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ASTRONOMY AND ASTROPHYSICS

On a Day-time Ionospheric Effect on some Radio Intensity Measurements and Interferometry

N. Meyer-Vernet

Observatoire de Paris-Meudon, Département de Recherche Spatiale*, F-92190 Meudon, France

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Summary. The effects of a sinusoidal phase-changing screen on radio-astronomical intensity measurements and interferometry are investigated analytically. The model is applied to predict the disturbing effects of an idealized travelling ionospheric disturbance of 'medium scale'. Since the phenomenon is important by day-time, it may disturb in particular solar observations, and should sometimes be taken into account when interpreting them. For observing wavelengths in the metric range (and below), it generally produces only slow temporal variations of the apparent source position, while in the decametric range it may produce more rapid intensity variations and modify the apparent source's size, position and shape.

Key words: radioastronomy – scintillation – solar radio bursts – travelling ionospheric disturbances – gravity waves

1. Introduction

While studying ionospheric effects on the observations of the 'type I' solar radio-bursts, Bougeret (1979) stresses the fact that solar observations at metric wavelengths often show important quasi-periodic variations of the angle of arrival (typical values: 5′ at 100 MHz, period 20 min, which can affect sources as large as one degree); he attributes them to refraction by travelling ionospheric disturbances of medium scale (which are due to gravity waves).

These variations, which are frequent by day-time and appear to maximise near 12^h local time (Bramley, 1974), are rather different from the usual random scintillations early studied by observing radiosources (Hewish, 1952), and later extensively analyzed (see Crane, 1977, for a review). The latter occur mainly by night, are ascribed to random electron density irregularities and generally involve much smaller spatial scales, and also smaller phase deviations. Here, we shall be concerned with the former, which have not been so much studied in an astrophysical context.

The travelling ionospheric disturbances (T.I.D.) of medium scale, which are a regular daily feature, generally appear as a train of quasi wave-like density disturbance [period $T \sim 20$ min, horizontal wave-length $2\pi d \sim 100$ km; see Yeh and Liu (1974) for a review]; the most important density disturbance occurs near the peak of the F layer (foF2) in the ionosphere.

This introduces a wave-like phase disturbance on an incident radio wave (typical value $\Phi_0 \sim 50$ rad at 100 MHz). Let us consider an observer who intends to study, through the disturbance, both the temporal intensity variation of a radio source, and its spatial location and shape (interferometry). If the observer is located near the disturbing region and the region is thin, he will essentially see quasi-periodic angle-of-arrival variations, as in Bougeret (1979). If the distance increases, focusing and interference effects must be considered; these may lead to temporal intensity variations and source shape modifications. It happens that the 'near zone' so defined, which is related to a focal length, is such that the observer is inside for observations in the metric range and lower wavelengths, and outside in the decametric range. In the latter case, simple geometric calculations are not expected to be valid, and it is of interest to calculate precisely how the T.I.D. will modify the observer's measurements.

It is worth noting that, for the practical cases relevant here, the observer is well inside the 'Fresnel distance' corresponding to a patch of dimension $2\pi d$. So, the limiting case of Fraunhofer conditions is not relevant in the present situation; it has been extensively studied in relation to such problems as diffraction by sound waves (see Gulyaev et al., 1978), or diffraction of X-rays in crystals (see Landau and Lifshitz, 1960).

We use here a simple model, and describe the T.I.D. by a one-dimensional (x) thin screen, which introduces a phase-change $\Delta\Phi(x,t) = \Phi_0 \cos[(x - vt)/d]$ on an incident radio-wave. So, unlike most scintillation problems, $\Delta\Phi(x,t)$ is not considered as a random function, and the only random process stems from the incoherent radio-source. We will return, in Sect. 4, to the expected validity of the hypotheses, and only note here that, in the present physical situation, the observer's integration time is well below the characteristic time variation due to the screen, and also, the parameters are such that (see Sect. 4) only a dimension $2\pi d$ (or less) on the screen actually contributes to the observer's results. The sinusoidal model was used by Hewish (1951), Ratcliffe (1956), and Buckley (1976), who did not perform all the present calculations, since they were more concerned with the random phase-change problem.

This model is convenient as it gives nearly exact analytic expressions for every useful observational parameter; this is of course not always the case for the usual random scintillations (see for example Salpeter, 1967; Cronyn, 1972).

In Sect. 2, we derive an analytic expression for the apparent visibility function corresponding to a given source in the general case; we apply the results in Sect. 3, and, in Sect. 4, we discuss as an example, the practical consequences for solar radio observations.

* Laboratoire associé au CNRS No. 264

Table 1. Notation

$f=\omega/2\pi$:	frequency (bandwidth Δf)
c:	velocity of light
k_0:	ω/c
(x,z):	two-dimensional coordinates (the screen is at $z=0$ and the observer at distance z
$\Delta\Phi(x)=\Phi_0 \cos\dfrac{x-x_0}{d}$:	phase change imposed by the screen at $(x,0)$
$x_0=vt$	
$E(x,z)$:	complex amplitude of the field ($e^{-i\omega t}$)
$\langle\ \rangle$:	time average [in practice, on a time $\tau\sim 1/\Delta f \ll \mathrm{Min}\,(d/\Phi_0 v, d/v)$] equivalent to an assembly average on the extended (incoherent and thus only quasi-monochromatic) source realizations
$\langle\ \rangle_\infty$:	time average: $\lim\limits_{L\to\infty}\dfrac{1}{2L}\int_{-L}^{+L} dt\ldots$ (in practice: $L\gg d/v$)
$V(a)$:	source's visibility when there is no disturbance
$V_{\mathrm{obs}}(a)$:	source's visibility in presence of the disturbance
$Z=z/(k_0 d^2)$	
$b=(x-x_0)/d$	
$\theta_s, 2\Delta\theta$:	angular position and 'size' of a 'Gaussian' source (with brillance distribution: $\exp[-(\theta-\theta_s)^2/\Delta\theta^2]$
$u=\Delta\theta z/2d$	
$T=2\pi d/v$	
$X(x,t)=\dfrac{x-vt+\theta_s z}{d}$	

2. Calculation

Notation is defined in Table 1.

2.1. The Starting Formula

We study the simple two-dimensional (x,z) problem: let a source be located at infinite distance $z_s\sim-\infty$, an infinitely thin diffracting screen at $z=0$, and an observer at distance z, who will study the source, both by total intensity measurement and interferometry near the frequency $f=\omega/2\pi$.

It is known that this can be reduced to a scalar problem (Born and Wolf, 1964); we define the complex amplitude of one field component ($\exp(-i\omega t)$), and assume that the screen acts on it such that:

$$E(x,0^+)=E(x,0^-)\exp(i\Delta\Phi(x)),$$

where

$$\Delta\Phi(x)=\Phi_0\cos((x-x_0)/d) \tag{1}$$

and

$$x_0=vt.$$

We assume $\Phi_0 v/d\ll\omega$, and neglect frequency shifts. Let $k_0=\omega/c$ satisfy $k_0 z\gg 1$ for all distances of interest, and $k_0 d\gg \sup(1, 1/\Phi_0)$ (these conditions are strongly satisfied here); also we suppose, for convenience, that the source's direction is near the z axis.

The source is assumed incoherent, and characterized by its visibility function when no screen does exist, or, equivalently, on the far side of the screen, i.e. the function (x and t independent):

$$V(a)=\langle E(x,0^-)E^*(x+a,0^-)\rangle$$

and we need to calculate the observed visibility function:

$$V_{\mathrm{obs}}(a)=\langle E(x,z)E^*(x+a,z)\rangle.$$

The angular brackets denote an assembly average over the source realizations, or an average over a time $\tau\sim 1/\Delta f\gg 1/\omega$. We stress that, as already noted, τ is in practice much smaller than the characteristic time of variation due to the screen itself, so that $V_{\mathrm{obs}}(a)$ is a priori x- and t-dependent.

Calculating the field in the Fresnel approximation gives (apart from inessential factors):

$$V_{\mathrm{obs}}(a)\propto\iint dx_1dx_2\exp\{ik_0[(x-x_1)^2-(x+a-x_2)^2]/2z\}\,V(x_2-x_1)\exp\{i\Phi_0[\cos((x_1-x_0)/d)-\cos((x_2-x_0)/d)]\}. \tag{2}$$

The integrations in Eq. (2) can be easily performed (Appendix 1), giving finally:

$$V_{\mathrm{obs}}(a)\propto\sum_{p=-\infty}^{+\infty}J_p[2\Phi_0\sin(pZ/2-a/2d)]\cdot\exp[-ip(b+a/2d)]\,V(a-pZd) \tag{3}$$

where:

$$Z=z/k_0d^2;\quad b=(x-x_0)/d;\quad x_0=vt.$$

The sum is in practice finite, owing to the rapid decrease in p of the function $J_p(u)\underset{p\to\infty}{\sim}(eu/2p)^p/\sqrt{2\pi p}$ when $p>eu/2$. This expression was not obtained in previous derivations (see for example Buckley, 1976), which gave only the Fourier series of the field $E(x,z)$.

2.2. Observational Quantities

The intensity measured at the location x, z and time t (with a small antenna and an integration time τ as already defined) is thus:

$$I_{\mathrm{obs}}=V_{\mathrm{obs}}(0)\propto V(0)+2\sum_{p=1}^{+\infty}J_p[2\Phi_0\sin(pZ/2)]\,\mathrm{Re}\{V(pZd)\exp(ipb)\} \tag{4}$$

where we have used the symmetry properties of the Bessel functions, and $V(-u)=V^*(u)$; the notation Re{ } means taking the real part.

Owing to the simplicity of this expression, every relevant intensity parameter can be easily obtained, often without any computation. For a purpose of comparison with random scintillations, we define the following time average:

$$\langle f(t)\rangle_\infty=\lim_{L\to\infty}\frac{1}{2L}\int_{-L}^{+L}f(t)\,dt$$

where, in practice, L must be (contrary to τ) much greater than the maximum characteristic time due to the screen. This yields $\langle I_{\mathrm{obs}}\rangle_\infty=1$ (as expected by energy conservation), and the scintillation index:

$$m=(\langle I_{\mathrm{obs}}^2\rangle_\infty-\langle I_{\mathrm{obs}}\rangle_\infty^2)^{1/2}/\langle I_{\mathrm{obs}}\rangle_\infty=\{2\sum_{p=1}^{\infty}J_p^2(2\Phi_0\sin(pZ/2))\,|V(pZd)|^2\}^{1/2}. \tag{5}$$

When studying the *spatial brightness distribution of the source*, the observer uses $V_{obs}(a)$ (a being the base line). We note that, contrary to $V(a)$, $V_{obs}(a)$ does not generally satisfy $V_{obs}(-a) = V^*(a)$, so that its Fourier transform is not real. Equation (3) also shows that the observed visibility takes the form: $V_{obs}(a) = V(a)G(a)$, where $G(a)$ is the observed visibility for a point-source ("isoplanetism") only if the source satisfies $V(a-p_{Max}Zd) \sim V(a)$, [where $p_{Max} \simeq \text{Sup}(1, e\Phi_0)$ in the general case].

We also note that time (or x) averaging Eq. (3) gives:

$$\langle V_{obs}(a)\rangle_\infty \propto J_0[2\Phi_0 \sin(a/2d)]\, V(a) \qquad (6)$$

(which reduces, for a point-source, to an expression equivalent to one given by Hewish, 1952). This could be used to deduce what would be seen by an observer with an integration time $L \sim \infty$, but is academic in the present context, since the T.I.D. will not in practice remain coherent over so long a time. It is interesting to remark that (if $\Phi_0 > 1$) the 'academic' observer looses source's structures finer than $\Phi_0/k_0 d$ (see Eq. 6), but, if there is isoplanetism, it is easy to show that $\langle |V_{obs}(a)|^2/V_{obs}(0)\rangle_\infty = |V(a)|^2$ (see Eq. 3); thus, the 'academic' observer could use speckle interferometry techniques (Labeyrie, 1970) to restore the visibility's modulus.

3. Results

The results found in Sect. 2 are in the form of analytic expressions which can be easily applied. The behavior of the expressions is controlled by a few parameters which have a clear physical significance. Indeed, if $\Phi_0 \ll 1$, the quantity $\exp(i\Phi_0 \cos((x-x_0)/d))$ in Eq. (1) can be linearized and the screen's spectrum (Appendix 2) has only the components $k=0, \pm 1/d$; in this case, the important non-dimensional parameter is $Z = z/k_0 d^2$, which is the ratio of the observer's distance to the Fresnel distance for a patch of dimension d. For the practical applications relevant here, one has $\Phi_0 > 1$ (and $Z < 1$); in this case, Eq. A.1 (where $p < e\Phi_0/2$), shows that the screen's spectrum has the maximum angular extension $\theta_s = e\Phi_0/2k_0 d$; thus, the important non-dimensional parameter will be $eZ\Phi_0/2$, which is the ratio of the observer's distance to the typical length d/θ_s; (note also that 'focal' conditions can be expected near $\Phi_0 Z \sim 1$, since, then, the phase differences due respectively to the screen and to the different path-lengths, nearly cancel each other).

We will not discuss here the two trivial limiting cases $\Phi_0 \ll 1$ and $Z \gg 1$, which are entirely academic in the present context; as previously noted, the latter is similar to the well-known Fraunhofer spectrum for diffraction by sonic waves in the so-called Raman and Nath conditions (Gulyaev et al., 1978). So, we will first study the rather simple case $eZ\Phi_0/2 < 1$ (and the limit $Z\Phi_0 \to 0$), and then the more general case $eZ\Phi_0/2 \gtrsim 1$.

For obtaining numerical values, the source must be defined explicitly. We choose, for convenience, a classical "gaussian" source (location θ_s, size $2\Delta\theta$). The corresponding explicit formulas are given in Appendix 3.

3.1. $e\Phi_0 Z/2 < 1$ (and $\Phi_0 \geqq 1$)

Let us first consider the limiting case $Z\Phi_0 \ll 1$. The high order terms in the series in Eq. (4) (or A.3), can be neglected (see the uniform asymptotic expansion of Bessel functions, Abramowitz and Segun, 1968); series developing the remaining yields, for the assumed Gaussian source:

$$I_{obs\,x,t} \propto 1 + \Phi_0 Z \cos[X(x,t)] \exp(-u^2) + O(\Phi_0 Z)^2$$

where:

$$u = k_0 Z d \Delta\theta/2 \; (= z\Delta\theta/2d)$$

and:

$$X(x,t) = \frac{x - vt}{d} + k_0 \theta_s Z d.$$

Thus, the observed intensity displays sinusoidal variations with the screen's period $T = 2\pi d/v$. The corresponding 'index': $m = \Phi_0 Z/\sqrt{2}$ reminds that obtained for random scintillations (with a single scale: Bramley and Young, 1967; Salpeter, 1967).

If, furthermore, the maximum interferometer's baseline a_M satisfies: $a_M/d \ll \min(1, 1/Z\Phi_0^2)$ and the source's size is such that $u < 1$, Eq. A.3 can be developed, giving:

$$V_{obs\,x,t}(a) \propto V(a) \exp\{-ia\Phi_0 \sin[X(x,t)]/d\}. \qquad (7)$$

Thus, the source displays an angular shift of amplitude $\Phi_0/k_0 d$ (varying sinusoidally in quadrature with the intensity) and negligible distorsions; this is consistent with geometrical optics results.

In the general case, an inspection of Eq. A.3 shows some results without any computation: the terms p that give a non-negligible contribution in Eq. A.3 (i.e. satisfying $p < e\Phi_0$), satisfy $pZ/2 < 1$; accordingly, except near $Z\Phi_0 \sim 1$, Eq. (4) or A.3 reduce, for a point-source (or a source satisfying $u\Phi_0 \ll 1$) to:

$$I_{obs\,x,t} \propto 1 + 2 \sum_{p=1}^{\infty} J_p[p\Phi_0 Z] \cos[pX(x,t)].$$

This is a Kapteyn series (Watson, 1966) formally identical to that giving the inverse of the radius vector in terms of the time t in the well-known Kepler's problem (the quantity $\Phi_0 Z$ plays the role of the eccentricity of the ellipse). This shows that I_{obs} varies monotonously with the time t (or x) during half a period T, and that the index is:

$$m = \{[1 - (\Phi_0 Z)^2]^{-1/2} - 1\}^{1/2} \qquad (8)$$

[of course, near $\Phi_0 Z \sim 1$, (8) must be replaced by the exact expression (5) which exhibits no singularity].

Numerical results (Eqs. A.2, A.3) are shown in Figs. 1 (upper part) and 2. In Fig. 1, the intensity is drawn in the time/frequency plane: this represents the so-called 'dynamic spectrum' often quoted by observers. The intensity-versus-time curves are similar to those given by Buckley (1976). Figure 2 shows examples of visibility results in conditions where Eq. (7) is a good approximation. Otherwise, source's distorsions may occur.

3.2. $e\Phi_0 Z/2 \gtrsim 1$ ($\Phi_0 \geqq 1$)

Some features can be inferred from inspection of Eq. (4) (or A.3). Let us consider first the terms p satisfying $pZ/2 < 1$; in these, the Bessel functions reduce, as in Sect. 3.1, to $J_p(p\Phi_0 Z)$. Incidentally we remind that the behavior of such terms for high p changes, in the vicinity of $\Phi_0 Z \sim 1$, from a nearly exponential decrease in p for $\Phi_0 Z < 1$, to a $p^{-1/3}$ decrease for $\Phi Z = 1$, and a $p^{-1/2}$ decrease for $\Phi_0 Z > 1$; this is consistent with the intuitive prediction since, as already noted, $\Phi_0 Z \sim 1$ corresponds to the vicinity of focal conditions. Indeed, the focusing appears clearly in Fig. 1, for $\Phi_0 Z$ slightly above 1; for this value of $\Phi_0 Z$, the scintillation index exhibits a maximum, which is sharper than the one obtained in the random case. For $e\Phi_0 Z/2 > 1$, there are also terms p satisfying $2/Z < p < e\Phi_0$; these are of the order of magnitude $J_p(2\Phi_0)$.

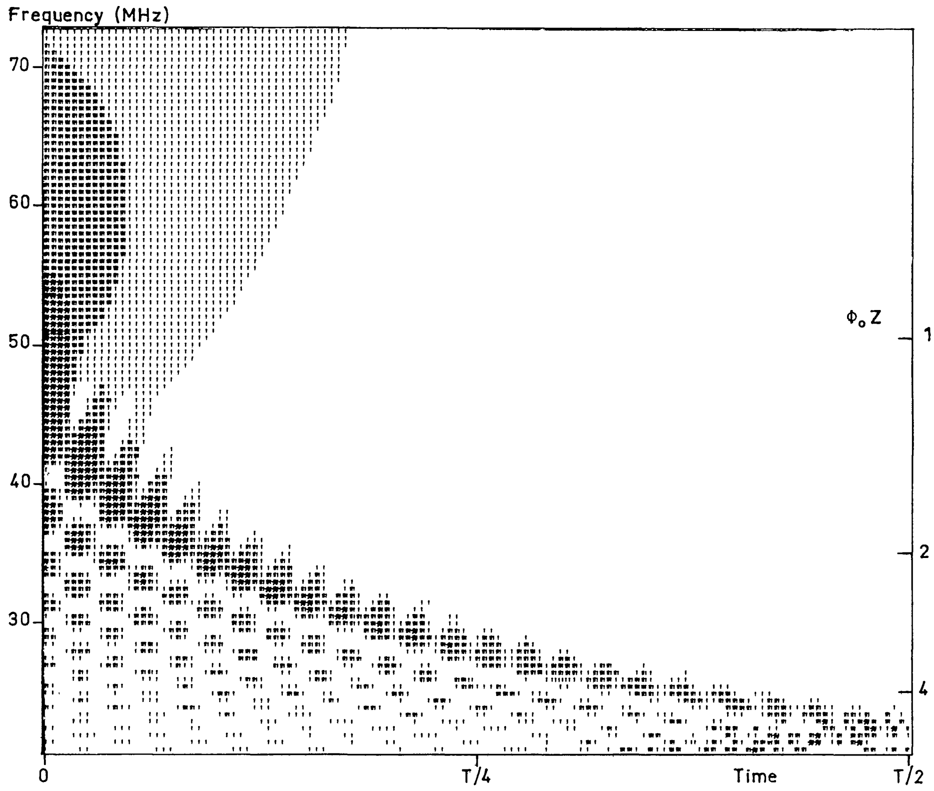

Fig. 1. Intensity in the time/frequency plane (i.e. the so-called 'dynamic spectrum') for parameters: $\Phi_0 = 10^3/f$ (MHz), $Z = 2.5/f$ (MHz), point-source. The pattern is symmetric with respect to the $t = 0$ and $t = T/2$ axes and has the temporal period T. Symbols used: ': $1 \leqq I_{obs} < 2$; ‴: $2 \leqq I_{obs} < 5$; ▪: $I_{obs} \geqq 5$. The values of $\Phi_0 Z$ are indicated on the right side

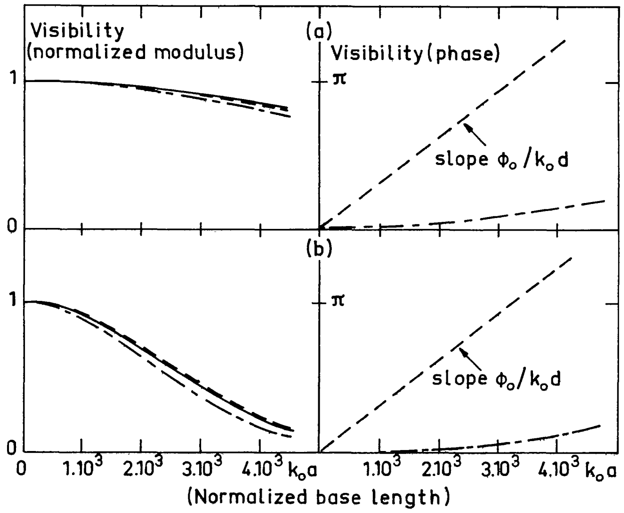

Fig. 2a and b. Visibility function (normalized) at times $t = 0$ (dotted line) and $t = T/2$ (dashed line) for $Z = 0.1/\Phi_0$, and two different source's sizes: $2\Delta\theta = 2/3'$ **a** and $2\Delta\theta = 4'$ **b**; other parameters: $\Phi_0 = 20$, $k_0 d = 2\ 10^4$. The visibility when there is no screen is shown for comparison (solid line). The screen introduces a variable apparent shift in source's position (amplitude $\delta\theta = \Phi_0/k_0 d$, period T, and few shape's distortions

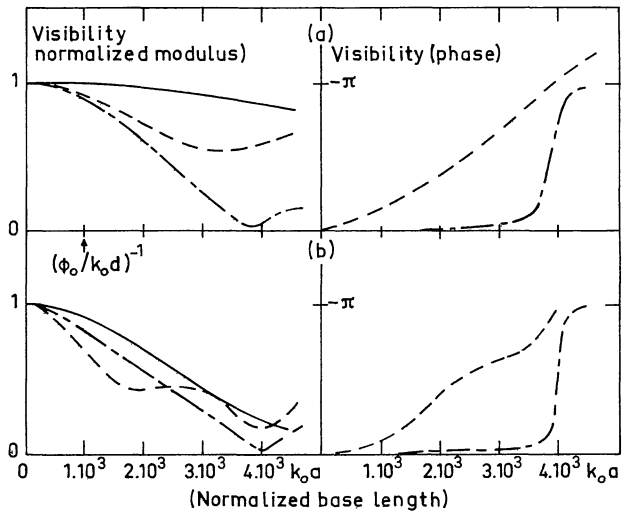

Fig. 3a and b. Same as Fig. 2 but with $Z = 4/\Phi_0$. The screen introduces an apparent variable source's broadening of order $\delta\theta \sim \Phi_0/k_0 d$, with a distorsion which could be possibly interpreted as a 'core-halo' structure, and a variable position shift

Concerning the terms $p > e\Phi_0$, they become rapidly negligible (in $(e\Phi_0/p)^p/\sqrt{p}$). The source's size acts in quenching the terms p satisfying $pu \gg 1$.

The corresponding temporal scales (or spatial) are T/p (or $2\pi d/p$). Of course the antenna dimension should be taken into account if it exceeds the lowest spatial scale.

The dynamic spectrum (Fig. 1, bottom part) exhibits regular features: these are due to the assumed sinusoidal phase variation on the screen; such features are not apparent in the individual intensity-versus-time curves, wich (except for the periodicity T) have an apparent random character, with a scale of temporal variation of order $T/\min(e\Phi_0, 1/u)$.

Likewise, an inspection of Eq. (3) (or A.2) shows that the visibility exhibits features corresponding in particular to a variable image's broadening of order $e\Phi_0/k_0d$. This can be seen in Fig. 3.

4. Consequences for Radio Observations Through a T.I.D. and Discussion

Owing to its characteristics, the phenomenon is expected to affect solar studies in particular; since it is very common, it is worth evaluating its effects.

First, let us determine plausible values for the disturbance's parameters. We use Bougeret (1979)'s Fig. 4, which shows measured 'peak to peak' angular deviations $2\delta\theta$, as function of frequency; we delete the low frequency part, and data with satellite beacons (which should be corrected to be compared with the others, since their distance to the screen (z_s) does not generally satisfy $z_s/z \gg 1$); the remaining data points are thus expected to satisfy: $\delta\theta = \Phi_0/k_0d$. This yields: $\delta\theta(') \sim 5\ 10^4/f^2$ (MHz), and thus: $\Phi_0/d \sim 10^6/\pi f$ (I.S. units). For the other parameters, we assume the likely values (Yeh and Liu, 1974):

$2\pi d \sim 10^5$ m; $\quad v \sim 10^2$ m s^{-1}; $\quad z \sim 3\ 10^5$ m.

Accordingly:

$\Phi_0 \sim 5\ 10^3/f$ (MHz); $eZ\Phi_0/2 \sim 4\ 10^2/f^2$ (MHz); $u \sim 3\ 10^{-3}\ \Delta\theta$ (').

These are typical values, expected to be valid within a factor two (or more), depending on such factors as the latitude, season and local time.

According to the above parameters, we expect very different results in the metric observing range (and shorter wavelengths), and in the decametric range. Let us take two typical examples.

(i) $f = 200$ MHz

Then, $\Phi_0 \sim 25$, $e\Phi_0Z/2 \sim 10^{-2}$, and the results of Sect. 3.1 apply. Accordingly, a typical source will exhibit small intensity time variations (amplitude $\Phi_0 Z \sim 1\%$ and period $T = 2\pi d/v \sim 10^3$ s). An interferometer with a base-line smaller than a few km, will measure a variable angular shift (period T and amplitude $\delta\theta = \Phi_0/k_0d \sim 1'$), and small distorsions of shape. This is similar to Bougeret's (1979) results.

(ii) $f = 20$ MHz

Then $\Phi_0 \sim 250$, $e\Phi_0Z/2 \gtrsim 1$, and the results of Sect. 3.2 apply.

The *intensity* expected to be measured is given by Eq. (4) (or A.3). An important point is that, at variance with the case (i), the result is very dependent on the parameter $\Phi_0 Z$, owing to the proximity of focal conditions. So, a behavior qualitatively analogous to that exhibited in the corresponding part of Fig. 1 is expected, namely, individual spikes or apparent random scintillations, depending on the parameter's values in the range given precedingly. Quantitatively, the minimum temporal scale is of order $T/\min(e\Phi_0, 1/u)$.

Thus, a typical source smaller than about 10′ could exhibit important intensity variations on temporal scales as low as 10 s or less (and also higher temporal scales). This could possibly play a role in the interpretation of recent experimental results on the scintillations of the 'type III' radio-bursts in the decametric range (Poquerusse and Steinberg, 1978). Likewise, it is worth noting that the middle part of the dynamic spectrum shown in Fig. 1 is reminiscent of some observations (Wild and Roberts, 1956: 'ridges with fine structure').

The expected observed *visibility* is given in Eq. (3) (or A.2). Referring to Fig. 3 (where the scale should be changed according to the different parameters' values), one expects in particular a variable source's broadening of order $\delta\theta = \Phi_0/k_0d \sim 10^{2\prime}$, with possibly some structure in it, and also a variable shift.

Owing to the finite maximal base-line and incomplete sampling that occur in actual observations, the visibility curves in Fig. 3 can be interpreted as representing a source with a core-halo structure. Indeed, as suggested by Steinberg et al. (1979), the present results could contribute to interprete recent observations by Chen and Shawhan (1977, 1978) who published visibility curves for solar radio-bursts, very similar to those in Fig. 3 (with the scale's change), and local-time dependent.

So far, the *frequency bandwidth* of the measurement has not been explicitly taken into account. It is easily seen from Sect. 3.1 (see also Fig. 1) that, with the usual bandwidths, the results found in the metric wavelengths range (and shorter wavelengths), will not change. In the decametric range, the terms corresponding to high p in Eqs. (3) and (4) can be altered: in the example (ii), a bandwidth $\Delta\omega$ only acts on the terms $p \geqq 1/(2\Phi_0 Z\Delta\omega/\omega)$. It is worth noting that, letting $p_{\max} \sim e\Phi_0$ gives a 'decorrelation bandwidth' (for the case $e\Phi_0 Z/2 > 1$, $\Phi_0 > 1$): $\Delta\omega \sim \omega^2 d^2/(2ecz\Phi_0^2) \propto \omega^4$; this formula is analogous to a known result for random scintillations (Bonazzola et al., 1978, for example).

Finally, let us return to the ability of the simple model used, to keep the physics of the actual situation.

An obvious limitation is that the actual ionospheric disturbance is not likely to remain coherent over many wavelengths ($2\pi d$) or periods (T). So, as explained in Sect. 1, the applications must be restricted to (typically) an observer with an integration time $\tau < T$ and whose 'seeing disk' is lower than a few wavelengths. Since $\Phi_0 > 1$, this last condition is equivalent to $\delta\theta z/2\pi d = \Phi_0 Z/2\pi \lesssim 1$, and it is indeed satisfied with the parameters used here.

An other limitation is linked to the modelling of the actual extended (width Δz in z direction) disturbing medium by an equivalent sinusoidal *thin* phase-changing screen. Since $\Phi_0 > 1$, this requires the condition $\Phi_0 \Delta z/k_0 d^2 \ll 1$, which ensures that (i) the intensity variation when emerging from the medium is small, (ii) the phase variation is proportional to the integrated electron density perturbation (see for example Salpeter, 1967), and (iii) the observer's results do not change very much when z is changed by Δz $\left[\text{since } \frac{\partial}{\partial z}(p_{\text{Max}} Z)\Delta z \ll 1\right]$. This is equivalent to $\Delta z/z \ll 1/(\Phi_0 Z)$, which is likely to be satisfied here.

5. Conclusion

We have calculated the effects of a thin sinusoidally phase-changing screen on the measurements of intensity (single antenna) and location and shape (interferometry) of an astronomical source. The simplicity of the problem makes it possible to easily perform nearly exact calculations of every useful observational parameter. We derive a simple expression for the visibility function and the intensity. This permits the dynamic spectrum and the visibility for a given source to be obtained, which were not explicitly studied in previous considerations of this model. As a by-product, this gives some results analogous (whenever a comparison is possible) to those found in random scintillations problems (phase screen with a single scale-size).

We have then applied the calculations to predict the effects of a 'travelling ionospheric disturbance of medium scale' on the measurements made by an Earth-bound radio-observer; of course, owing to the simplified model of the T.I.D., the results have only an indicative value.

The phenomenon is important near local noon, and the present calculations concern an observer using a short integration time; thus, the results are relevant in particular for solar observations.

For observations in the metric range (and lower wavelengths), the main expected effect is an apparent slow periodic ($T \sim 20$ min) source displacement; on the other hand, in the decametric range, one should observe important and rapid (scales as low as a few seconds) intensity variations, as also variable image's distorsions (for example a broadening to about 1° at 20 MHz with possibly a core-halo-like structure) and shifts.

These effects could, in some incomplete experimental configurations, be confused with the usual ionospheric scintillations which are ascribed to random irregularities with smaller spatial scales and phase variations.

The present results could play a part in the interpretation of recent experimental results on the scintillations of solar radio-bursts and their local-time dependent size and 'core-halo' structure in the decametric range. They show in particular that ionospheric effects can be more important in the decametric radio observing range than is generally assumed by solar observers (see, for example Chen and Shawhan, 1977).

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Appendix 1: Derivation of Eq. (3)

Transform the sum of cosine in product, in the integral (2), and make the change of variables $u = x_2 - x_1$, $v = x_2 + x_1$. Then, use the standard development:

$$\exp(iA \sin B) = \sum_{-\infty}^{+\infty} J_p(-A) \exp(-ipB)$$

(where J_p denotes the Bessel function of the first kind, of integer order p), with the parameters:

$A = 2\Phi_0 \sin(u/2d)$

$B = (v - 2x_0)/2d$

Performing the u and v integrations then yields Eq. (3).

We note that this expression could also be obtained from the standard expression of the field $E(x,z)$ (given in Ratcliffe, 1956) by transforming the double series into a simple one by using the standard addition theorems for Bessel functions.

Appendix 2: The Screen's "Spectrum"

The screen's angular spectrum (or the Fourier transform of the function $\exp(i\Phi_0 \cos((x-x_0)/d))$ is easily obtained by replacing it by its Fourier series. This gives, (see Ratcliffe, 1956):

$$\tilde{\Phi}(k) = \int_{-\infty}^{+\infty} \exp\left[i\Phi_0 \cos\left(\frac{x-x_0}{d}\right) - ikx\right] dx$$

$$= \sum_{p=-\infty}^{+\infty} (i)^p J_p(\Phi_0)\, \delta(k - p/d) \exp(-ipx_0/d). \quad \text{(A.1)}$$

Appendix 3

Explicit formulas for a "Gaussian" source with brillance distribution: $B(\theta) \propto \exp[-(\theta-\theta_s)^2/\Delta\theta^2]$.

The visibility on the far side of the screen is:

$V(a) \propto \exp[ik_0\theta_s a - (k_0 a \Delta\theta/2)^2]$.

Equation (3) yields thus the following observed visibility:

$$V_{\mathrm{obs}\,x,t}(a) \propto \exp(ik_0\theta_s a) \sum_{p=-\infty}^{+\infty} J_p[2\Phi_0 \sin(pZ/2 - a/2d)]$$

$$\cdot \exp - \{ip(X(x,t) + a/2d) + [k_0 a \Delta\theta/2 - pu]^2\} \quad \text{(A.2)}$$

where $X(x,t)$ is defined in Sect. 3.1, and the intensity:

$$I_{\mathrm{obs}\,x,t} \propto 1 + 2 \sum_{p=1}^{+\infty} J_p[2\Phi_0 \sin(pZ/2)] \cos[pX(x,t)] \exp - (pu)^2. \quad \text{(A.3)}$$

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