and

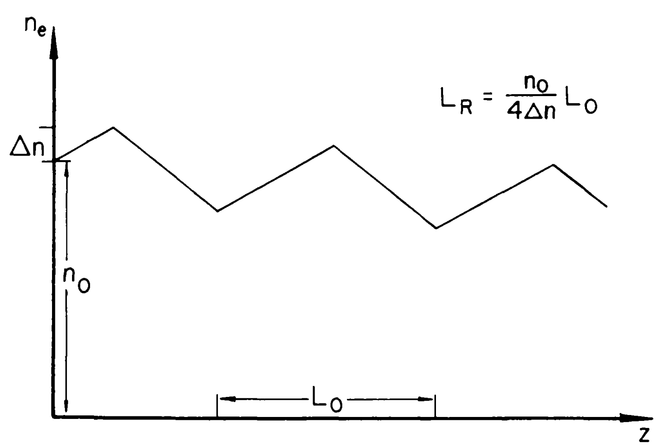

Fig. 2. An idealized 'sawtooth' variation of amplitude Δn in the local electron density is shown superimposed on a gradient in the mean density n_0 with height z. L_0 is the wavelength of the sawtooth and L_R is the characteristic distance associated with the sawtooth gradient.

falls (Figure 2). Let L_R be the characteristic distance over which the density changes in each inhomogeneity, and define an optical depth, cf. (29) and (30),

$$\tau_R = \frac{V_i}{v_\phi}\frac{L_R}{2l_0}. \tag{35}$$

One then has $\bar{\tau} = (L_N/L_R)\tau_R$. The relevant solution of the transfer equation is

$$T^t = 2T_0 \frac{L_N}{L_R}\phi(\sqrt{\tau_R}/2), \tag{36}$$

where the variations in the density in each sawtooth is assumed to be much greater than the intrinsic bandwidth of the emission. The generation of the fundamental occurs in a sequence of $2L_N/L_R$ separate sections of path length, with half of these corresponding to locally increasing density and half to locally decreasing density. According to (36) the maximum brightness temperature occurs when each sawtooth contributes $2T_0$ and the contributions from the L_N/L_R teeth add to give $T^t = 2(L_N/L_R)T_0$.

5.4. Difficulties with the 'standard' version

It is not intended here to give a detailed discussion of the application to type III bursts, or to other types of burst, but it is appropriate to point out the difficulties implied by the foregoing results. First, brightness temperatures up to about 3×10^{11}K($\approx 10^2\times 2T_0$) are relatively common for fundamental type III bursts at 43 MHz (Dulk and Suzuki, 1979); the bursts are less bright at 80 MHz and less still at 160 MHz. One requires $\bar{\tau} \gg 1$ to explain the brightest bursts in terms of scattering by thermal ions. For $L_N \approx 10^5$ km, $v_\phi \approx 30V_e$ and $f_p \approx 100$ MHz, one estimates $\bar{\tau} \approx 10^6(W^l/n_i\kappa T_i)$ from (28), (29), and (30). To have $\tau \gg 1$ and hence $W^l \gg 10^{-6}\, n_i\kappa T_i$ uniformly across the stream implies excessive energy losses by the stream (Melrose, 1977) except if this energy is reabsorbed by the stream. To have $\bar{\tau} \gg 1$ and not to violate energy constraints requires that the Langmuir waves be in intense clumps. Such a clumpy distribution of Langmuir turbulence is observed in the interplanetary medium (Gurnett and Anderson, 1977; Gurnett, 1979). Second, even if values of $\bar{\tau} \gg 1$ are possible, the source models considered above imply that it is difficult to obtain values of $T^t \gg 2T_0$. Quite special source models are required. For example, if the sawtooth model is invoked, one would need a ray to pass through $\approx 10^2$ optically thick regions where ω_p changed by an amount in excess of the bandwidth $\Delta\omega$. This would required intense clumpy Langmuir waves in a region of rapidly quasi-periodically varying plasma density. Another type of model could invoke edge effects associated with the clumps of Langmuir waves. A sharply-edged clump would act like a slab and for $\bar{\tau} \gg 1$ one could have $T^t \gg T_0$, as implied by (32).

It seems that we have three alternative ways of overcoming these difficulties. (i) We could accept scattering by ions as the relevant emission mechanism and address the problem of identifying source models which can produce brightness temperature as high as those observed without violating energy constraints. (ii) We could appeal to reabsorption of the Langmuir waves by the stream itself. Although possible in principle, this alternative does not seem favourable because a very high efficiency of reabsorption is required to alleviate the difficulty with excessive energy losses. (iii) We could seek alternative mechanisms which are more efficient in converting

Langmuir waves into transverse waves. Two possible alternative conversion mechanisms are discussed in the next two sections.

6. Coalescence with Low-Frequency Waves

The coalescence of Langmuir waves (ω', $\mathbf{k}'$) with low-frequency waves (ω'', $\mathbf{k}''$) into fundamental transverse waves (ω, $\mathbf{k}$) is possible when the parametric conditions

$$\omega = \omega' \pm \omega'', \qquad \mathbf{k} = \mathbf{k}' \pm \mathbf{k}'' \tag{37a, b}$$

are satisfied. The plus signs in (37a, b) correspond to the coalescence process and the minus signs to a closely related decay process.

6.1. The Analogy with Scattering by Ions

The rate at which the conversion $l \to t$ occurs due to the low-frequency turbulence can be estimated simply for the particular case of ion-sound turbulence. The fluctuations in the electron density due to the ion-sound waves may be regarded as propagating enhanced versions of the density inhomogeneities associated with the shielding of thermal ions. If the ion-sound turbulence is at a thermal level, i.e. if the effective temperature $T^s(\mathbf{k}'')$ of the ion-sound waves is equal to the electron temperature T_e, then the conversion $l \to t$ due to the ion-sound turbulence should be closely analogous to that due to scattering by thermal ions. That is (11) should apply to the conversion processes when the ion-sound waves are thermal. For an enhanced level of ion-sound turbulence, the rate of conversion is enhanced by the ratio $T^s(\mathbf{k}'')/T_e$, with $\mathbf{k}''$ determined by (37a, b). Thus for non-thermal ion-sound waves (11) is replaced by (cf. Appendix B)

$$\frac{\mathrm{d}W^t}{\mathrm{d}l} = \sigma_T n_e W^l \frac{T^s}{T_e}, \tag{38}$$

where the ions are assumed singly charged and where the factor (9) has been omitted for the reason given. The maximum value of T^t is given by

$$T^t < \frac{T^l T^s}{T^s \pm \dfrac{v_s}{v_\phi} T^l} \approx T^l. \tag{39}$$

The plus and minus signs in (39) are for the processes as defined by (37a, b) and the final approximate equality applies for $T^s \gg (v_s/v_\phi)T^l$ with $v_\phi = \omega_p/k'$ and $k'' \approx k'$. The case in which the denominator $T^s - (v_s/v_\phi)T^l$ in (39) is negative requires further comment. For the present let us assume that the denominator is positive and return to this point later.

6.2. Suitable Types of Low-Frequency Turbulence

The coalescence and decay processes described by (37a, b) require low-frequency turbulence with large wavenumbers, specifically with wavevectors approximately equal and/or opposite to those of the Langmuir waves with which the coalescence occurs. Lower-hybrid and ion-Bernstein waves are plausible alternatives to ion-sound waves. Other possibilities include ion-cyclotron waves and certain drift modes. For all such waves (37a, b) apply and as in (39) one has $T^t \lesssim T^l$ for $T''/\omega'' \gg T^l/\omega_p$. The only modification required is to (38) which needs to be multiplied by a factor which is of order unity and is discussed in Appendix B.

6.3. Line splitting

A notable feature of coalescence with low-frequency turbulence is that it can cause a frequency-splitting of fundamental radiation. For monochromatic Langmuir waves with phase speed v_ϕ the splitting for ion-sound waves is such that the two escaping lines (at ω_+ and ω_-) satisfy

$$\frac{\omega_+ - \omega_-}{\omega_+ + \omega_-} = \frac{v_s}{v_\phi} = \frac{1}{43}\frac{V_e}{v_\phi}. \tag{40}$$

For other waves (40) is replaced by its more general form

$$\frac{\omega_+ - \omega_-}{\omega_+ + \omega_-} = \frac{\omega''}{\omega_p}, \tag{40'}$$

where ω'' is the frequency of the low-frequency waves, e.g. the lower-hybrid frequency, an harmonic of the ion gyrofrequency for ion Bernstein waves or the ion gyrofrequency itself for ion-cyclotron waves.

6.4. Required level of low-frequency turbulence

The parameter T^s is not as convenient for semi-quantitative purposes as the energy density W^s in ion-sound turbulence. Due to the fact that only ion-sound waves of particular wavevectors can coalesce with given Langmuir waves, any estimate of T^s in terms of W^s is sensitive to the detailed form of the spectrum of ion-sound waves. Assuming the waves are isotropic and confined to a range $\Delta k'' \approx k''$ one finds

$$\kappa T^s \approx 2\pi^2 W^s \left(\frac{v_\phi}{\omega_p}\right)^3 \tag{41}$$

or

$$\frac{T^s}{T_e} \approx \frac{\pi}{2}\left(\frac{v_\phi}{c}\right)^3 \frac{c}{r_0\omega_p}\frac{W^s}{n_e \kappa T_e}, \tag{42}$$

where v_ϕ is the phase speed of the Langmuir waves and r_0 is the classical radius of the electron.

Emission in a bandwidth $\Delta\omega$ must come from a range of heights less than $(\Delta\omega/\omega_p)L_N/2$ in a smoothly varying corona. Assuming the low-frequency turbulence to be uniformly excited over this height, we can estimate the minimum value of its energy density required for the conversion process to saturate at $T^t = T^l$. It is shown in Appendix C that one requires

$$\frac{W^s}{n_e\kappa T_e} \gtrsim \frac{6\sqrt{3}}{\pi}\,\frac{c}{\omega_p L_N}\,\frac{V_e}{v_\phi}. \tag{43}$$

The right hand side of (14) is of order 10^{-9} for reasonable parameters in the lower solar corona. It is interesting that the value of W^s required for the process to saturate is independent of the value T^l at which this saturation occurs. The result (43) applies without modification to lower-hybrid turbulence.

6.5. The possibility of amplication

So far we have been concerned with the case where the denominator in (39) is positive. Suppose it is negative, which corresponds specifically to $N^s < N^l$. From Equation (B1) below, the rate at which N^t increases is proportional to $N^l N^s + N^t(N^l - N^s)$. For $N^s < N^l$ amplification is possible. This is an important new result which has not been recognized previously.

The situation with the interaction with low-frequency turbulence (assumed to be ion-sound turbulence for the discussion here) interacting with Langmuir turbulence can be summarized as follows. (i) The process $l + s \to t$ can never lead to amplification of the transverse waves. For $N^s \gg N^l$ the processes $l + s \to t$ and $l \to t + s$ are similar and lead to a line-split emission. This emission saturates at $N^t \approx N^l$, which implies $T^t \approx T^l$. (ii) For $N^s \ll N^l$ the processes $l + s \to t$ and $l \to t + s$ are similar and lead to a line-split emission only for $N^t \ll N^s$, i.e. for

$$T^t \ll T^s \frac{v_\phi}{v_s}. \tag{44}$$

The process $l + s \to t$ saturates at $T^t \approx T^s(v_\phi/v_s)$. However, the process $l \to t + s$ becomes an amplifying one for $T^t \gtrsim T^s(v_\phi/v_s)$, and it saturates only for $T^t \approx T^l$.

The situation here is closely analogous to that of scattering by ions. In the scattering case, spontaneous scattering generates t-waves with $\omega > \omega^l$ and with $\omega < \omega^l$. The former are absorbed and the latter are amplified when induced scattering becomes important. In the interaction with low-frequency turbulence for $N^s \ll N^l$, two lines at $\omega = \omega^l \pm \omega^s$ are generated; the higher frequency one is absorbed and the low-frequency one is amplified as N^t approaches N^s.

6.6. Significance of the amplification

The growth rate for the amplification of t-waves due to the process $l \to t + s$ for $N^s \ll N^l$ is the same as the growth rate due to induced scattering by ions $l \to t$ to within a factor of order unity. Nevertheless the process $l \to t + s$ is more effective than induced scattering by thermal ions in one important way: the threshold for amplification is reached more rapidly. The analogue of spontaneous scattering is the process $l \to t + s$ for $N^t \ll N^s$, and this occurs at a rate faster than that of spontaneous scattering by a factor $\approx T^s/T_i$. Hence the difficulty in building up the brightness temperature of the transverse waves to the threshold for amplification can be overcome for say $T^s/T_i \approx 10$ to 100.

The amplification process encounters the same difficulty as induced scattering in that absorption occurs in an overlying layer. Specifically, the t-waves generated by the process $l \to t+s$ must pass through a layer where the process $l+s \to t$ can cause their reabsorption. However, in this case the process $l \to t+s$ requires low-frequency waves with wavevectors nearly parallel to those of the Langmuir waves while the process $l+s \to t$ requires nearly anti-parallel wavevectors. Hence no reabsorption would occur if both the Langmuir waves and the low-frequency waves were anisotropic and favouring the same direction.

The implications of amplification due to the process $l \to t+\sigma$ for $N^\sigma < N^l$ on the theories of various types of solar radio bursts requires further investigation.

7. Direct Emission

7.1. Historical remarks

Langmuir waves can be converted into escaping transverse waves due to inhomogeneities in the plasma. From an historical viewpoint this mechanism was proposed well before scattering by ions (Shklovsky, 1946; Martyn, 1947; and the review by Pawsey and Smerd, 1953; Martyn (1947) also argued that the emission should be polarized in the sense of the o-mode). The first attempt at a quantitative treatment was given by Field (1956). The mechanism was generally disregarded following Ginzburg and Zheleznyakov's (1959) unfavourable comparison between it and scattering by ions. However Denisse (1960) still invoked it in another connection. I argue elsewhere (Melrose, 1979c) that Ginzburg and Zheleznyakov's unfavourable comparison of the two mechanisms is based on an underestimate of the efficiency of direct conversion. The underestimate is due to their assumption that the appropriate density gradient is the mean density gradient in the corona, whereas local gradients can be much more effective.

7.2. Quantitative description of the conversion

The coupling between Langmuir waves and o-mode waves is a two stage process: propagation of the Langmuir waves to the coupling point and tunneling through a region of evanescence into the o-mode. The coupling point is illustrated in Figure 1 on the refractive index curves. A more informative, but less familiar description is in terms of solutions of the Booker quartic equation in which the component of $\mathbf{k}$ orthogonal to the density gradient is chosen as one of the independent variables, thereby incorporating Snell's law (Figure 3).

Langmuir waves propagating in the direction of decreasing plasma density have increasing k' and are ultimately Landau damped. Langmuir waves propagating in the direction of increasing n_e have k' decreasing and when k' becomes less than about ω_p/c the waves evolve into z-mode waves (Melrose, 1976). Waves propagating

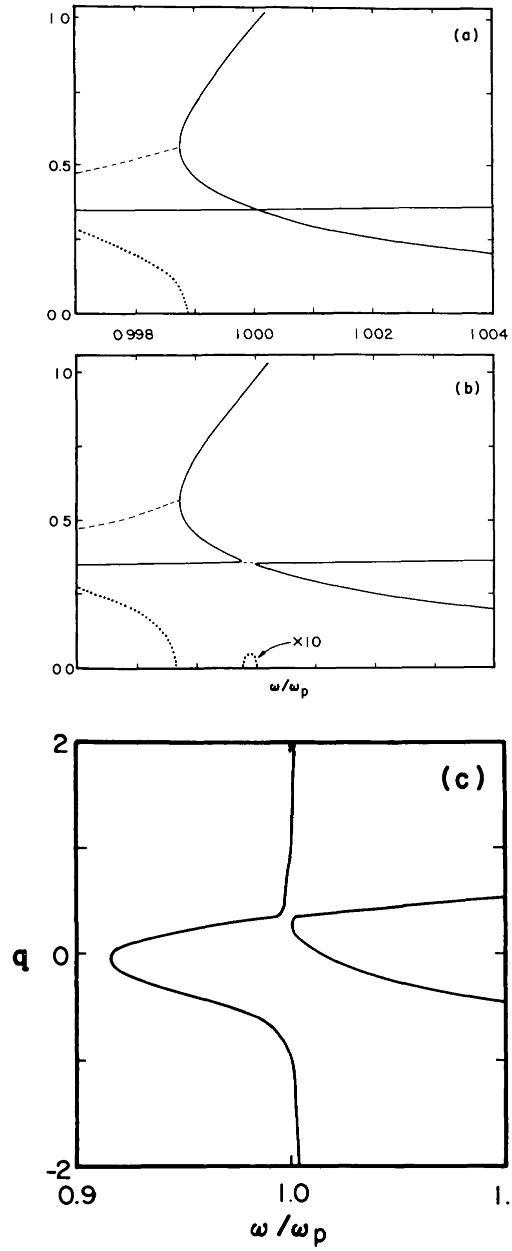

Fig. 3 (a–c). Solutions of the Booker quartic equation for $q = n \cos \rho$, where ρ is the angle between **k** and the direction of the density gradient, as a function of ω/ω_p are plotted for $\Omega_e/\omega_p = 0.2$ and for the magnetic field in the same plane as **k** and the density gradient and at an angle $\psi = 30°$ to the latter. Solid curves denote real solutions and dashed and dotted curves are real and imaginary parts, respectively, of complex solutions. (a) For $r = r_0$, with $r = n \sin \rho$ and $r_0 = \{\omega_p/(\omega_p + \Omega_e)\}^{1/2} \sin \psi$: the coupling point is where the two curves cross. (b) For $r = 0.99\, r_0$: the two curves do not cross but there are two oppositely directed 'tongues' in the z-mode and the o-mode on either side of the coupling point. (c) For $r = 0.9\, r_0$; the right-pointing 'tongue' in the z-mode has disappeared; the complex solutions are not plotted. On a larger scale (a) and (b) are similar to (c).

sufficiently nearly along the direction of increasing n_e move along a path which can be inferred from Figure 3: in order to approach the coupling point the Langmuir waves entering the figure from the top must move down the curve (decreasing ω/ω_p and hence increasing ω_p) until they are reflected and start moving towards decreasing n_e along the 'tongue' in the z-mode curve. At the tip of this 'tongue' the waves either reflect again or tunnel through into the o-mode. The waves which are reflected are subsequently reflected again and ultimately propagate towards decreasing plasma densities and are Landau damped.

Tunneling into the o-mode is effective provided the imaginary part of the wavenumber (in the region of evanescence between the 'tongue' in the z-mode and the o-mode) times the spatial distance over which the waves are evanescent is less than about unity. Otherwise the waves are reflected. Hence by estimating the wave properties in the 'tongue' region one may treat the coupling semi-quantitatively (Melrose, 1979c).

Coupling to the x-mode (which is way off Figure 3 to the right) is possible in principle but is entirely negligible for parameters appropriate to the solar corona. Hence the escaping radiation is entirely in the o-mode.

7.3. The efficiency of conversion

The result used by Ginzburg and Zheleznyakov (1959) in their discussion of direct conversion is one derived earlier by Zheleznyakov (cf. his book 1970, p. 385). Under more general conditions I obtained essentially the same result (Melrose, 1979c).

Let Langmuir waves with phase speed v_ϕ be confined to a cone of solid angle $\Delta\Omega \leqslant 4\pi$) which includes the direction of increasing plasma density. The average efficiency of conversion Q_{av} is identified as the ratio of the cone $\Delta\Omega_c$, of waves which can tunnel through the evanescent region, to the total solid angle $\Delta\Omega$ filled by the Langmuir waves. Detailed calculations give, to within a factor of order unity,

$$Q_{\mathrm{av}} \approx \frac{v_\phi^2}{c\omega_p L_N \Delta\Omega}\left(\frac{\omega_p + \Omega_e}{\Omega_e}\right)^{1/2} . \tag{45}$$

7.4. Numerical estimates

For $\omega_p/2\pi \approx 100$ MHz, $v_\phi \approx c/3$, and (from Melrose *et al.*, 1978) $\Delta\Omega \approx 0.1$ and $\Omega_e/\omega_p \approx \frac{1}{5}$, (45) implies $Q_{\mathrm{av}} \approx 10^{-3}/L_N$ with L_N in kilometers. For the average density gradient in the corona, e.g. for $L_N = 10^5$ km, one has $Q_{\mathrm{av}} \approx 10^{-8}$. Ginzburg and Zheleznyakov (1959) obtained a slightly smaller value due to a slightly different choice of parameters.

Note however that if there are small-scale density gradients, e.g. $L_N \approx 10\text{–}10^2$ km, then the efficiency of conversion becomes $Q_{\mathrm{av}} \approx 10^{-4}\text{–}10^{-5}$. Such efficiencies would be significant.

7.5. Possible significance of diirect conversion

I do not intend to argue that direct conversion is important in solar radio bursts, but it does seem that the mechanism deserves more serious attention than it has been given. In this connection I would like to make two remarks. The first is that on the basis of radio data the suggestion has been made that the corona is inhomogeneous in various ways, e.g. Steinberg *et al.* (1971), Riddle (1972, 1974), Melrose (1975), Bourgeret and Steinberg (1977), and Duncan (1979). Although opinions may differ on this point, it seems possible that the corona may be inhomogeneous on a scale $L_N \approx 10-100$ km. The second point is that in attempting to formulate a model based on the 'standard' version it seems that we are forced to an inhomogeneous model involving clumpy Langmuir waves. Presumably such clumps would be associated with density inhomogeneities and direct conversion could occur due to this inhomogeneity itself. The conversion requires Langmuir waves propagating towards increasing density, and therefore requires that the clumps be low-density regions. Clumps formed by soliton collapse ('cavitons') would be low-density regions.

8. Second Harmonic Plasma Emission

Second harmonic plasma emission is attributed to the coalescence of two Langmuir waves into a transverse wave. The parametric equations (37a, b) are satisfied with the + sign for any coalescence process. A quantitative treatment of second harmonic emission is outlined in Appendix D. In the discussion here I shall concentrate on the similarities and differences between fundamental and second harmonic plasma emission.

8.1. The analogy with Thomson scattering

Second harmonic emission is of the same order of nonlinearity in plasma theory as is fundamental plasma emission. Consequently it must be possible (on dimensional grounds) to describe it in a form analogous to the form (11) for Thomson-like scattering. Assuming the Langmuir waves are isotropic and that the 'head-on' approximation is valid (see below) one finds

$$\begin{aligned}\frac{\mathrm{d}W^t}{\mathrm{d}l} &= \frac{6}{5}\frac{\sigma_T n_e}{m_e c^2}\int \frac{\mathrm{d}k' k'^2}{2\pi^2}\{\kappa T^l(k')\}^2 \\ &= \frac{6}{5}\sigma_T n_e W^l\left(\frac{\kappa T^l}{m_e c^2}\right),\end{aligned} \tag{46}$$

where in the second form T^l is assumed independent of the wavenumber k' of the Langmuir waves.

Comparison of (46) and (11) shows that generation of the second harmonic is more efficient than generation of the fundamental due to scattering by ions for

$$T^l \gtrsim m_e c^2/\kappa = 6\times 10^9\ \mathrm{K} \tag{47}$$

or for

$$W^l \gtrsim n_e \kappa T_e \frac{r_0 \omega_p c^4}{V_e^2 v_\phi^3}, \tag{48}$$

where r_0 is the classical radius of the electron, V_e is the thermal speed of electrons and where T^l is assumed constant over a range $\Delta k' \approx k' = \omega_p/v_\phi$. The value (48) is relatively modest, e.g. it implies $W^l \gtrsim 10^{-9} n_e \kappa T_e$ for plausible values in the solar corona. Hence, on the basis of this comparison one would always expect the second harmonic to dominate the fundamental (e.g. Melrose, 1970b). That this is not so is a further argument that the 'standard' version of fundamental plasma emission in its simplest form is not efficient enough to account for the observations.

8.2. Saturation

The second harmonic saturates at $T^t \approx T^l$ for isotropic Langmuir waves (Melrose, 1970a). If the waves are not isotropic but involve a primary forward component with effective temperature T^l and a secondary backward component with effective temperature $T^{l'} \ll T^l$, then saturation occurs at $T^t \approx 2T^{l'}$.

One can apply an argument analogous to that leading to (42) to the second harmonic. Using (46), one finds the second harmonic saturates at $T^t = T^l$ for

$$\frac{\kappa T^l}{m_e c^2} \gtrsim 5\sqrt{3}\,\frac{v_\phi^3}{\omega_p^2 L_N r_0 c}\,. \tag{49}$$

For $v_\phi = c/3$, $\omega_p/2\pi = 100$ MHz, and $L_N = 10^5$ km, (49) implies saturation for $T^l \gtrsim 10^{15}$ K in the corona.

The foregoing result is a significant one. For all second harmonic emission one must have $T_b < T^t < T^l$, where T_b is the observed brightness temperature. If the actual size of the source is much less than its apparent size, due to coronal scattering for example, then one has $T_b \ll T^t$. The foregoing result implies that if T^t is greater than about 10^{15} K, then one has $T^t \approx T^l$. The brightest type III bursts have $T_b \approx 10^{12}$ K (Dulk and Suzuki, 1979). If the effective radiating area is much less than about 10^{-3} of the apparent area of the source, as implied by models involving clumpy Langmuir waves, then one would have $T^t \approx T^l$ for each clump.

8.3. Polarization

The polarization of second harmonic plasma emission was derived by Melrose and Sy (1972) and the theory and its applications to type III bursts were discussed by Melrose *et al.* (1978). The polarization is expected to be in the sense of the *o*-mode, i.e. in the same sense as the fundamental, only for Langmuir waves highly collimated along the magnetic field. Observed harmonic pairs of type III bursts are polarized in the same sense (Suzuki and Sheridan, 1977; Dulk and Suzuki, 1979) and this implies that the Langmuir waves are highly collimated at least in those type III bursts which show harmonic structure. The degree of polarization of the second harmonic provides probably the best available estimate of the magnetic field strength in the corona (Melrose *et al.*, 1978; Dulk and McLean, 1978).

For Langmuir waves which are not highly collimated, the expected polarization is in the sense of the *x*-mode. This is the same sense as for gyro-synchrotron emission and adds to the difficulty of distinguishing between gyro-synchrotron and second harmonic plasma emission for type IV emission, e.g. for the flare continuum.

Note added in proof: It has been pointed out to me by Dr E. Ya. Zlotnik that in the formula derived by Melrose and Sy (1972) for the polarization of the second harmonic certain terms were unjustifiably neglected. Preliminary calculations (by G. A. Dulk and D. Gary) based on corrected formulae give results which are qualitatively similar to those found by Melrose *et al.* (1978), but are smaller in the important relevant range by a factor ≈ 3.

8.4. The 'head-on' approximation

The assumption that the wavenumber of the transverse waves ($k = \sqrt{3}\,\omega_p/c$ at the second harmonic) is negligible in comparison with those of the Langmuir waves ($k' = \omega_p/v_\phi$) implies, through (37b), that the two Langmuir waves must have equal and opposite wavevectors. That is, the coalescence then occurs for Langmuir waves meeting head-on. The emission pattern is then quadrupolar, i.e. like $\sin^2\theta\cos^2\theta$ where $\theta = 0$ and $\theta \doteq \pi$ correspond to the direction of the two Langmuir waves.

The validity of the 'head-on' approximation has been investigated by Melrose and Stenhouse (1979). It is a relatively poor approximation for $v_\phi \approx c/3$, which is thought relevant to type III bursts; for $v_\phi \approx c/3$ it leads to an overestimate of the power at the second harmonic by a factor of two or so. The approximation becomes rapidly worse with increasing $v_\phi \gtrsim c/3$. For $v_\phi \approx c$ it leads to an overestimate of the power radiated by about four orders of magnitude.

Second harmonic emission is ineffective for $v_\phi \gtrsim c$. Even for $v_\phi \approx c/3$ it should be used only for semi-quantitative purposes.

8.5. Alternative second harmonic mechanisms

There are no plausible alternative emission mechanisms for second harmonic plasma emission. One possible mechanism is coalescence of a fundamental transverse wave with a Langmuir wave. This is possible only for Langmuir waves with small k', specifically with $k' \approx \sqrt{3}\,\omega_p/c$. Another possible alternative is that the second harmonic can be amplified in the presence of intense low-frequency turbulence (Kamilov *et al.*, 1974). However, the conditions for amplification to occur are extreme and one would not expect them to be satisfied over a sufficient volume of the corona to produce detectable emission. Under expected conditions amplification of the second harmonic is impossible; it is inconsistent with the basic equations written down in Appendix D.

8.6. Discussion

On the one hand second harmonic emission is simpler to treat than fundamental plasma emission in the sense that the process itself involves fewer complications. On the other hand however, the generation of the second harmonic involves a secondary distribution of Langmuir waves and hence involves, in the case of type III emission, an additional step (scattering $l \to l$) in the generation process. Thus, although quantitative estimates of second harmonic emission could be made with confidence if the spectrum of Langmuir waves were well known, in practice quantitative estimates are subject to considerable uncertainty.

I have attempted to treat second harmonic type III emission semi-quantitatively elsewhere (Melrose, 1979b, p. 229), and found that some quantitative difficulties are encountered. These are not as severe as for the fundamental and may be overcome by appealing to a clumpy distribution of Langmuir waves. Nevertheless it requires special assumptions to account for the brightest type III bursts observed.

9. Gyro-Synchrotron Emission

Gyro-synchrotron emission is the accepted mechanism for microwave bursts. It has also been the accepted mechanism for moving type IV bursts, but this has been questioned recently for reasons discussed below. Most treatments of gyro-synchrotron have been numerical. Our current understanding is based partly on numerical results and partly on extensions from the non-relativistic ('cyclotron') and ultra-relativistic ('synchrotron') into the intermediate ('gyro-synchrotron') range. Specific numerical calculations have been presented by Ramaty (1969), Holt and Ramaty (1969), Takakura and Scalise (1970), Ramaty and Petrosian (1972), and Takakura (1972) for microwave bursts and by Schmahl (1972), Dulk (1973), and Robinson (1974, 1977) for moving type IV bursts. Here I shall concentrate on approximate analytic methods and the application to moving type IV bursts.

9.1. The cyclotron and synchrotron approximations

A general theory for gyromagnetic emission is presented in Appendix E. Emission from a single electron at the sth harmonic is at the Doppler-shifted gyrofrequency

$$\omega = \frac{s\Omega_e(1-\beta^2)^{1/2}}{1-n\beta\cos\alpha\cos\theta}, \tag{50}$$

where $n(\equiv n_\sigma(\omega,\theta)$ for waves in the mode $\sigma)$ is the refractive index, θ is the wave angle, βc is the speed of the electron and α is its pitch angle. The only waves which can escape directly from a plasma have $n\leqslant 1$, and emission at $s\leqslant 0$ is then not possible according to (50). The emissivity at the sth harmonic involves the Bessel functions $J_s(sx)$, and its derivative $J_s'(sx)$, with

$$x = \frac{n\beta\sin\alpha\sin\theta}{1-n\beta\cos\alpha\cos\theta}. \tag{51}$$

One has $0\leqslant x<1$.

In the non-relativistic approximation (50) implies $\omega = s\Omega_e(1+n\beta\cos\alpha\cos\theta)$. Thus emission at the sth harmonic from a distribution of electrons is centered on $\omega = s\Omega_e$ to $0(\beta^2)$. One has $x\ll 1$ and then the power series expansions of the Bessel functions converge rapidly. Retaining only the leading terms give

$$J_s(sx)\approx\frac{1}{s!}\left(\frac{sx}{2}\right)^s,\qquad J_s'(sx)\approx\frac{J_s(sx)}{x}. \tag{52a, b}$$

The angular distribution at the sth harmonic is that of a 2^s-multipole, i.e. the angular pattern is as $\cos^2\theta(\sin\theta)^{2(s-1)}$ with $s = 1, 2, 3, \ldots$ being dipolar, quadrupolar, octupolar, ...

In the ultrarelativistic limit ($\gamma = (1-\beta^2)^{-1/2} \gg 1$) the angular pattern is sharply peaked about $\theta = \alpha$. The emission at the sth harmonic is then centered on

$$\omega = \frac{s\Omega_e}{\gamma \sin^2\theta}. \tag{53}$$

In this case we have $1 - x \ll 1$ and the Bessel functions may be approximated by Airy functions. The basic approximations are

$$J_s(sx) \approx \frac{1}{\pi\sqrt{3}y} K_{1/3}(z), \qquad J'_s(sx) \approx \frac{1}{\pi\sqrt{3}y^2} K_{2/3}(z), \tag{54a, b}$$

with

$$y := \left\{\frac{x}{2(1-x)}\right\}^{1/2}, \qquad z = \frac{sx}{3y^3}.$$

The emission from electrons with a range of pitch angles occurs over a broad band centred on about $\omega \approx 0.3\omega_c$, with

$$\omega_c = \frac{3}{2}\Omega_e \gamma^2 \sin\theta. \tag{55}$$

Thus a range of harmonics centred on $s \approx (\gamma \sin\theta)^3$ is involved.

One can carry out the integrals over a distribution of electrons in certain special cases. These include a non-relativistic Maxwellian and its generalization to a non-relativistic bi-Maxwellian streaming distribution (e.g. Melrose, 1973). In the ultra-relativistic limit one can integrate over a power-law energy distribution explicitly (e.g. Ginzburg and Syrovatskii 1965).

9.2. The Carlini Approximation

The Carlini approximation to the Bessel functions was used by Trubnikov (1958) to treat the intermediate case relevant to gyro-synchrotron emission. The Carlini approximation is (e.g. Watson, 1944, p. 6)

$$J_s(sx) \approx \frac{x^s \exp\{s(1-x^2)^{1/2}\}}{(2\pi s)^{1/2}(1-x^2)^{1/4}\{1+(1-x^2)^{1/2}\}^s} \tag{56a}$$

$$J'_s(sx) \approx \frac{(1-x^2)^{3/4}}{x} J_s(sx). \tag{56b}$$

Trubnikov applied this approximation to the special case $\theta = \pi/2, \alpha = \pi/2$ for emission *in vacuo*. The resulting emissivity is written down in Appendix E below. He also used the Carlini approximation to treat emission by a mildly relativistic Maxwellian (Section 9.4 below).

9.3. Wild and Hill's approximation

Wild and Hill (1971) developed approximate expressions for the Bessel functions. Their expressions are related to the Carlini approximation but are of much wider validity. Their expressions are

$$J_s(sx) \approx \frac{e^{-s/2s_1}}{(2\pi s)^{1/2}} \left(\frac{3}{2s_2} + \frac{a}{s}\right)^{-1/6}, \tag{57a}$$

$$J'_s(sx) \approx \frac{e^{-s/2s_1}}{(2\pi s)^{1/2}} \frac{1}{x} \left(\frac{3}{2s_2} + \frac{b}{s}\right)^{1/6} \left(1 - \frac{1}{5s^{2/3}}\right), \tag{57b}$$

with

$$\frac{1}{2s_1} := \ln\{1 + (1 - x^2)^{1/2}\} - \ln x - (1 - x^2)^{1/2},$$
$$s_2 = \tfrac{3}{2}(1 - x^2)^{-3/2}, \tag{58}$$
$$a = 0.503\,30, \qquad b = 1.193\,00.$$

These approximations are useful for numerical treatment of gyro-synchrotron emission (e.g. Tarnstrom, 1976) and they may also be used to treat the non-relativistic and ultrarelativistic limits.

9.4. Analytic results for thermal distribution

Analytic approximations are available for gyro-synchrotron emission from a Maxwellian distribution of electrons. When relativistic effects are ignored the integrals may be evaluated explicitly for emission at an arbitrary direction in a plasma. When relativistic effects are included the only relevant results are two approximations derived by Trubnikov (1958) using the Carlini approximation. One of Trubnikov's approximations overlaps the non-relativistic approximation. Dulk *et al.* (1979) have used this fact to write down an approximate form for the absorption coefficient (Appendix E). Their result applies for

$$\left(\frac{\omega}{\Omega_e}\right)^2 \beta_0^2 \ll 1, \qquad \beta_0^2 = \frac{\kappa T_e}{m_e c^2} \tag{59}$$

and is, for $|(\pi/2) - \theta| \ll \Omega_e/2\omega \ll 1$,

$$\gamma^\sigma(s, \theta) = \frac{\omega_p^2}{\Omega_e} \left(\frac{\pi \Omega_e}{2\omega}\right)^{1/2} \frac{1}{\beta_0^2 \sin^2\theta} \left(\frac{e\beta_0^2 \omega \sin^2\theta}{2\Omega_e}\right)^{\omega/\Omega_e} (1 - \sigma|\cos\theta|)^2. \tag{60a}$$

and, for $\theta = \pi/2$,

$$\gamma^x(x, \theta) = \frac{\omega_p^2}{\Omega_e} \left(\frac{\pi \Omega_e}{2\omega}\right)^{1/2} \frac{1}{\beta_0^2} \left(\frac{e}{2} \beta_0^2 \frac{\omega}{\Omega_e}\right)^{\omega/\Omega_e}. \tag{60b}$$

$$\gamma^o(s, \theta) = \beta_0^2 \gamma^x(s, \theta). \tag{60c}$$

(Note $e = \exp[1]$ in (60a, b).) In (60a) one has $\sigma = 1$ for the o-mode and $\sigma = -1$ for the x-mode.

Trubnikov's other approximation is for

$$\rho := \frac{9}{2}\frac{\omega}{\Omega_e}\beta_0^2 \gg 1 \tag{61}$$

and is

$$\gamma^x(\omega, \pi/2) = \frac{\omega_p^2}{\Omega_e}\frac{3\pi^{1/2}}{e\beta_0\rho}\exp\left[-\frac{1}{\beta_0^2}\left\{\rho^{1/3} - 1 + \frac{9}{20\rho^{1/3}}\right\}\right] \tag{62a}$$

$$\gamma^o(\omega, \pi/2) = \frac{\beta_0^2}{\rho^{1/3}}\gamma^x(\omega, \pi/2). \tag{62b}$$

These approximations overlap (Dulk *et al.*, 1979).

Attempts (by S. M. White and myself) to use these techniques to treat gyro-synchrotron emission and absorption analytically for a power-law distribution have proved unsuccessful so far.

9.5. The Razin Effect

Gyro-synchrotron emission, like any other emission, cannot escape from the corona if its frequency is less than the plasma frequency. For frequencies between ω_p and about $2\omega_p$ gyro-synchrotron emission is suppressed due to the difference of the refractive index from unity. This effect is called 'medium suppression' or the 'Razin effect' by analogy with its counterpart for relativistic electrons.

In the non-relativistic limit, where emission at the sth harmonic is 2^s-electric multipole emission, the refractive index appears to the power n^{2s+1}, i.e.

$$\eta_s(\omega, \theta) \propto \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{s+1/2}. \tag{63}$$

Thus, for example, for $s = 5$ and $\omega = 1.5\omega_p$ the suppression over emission *in vacuo* is by a factor 4×10^{-2}. Wild and Hill (1971) presented an analytic treatment of the Razin effect which is of wider validity than the result (63).

Semi-quantitatively, gyromagnetic emission from an individual electron in a plasma is suppressed for

$$\omega \lesssim (1 + \gamma)\omega_p, \tag{64}$$

and the emission from a distribution of electrons is suppressed for

$$\omega \lesssim \begin{cases} 2\omega_p & \text{for} \quad \gamma \approx 1 \\ \dfrac{3}{2}\dfrac{\omega_p^2}{\Omega_e \sin\theta} & \text{for} \quad \gamma \gg 1. \end{cases} \tag{65}$$

The suppression at $\omega \lesssim 2\omega_p$ make it difficult to distinguish between gyro-synchrotron and second harmonic plasma emission for continuum sources. This difficulty is exacerbated by the fact that one expects the polarization to be x-mode in both cases. Only when the degree of polarization is high ($\gtrsim 50\%$) is gyro-synchrotron emission implied.

9.6. Self-absorption in moving type IV bursts

Self-absorption (in the absence of maser action) limits the brightness temperature of the source to less than about the energy of the emitting electrons. Hence, for gyro-synchrotron emission from 100 keV electron one would expect a low-frequency turnover at the frequency where the brightness temperature reaches about 3×10^8 K. This explanation seemed quite satisfactory based on data prior to 1978 (e.g. Dulk, 1973). However, over the last year or so several moving type IV bursts observed at 43 MHz with the Culgoora radioheliograph have shown high brightness temperature, Stewart *et al.* (1978) estimated $\gtrsim 10^{10}$ K for one burst. G. A. Dulk (private communication) has re-estimated the brightness temperature of that burst and found it more likely to be 3 to 5×10^9 K at maximum; he found only one or two other bursts with $T_b \approx 10^9$ K. It does not seem possible to explain $T_b \approx 10^{10}$ K in terms of gyro-synchrotron emission from $\gtrsim 1$ MeV electrons because, in later phases of the bursts, the high degrees of polarization ($\gtrsim 0.5$) observed can be explained only by emission from electrons $\ll 1$ MeV.

One suggestion for the interpretation of this emission is that a maser action in gyro-synchrotron emission is involved (Melrose and White, 1978). A more likely explanation is that in the early phase, when the brightness temperature is high and the polarization is low, the emission is second harmonic plasma emission. Then as the burst fades the gyrosynchrotron emission becomes dominant and causes the high degree of polarization and the frequency turnover in the late phase.

10. Discussion and Conclusions

The quantitative theory of meter $-\lambda$ solar radio bursts is in an unsatisfactory state. The only cases in which the observations are not obviously unexplained by theory are second harmonic type III emission and moving type IV emission, and even for these cases the simplest theories seem inadequate (Sections 8.6 and 9.5).

For fundamental plasma emission the quantitative difficulties are quite severe for the 'standard' version of scattering by thermal ions. There are also qualitative difficulties. First, it is not clear whether one should retain the 'standard' version and invoke the special conditions required for it to account for the high brightness temperatures observed, or abandon it. Second, if one abandons the 'standard' version it is not clear which of several alternatives to choose. Coalescence with low-frequency turbulence is a more complicated process in that it requires two distinct nonthermal wave distributions to be excited in the source region, and direct conversion requires local inhomogeneities of a specific kind throughout the source region. Third, for type III bursts it is not clear (a) why the fundamental is present only around decametric frequencies and is absent at high frequencies ($\gtrsim 100$ MHz) and at low frequencies ($\lesssim 1$ MHz), (b) why the polarization of the fundamental is often rather low ($\approx 35\%$) whereas simply theory predicts 100% (cf. Melrose, 1975), and (c) what causes the directivity of fundamental emission so that it is often not seen, i.e. in structureless bursts (Dulk and Suzuki, 1979). Fourth, if type I emission is fundamental plasma emission then it is not clear why its properties are so different from those of fundamental type II and type III emission.

If the fundamental is due to coalescence with low frequency waves, then for $T^l \gtrsim 10^{15}$ K both the fundamental and second harmonic should both saturate at $T^t \approx T^l$ (Sections 6.1 and 8.2). If the ratio η of the actual radiating surface area (e.g. the sum of the surface areas of the clumps in a clumpy model) to apparent surface area is less than about 10^{-3}, then one encounters no energetic difficulties in the theory of type III bursts (e.g. Melrose, 1979b, p. 233). One would then expect fundamental and second harmonic emission of comparable brightness with $T_b = \eta T^t = \eta 10^{15}\,\mathrm{K} \lesssim 10^{12}$ K. In this way one can account for the approximate equality of the observed brightness temperatures of harmonic pairs.

In my opinion some basic ingredients are still missing in our current ideas on the emission of meter $-\lambda$ solar radio bursts. Although further work on the emission mechanisms is desirable, it seems to me that substantial progress requires new data. The observation that Langmuir waves are clumpy in the interplanetary medium (Gurnett and Anderson, 1977) is an obvious example of the type of additional data which can alter our views on the emission mechanisms.

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Appendix A: Nonlinear Scattering by Thermal Ions

The kinetic equation for scattering $l \leftrightarrow t$ due to an arbitrary distribution of particles is (Melrose, 1979a, p. 172)

$$\frac{\mathrm{d}N^t(\mathbf{k})}{\mathrm{d}t} = \int \mathrm{d}^3\mathbf{p} \int \frac{\mathrm{d}^3\mathbf{k}'}{(2\pi)^3}\, w^{tl}(\mathbf{p},\mathbf{k},\mathbf{k}') \Big[f(\mathbf{p})\{N^l(\mathbf{k}') - N^t(\mathbf{k})\} + + \hbar(\mathbf{k}'-\mathbf{k})\cdot \frac{\partial f(\mathbf{p})}{\partial \mathbf{p}} N^l(\mathbf{k}')N^t(\mathbf{k}) \Big]. \tag{A.1}$$

For scattering by thermal ions one has (Melrose, 1979b, p. 215)

$$\sum \int \mathrm{d}^3\mathbf{p} f(\mathbf{p}) w^{tl}(\mathbf{p},\mathbf{k},\mathbf{k}') = \sum_i \frac{(2\pi)^{5/2} e^4 Z_i^2 n_i}{m_e^2 |\mathbf{k}-\mathbf{k}'| V_i} \left(\frac{T_i}{T_e + T_i}\right)^2 \sin^2\theta \times \times \exp\left[-\frac{\{\omega^t(\mathbf{k}) - \omega^l(\mathbf{k}')\}^2}{2|\mathbf{k}-\mathbf{k}'|^2 V_i^2} \right], \tag{A.2}$$

where the sums are over all species of ion and where θ is the angle between $\mathbf{k}$ and $\mathbf{k}'$.

Equation (11) may be obtained from (A.1) by neglecting the term $-N^t(\mathbf{k})$, which describes spontaneous scattering $t\to l$, and the final term, which describes induced scattering, and evaluating

$$\frac{\mathrm{d}W^l}{\mathrm{d}s}=\int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\hbar\omega^t(\mathbf{k})\frac{\mathrm{d}N^t(k)}{\mathrm{d}t}\Big/\frac{\partial\omega^t(\mathbf{k})}{\partial k}. \qquad (A.3)$$

Equations (8), (13a, b), (14), (15a, b), and $\partial\omega/\partial k=cn$ are used and T_e and T_i are set equal as was done in deriving (11).

Equation (18) with (19) or (20) and with (26) is an approximation to that obtained from (A.1) by multiplying by $\hbar\omega^t(\mathbf{k})$, cf. (16a), and evaluating the right-hand side for a specific distribution of Langmuir waves. For an isotropic distribution confined to a narrow range Δk of wavenumbers about k_0, i.e. for

$$T^l(\mathbf{k}')=\begin{cases}\text{constant} & \text{for}\quad k_0-\tfrac{1}{2}\Delta k<k'<k_0+\tfrac{1}{2}\Delta k\\ 0 & \text{otherwise},\end{cases} \qquad (A.4)$$

one finds, for $\Delta k \ll V_i v_\phi/3V_e^2$ with $v_\phi := \omega_p/k_0$,

$$\frac{\mathrm{d}T^t}{\mathrm{d}t}\approx\left(\frac{\pi}{2}\right)^{1/2}\frac{\omega_p v_\phi W^l}{12n_eV_i}\left[1-\left\{\frac{\omega-\omega_p}{\omega_p}-\frac{3V_e^2}{2v_\phi^2}\right\}\frac{T^t}{T_i}\right]. \qquad (A.5)$$

Appendix B: Coalescence of Langmuir with Low-Frequency Waves

The kinetic equations for the processes $l+\sigma\to t$ and $l\to\sigma+t$, where σ denotes a low-frequency mode, are (Melrose, 1979a, p. 173)

$$\frac{\mathrm{d}N^t(\mathbf{k})}{\mathrm{d}t}=\int\frac{\mathrm{d}^3\mathbf{k}'}{(2\pi)^3}\int\frac{\mathrm{d}^3\mathbf{k}''}{(2\pi)^3}u_+^{tl\sigma}(\mathbf{k},\mathbf{k}',\mathbf{k}'')[N^l(\mathbf{k}')N^\sigma(\mathbf{k}'')- -N^t(\mathbf{k})\{N^l(\mathbf{k}')+N^\sigma(\mathbf{k}'')\}] \qquad (B.1)$$

and

$$\frac{\mathrm{d}N^t(\mathbf{k})}{\mathrm{d}t}=\int\frac{\mathrm{d}^3\mathbf{k}'}{(2\pi)^3}\int\frac{\mathrm{d}^3\mathbf{k}''}{(2\pi)^3}u_-^{tl\sigma}(\mathbf{k},\mathbf{k}'\,\mathbf{k}'')[N^l(\mathbf{k}')N^\sigma(\mathbf{k}'')+ +N^t(\mathbf{k})\{N^l(\mathbf{k}')-N^\sigma(\mathbf{k}'')\}] \qquad (B.2)$$

with

$$u_\pm^{tl\sigma}(\mathbf{k},\mathbf{k}',\mathbf{k}'')=\frac{(2\pi)^5}{2}\frac{\hbar e^2}{m_e^2}\frac{\omega^\sigma(\mathbf{k}'')}{k''^2V_e^4}\left(\frac{\omega_p}{k'}\right)^2|\mathbf{e}\cdot\mathbf{k}'|^2R_{\mathrm{E}}^\sigma(\mathbf{k}'')\delta^3(\mathbf{k}-\mathbf{k}'\mp\mathbf{k}'')\times \times\delta(\omega^t(\mathbf{k})-\omega^l(\mathbf{k}')\mp\omega^\sigma(\mathbf{k}'')). \qquad (B.3)$$

The derivation of (B.3), in which the low-frequency waves are assumed longitudinal, is analogous to that given by Melrose (1979b, p. 172) for ion-sound waves. For ion sound waves one has

$$R^s_{\mathrm{E}}(\mathbf{k}'') = \frac{1}{2}\left(\frac{\omega^s(\mathbf{k}'')}{\omega_{pi}}\right)^2 \tag{B.4}$$

with ($v_s := (\omega_{pi}/\omega_p)V_e$)

$$\omega^s(\mathbf{k}'') = \frac{k''v_s}{(1+k''^2\lambda^2_{\mathrm{D}e})^{1/2}} \approx k''v_s \tag{B.5}$$

in the range of relevance here.

The result (38) follows from (B.1) and (B.2) by multiplying by $\hbar\omega^t(\mathbf{k})$ and dividing by $\partial\omega^t(\mathbf{k})/\partial k$ and then integrating over $\mathrm{d}^3\mathbf{k}/(2\pi)^3$. For $N^s \gg N^l$ the contributions from (B.1) and (B.2) are equal, and the negative terms on the right-hand sides limit N^t to less than N^l, i.e. $T^t < T^l$ in accord with (39).

For other longitudinal waves one has

$$R^\sigma_{\mathrm{E}}(\mathbf{k}'') = 1\bigg/\left[\frac{\partial}{\partial\omega''}\{\omega''\varepsilon^l(\mathbf{k}'')\}\right]_{\varepsilon^l(\mathbf{k}'',\,\omega)=0} \tag{B.6}$$

In the case of lower-hybrid waves (B.6) gives

$$R^{\mathrm{LH}}_{\mathrm{E}}(\mathbf{k}'') = \frac{1}{2}\left(\frac{\omega^{\mathrm{LH}}(\mathbf{k}'')}{\omega_{pi}}\right)^2. \tag{B.7}$$

On carrying through the derivation in this case, the result (38) follows with T^s replaced by T^{LH}. For the other modes mentioned in the text (ion-Bernstein, ion-cyclotron and drift waves) it is more difficult to satisfy (37b), e.g., ion-Bernstein waves have large k and are confined to nearly perpendicular propagation. Nevertheless, if (37b) can be satisfied then one would expect (38) to be valid to within a factor of order unity, which factor is actually $2R^\sigma_{\mathrm{E}}(\mathbf{k}'')\omega^2_{pi}/(\omega^\sigma(\mathbf{k}''))^2$.

Appendix C: Energy Density Required in Low-Frequency Waves

In deriving (43) the Langmuir waves and ion-sound waves have been assumed isotropic and confined to ranges $\Delta k' \approx k'$ and $\Delta k'' \approx k''$ with $k'' \approx k' \approx \omega_p/v_\phi$. Then one has, cf. (15a),

$$W^l \approx \left(\frac{\omega_p}{v_\phi}\right)^3 \frac{\kappa T^l}{2\pi^2} \tag{C.1}$$

and

$$W^s \approx \left(\frac{\omega_p}{v_\phi}\right)^3 \frac{\kappa T^s}{2\pi^2}. \tag{C.2}$$

The transverse waves are also assumed isotropic and confined to a range $\Delta k = \Delta\omega\omega_p/kc^2$. Then (15b) gives

$$W^t \approx \left(\frac{\omega_p}{c}\right)^3\left(\frac{\sqrt{3}\,V_e}{v_\phi}\right)\left(\frac{\Delta\omega}{\omega_p}\right)\frac{\kappa T^t}{2\pi^2}. \tag{C.3}$$

Transverse waves in a range $\Delta\omega$ can be generated only over a height range $(\Delta\omega/\omega_p)L_N/2$ in a smoothly varying corona. Over this height (38) implies that W^t builds up to

$$W^t \approx \sigma_T n_e W^l \frac{T^s}{T_e}\frac{\Delta\omega}{\omega_p}\frac{L_N}{2}. \tag{C.4}$$

However, (38) and (C.4) apply only for $T^t \ll T^l$ and when T^t approaches T^l the conversion process saturates. Using (C.1) and (C.3) one can determine the ratio W^t/W^l which corresponds to $T^t = T^l$, and then from (C.4) one can determine the lower limit and T^s/T_e required to achieve saturation. It is

$$\frac{T^s}{T_e} \gtrsim 3\sqrt{3}\frac{V_e v_\phi^2}{r_0 L_N \omega_p^2 c}. \tag{C.5}$$

Using (C.2), (C.5) implies (43).

Appendix D: Second Harmonic Emission

The kinetic equations for second harmonic plasma emission are

$$\frac{dN^t(\mathbf{k})}{dt} = \int\frac{d^3\mathbf{k}'}{(2\pi)^3}\int\frac{d^3\mathbf{k}''}{(2\pi)^3}u^{tll}(\mathbf{k},\mathbf{k}',\mathbf{k}'')[N^l(\mathbf{k}')N^l(\mathbf{k}'') - \\ - N^t(\mathbf{k})\{N^l(\mathbf{k}') + N^l(\mathbf{k}'')\}], \tag{D.1}$$

$$\frac{dN^l(\mathbf{k}')}{dt} = \int\frac{d^3\mathbf{k}}{(2\pi)^3}\int\frac{d^3\mathbf{k}''}{(2\pi)^3}u^{tll}(\mathbf{k},\mathbf{k}',\mathbf{k}'')\times \\ \times[N^t(\mathbf{k})\{N^l(\mathbf{k}') + N^l(\mathbf{k}'')\} - N^l(\mathbf{k}')N^l(\mathbf{k}'')]. \tag{D.2}$$

In the absence of a magnetic field we have

$$u^{tll}(\mathbf{k},\mathbf{k}',\mathbf{k}'') = \frac{(2\pi)^5}{16}\frac{\hbar e^2}{m_e^2\omega_p}\frac{(k'^2 - k''^2)^2}{k^2}|\boldsymbol{\kappa}'\times\boldsymbol{\kappa}''|^2\times \\ \times\delta^3(\mathbf{k}-\mathbf{k}'-\mathbf{k}'')\delta(\omega^t(\mathbf{k}) - \omega^l(\mathbf{k}') - \omega^l(\mathbf{k}'')), \tag{D.3}$$

$$\approx \frac{\sqrt{3}\,(2\pi)^5}{2}\frac{\hbar e^2\omega_p}{m_e^2 c^3}|\boldsymbol{\kappa}\cdot\boldsymbol{\kappa}'|^2|\boldsymbol{\kappa}\times\boldsymbol{\kappa}'|^2\delta^3(\mathbf{k}-\mathbf{k}'-\mathbf{k}'')\times \\ \times\delta\left(k - \sqrt{3}\frac{\omega_p}{c}\left\{1 + \frac{2k'^2V_e^2}{\omega_p^2}\right\}\right), \tag{D.3'}$$

where the approximate result (D.3′) applies in the 'head-on' approximation, and with

$$\boldsymbol{\kappa} = \mathbf{k}/k\,, \qquad \boldsymbol{\kappa}' = \mathbf{k}'/k'\,.$$

The expression for $u^{llt}(\mathbf{k}, \mathbf{k}', \mathbf{k}'')$ in the presence of a magnetic field was given by Melrose and Sy (1972).

Saturation of the process $l + l \to t$ occurs when the arguments in (D.1) and (D.2) vanish. In the 'head-on' approximation for $N^l(\mathbf{k}') = N^l(-\mathbf{k}')$ saturation occurs for $N^t(\mathbf{k}) = N^l(\mathbf{k}')/2$ and hence for $T^t(\mathbf{k}) = T^l(\mathbf{k}')$. For $N^l = N^{l'} + N^{l''}$ with $N^{l'}$ a forward distribution and $N^{l''}$ a backward distribution, and for $N^{l''} \ll N^{l'}$, saturation occurs for $N^t \approx N^{l''}$, i.e. $T^t \approx 2T^{l''}$.

The result (46) is obtained from (D.1) by inserting (D.3′), multiplying by $2\hbar\omega_p$ and integrating over $\mathrm{d}^3\mathbf{k}/(2\pi)^3$. The assumed isotropy allows one to carry out the angular integrals explicitly, so that $|\boldsymbol{\kappa} \cdot \boldsymbol{\kappa}'|^2 |\boldsymbol{\kappa} \times \boldsymbol{\kappa}'|^2$ is replaced by its average of $\frac{2}{15}$.

Appendix E: Gyromagnetic Emission

General formula for the emission and absorption of waves in the mode σ due to particles with charge $q(=: \epsilon\,|q|)$ mass m and distribution function $f(p_\perp, p_\parallel)$ are, e.g. Melrose (1979a, §§5.2 and 5.3).

$$\frac{\mathrm{d}N^\sigma(\mathbf{k})}{\mathrm{d}t} = \alpha^\sigma(\mathbf{k}) - \gamma^\sigma(\mathbf{k}) N^\sigma(\mathbf{k})\,, \tag{E.1}$$

with

$$\begin{bmatrix} \alpha^\sigma(\mathbf{k}) \\ \gamma^\sigma(\mathbf{k}) \end{bmatrix} = \sum_{s=-\infty}^{\infty} 2\pi \int_0^\infty \mathrm{d}p_\parallel \int_0^\infty \mathrm{d}p_\perp p_\perp w^\sigma(s, \mathbf{p}, \mathbf{k}) \times \\ \times \begin{bmatrix} f(p_\perp, p_\parallel) \\ -\hbar\left\{\dfrac{s\Omega}{v_\perp}\dfrac{\partial}{\partial p_\perp} + k_\parallel \dfrac{\partial}{\partial p_\parallel}\right\} f(p_\perp, p_\parallel) \end{bmatrix} \tag{E.2}$$

with

$$w^\sigma(s, \mathbf{p}, \mathbf{k}) = \frac{8\pi^2 q^2 R_{\mathrm{E}}(\mathbf{k})}{\hbar\omega^\sigma(\mathbf{k})} |\mathbf{e}^{\sigma *}(\mathbf{k}) \cdot \mathbf{V}(s, \mathbf{p}, \mathbf{k})|^2\, \delta\{\omega^\sigma(\mathbf{k}) - s\Omega - k_\parallel v_\parallel\}\,, \tag{E.3}$$

$$\mathbf{V}(s, \mathbf{p}, \mathbf{k}) = \left(v_\perp \frac{s}{z} J_s(z), -i\,\epsilon\, v_\perp J_s'(z), v_\parallel J_s(z)\right), \tag{E.4}$$

$$v_\perp = \frac{p_\perp}{\gamma m c}\,, \qquad v_\parallel = \frac{p_\parallel}{\gamma m c}\,, \qquad z = \frac{k_\perp v_\perp}{\Omega}\,, \qquad \Omega = \frac{|q| B}{\gamma m c}\,. \tag{E.5}$$

and where the magnetic field is along the 3-axis with

$$\mathbf{k} = (k_\perp, 0, k_\parallel)\,. \tag{E.6}$$

For the magnetoionic waves ($\sigma = +1$ for o-mode, $\sigma = -1$ for x-mode) one has, e.g. Melrose (1979b, §12.1),

$$\mathbf{e}^{\sigma} = \frac{(K_\sigma \sin\theta + T_\sigma \cos\theta,\, i,\, K_\sigma \cos\theta - T_\sigma \sin\theta)}{(1+K_\sigma^2+T_\sigma^2)^{1/2}},$$

$$n_\sigma^2 = 1 - \frac{XT_\sigma}{T_\sigma - Y\cos\theta}, \qquad K_\sigma = \frac{XY\sin\theta}{1-X}\,\frac{T_\sigma}{T_\sigma - Y\cos\theta} \tag{E.7}$$

$$R_{\mathrm{E}}^{\sigma} = \frac{1+K_\sigma^2+T_\sigma^2}{(1+T_\sigma^2)2n_\sigma \dfrac{\partial}{\partial\omega}(\omega n_\sigma)},$$

$$T_\sigma = \frac{Y(1-X)\cos\theta}{\frac{1}{2}Y^2\sin^2\theta - \sigma\Delta},$$

with

$$\Delta^2 := \tfrac{1}{4}Y^4\sin^4\theta + (1-X)^2\,Y^2\cos^2\theta,$$

$$X := \frac{\omega_p^2}{\omega^2}, \qquad Y := \frac{\Omega_e}{\omega}, \qquad \tan\theta := \frac{k_\perp}{k_\parallel}. \tag{E.8}$$

The result (60a) is obtained by integrating over a Maxwellian distribution, expanding the Bessel functions and approximating the magnetoionic properties by $T_\sigma = -\sigma$ for $Y \ll 1$. Then one uses Stirling's formula and replaces s by ω/Ω_e by comparison with Trubnikov's result.

The emissivity at the sth harmonic *in vacuo* for $\theta = \pi/2$, $\alpha = \pi/2$ is

$$\eta_s(\omega, \pi/2) = \frac{e^2\omega^2}{2\pi c}\,\beta^2 J_s'^2(s\beta)\,\delta(\omega - s\Omega_e/\gamma). \tag{E.9}$$

Making the Carlini approximation (56b) gives

$$\eta_s(\omega, \pi/2) = \frac{e^2}{(2\pi)^3}\,\frac{s\Omega_e^2}{\gamma^4}\left(\frac{\gamma-1}{\gamma+1}\right)^s e^{2s/\gamma}\,\delta(\omega - s\Omega_e/\gamma). \tag{E.10}$$

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