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SOME SECONDARY INDICATIONS OF GRAVITATIONAL COLLAPSE

D. K. NADYOZHIN
Institute of Theoretical and Experimental Physics, Moscow, U.S.S.R.

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Abstract. A stellar core becomes somewhat less massive due to neutrinos radiated away during its collapse in a neutron star or a black hole. The paper deals with the hydrodynamic motion of stellar envelope induced by such a mass loss. Depending on the structure of the outer stellar layers, the motion results either in ejection of an envelope with mass and energy proper for Nova outbursts; or nearly instantaneous excitation of strong pulsations of the star; or lastly in a slow slipping away of the whole stellar envelope. These phenomena are of importance when more powerful events, like supernova outbursts presumably associated with gravitational collapse, are absent. Such secondary indications of gravitational collapse are of special interest, since they may be a single observable manifestation (besides neutrinos and gravitational waves) of massive black hole formation.

1. Introduction

It is well known that a gravitational collapse of the central regions of massive stars results in the formation of neutron stars and possibly of black holes. There is at present no doubt that this process will be followed by a powerful surge of neutrino radiation. Under favourable conditions, the gravitational collapse may be accompanied by a supernova outburst. There are, in fact, strong arguments in favour of the view that Type I supernovae are related to the birth of the neutron stars (Shklovsky, 1975, 1978; Nadyozhin and Utrobin, 1977; Utrobin, 1977). It is not excluded, however, that under certain conditions the gravitational collapse may not be associated with so violent an event as supernova outburst. This is especially the case when massive stellar core collapses into a black hole. When such a quiet collapse occurs, some secondary concomitant effects become of prime importance and can identify it from the observations.

The present paper deals with one of such weak manifestations of the collapse, consisting in the response of stellar envelope to the mass loss by neutrino radiation. Some attention has been already paid to this problem by Wilson (1971) and Barkat *et al.* (1974). However, in our opinion it deserves a further and more detailed investigation, since any additional evidences of gravitational collapse may be of great interest – especially if it relates to the formation of black holes.

2. An Outline of the Problem and Main Assumptions

The following discussion is based primarily on the fact that, in accordance with the theory of evolution of single stars, the star immediately before the loss of its stability

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was of an extremely heterogeneous structure: the central stellar core, which is just going to collapse, has dimensions characteristic of the white dwarfs ($\sim 10^{-2} R_\odot$) and is embedded in an extended envelope with radius as large as 10^3–$10^4 R_\odot$. When a star evolves in a close binary system, it can hardly develop so extended an envelope. However, the external radius of the star can nevertheless exceed considerably the radius of its core.

Such a heterogeneous structure gives an opportunity to make use of simple models in order to evaluate the effect in question. Let us assume the star immediately prior to the collapse to consist of a point central core of mass M_c and of an envelope of mass M_e; the latter being in hydrostatic equilibrium against its own gravity and that of the point central core. In addition, assume the envelope density law in the form

$$\varrho = \varrho_n \left(1 - \frac{r}{R}\right)^n, \tag{1}$$

where R denotes the radius of the star. Equation (1) enables us to calculate easily the mass and the gravitational energy of the envelope: i.e.,

$$M_e = \frac{8\pi R^3 \varrho_n}{(n+1)(n+2)(n+3)}, \tag{2}$$

$$E_g = -\frac{n+3}{2}\frac{GM_e}{R}\left[M_c + \frac{(n+3)(5n+8)}{4(2n+3)(2n+5)} M_e\right]. \tag{3}$$

Furthermore, it can be shown that the virial theorem asserts for the envelope that

$$-E_g = 3\int_0^R 4\pi P r^2 \, \mathrm{d}r \equiv V, \tag{4}$$

where P is the pressure. Therefore, the total energy of the envelope is given for constant adiabatic index γ by

$$\mathscr{E} = E_g + E_T = \frac{3\gamma - 4}{3(\gamma - 1)} E_g. \tag{5}$$

Let the radius of the envelope be taken large enough for the hydrodynamic time scale of the envelope t_H to exceed the time scale of neutrino radiation considerably: i.e., that

$$t_{\mathrm{H}} \gg t_\nu \quad \text{or} \quad R\sqrt{\frac{R}{GM}} \gg t_\nu, \tag{6}$$

where $M = M_e + M_c$ is the total mass of the star. In this case, the stellar core can be considered to experience an instantaneous loss of its mass by neutrino radiation, at

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least in so far as it concerns hydrodynamic motions induced in the envelope. Recourse to Equation (6) with typical value $t_\nu = 20$ s (Nadyozhin, 1978) yields

$$R/R_\odot \gg 0.054\,(M/M_\odot)^{1/3}. \tag{7}$$

Thus for $R \geqslant 0.5\,R_\odot$, the assumption of instantaneous nature of neutrino radiation is justified for all reasonable values of stellar mass ($M \leqslant 30\,M_\odot$).

The mass loss by collapsing stellar core results in an increase of the envelope gravitational energy which, in accordance with Equation (3), is given by

$$\Delta E_g = \frac{n+3}{2}\frac{GM_e}{R}qM_c, \tag{8}$$

where qM_c denotes the mass lost by the stellar core. Considering Equations (3), (5), and (8), one can conclude that if γ is sufficiently close to its critical value $\frac{4}{3}$, the total envelope energy can become positive. This occurs when the following inequality takes place

$$\gamma - \frac{4}{3} < \frac{1}{3}\frac{qM_c}{(1-q)M_c + [(n+3)(5n+8)]/[4(2n+3)(2n+5)]}M_e \tag{9}$$

If Equation (9) holds, a large fraction of the envelope mass may be thrown off into interstellar space.

The relative mass loss, q, is equal to about 0.1 for $M_c = 1.4\,M_\odot$ and $\sim$0.14 for $M_c = 2\,M_\odot$ (Cohen and Cameron, 1971); in addition, rough estimates yield $q =$ 0.05–0.08 for collapse of massive stellar core, $M_c = 10\,M_\odot$, into a black hole (Zeldovich and Novikov, 1965; Nadyozhin, 1978). Thus, the variation of q is not very large for the collapsing core masses of interest. Therefore, we specify hereafter that $q = 0.1$.

Let now the mass of the point-like core of hydrostatic stellar model, described above, be instantly decreased by qM_c. The problem is to calculate the hydrodynamic motions caused in stellar envelope by a corresponding decrease of gravity. First of all, it is necessary to specify a boundary condition in the centre of the star. We confine ourselves to a simple condition imposed on the velocity of stellar material: $u = 0$ at $r = 0$. This boundary condition implies that the hydrodynamic unloading of the internal layers of the stellar envelope by the infall of the collapsing core is entirely absent. However, in a rough treatment of the problem when the effect is expected to be evaluated only by the order of magnitude, such a neglect of hydrodynamic unloading may be justified by the following arguments: (i) the time-scale of the unloading is at least several times the core hydrodynamic time t_H*, while the response of the very internal layers of the envelope to the instant decrease of the core mass develops in time scale equal to t_H; (ii) the not very fast initial rotation of the internal envelope layers is bound to prevent their deep collapse and thereby to slacken the hydrodynamic unloading considerably.

* In the straightforward calculation of the $2\,M_\odot$ star collapse, the very external layers of the star were involved in the collapse in time of $\sim 5t_H$ (Nadyozhin, 1978).

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3. Hydrodynamic Response of the Envelope

In order to estimate the motion of stellar envelope induced by the mass loss of the collapsing core, the following system of hydrodynamic equations in spherical symmetry should be solved numerically:

$$\frac{\partial r}{\partial t} = u, \tag{10}$$

$$\frac{\partial u}{\partial t} = -4\pi r^2 \frac{\partial P}{\partial m} - \frac{G(M_c' + m)}{r^2}, \tag{11}$$

$$\frac{\partial r^3}{\partial m} = \frac{3}{4\pi\varrho}, \tag{12}$$

$$\frac{\partial E}{\partial t} + P\frac{\partial}{\partial t}\left(\frac{1}{\varrho}\right) = 0, \tag{13}$$

$$E = \frac{P}{(\gamma - 1)\varrho}, \tag{14}$$

where $M_c' = (1 - q)M_c$ is the reduced mass of the stellar core. Equations (10)–(14) are written out in the Lagrangian form, with t and m chosen as independent variables. According to the preceding section, the boundary condition in the centre of the star is taken in the form

$$r = 0 \quad \text{and} \quad u = 0 \ \text{ for } m = 0. \tag{15}$$

At the stellar surface, the common boundary condition to be employed is that

$$P = 0 \quad \text{for } m = M_e. \tag{16}$$

The dependences of r, P, and ϱ on current mass m at the initial time $t = 0$ are defined in accordance with above simple hydrostatic model; besides the zero initial velocity is chosen all over the star: $u = 0$.

It is convenient to change over to dimensionless variables (Nadyozhin and Frank-Kamenetsky, 1964) specifying the units of length, mass, velocity, time etc. as follows (the units are supplemented with subscript 0):

$$\begin{aligned} &R_0 = R, \qquad M_0 = M = M_c + M_e, \qquad u_0 = \sqrt{\frac{GM}{R}}, \\ &t_0 = \frac{R_0}{u_0} = R\sqrt{\frac{R}{GM}}, \qquad \varrho_0 = \frac{M}{4\pi R^3}, \qquad P_0 = \frac{GM^2}{4\pi R^4}, \\ &E_0 = \frac{GM}{R}, \qquad \mathscr{E}_0 = M_0 u_0^2 = \frac{GM^2}{R}, \end{aligned} \tag{17}$$

where M and R is the total mass and radius of the star, and the energy unit is denoted by $\mathscr{E}_0$. It can be easily proved that the hydrodynamic problem in question is com-

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pletely defined by only four dimensionless parameters: n, γ, q and M_c/M. The results obtained for a fixed set of these parameters can always be used for any values of stellar radius R and total mass M of interest.

Equations (10)–(14) describe adiabatic motions when both the radiative heat transfer and energy released in thermonuclear reactions are neglected. This approximation is adequate since, within a short hydrodynamic time-scale considered here, the transfer of heat by radiation is negligible for the bulk of the envelope (it should, of course, be allowed for in the very external layers of the envelope in order to elucidate the time-behavior of luminosity of the star; but this poses quite a different problem). Moreover, Barkat *et al.* (1974) have found that the shock-wave induced in the stellar envelope by the neutrino mass loss is not so strong as to ignite thermonuclear fuel.

The main results for all four calculated versions are presented in Table I. The star was divided into 151 mass zones, the division being uniform over the initial radius r. The artificial viscosity (Richtmyer, 1957) was introduced in Equations (11) and (13) to calculate the shock waves properly.

In the first columns of Table I, the above parameters of initial models are presented including an increase of the envelope gravitational energy, $\Delta E_g/\mathscr{E}_0$, given by Equation (8) and its ratio to the total gravitational energy $\Delta E_g/|E_g|$. The last two columns give the dimensionless mass M_{ex}/M, and kinetic energy $E_{\text{ex}}/\mathscr{E}_0$, of the expelled envelope resulting from the hydrodynamic calculations.

The disturbance caused by the mass loss of the collapsing core results in a compression wave appearing in the internal layers of the envelope and propagating to the stellar surface (Figure 1, Moments 1–3). Such a kind of hydrodynamic flow can be easily understood if one takes into account the fact that, first, the speed of sound increases from the surface to the centre of the star (and, therefore, the internal layers of the envelope reveal the most quick response to decrease of gravity); and, second, that since the mass of the envelope does not differ considerably from that of the collapsing core, a relative decrease of gravity appears to be much greater in the in-

TABLE I

The main dimensionless parameters of models calculated

Version	q	γ	n	$\frac{M_c}{M}$	$\frac{M_e}{M}$	$\frac{\Delta E_g}{\mathscr{E}_0}$	$\frac{\Delta E_g}{\lvert E_g\rvert}$	$\frac{M_{\text{ex}}}{M}$	$\frac{E_{\text{ex}}}{\mathscr{E}_0}$
I	0.1	$\frac{5}{3}$	3	$\frac{2}{3}$	$\frac{1}{3}$	0.0667	0.0852	9.8×10^{-5}	3.9×10^{-5}
II	0.1	$\frac{5}{3}$	3	$\frac{1}{3}$	$\frac{2}{3}$	0.0667	0.0589	2.4×10^{-5}	7.7×10^{-6}
III	0.1	$\frac{4}{3}$	0	$\frac{1}{7}$	$\frac{6}{7}$	0.0184	0.0294	$\frac{6}{7}$	1.84×10^{-2}
IV	0.1	$\frac{5}{3}$	0	$\frac{2}{3}$	$\frac{1}{3}$	0.0333	0.0833	There is no ejection of an envelope, but pulsations are excited with period $p \approx 5.5t_0$.	

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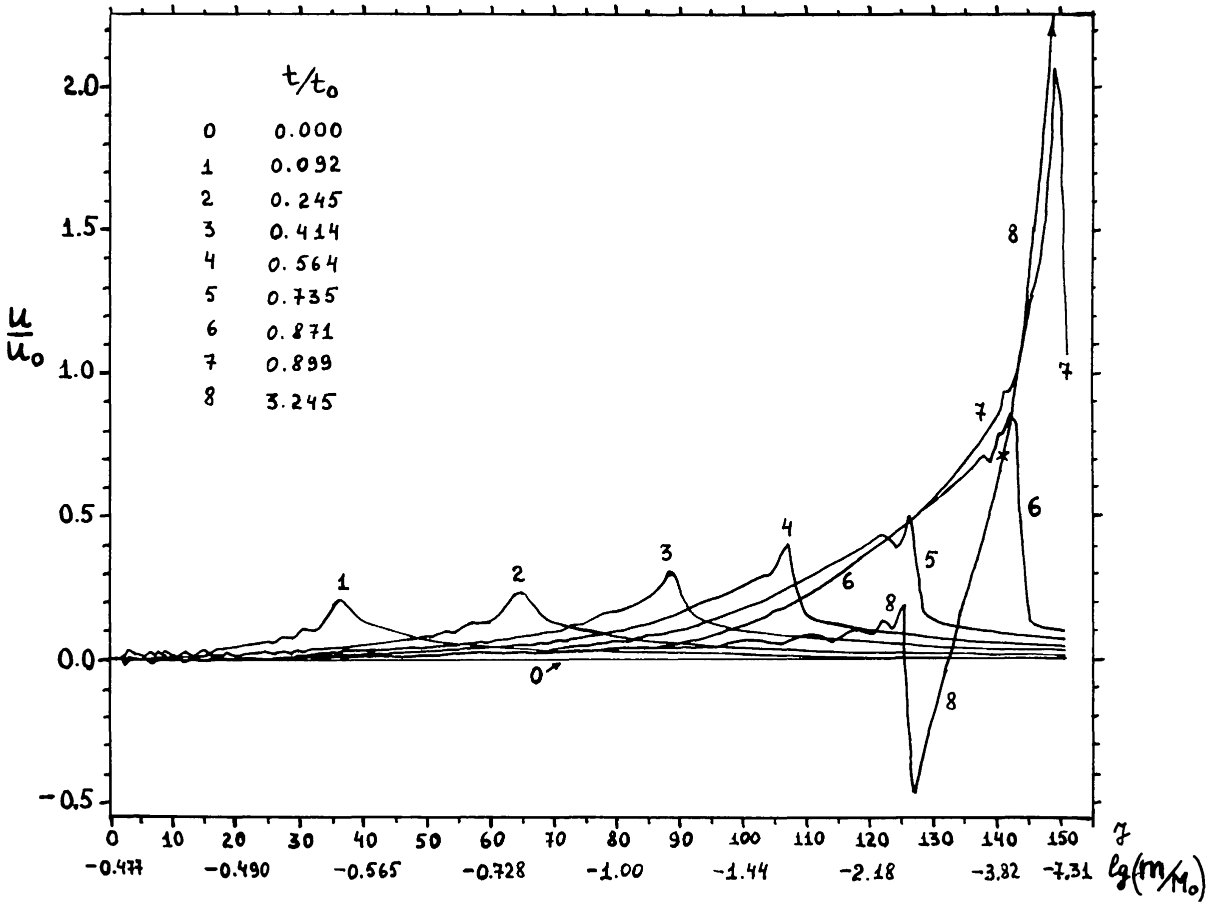

Fig. 1. The velocity distribution in the stellar envelope after the core of the star has collapsed (Version I). The number of mass zone J and logarithm of mass measured from the stellar surface are indicated under horizontal axis. A cross on the velocity distribution at Moment 8 stands to indicate a point where the velocity of matter is equal to its local parabolic (escape) value. The units of time (t_0), velocity (u_0) and mass (M_0) are expressed through the mass and radius of the star by means of Equations (17).

ternal layers of the envelope. Versions I and II indicate that for a fairly steep density gradient in the stellar envelope, the compression wave turns eventually into a shock wave which becomes stronger when propagating along decreasing density (Figure 1, Moments 4–7) and results in an expulsion of the envelope.

The mass and energy of the envelope expelled prove to be of the same order of magnitude as in the case of the Nova outbursts. However, the time-behavior of the light emitted by the outgoing envelope should differ considerably from the typical light curve characteristic of the Nova outburst. A quick adiabatic cooling of expanding envelope makes, in fact, the total radiated energy to be 2 or 3 orders of magnitude smaller than the kinetic energy E_{ex} presented in Table I. There is so obvious a disagreement with observations an examination of which shows that the energy lost by Novae in the form of light can exceed sometimes the kinetic energy of the ejected envelope (Vorontsov-Velyaminov, 1948).

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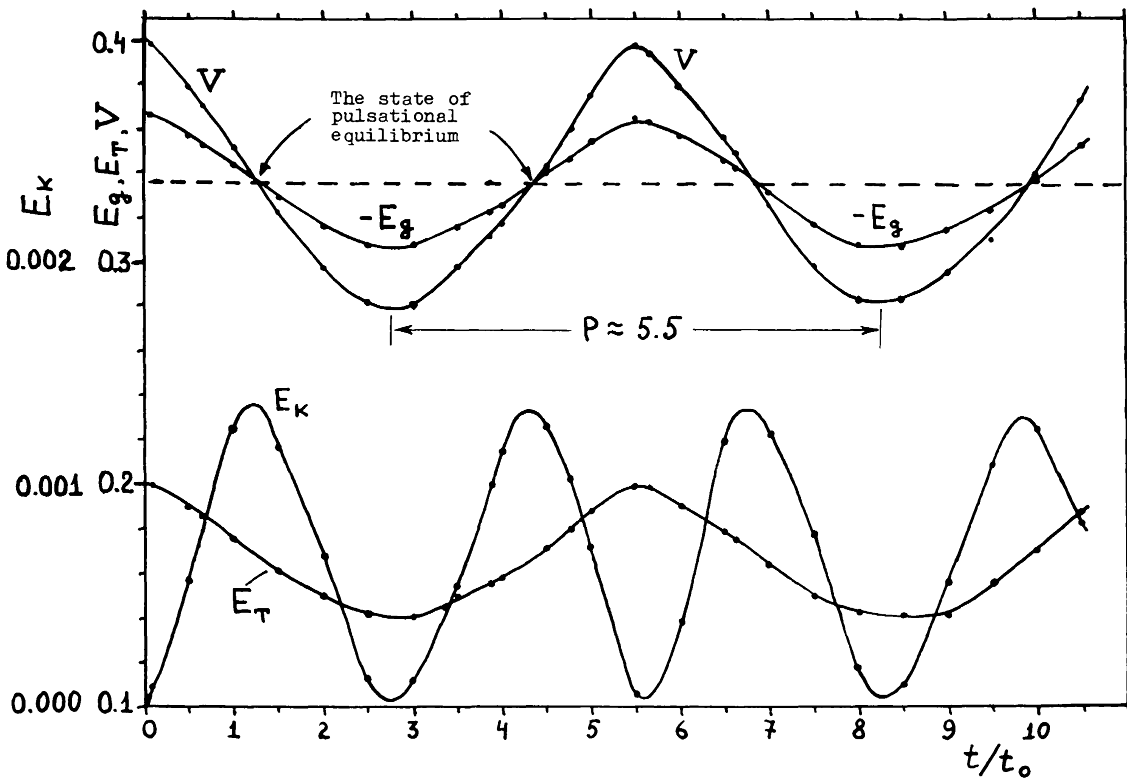

Fig. 2. The time-behavior of gravitational (E_g), internal (E_T) and kinetic (E_k) energies and of integral V (Equation (4)) in the course of pulsations of the star induced by the mass loss of the core which has collapsed at moment $t = 0$ (Version IV). Note a highly enlarged scale used for kinetic energy. A dashed horizontal line indicates the state of new equilibrium relative to which the star pulsates. The energies are measured in unit $\mathscr{E}_0$ given by Equation (17).

If, in the outer layers of the star, a steep density gradient is absent, the compression wave does not turn into the shock wave and the envelope is never ejected. However, strong pulsations of the star are excited at once. This effect is illustrated by Version IV (Table I, Figure 2), which is characterized by a homogeneous initial density in the envelope (see details in the Appendix).

Stellar models consisting of the homogeneous envelope and point mass as the core, are suitable for a rough description of structure exhibited by the massive red supergiants at advanced stages of their evolution (cf., for example, Uus, 1970). The motion of the supergiant envelope, induced by decrease of the collapsing core mass, can display itself not only in the form of violent pulsations but can also result in the outflow of a considerable part – if not all – of the envelope into interstellar space. The point is that internal energy (the sum of thermal and recombination energy) of the red supergiant envelopes is close to their gravitational energy (Paczynski and Ziolkowski, 1968). Therefore, a decrease in absolute value of the envelope gravitational energy, due to the collapse of the core, can make the total envelope energy positive. This undoubtedly occurs when inequality given by Equation (9) holds for the envelope adiabatic index γ.

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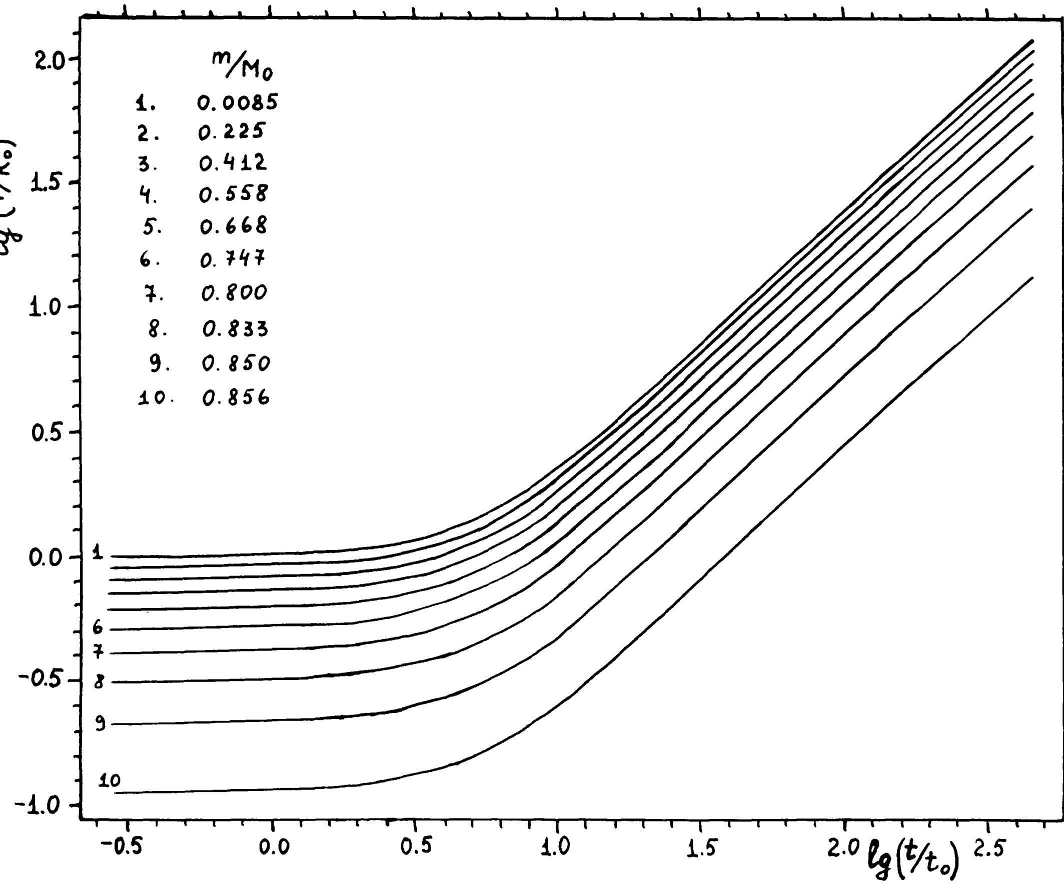

Fig. 3. Total disruption of the stellar envelope triggered by a decrease of the mass of the collapsing stellar core ($\gamma = \frac{4}{3}$, Version III). The time-dependence of radii of various Lagrangian layers is presented. The masses enclosed between the stellar surface and the numbered layers are also indicated.

Version III (Figure 3) illustrates the outflow of the envelope in case of $\gamma = \frac{4}{3}$. The total envelope energy is here positive and equals to ΔE_g; it turns ultimately into the kinetic energy of expansion. The radiation law of the outflowing envelope (the light curve) is of special interest. In order to obtain the resulting light curve and a more reliable evaluation of the velocity and mass of the envelope expelled, it is necessary to perform an extensive calculation based on real initial models of the red supergiant envelopes resulting from the theory of stellar evolution. It requires an incorporation of radiative heat transfer at various stages of ionization of stellar matter in the hydrodynamic calculations.

4. Conclusions

The aim of the present paper has been to draw attention to various manifestations of the gravitational collapse which can be identified by the observations. There are, as demonstrated above, two natural circumstances: namely, the mass loss of a collapsing stellar core due to neutrino radiation, and a heterogeneous structure of the star at

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advanced evolutionary stages, which can lead (depending on the envelope structure and in absence of more powerful events like supernova outbursts) to the following three and in principle observable, effects:

(i) ejection of an envelope of mass and kinetic energy appropriate for Nova outbursts, but characterized by light curves of much shorter duration;
(ii) instantaneous excitation of strong ($\Delta R/R = 0.1$–0.2) pulsations of the star (the time-scale of the development of stellar pulsations used to exceed the period of pulsations by several orders of magnitude);
(iii) outflow of massive envelopes with comparatively low velocity.

The third effect is of special interest since it may be related to the formation of black holes. In this case, the kinetic energy of outflowing envelope should be approximately of the same order of magnitude as the change of gravitational energy ΔE_g given by Equation (8). Specifying parameters of Equation (8), say, as $q = 0.1$, $n = 0$, $M_c = 10\ M_\odot$, $R = 1000\ R_\odot$, we can find that the birth of a black hole of mass 10 $M_\odot$ may be followed by appearance of expanding envelope with specific kinetic energy $\varepsilon_k \approx \Delta E_g/M_e \approx 2.8 \times 10^{12}$ erg g^{-1} and average expansion velocity $\bar{u} = \sqrt{2\varepsilon_k} \approx$ 24 km s^{-1}. The mass of outflowing envelope may attain 20–30 $M_\odot$.

APPENDIX: Radial Pulsations of Homogeneous Star with a Point Mass in its Centre

The small-amplitude pulsations of stellar model, having a point mass M_c in the centre of homogeneous envelope ($n = 0$), are described by the equation

$$\frac{\mathrm{d}}{\mathrm{d}r}\left\{\gamma r^3[M_c(1 - r) + \tfrac{1}{2}(1 - M_c)r(1 - r^2)]\frac{\mathrm{d}f}{\mathrm{d}r}\right\} + \\ +\{\omega^2 r^4 - (3\gamma - 4)r[r^3 + (1 - r^3)M_c]\}f = 0, \tag{1A}$$

where $f = \delta r/r$ is a relative amplitude of displacement. Equation (1A) is written down in units used in Equation (17); and besides, t_0^{-1} is chosen as a unit of the frequency ω of oscillation. An analysis of Equation (1A) shows that f behaves for $r \to 0$ as

$$f \sim r^\alpha \quad \text{with} \quad \alpha = -1 + 2\sqrt{(\gamma - 1)/\gamma}. \tag{2A}$$

Integrating Equation (1A) by r from 0 to 1 with f given by Equation (2A), we obtain an approximate expression for frequency of fundamental mode in form

$$\omega_0^2 = (3\gamma - 4)\left(1 + \frac{3M_c}{2 + \alpha}\right). \tag{3A}$$

Following the technique developed in Rosseland's 1949 book, one finds that, in a special case of $M_c = \frac{1}{3}$, Equations (2A) and (3A) are exact for the fundamental mode.

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Moreover, the whole pulsational spectrum is represented in an explicit form

$$\omega_k^2 = (3\gamma - 4)\left[1 + \frac{\alpha + k(3k + 2\alpha + 5)}{\alpha(\alpha + 2)}\right], \quad (k = 0, 1, \ldots). \tag{4A}$$

The corresponding eigenfunctions are expressed as $f_k = r^\alpha P_k(r^3)$, where P_k denotes a polynomial of power k.

Thus it can be concluded that Equation (3A) is exact both for $M_c = 0$ (Rosseland, 1949) and for $M_c = \frac{1}{3}$. It should be noted that, although in other cases Equation (3A) is only approximate, it is nevertheless more accurate than the well-known formula expressing the fundamental frequency through gravitational energy and central moment of inertia (Rosseland, 1949), which can be derived by putting $\alpha = 0$ in Equation (3A).

The use of Equation (3A) with parameters characteristic of Version IV: i.e., $\gamma = \frac{5}{3}$ and $M_c = \frac{2}{3}$, yields the period of pulsations $p = 2\pi/\omega = 4.6$. This quantity differs considerably from the period $p \approx 5.5$ with which stellar model pulsates according to more elaborate calculations. The fact is that Equation (3A) has been applied to *the initial* stellar model, and we fully ignored that the star expands immediately after the collapse and, therefore, pulsates around *a new* state of equilibrium (Figure 2). Allowing for the law of conservation of energy and an adiabatic nature of the transition into the new state of equilibrium, one can calculate that, at the moments of *pulsational equilibrium* when $V = -E_g$ while E_k attains its maximum value (the notion of pulsational equilibrium has been introduced by D. A. Frank-Kamenetsky, 1959), the following relations are valid:

$$R \approx (1 - \Delta E_g/|E_g|)^{-1} = 1.09, \tag{5A}$$

$$E_{k\,\max} \approx \frac{1}{2}\frac{\Delta E_g}{\mathscr{E}_0}\frac{\Delta E_g}{|E_g|} = 0.0014, \tag{6A}$$

where the numerical values are obtained with $\Delta E_g/\mathscr{E}_0$ and $\Delta E_g/|E_g|$ from Table I. The value of $E_{k\,\max}$ is in fair agreement with Figure 2.

Let us apply now Equation (3A) to the new state of equilibrium. First of all, it should be taken into account that Equation (3A) is written out in units for which total mass and radius of the star just before the collapse are equal to 1. In the new state of equilibrium, the mass of stellar core and the total mass of the star become $M_c' = (1 - q)M_c = 0.6$ and $M = M_c' + \frac{1}{3} = 0.9333\ldots$, respectively, while the radius of the star is equal to 1.09 according to Equation (5A). Introducing $M_c = 0.6/0.9333 = 0.643$ into Equation (3A) and multiplying the right-hand side of Equation (3A) by $M/R^3 = 0.9333/(1.09)^3 = 0.721$ in order to change over to units in which radius and mass of *initial* model are equal to 1 (Figure 2 has been plotted in these units), one gets $p = 5.44$, in excellent agreement with Figure 2.

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