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THE HEATING OF GAS IN CLUSTERS OF GALAXIES BY RELATIVISTIC ELECTRONS: COLLECTIVE EFFECTS

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ABSTRACT

We show that the rate at which gas is heated in X-ray clusters of galaxies by streaming relativistic electrons can be much greater than the Coulomb heating rate because of the stimulated growth of a high level of electrostatic turbulence and its subsequent collapse to shorter wavelengths. This enhanced heating (and, hence, energy loss) rate allows the X-ray emitting gas to be heated by those particles which are observable through their synchrotron emission at low radio frequencies and yields a radio source size consistent with the observed radio halo sizes in the Coma cluster. The heating of gas in clusters of galaxies by relativistic electrons will significantly affect the cluster gas dynamics.
Subject headings: galaxies: clusters of — galaxies: intergalactic medium — X-rays: sources

I. INTRODUCTION

It has been suggested that relativistic electrons contribute significantly to the heating and dynamics of gas in clusters of galaxies (Sofia 1973; Lea and Holman 1978). Radio observations (primarily decametric) indicate that large numbers of such electrons are present in X-ray clusters (Owen 1974; Erickson, Matthews, and Viner 1978; Hanisch, Matthews, and Davis 1980). In the works just cited, the heating of the intracluster gas was assumed to occur at a rate given by the usual Coulomb formula. Lea and Holman showed that such a heating rate is sufficient to explain the X-ray luminosity of the clusters if one assumes that the distribution of relativistic electron energies extends down to energies $E_{\rm min} \sim 10$–$100 m_e c^2$. Thus the energetics of simple Coulomb heating entails an assumption that there exist in the clusters 10^4–10^6 times as many low-energy electrons ($\sim 10 m_e c^2$) as there are directly observable higher-energy electrons. (The lowest-energy electrons which can be observed through their decametric synchrotron radiation have energies $\sim 1000 m_e c^2$.)

The low-energy electrons responsible for the Coulomb heating in the Lea-Holman picture emit frequencies too low to be observed. Therefore, one does not know whether the power-law distribution of electron energies [$N(E) \alpha E^{-3}$, as inferred from the decametric observations] extends to energies low enough to provide the requisite heating (as calculated by Lea and Holman 1978).

However, as the previous authors suggest, the simple, individual-particle Coulomb heating rate represents the minimum possible transfer of energy between the relativistic electrons and the gas. In fact, as we demonstrate below, collective (i.e., plasma) effects can enhance the heating rate far beyond that given by the Coulomb formula. A higher heating rate decreases the total number of electrons needed to provide the requisite heating and, hence, increases the value of $E_{\rm min}$. We demonstrate here that because of the increased heating rate provided by collective interactions the X-ray emitting gas can in fact be heated by those particles which are directly observed through their synchrotron radio emission.

We assume, as did Sofia (1973) and Lea and Holman (1978), that the source of particles is an active galaxy at the center of the cluster (such as NGC 1275 in the Perseus cluster). We show that, with the higher energy-loss rate obtained here, the predicted length scale of relativistic electron energy loss corresponds to the actual observed radio halo sizes in the Perseus and Coma clusters.

Although this paper is devoted to the heating of gas in clusters of galaxies, it should be noted that the basic physics presented here is applicable to several problems of astrophysical interest—i.e., the heating of the interstellar medium by the streaming of cosmic rays away from pulsars and supernova remnants. This problem is under investigation by the authors.

II. PLASMA THEORY

For the purposes of this paper, we will assume that the magnetic field in the cluster is uniform and, hence, not significantly tangled. This gas in clusters is a very high β $(nkT/B^2/8\pi)$ plasma. Small-scale kinks in the magnetic field (i.e., magnetosonic waves) will thus be effectively damped by proton transit time damping. If the field were tangled on large scales, then the heating mechanism discussed here would still operate, but the energy would be deposited in a smaller spatial fraction of the cluster; i.e., the path length along the field would be the same as that calculated below, but the radial distance away from the central particle source would be smaller than our predicted value.

We therefore assume that in such a cluster of galaxies particles are free to stream away from the central source along the field lines at speeds $v_s \sim c3^{-1/2}$. The particles are of course coupled to the background gas through their interaction with Alfvén waves which are generated by the particles' passage. This coupling is weak, however, since the ratio β of thermal energy density $(U_{\rm th})$ to magnetic-field energy density $(B^2/8\pi)$ is very large. We have demonstrated (Holman, Ionson, and Scott 1979) that in such a case self-generated Alfvénic turbulence does not significantly slow a relativistic particle beam. The particle pitch angles are merely distributed over the forward-moving hemisphere in phase space ($\theta < \frac{1}{2}\pi$, $\Delta\theta \sim \frac{1}{2}\pi$, where $\Delta\theta$ is the angular spread of the beam).

In any case, observations (Hanisch *et al.* 1980) indicate that the particles in the Coma cluster travel at speeds much greater than the Alfvén speed, $V_{\rm A}$, since the particles are observed out to radii, R_p, such that $R_p/\tau \sim 10^4 V_{\rm A}$, where τ is the particle lifetime (against synchrotron and inverse Compton losses). The inference that the particles must travel at high speeds is only slightly weakened by the possibility of particle reacceleration, since the amount of energy required to reaccelerate the particles enough times to allow them to reach R_p (if they were traveling at $V_{\rm A}$) is equal to the total inverse-Compton and synchrotron losses and is of the same order of magnitude as the total cluster binding energy!

The rapidly streaming relativistic electrons are, however, strongly interacting with the background plasma through the thermal electrostatic waves. Such waves (with wave number k_1) are in fact amplified by the particles with a linear growth rate, $\Gamma_{\rm L1}$ (eq. [12.62] of Kaplan and Tsytovich 1973):

$$\Gamma_{\rm L1} \approx \frac{n_b}{n} \frac{\omega_e}{\gamma_{\rm min}} (\Delta\theta)^{-2} , \tag{1}$$

where ω_e is the electron plasma frequency of the background plasma, n is the background plasma density, n_b is the number density of relativistic beam electrons, and $\gamma_{\rm min}$ is the energy (in units of $m_e c^2$) of the lowest-energy relativistic electrons. This is the well-known relativistic, warm-beam, two-stream instability. It should be noted that this growth rate (1) is valid only for $v_s \sim c$ and may be smaller for $v_s \ll c$ (see, e.g., Ginsburg, Ptuskin, and Tsytovich 1973). Hence the heating rate due to the stimulated growth of electrostatic waves may be significantly diminished if v_s falls significantly below c, depending upon the explicit form of the particle distribution function.

Following Papadopoulos (1975) the energy density in these waves W_1 (in units of the background thermal energy density, $nk_B T$) continues to grow until $W_1 \gtrsim (k_1\lambda_{\rm D})^2$, where $\lambda_{\rm D}$ is the Debye length. This energy is the threshold for the onset of the familiar oscillating two-stream instability (OTS). When the OTS threshold has been reached the waves W_1 begin to scatter nonlinearly or collapse to higher values of k (i.e., short wavelengths), at a rate

$$\Gamma^{\rm OTS} = \left(\frac{m_e}{m_i}\right)^{1/2} W_1{}^{1/2}\omega_e \, . \tag{2}$$

[Note: We have used the formulae appropriate to one-dimensional caviton formation, since our values of ω_e/Ω_e are the same as the case considered by Goldstein, Papadopoulos, and Smith (1980), where the one-dimensional form was shown to be appropriate.] The "collapse" of the turbulence from k_1 to k_2 (with corresponding energy density W_2) produces concomitantly a level W_s of ion density fluctuations ($\boldsymbol{k}_1 = \boldsymbol{k}_2 + \boldsymbol{k}_s$, but $k_1 \ll k_2$, k_s and therefore $k_2 \approx -k_s$). It is these sound "waves" which provide the anomalous DC restivity which in turn enhances the heating rate above the Coulomb rate given by Lea and Holman (1978). As a *demonstration* of how large this heating enhancement can be, we will calculate a steady-state value of W_s.

The aforementioned OTS instability increases the level of W_s and W_2 and decreases W_1 below the threshold. A quasi-steady state is obtained when the once again linearly growing waves, W_1, nonlinearly interact with the sound waves (with their OTS enhanced energy density) and further amplify W_2. This interaction, commonly known as Dawson-Oberman AC resistivity, results in the following growth rate for W_2 (Dawson and Oberman 1962, 1963):

$$\Gamma_{\rm NL}{}^{\rm D\text{-}O} = (k_s\lambda_{\rm D})^{-2} W_s\omega_e \, , \tag{3}$$

where k_s is the wave number of the ion density fluctuations. The quasi-steady state is obtained when

$$\Gamma_{\rm L1} = \Gamma_{\rm NL}{}^{\rm D\text{-}O}(W_s) \, . \tag{4}$$

A true steady state cannot be obtained at this point, however, since the sound "waves," W_s, are being damped in the Dupree-Weinstock sense, i.e., by the resonance-broadening phenomena (cf. eq. [16] of Smith, Goldstein, and Papadopoulos 1976), at a rate

$$\Gamma_{\mathrm{NLS}}{}^{\mathrm{D\text{-}W}} = \left(\frac{m_e}{m_i}\right)^{1/2} (k_s \lambda_{\mathrm{D}})^{1/2} W_s{}^{1/4} \omega_e . \tag{5}$$

On the other hand, in the cases of interest here, the level of W_s (and W_s determines directly the anomalous heating rate) is maintained by the onset of a secondary OTS which occurs when the wave level satisfies

$$W_2 > 4\left(\frac{\Gamma_{\mathrm{L2}}{}^d}{\omega_{pe}}\right), \tag{6}$$

where $\Gamma_{\mathrm{L2}}{}^d$ is the linear Landau damping rate for the waves W_2:

$$\Gamma_{\mathrm{L2}}{}^d = \frac{v_e{}^2 \omega_e}{(k_2 \lambda_{\mathrm{D}})^2} \left(\frac{\partial f_{\mathrm{ST}}}{\partial v}\right)\bigg|_{v=\omega_e/k_2} \tag{7}$$

Here f_{ST} is the distribution of superthermal particles created by the Landau damping of W_2. The function f_{ST} is determined by the detailed spectrum of $W_2(k_2)$. However, as in ionospheric settings where convective and collisional losses are small, we expect a typically power-law distribution (cf. Papadopoulos and Coffey 1974*a*):

$$f_{\mathrm{ST}} = A\left(\frac{1}{v}\right)^{\tau}, \tag{8}$$

where $\tau \approx 2$ and A is determined by normalization, i.e., one assumes the number of particles in the superthermal tail (i.e., particles with $v > v_{\min}$) is equal to the number of particles which would have $v > v_{\min}$ if the particle distribution were Maxwellian:

$$\int_{v_{\min}}^{\infty} \left(\frac{m_e}{2\pi k_B T_e}\right)^{1/2} \exp\left(-v^2/2v_e{}^2\right) dv = \int_{v_{\min}}^{\infty} \frac{A}{v^{\tau}}\, dv ,$$

$$A = (\tau - 1)(v_{\min})^{\tau-1}\left(\frac{1}{2\pi}\right)^{1/2}\left(\frac{v_e}{v_{\min}}\right) e^{-(v_{\min}/v_e)^2/2}, \quad \text{or} \quad f_{\mathrm{ST}} = \eta_{\mathrm{tail}}(\tau - 1)\left(\frac{v_{\min}}{v}\right)^{\tau}\left(\frac{1}{v_{\min}}\right), \tag{9}$$

with $\eta_{\mathrm{tail}} \equiv$ fraction of particles in the superthermal tail:

$$\eta_{\mathrm{tail}} \equiv n_{\mathrm{ST}}/n_e = \left(\frac{1}{2\pi}\right)^{1/2} (k_2{}^{\max} \lambda_{\mathrm{D}}) \exp\left(-\frac{1}{k_2{}^{\max}\lambda_{\mathrm{D}}}\right)^2 ,$$

where $k_2{}^{\max}$ is the wavenumber resonant with particles having $v = v_{\min}$.

We may now write all relevant rate equations:

$$\frac{\partial W_1}{\partial t} = 2[\Gamma_{\mathrm{L1}} - \Gamma_{\mathrm{NL}}{}^{\mathrm{D\text{-}O}}(W_s)] W_1 - 2\Gamma_{\mathrm{NL}}{}^{\mathrm{OTS}}(W_1) W_2 \mathrm{H}(W_1 - k_1{}^2 \lambda_{\mathrm{D}}{}^2) , \tag{10a}$$

$$\frac{\partial W_2}{\partial t} = 2\Gamma_{\mathrm{NL}}{}^{\mathrm{D\text{-}O}}(W_s) W_1 + 2\Gamma_{\mathrm{NL}}{}^{\mathrm{OTS}}(W_1) W_2 \mathrm{H}(W_1 - k_1{}^2 \lambda_{\mathrm{D}}{}^2) - 2\Gamma_{\mathrm{L2}}{}^d W_2 - 2\Gamma_{\mathrm{NL}}{}^{\mathrm{OTS}}(W_2) W_s , \tag{10b}$$

$$\frac{\partial W_s}{\partial t} = \left[2\Gamma_{\mathrm{NL}}{}^{\mathrm{OTS}}(W_1) \mathrm{H}(W_1 - k_1{}^2\lambda_{\mathrm{D}}{}^2) + 2\Gamma_{\mathrm{NL}}{}^{\mathrm{OTS}}(W_2) \mathrm{H}\left(W_2 - \frac{4\Gamma_{\mathrm{L2}}{}^d}{\omega_e}\right) - 2\Gamma_{\mathrm{NLS}}{}^{\mathrm{D\text{-}W}}\right] W_s , \tag{10c}$$

where H is the Heavyside step function.

For times after the onset of the original OTS (W_1), $\mathrm{H}(W_1 - k_1 \lambda_{\mathrm{D}}{}^2) = 0$ since W_1 will be kept below threshold by Dawson-Oberman scattering. Then, assuming that the secondary OTS (W_2) occurs (i.e., $W_2 > 4\Gamma_{\mathrm{L2}}{}^d/\omega_{pe}$), we may obtain a unique steady-state solution, $\partial W_1/\partial t = \partial W_2/\partial t = \partial W_s/\partial t = 0$:

$$W_s = (k_s \lambda_{\mathrm{D}})^2 \left(\frac{\Gamma_{\mathrm{L1}}}{\omega_e}\right), \tag{11a}$$

$$W_1 = \frac{\Gamma_{\mathrm{L1}}{}^d W_2 + \Gamma_{\mathrm{NL}}{}^{\mathrm{OTS}}(W_2) W_s}{\Gamma_{\mathrm{NL}}{}^{\mathrm{D\text{-}O}}} = \Gamma_{\mathrm{L2}}{}^d (\Gamma_{\mathrm{L1}}{}^g \omega_e)^{-1/2} (k_s \lambda_{\mathrm{D}})^2 = \left(\frac{m_e}{m_i}\right)^{1/2} (k_s \lambda_{\mathrm{D}})^{1/2} . \tag{11b}$$

$$W_2 = (k_s \lambda_{\mathrm{D}}) W_s{}^{1/2} = (k_s \lambda_{\mathrm{D}})^2 \left(\frac{\Gamma_{\mathrm{L1}}}{\omega_e}\right)^{1/2}, \tag{11c}$$

We need find only $(k_2^{\max}\lambda_D)$ in order to obtain the numerical solution. Since $k_2^{\max}$ is the smallest wavelength which the OTS can cause to grow (in the presence of the Landau damping) we may find the limits by

$$\Gamma_{NL}^{OTS}(W_1)W_2 > \Gamma_{L2}^{d}W_2|_{t=0} . \tag{12}$$

At time $t = 0$ the damping, Γ_{L2}^{d}, is due to a Maxwellian thermal background, so

$$\Gamma_{L2}^{d} = (k_2^{\max}\lambda_D)^{-3} \exp[-(1/k_2^{\max}\lambda_D)^{1/2}]\omega_{pe} . \tag{13}$$

Therefore, rewriting (12),

$$(k_2^{\max}\lambda_D) \lesssim \left\{\frac{1}{\ln[(m_e/m_i)^{-1}\omega_1^{-1}(k_2^{\max}\lambda_D)^6]}\right\}^{1/2} . \tag{14}$$

We now obtain the steady-state level W_s. These ion density fluctuations provide an anomalous resistivity, η_r, which causes the direct dissipation of current energy into heat. In particular, the return current generated in the plasma by the passage of the cosmic-ray beam dissipates its energy into thermal electron motions. (Note that the return current speed $v_{rc} = cn_b/n_e$ is much smaller than the ion-acoustic speed, and therefore no other instabilities are introduced into the plasma by the return current.) The joule heating rate Q_j is

$$Q_j = \tfrac{1}{3}(n_b ec)^2 n_r , \tag{15}$$

where (e.g., Papadopoulos and Coffey 1974*b*)

$$\eta_r = \frac{4\pi}{\omega_{pe}}(k_2^{\max}\lambda_D)W_s , \tag{16}$$

or

$$Q_j = \left(\frac{n_b}{n_e}\right)^2 (k_2^{\max}\lambda_D)^3(n_e m_e c^2)\Gamma_{L1} . \tag{17}$$

The stopping distance L of the beam (the scale size over which the beam has dissipated its energy) may be found by setting Q_j equal to the divergence of the beam energy flux $F_b \sim \gamma n_b m_e c^3 3^{-1/2}$,

$$\nabla \cdot F_b = Q_j \Rightarrow L \sim F_b/Q_j . \tag{18}$$

One may also find the direct heating rate Q_L due to the Landau damping of the waves W_2 directly on the thermal background. The rate is *highly* uncertain and depends very sensitively upon assumptions about the shape of the superthermal tail and therefore upon $(k_2^{\max}\lambda_D)$ (e.g., a factor of 2 change in $k_2^{\max}\lambda_D$ changes the calculated *direct* heating rate by many orders of magnitude). In any case, direct heating is an addition to the Joule heating rate, and we are therefore being conservative by considering only the Joule dissipation here.

III. CLUSTERS OF GALAXIES

For clusters of galaxies we use the following "typical" parameters to calculate our sample collective heating rate:

$$n_e = 10^{-3}\ \text{cm}^{-3} , \quad B = 3 \times 10^{-7}\ \text{gauss} , \quad T = 10^8\ \text{K} , \quad L_{\text{sync}} = 3 \times 10^{41}\ \text{ergs s}^{-1} .$$

Therefore, at (e.g.) $\gamma_{\min} = 10^3$, $n_b(\gamma_{\min})/n \sim 4 \times 10^{-6}$ and

$$Q_j = 4 \times 10^{-27}\ \text{ergs cm}^{-3}\text{-s}^{-1} .$$

This heating rate is a factor of 10^5 larger than the normal Coulomb rate! We then find for the beam stopping distance L,

$$L = 10^{25}\ \text{cm} ,$$

whereas the typical observed radio halo size is $r_{\text{rad}} \approx 10^{24}$ cm, in good agreement (recall that the magnetic field will not actually be perfectly uniform and that our heating rate is a lower limit). Furthermore, over the volume of the radio source ($\sim 10^{72}$ cm^3) the total heating input rate Q_j(TOT) is of order

$$Q_j(\text{TOT}) \sim 4 \times 10^{45}\ \text{ergs s}^{-1} ,$$

again in good agreement with bright cluster X-ray source luminosities of $L_x \sim 10^{45}$ ergs s^{-1}. Indeed, it is easy to pick parameters such that $Q_j(\text{TOT}) \gg L_x$, indicating that much of the relativistic particle energy is going into hydrodynamic expansion of the cluster gas (see Lea and Holman 1978).

Of course our analysis implies that the central radio source (or succession of sources) must supply $\sim 4 \times 10^{45}$ ergs s^{-1} in the form of relativistic particles. Over the lifetime of a cluster of galaxies ($\sim 3 \times 10^{17}$ s) this implies a total energy input of $\sim 10^{63}$ ergs. Such energies are typical of the estimates of the total energy available in large radio sources (see Christiansen, Pacholczyk, and Scott 1977). Furthermore, the energy need not be supplied by a single source. Strong X-ray clusters inevitably have central radio sources. Since the lifetimes of the individual radio sources are thought to be much shorter than the cluster age, this fact implies that radio sources are continuously turned on in X-ray clusters. The total energy required (10^{63} ergs) is of the same magnitude as the gravitational energy of the cluster. This may not be accidental since the most likely source of energy for the "turn on" of radio source is the gravitational energy in the cluster gas. One is tempted to speculate that when the additional heating and momentum support which is provided by a radio source is turned off, the gas has a tendency to collapse back on the central galaxy, thereby imitating a new radio source.

At this point we wish to emphasize that the numbers calculated above should only be considered as an example, because of the sensitivity of the results to uncertain observational parameters, primarily B (which determines n_b/n_e, γ, etc.). Observations of rotation measures of background radio source can be combined with improved X-ray data (*HEAO 2*) to determine the value of B much more accurately. Such observations are planned by a number of groups.

IV. SUMMARY

We have pointed out that the heating rate of astrophysical plasmas by relativistic electrons can be many orders of magnitude larger than the normal Coulomb heating rate when collective effects (e.g., the growth of electrostatic waves, OTS instability, and anomalous resistivity) are important. We have also shown that the heating of gas in clusters of galaxies can easily be accomplished by the relativistic particles known to be present therein. Indeed, it seems likely that the dynamics of gas in some clusters of galaxies are determined *primarily* by the interaction between the gas and the relativistic particles. Finally, we point out that the very large heating rates which are expected relax previously calculated limits on the total amount of gas present in clusters (see, e.g., Lea *et al.* 1973). We feel it is probable that, given the expected particle heating rates, even as much material as the "missing mass") ($\sim 10^{15}$ g) may be supported in clusters in the form of a large, hot X-ray emitting gas halo. This question is under investigation by the authors.

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