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Multicolour photoelectric photometry of Magellanic Cloud cepheids – II. An analysis of *BVI* observations in the LMC

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Summary. Photoelectric *BVI* observations of LMC cepheids (Martin & Warren) are combined with previous photoelectric *BV* values and analysed. It is shown that a P–L–C relation gives a much better fit to the data than a P–L relation at least in the period range $0.9 < \log P < 1.7$. Individual corrections for interstellar reddening are derived from the *BVI* results. The colour term in the P–L–C relation is shown to be intrinsic rather than due to differential interstellar reddening. We find

$$\langle V_0 \rangle = -3.80 \log P + 2.70 \ \{\langle B_0 \rangle - \langle V_0 \rangle\} + 16.41.$$

This relation is fitted to galactic cepheids in clusters and associations and also to cepheids with absolute magnitudes from radii determinations. We find

$$M_{\langle V \rangle} = -3.80 \log P + 2.70 \ \{\langle B_0 \rangle - \langle V_0 \rangle\} - 2.39.$$

At a given period there is a spread of 0.3 to 0.4 mag in $(\langle B \rangle - \langle V \rangle)$, most of which is intrinsic. In the mean the LMC cepheids are bluer at a given period than those in the Galaxy. This and the *BVI* results lead to a $B-V$ excess $\Delta(B-V) \sim 0.05$, presumably due to metal deficiency. The true distance modulus of the LMC derived from the P–L–C relation and corrected for abundance effects is 18.69 (±0.15). The reasons for the smaller modulus derived recently by de Vaucouleurs are briefly discussed.

1 Introduction and observations

It has long been realized that the study of cepheid variables in the Magellanic Clouds is fundamental to our knowledge of cepheids and their use as extragalactic distance indicators. However, despite the considerable efforts of earlier workers, there is still too small a sample of Magellanic Cloud cepheids available with good photoelectric light and colour curves. In addition, it is now highly desirable to obtain multicolour photometry in a system that will allow us to estimate the interstellar reddening of individual cepheids. In a recent paper, Dean, Warren & Cousins (1978) have demonstrated the usefulness of the *BVI* photometric system in determining reddenings of galactic cepheids. The *BVI* system is described by

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Cousins (1976). Throughout this paper, we write I for the I_{KC} of Cousins (I in the Cape–Kron system).

A programme to observe Magellanic Cloud cepheids (especially those with periods in the range 8 to 20 days) was carried out at the SAAO Sutherland Observatory by the first two writers. *UBVI* observations of cepheids in both clouds were obtained with the 1.0-m reflector and the St Andrews photoelectric photometer. However, so far rather few I

Table 1. SAAO mean results LMC cepheids.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
HV	Log P	$\langle V \rangle$	$\langle B \rangle - \langle V \rangle$	No. of pairs B, V	$\overline{V-I}$	$\overline{B-V}-\overline{V-I}$	No. of sets BVI	E_{B-V}
2353	0.492	15.37	0.53	10				
12765	0.535	15.29	0.57	10				
12700	0.911	14.85	0.73	18	0.75	−0.01	8	0.02
12823	0.919	14.57	0.55	10				
2854	0.936	14.64	0.72	17	0.78	−0.02	6	0.05
2733	0.941	14.69	0.62	20	0.76	−0.09	9	0.13
12816	0.960	14.51	0.57	22	0.72	−0.09	11	0.11
971	0.968	14.43	0.68	9				
2301	0.978	13.94	0.84	10				
6105	1.019	14.91	0.79	8				
2864	1.041	14.70	0.79	17	0.89	+0.01	9	0.07
2527	1.112	14.59	0.80	8				
2260	1.114	14.86	0.89	19	1.00	−0.02	9	0.16
997	1.119	14.52	0.85	21	0.95	−0.02	11	0.14
2579	1.128	13.95	0.66	9				
2352	1.134	14.17	0.75	24	0.86	−0.06	12	0.14
955	1.138	14.07	0.73	9				
5655	1.153				0.95	0.00	2	0.11
2324	1.160	14.35	0.85	21	0.92	+0.02	11	0.07
2262	1.200				0.92	+0.04	1	0.05
2549	1.209	13.70	0.72	9				
2580	1.229	13.96	0.75	21	0.86	−0.05	11	0.13
2836	1.246	14.62	1.01	17	1.13	0.00	9	0.20
1005	1.272	13.96	0.82	9				
2793	1.283	14.09	1.01	11	1.08	+0.06	9	0.10
1013	1.382	13.83	1.04	34	0.96	−0.02	7	0.14
12815	1.418	13.46	0.95	22	1.03	+0.05	12	0.09
1023	1.425	13.74	1.04	19	1.06	+0.08	11	0.07
1002	1.484				0.76	+0.08	6	−0.08
899	1.492				1.10	+0.06	6	0.11
2294	1.563	12.70	0.83	38	0.99	+0.06	9	0.06
879	1.566	13.35	1.03	31	1.06	+0.09	10	0.06
909	1.575				1.01	+0.04	3	0.09
2257	1.594				1.04	+0.08	4	0.06
2338	1.625	12.76	0.92	30	0.95	+0.05	8	0.05
877	1.655	13.36	1.14	34	1.19	+0.12	11	0.10
900	1.677				0.92	−0.01	3	0.11
953	1.680				0.91	−0.01	6	0.10
2369	1.684	12.62	0.96	36	1.05	+0.05	8	0.10
2827	1.897	12.30	1.24	29	1.08	+0.08	11	0.08
5497	1.995	11.92	1.20	38	1.15	+0.11	14	0.09
2883	2.033				1.10	+0.19	10	0.01
2447	2.007	12.02	1.29	38	1.10	+0.15	12	0.03
883	2.127	12.12	1.20	42	1.22	+0.16	16	0.08

measurements have been made of SMC cepheids. The individual observations are published separately (Martin & Warren 1979). In this paper we discuss the *BVI* observations in the LMC.

The SAAO data have been used to derive various photometric parameters which are listed in Table 1. The columns of Table 1 contain the following:

(1) Harvard Variable Number.

(2) Log *P*. These are generally standard values from the literature. In a few cases modifications to the standard values have been made on the basis of extensive photographic photometry (Martin, in preparation).

(3), (4) $\langle V\rangle$ and $(\langle B\rangle - \langle V\rangle)$, where the brackets denote intensity means in the usual way.

(5) The number of *B*, *V* pairs that were used in deriving the quantities in (3) and (4). It will be noted that for many of the cepheids a much larger number of observations have been obtained than has been customary in the past.

(6), (7) $(\overline{V-I})$ and $\{\overline{(B-V)-(V-I)}\}$. These are straight (magnitude) means of all the simultaneous $(V-I)$ and $(B-V)$ pairs. The use of these values is described later.

(8) The number of *BVI* sets which were used in deriving the quantities in (6) and (7).

(9) Values of the reddening E_{B-V} derived from the *BVI* observations as described later.

Table 2. LMC cepheids in common between different observers.

HV	$\Delta(\langle V\rangle)$	$\Delta(\langle B\rangle-\langle V\rangle)$	Source
877	0	+3	E
	+3	+2	M
	−2	−6	S
881	−1	+4	E
	+2	−5	M
883	+4	−1	E
	+1	−1	M
	−3	+1	G
	−3	+1	S
886	+3	+1	M
	−3	−1	G
900	+4	+4	E
	−1	+3	M
	−2	−6	G
909	+2	+3	M
	−3	−3	G
953	+2	+2	E
	0	+1	M
	−1	−2	G
1002	+3	+2	M
	−2	−2	G
2257	+2	+2	E
	−3	−1	M
2294	+3	+4	E
	−4	−1	M
	+1	−4	G
	+2	+2	S
2338	+1	+2	E
	+1	+1	M
	−2	−2	S
2369	+2	+2	E
	−1	−2	M
	−3	−1	G
	+2	0	S
2447	+2	−7	E
	0	+4	M
	−4	−2	G
	+1	+4	S
5497	−2	0	E
	+4	−1	M
	−3	0	S
12700	+1	−1	S
	0	+1	G

Units 0.01 mag.

		$\Delta(\langle B\rangle-\langle V\rangle)$	$\Delta(\langle V\rangle)$
E = Eggen (1977)	E	+0.014 ± 0.010	+0.016 ± 0.006
M = Madore (1975)	M	+0.004 ± 0.006	+0.007 ± 0.006
G = Gascoigne (1969)	G	−0.019 ± 0.007	−0.020 ± 0.005
S = SAAO (present paper)	S	−0.002 ± 0.010	−0.005 ± 0.008

Photoelectric B, V observations of LMC cepheids have been published by Gascoigne & Kron (1965), Gascoigne (1969), Madore (1975), Eggen (1977) and Gascoigne & Shobbrook (1978). Table 2 compares values of $\langle V\rangle$ and $\langle B\rangle - \langle V\rangle$ for stars in common between the various investigators. Note that the work of Gascoigne & Kron has been combined by Gascoigne with his later work and is here referred to simply as 'Gascoigne', as is the recent work of Gascoigne & Shobbrook. The numbers quoted are the residuals from the mean (in units of 0.01 mag). Mean values are given at the bottom of the table. The residuals are

Table 3. Mean results.

HV	Log P	<V>	<B> - <V>	HV	Log P	<V>	<B> - <V>
ROB 44	0.408	16.36	0.58	12471	1.200	14.68	0.99
24	0.429	16.22	0.51	2549	1.209	13.70	0.72
33	0.478	16.22	0.66	2580	1.229	13.96	0.75
HV2353	0.492	15.37	0.53	2836	1.246	14.62	1.01
ROB 25	0.529	16.03	0.60	1005	1.272	13.96	0.82
HV12765	0.535	15.29	0.57	2793	1.283	14.09	1.01
ROB 10	0.556	15.76	0.56	U 11	1.303	13.99	0.86
29	0.569	15.80	0.58	U 1	1.353	14.10	0.98
HV12720	0.635	15.74	0.63	HV2749	1.364	14.73	1.24
12869	0.653	15.06	0.62	878	1.367	13.57	0.85
ROB 22	0.669	15.55	0.70	886	1.380	13.33	0.78
HV12231	0.677	15.43	0.64	1013	1.382	13.83	1.04
6093	0.680	15.35	0.64	1003	1.387	13.25	0.71
12077	0.703	15.30	0.61	889	1.412	13.71	0.95
12079	0.841	14.94	0.66	12815	1.418	13.46	0.95
12976	0.895	14.99	0.74	902	1.421	13.22	0.70
12700	0.911	14.84	0.74	1023	1.425	13.74	1.04
12823	0.919	14.57	0.55	2251	1.447	13.10	0.75
2854	0.936	14.64	0.72	8036	1.453	13.60	0.90
2733	0.941	14.69	0.62	875	1.482	13.04	0.78
12452	0.941	14.79	0.71	1002	1.484	12.94	0.75
12717	0.947	14.69	0.70	899	1.492	13.43	0.94
12816	0.960	14.51	0.57	882	1.503	13.42	0.89
971	0.968	14.43	0.68	873	1.536	13.52	1.09
2301	0.978	13.94	0.84	881	1.553	13.12	0.88
12474	0.993	14.65	0.69	2294	1.563	12.68	0.81
6105	1.019	14.91	0.79	879	1.566	13.35	1.03
2432	1.038	14.23	0.52	909	1.575	12.74	0.80
12248	1.038	14.47	0.75	2257	1.592	13.06	0.96
2864	1.041	14.70	0.79	2338	1.625	12.78	0.94
12716	1.051	14.70	0.76	877	1.655	13.38	1.20
12253	1.099	14.41	0.76	900	1.677	12.78	0.92
2527	1.112	14.59	0.80	953	1.680	12.28	0.87
2260	1.114	14.86	0.89	2369	1.684	12.60	0.96
997	1.119	14.52	0.85	2827	1.897	12.30	1.24
2579	1.128	13.95	0.66	5497	1.995	11.95	1.20
2352	1.134	14.17	0.75	2883	2.033	12.41	1.23
955	1.138	14.07	0.73	2447	2.077	12.01	1.25
2324	1.160	14.35	0.85	883	2.127	12.15	1.19

satisfactorily small, especially when one considers that they include not only any systematic or random errors in the photometry, but also uncertainty in drawing mean intensity curves.

Table 3 lists the adopted values of $\langle V \rangle$ and $(\langle B \rangle - \langle V \rangle)$. Where a cepheid has been observed by more than one observer, straight means of the various $\langle V \rangle$ and $(\langle B \rangle - \langle V \rangle)$ values have been taken. The discussion of the present paper rests on the $\langle V \rangle$ and $(\langle B \rangle - \langle V \rangle)$ values of Table 3 and the $(\overline{V-I})$ and $(\overline{B-V})-(\overline{V-I})$ values of Table 1. Notes on some individual cepheids follow Table 3.

2 Period–luminosity and period–luminosity–colour relations

Fig. 1 is a $\langle V \rangle$–$\log P$ plot for all the cepheids of Table 3. Different symbols distinguish different observers. The considerable increase in the number of cepheids due to the SAAO work is evident.

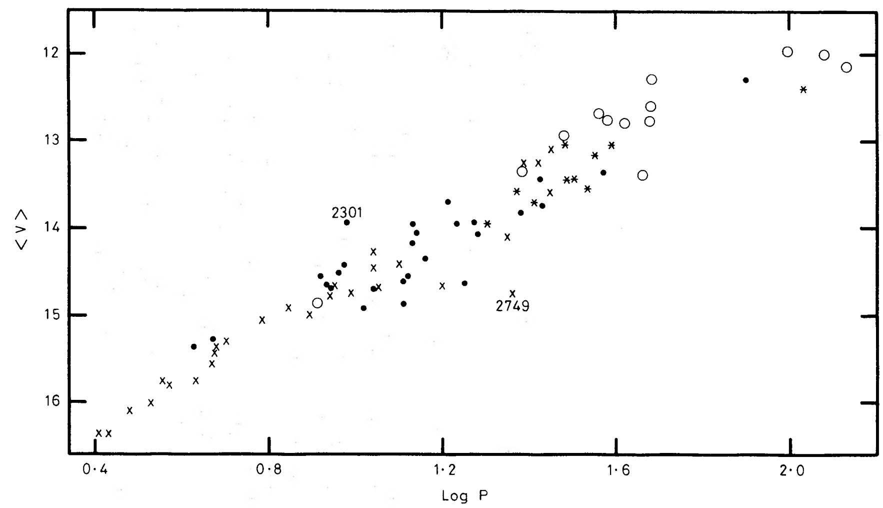

Figure 1. The $\langle V \rangle$ against $\log P$ plot for all cepheids of Table 3. Different symbols denote different observers. Filled circles, SAAO; crosses, Gascoigne; asterisks, Madore; open circles, more than one observer.

Notes to Table 3:

HV 2353. This cepheid has low amplitude and sinusoidal light curves and is too bright for the normal P–L–C relation at the tabulated period. It is assumed to be pulsating in the first harmonic. In the analysis $\log P = 0.624$ was adopted. This value for the log of fundamental period was calculated according to equation (7) of Iben & Tuggle (1975).

HV 12765. The remark to HV 2353 applies to this cepheid also. $\log P = 0.668$ was adopted in the analysis.

HV 12869. The remark of HV 2535 applies to this cepheid also (see Gascoigne & Shobbrook 1978), $\log P = 0.786$ was adopted in the analysis.

HV 2301. This cepheid was omitted in the final analysis. It deviates from the P–L–C relations (*cf.* Fig. 2).

HV 2749. This cepheid was omitted in the final analysis as it is a heavily reddened cepheid in the 30 Doradus region (*cf.* Gascoigne 1969 and Fig. 1).

HV 2432. Of the stars included in the general solutions (Table 4) HV 2432 has by far the largest residual (0.37 in solution 2b). Gascoigne (private communication) notes that he suspects a blue companion. In view of the number of stars involved, this star will not have affected the solutions significantly. Since no I observations of HV 2432 have been made, it is not included in the final solution. In the $\log P$ versus $(\langle B \rangle - \langle V \rangle)$ plot of Fig. 5, HV 2432 lies just below the lower dotted line.

Table 4. Coefficients of P–L and P–L–C relations.

Solution	No. of stars	ϵ	δ	σ	α	β	γ	σ
1 (a) All cepheids	77	-2.59 ± 0.07	17.22 ± 0.09	0.26 ± 0.02	-3.45 ± 0.08	2.21 ± 0.17	16.46 ± 0.08	0.14 ± 0.01
(b)					-3.79	3.08	16.16	0.16 ± 0.01
2 (a) All cepheids	72	-2.70 ± 0.09	17.32 ± 0.10	0.25 ± 0.02	-3.49 ± 0.07	2.28 ± 0.15	16.44 ± 0.08	0.13 ± 0.01
(b) $\log P < 1.7$					-3.72	2.95	16.18	0.14 ± 0.01
3 (a) All cepheids with *BVI*	26	-2.90 ± 0.22	17.62 ± 0.31	0.30 ± 0.04	-3.70 ± 0.13	2.35 ± 0.24	16.65 ± 0.17	0.13 ± 0.02
(b) $\log P < 1.7$					-3.88	2.85	16.44	0.14 ± 0.02
4 (a) As solution 3 with individual	26	-2.79 ± 0.20	17.17 ± 0.27	0.27 ± 0.04	-3.60 ± 0.13	2.18 ± 0.23	16.55 ± 0.14	0.12 ± 0.02
(b) reddenings applied					-3.80	2.70	16.41	0.14 ± 0.02

$\langle V \rangle = \epsilon \log P + \delta$.
$\langle V \rangle = \alpha \log P + \beta (\langle B \rangle - \langle V \rangle) + \gamma$.
σ = Standard deviation of $\langle V \rangle$ for a single cepheid from the solution.
(a) Least squares fit (standard errors quoted).
(b) Maximum likelihood fit.

Table 4 contains some solutions of the equations

$$\langle V \rangle = \epsilon \log P + \delta$$

and

$$\langle V \rangle = \alpha \log P + \beta(\langle B \rangle - \langle V \rangle) + \gamma.$$

σ is the standard deviation of a cepheid from the corresponding solution.

Solutions were performed for the cepheids of Table 3 divided into various groups. The solutions listed are:

(1) All LMC cepheids.

(2) All LMC cepheids with $\log P < 1.7$. This omits the very long-period cepheids which have no known galactic counterparts.

(3) All cepheids with $\log P < 1.7$ which also have *BVI* measures.

(4) The same sample as (3) but with individual reddening corrections applied. Thus this solution refers to $\langle V_0 \rangle$ and $\langle B_0 \rangle - \langle V_0 \rangle$. The adopted reddenings are listed in Table 1.

For each of these groupings two solutions were made:

(a) A conventional least-squares solution.

(b) A maximum likelihood solution.

In carrying out the maximum likelihood solution, the following standard errors for the observational quantities were adopted; for $\langle V \rangle$, 0.03; for $(\langle B \rangle - \langle V \rangle)$, 0.04; for $\log P$, 0.001. The standard error for $\langle V \rangle$ and $(\langle B \rangle - \langle V \rangle)$ follow from an analysis of the independent values of these quantities for stars in common between different observers (i.e. an analysis of the data of Table 2). The standard error of $\log P$ is a conventional value. This is in any case so much smaller than the errors in the other two quantities that its exact value is of little importance in these solutions.

For the $\langle V \rangle - \log P$ relation with essentially all the error residing in $\langle V \rangle$ we expect the method of least squares to produce the same results as the method of maximum likelihood. This was found to be the case and only one value is entered in Table 4 (i.e. the same values apply to solutions (b) as are entered under solutions (a)). Again, as we might anticipate, the methods of maximum likelihood and least squares give somewhat different values for the coefficients in the P–L–C relation. In principle the maximum likelihood results are preferable provided the adopted standard errors are reliable. It should be stressed that despite the differences between the coefficients determined by the two methods, the results by either method reproduce the observations well. This is shown by the values of σ listed. Since there is a (statistical) correlation between $(\langle B \rangle - \langle V \rangle)$ and $\log P$ (*cf.* Section 5), the constants in the P–L–C relation are not completely separable in the solutions. Thus, in representing the observations any one set of constants (α, β, γ) should be used in its entirety.

A striking feature of Table 4 is that the values of σ are all considerably less (for the more restricted solutions a factor of about 2 less) in the case of the P–L–C relations than for the P–L relation for the same stars. This shows rather convincingly that the introduction of a colour term greatly improves the representation of the observations. This is illustrated graphically in Fig. 2. Here the quantity, $\langle V \rangle - 2.70\,(B-V)$ is plotted against $\log P$ for all the stars in Table 3. The colour coefficient is that finally adopted (Section 4). Fig. 2 should be compared with the $\langle V \rangle - \log P$ plot of the same stars in Fig. 1. It is clear that the introduction of a colour term has greatly reduced the scatter. This is particularly noticeable for $0.9 < \log P < 1.7$ where most of the results are concentrated and where the sample of cepheids used shows a wide range of $B-V$ at a given period (see Fig. 5 and Section 5). Further work is probably required firmly to establish the superiority of a P–L–C relation over a P–L

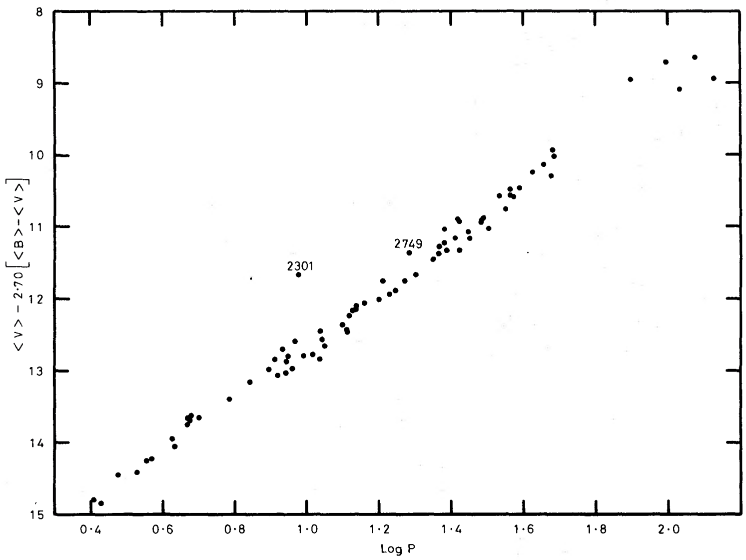

Figure 2. A plot of log P against $\{\langle V\rangle - 2.70\ (\langle B\rangle - \langle V\rangle)\}$ for the same cepheids as in Fig. 1.

relation for cepheids with $\log P < 0.7$ and those with $\log P \sim 2.0$. HV 2301 (which is marked in Fig. 2) is anomalous (see notes to Table 3) and obviously requires further study.

We believe that the present results leave no doubt as to the need to introduce a colour term into the discussion. However, such a term *might* arise if there were large amounts of differential interstellar reddening between different cepheids in the LMC. The coefficient R in the expression $A_V = R \cdot E_{B-V}$ is about 3 for a normal reddening law. This is close to the values of β in Table 4. In these circumstances it is easy to show that it is difficult to distinguish between a spread in the P–L relation due to differential reddening and a spread due to an intrinsic colour term unless reddening corrections for individual stars are available. We therefore now discuss the use of the *BVI* measures to deduce individual reddenings.

3 Reddening estimates for LMC cepheids from *BVI* measures

In a recent paper Dean *et al.* (1978) have shown that the *BVI* system allows good discrimination between temperature reddening and interstellar reddening for cepheids. They derive an intrinsic line and a reddening line in the $(V-I)$ against $\{(B-V)-(V-I)\}$ diagram. In Fig. 3 the line AA′ is the intrinsic line derived by Dean *et al.* However, as they note, this is

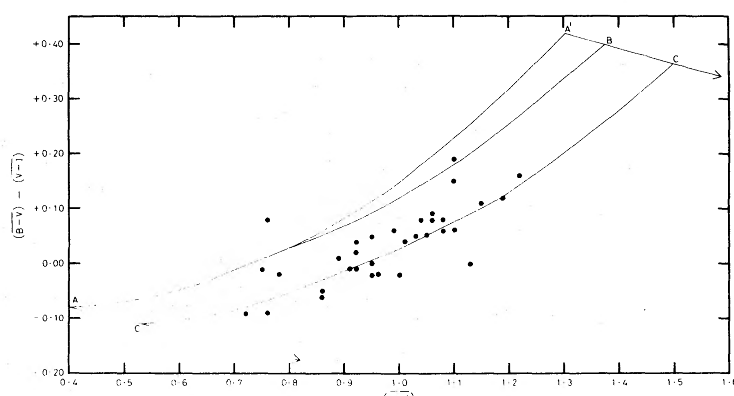

Figure 3. A $(\overline{V-I})$ against $(\overline{B-V})-(\overline{V-I})$ plot for the cepheids of Table 1. The line AA′ is the intrinsic line of Dean *et al.* and AB is the intrinsic line now adopted (see text). The line CC shows the effect of a reddening, $E_{B-V} = 0.10$, on AB. Reddening trajectories are shown (arrowed).

essentially an extrapolation for $(V-I) > 0.9$. The points for galactic cepheids at minimum shown in Fig. 3 of Dean *et al.* strongly suggest that the line AA′ is drawn too high for $(V-I) > 0.9$ and indicates that the line AB of our Fig. 3 is a better approximation to the intrinsic line. The line AB is also consistent with the line for late-type supergiants of luminosity class Ib recently derived by Cousins (1978). We have adopted AB as the intrinsic line in what follows. However, the difference between the values of E_{B-V} for LMC cepheids using curve AB instead of AA′ is very slight. It should also be noted that our primary concern is to determine whether or not there is any large differential reddening amongst LMC cepheids rather than to determine absolute reddenings (see also the discussion of abundance effects in Section 7). Also shown in Fig. 3 are reddening trajectories as derived by Dean *et al.* and a line CC showing the effect of reddening the intrinsic line AB by $E_{B-V} = 0.10$.

Dean *et al.* have shown that the $(B-V)$, $(V-I)$ loops of galactic cepheids are approximately straight lines. This means that it is convenient to use the quantities $(\overline{V-I})$ and $\{(\overline{B-V})-(\overline{V-I})\}$ as defined in Section 1 and listed in Table 1 to determine the reddening. The points are plotted in Fig. 3 and the derived reddenings (with respect to the curve AB) listed in Table 1. It may be worth noting here that if X, Y, Z are three simultaneous magnitudes of a variable star at different wavelengths, the pair of colours $(\langle X\rangle-\langle Y\rangle)$, $(\langle Y\rangle-\langle Z\rangle)$ is not in general appropriate for a reddening determination since the variable may never pass through such a point ($\langle X\rangle$, $\langle Y\rangle$, $\langle Z\rangle$, do not necessarily refer to the same phase).

It is clear from either Table 1 or Fig. 3 that the spread in E_{B-V} values is small for this sample of LMC cepheids. The mean value is $E_{B-V} = 0.086$ mag with a standard deviation for an individual cepheid of 0.05 mag. These values include HV 2836 which appears to have a distinctly higher reddening (0.2 mag) than the other cepheids in the sample. We discuss the absolute value of the reddening further in Section 7.

4 Reddening corrected P–L–C relationship

The small spread in reddenings derived in the last section makes it at once unlikely that the observed spread in the P–L relationship could be due to differential interstellar reddening. This is borne out by the plots of Fig. 4(a), (b), (c) and by the solutions of Table 4. Fig. 4 shows:

(a) A $\langle V\rangle$ against $\log P$ plot for stars with individual *BVI* reddening (i.e. before reddening corrections are applied).

(b) A $\langle V_0\rangle$ against $\log P$ plot for the same stars (i.e. individual reddenings applied, adopting $A_V = 3.3\ E_{B-V}$ which is appropriate for stars of this colour (*cf.* de Vaucouleurs 1978a).

(c) A $\{\langle V_0\rangle - 2.70\ (\langle B_0\rangle-\langle V_0\rangle)\}$ against $\log P$ for these same stars. The colour coefficient is the one finally adopted below.

It will be seen that there is some reduction in scatter between (a) and (b). However, there is a much more marked decrease in scatter in (c). Table 4 provides quantitative data on this effect. As above, it seems desirable to leave the few very long-period cepheids ($\log P \sim 2.0$) out of the discussion which then restricts the sample to $\log P < 1.7$. Making this restriction, the values of σ (the standard deviation of one star) corresponding to the fits to the data of Fig. 4(a), (b), (c) are: (a) 0.30 ± 0.04, (b) 0.27 ± 0.04, (c) 0.14 ± 0.02. There is thus a marginal improvement between (a) and (b) but a very considerable one between (b) and (c). We believe that these results demonstrate convincingly that a P–L–C relation (with an intrinsic colour term) is a distinct improvement on a P–L relation in this period range.

The individual corrections for reddening do reduce the scatter in the P–L relation slightly. For instance the two most reddened cepheids in the sample HV 2836 and HV 2260 are brought into better agreement with the rest of the sample.

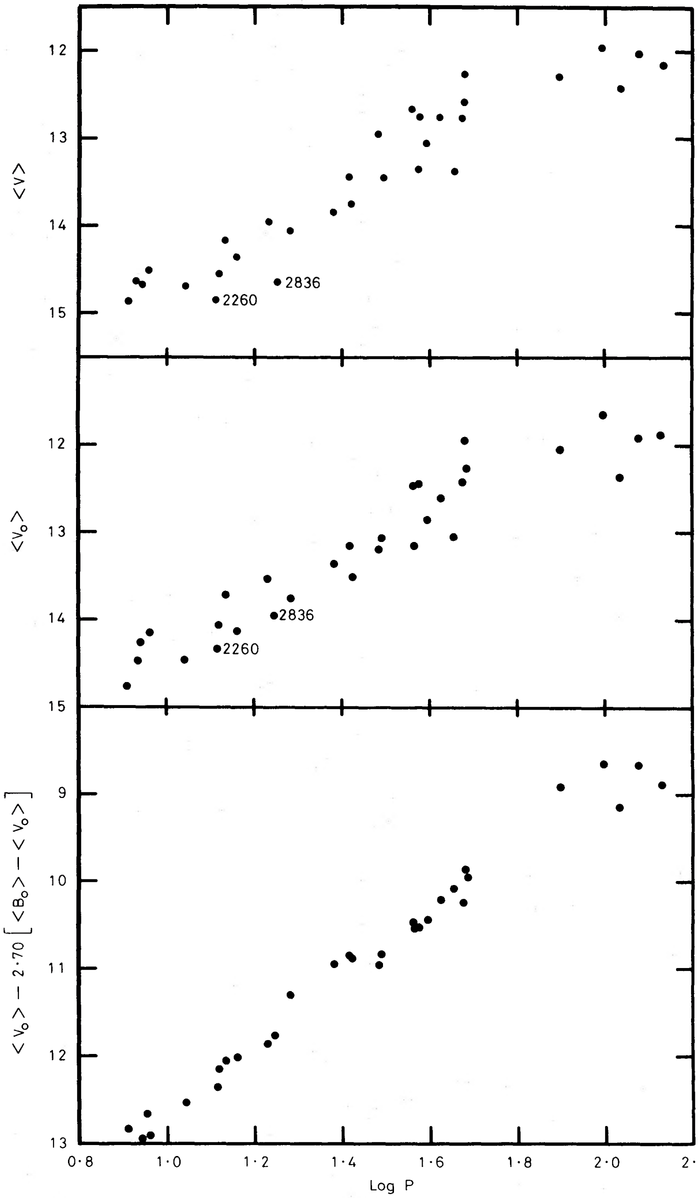

Figure 4. Plots for the 31 cepheids with *VBI* reddenings. (a) A $\langle V\rangle$ against $\log P$ plot. (b) A $\langle V_0\rangle$ against $\log P$ plot. (c) A $\{\langle V_0\rangle - 2.70\ (\langle B_0\rangle - \langle V_0\rangle)\}$ against $\log P$ plot.

We adopt as the best present P–L–C relationship for the LMC, solution 4b of Table 4. That is:

$$\langle V_0\rangle = -3.80 \log P + 2.70\ \{\langle B_0\rangle - \langle V_0\rangle\} + 16.41.$$

This should reproduce values of $\langle V_0\rangle$ for LMC cepheids in the range $0.9 < \log P < 1.7$ with a standard deviation of 0.14 mag (the σ of Table 4). For LMC cepheids for which no reddenings are available presumably the P–L–C relation of Table 4 (2b) is the most suitable. As might be expected, the difference in the predicted magnitudes from solutions 2b or 4b is very small.

In the above work we have been concerned to show empirically that the LMC results demand a P–L–C relationship. The necessity of an intrinsic colour term is of course to be expected on theoretical grounds (e.g. Sandage 1972) if the cepheids considered occupy a region of finite width in the HR diagram. It is gratifying therefore to see that the present

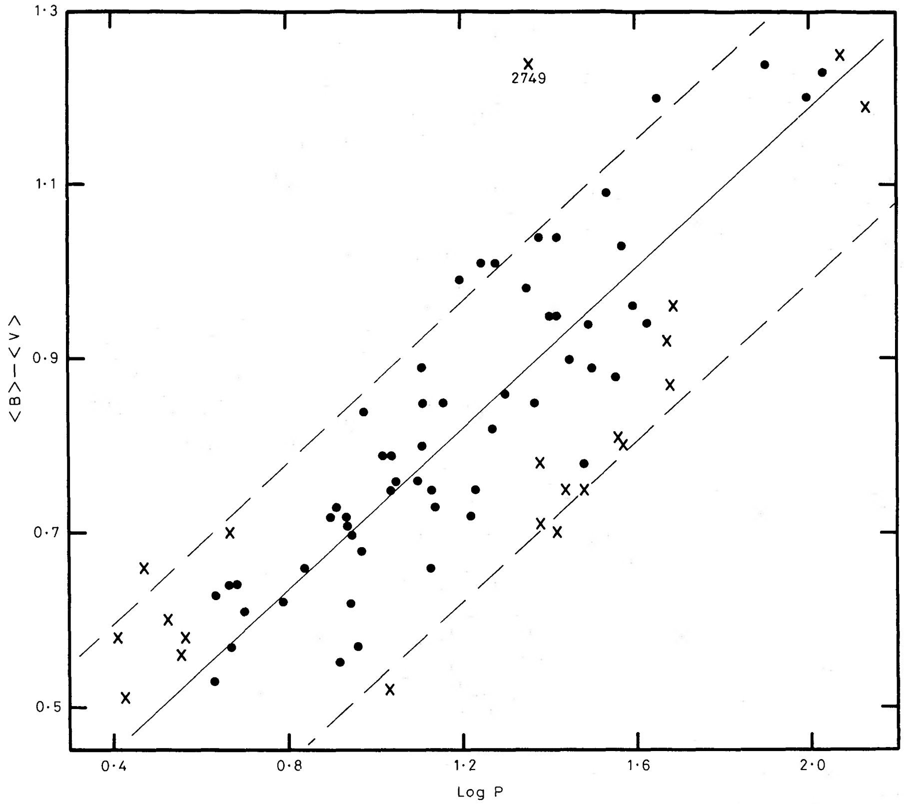

Figure 5. A (⟨*B*⟩–⟨*V*⟩) against log *P* plot for the cepheids of Table 3.

results strongly support the theoretical expectation. Indeed our coefficient of $(B-V)$ (= 2.70) is close to the theoretical value (= 2.647) (Tammann 1970).

5 Period–colour relation

The observed $\log P - \{\langle B\rangle - \langle V\rangle\}$ diagram is shown in Fig. 5. There is evidently a wide range of colour at a given period. Since the spread in the reddenings for stars with *BVI* (Section 3) is small, we anticipate that most of the spread in colour at a given period must be intrinsic. The LMC cepheids therefore populate an instability strip between 0.3 and 0.4 mag wide in $B-V$. Fig. 5 contains all the stars from Table 3. However, those previously observed by Gascoigne (1969) are shown as crosses. The way Gascoigne's stars in the range $0.9 < \log P < 1.7$ lie to the blue of most of the other cepheids is remarkable. Since the overlaps discussed in Section 1 show quite good agreement between different observers, it seems unlikely that this behaviour can be due to systematic errors. The most likely explanation is that this is due simply to selection effects and/or chance. It illustrates the fact that with the cepheids occupying such a wide band in the P–C diagram, it is necessary to have a very large number of observations before the band can be delineated with any certainty. The present number of cepheids, although a great improvement on previous work, is hardly sufficient to define the band accurately. This problem will be further discussed by Martin (in preparation) in connection with the extensive SAAO photographic work on Magellanic Cloud cepheids. It is clear that unless one has large numbers of cepheids in both the LMC and the galaxy, one cannot meaningfully compare P–C relations in the two galaxies. Nevertheless, the present results allow some progress in this direction. The line in Fig. 5 is the galactic relation of Dean *et al.* (1978).

$$\langle B_0\rangle - \langle V_0\rangle = 0.46 \log P + 0.27.$$

A band of this slope (dotted lines) would seem to fit the LMC observations quite well a least in the range $0.9 < \log P < 1.7$ where there are sufficient cepheids to be tolerabl certain of the distribution. However, the centre of the LMC band lies somewhat to the blu of the galactic relation. The dotted lines define a centre about 0.025 mag to the blue of th galactic relation. This is before any reddening is taken into account for the LMC stars Although more cepheids are required to define the band accurately and although the galacti relation is not error free, it would seem that the LMC cepheids are bluer at a given perio than their galactic counterparts. Although by no means as blue in the range $0.9 < \log P < 1.$ as would be implied by Gascoigne's (1969) observations alone. This matter is furthe discussed in Section 7.

6 The calibration of the cepheid distance scale

The absolute calibration of the cepheid distance scale remains a matter of some difficulty We consider here the calibration derived from galactic cepheids of known distance (membe of clusters or associations) and distances derived from recent radii determinations (Balon 1977).

The available data on galactic cepheids in clusters and associations have recently bee summarized by van den Bergh (1977a). His list differs from earlier ones (e.g. Sandage & Tammann 1969) in that reasons are given for omitting some cepheids (e.g. RS Pup *cf.* Egge 1977). Furthermore van den Bergh includes a number of new calibrating cepheids whic have been recently discussed as members of clusters or associations. We have adopted va den Bergh's list (his table I) and data, except for the colours of the seven cepheids whic he takes from Sandage & Tammann (1969). The value of E_{B-V} previously used for thes seven stars needs changing to allow for the variation of E_{B-V}/A_V with intrinsic colou (Schmidt-Kaler 1961) (*cf.* Dean *et al.* 1978, table II). This effect has already been taken int account for the other cepheids in van den Bergh's list.

From the discussion of Section 4 we adopt a P–L–C relation of the form

$$M_{\langle V \rangle} = -3.80 \log P + 2.70\,(\langle B_0\rangle - \langle V_0\rangle) + \phi.$$

Individual values of ϕ were then derived for the 14 cepheids in clusters and associatio and are plotted against period in Fig. 6 (crosses and open circles). The crosses denote wh may perhaps be regarded as the 'classical' calibrators. These are the seven cepheids in cluste (EV Sct, CEb Cas, CF Cas, CEa Cas, U Sgr, DL Cas, S Nor) which are in common betwee

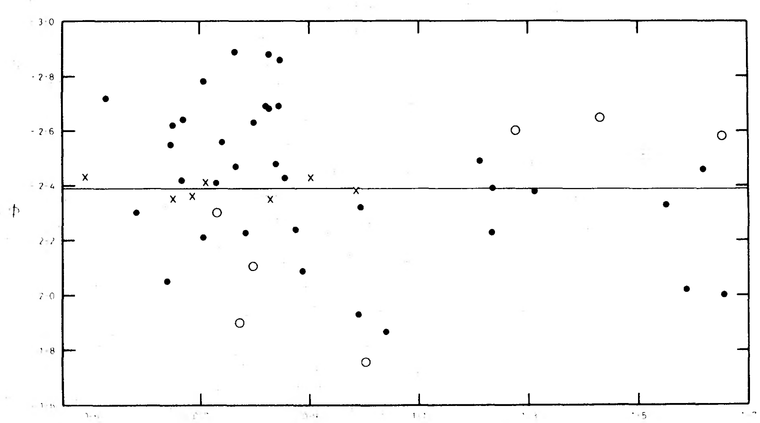

Figure 6. Values of the constant $\phi\{= M_V + 3.80 \log P - 2.70\,(\langle B_0\rangle - \langle V_0\rangle)\}$ for galactic calibrati cepheids. Crosses, 'classical' calibrators (see text); open circles, other cepheids in clusters and associatio filled circles, cepheids with radii (Balona 1977).

the lists of Sandage & Tammann (1969) and van den Bergh (1977a). It is rather striking in Fig. 6 that the scatter in ϕ values is much less for these seven stars than for the other cepheids in van den Bergh's list (open circles). This suggests that further work to improve the data on these additional calibrators would be profitable. It may however be noted that the three open circles with $\log P > 1.2$ come from cepheids in associations and one might therefore perhaps expect more scatter in this case than for cepheids in clusters.

Balona (1977) has recently applied the method of maximum likelihood to the problem of determining cepheid radii from radial velocity measurements and photometry. He shows that his results are a considerable improvement in accuracy over previous work. It is possible to derive absolute magnitudes from his radii. These absolute magnitudes are not expected to have the individual accuracy that can be achieved from cepheids in well-studied clusters. However, the method has the great advantage that it can in principle be applied to a large number of galactic cepheids in the general field so that lower individual accuracy will be compensated by the large number of objects. In addition, it is possible in principle to cover any desired period range.

We proceed in the following way. Since the mean radius of a cepheid refers to a point near maximum light, we apply Balona's equation 1.3 at maximum light. Hence:

$$M_V^{\max} = A(B-V)_0^{\max} - 5 \log R_0 + C.$$

Here, $M_V^{\max}$ and $(B-V)_0^{\max}$ are the visual absolute magnitude and intrinsic colour at maximum and R_0 is the mean radius derived by Balona (his table I). A is a constant also derived for each cepheid by Balona. We adopt his mean value ($A = 2.15$) for all stars.

$$C = M_{\mathrm{BOL}\odot} + 10 \log T_{\mathrm{e}\odot} - (10a_0 + b_0)$$

where a_0 and b_0 are constants in the relations

$$\log T_{\mathrm{e}} = a_0 + a_1(B-V)$$

$$M_{\mathrm{BOL}} = M_V + b_0 + b_1(B-V)_0.$$

Following Balona, we adopt $a_0 = 3.886$ $b_0 = 0.145$ (Parsons 1971; Kraft 1961). This leads to $C = 3.40$.

For each cepheid we then compute $\langle V\rangle - V^{\max}$ and derive

$$M_{\langle V\rangle} = M_V^{\max} + (\langle V\rangle - V^{\max}).$$

Using these values of $M_{\langle V\rangle}$ we derive values of ϕ for the 36 cepheids for which Balona gives radii and for which data on reddening are given by Dean *et al.* (1978). These values of ϕ are shown as dots in Fig. 6. As anticipated there is considerable scatter. Taken alone, the dots in Fig. 6 show a slight dependence of ϕ on $\log P$ but the open circles show a slight opposite trend. We conclude that there is no good evidence for a trend of ϕ with period and that the adopted P–L–C relation fits the galactic cepheids satisfactorily.

Values of ϕ may be derived as follows:

(1) Seven 'classical' cluster calibrators = -2.39 ± 0.015.

(2) All 14 cepheids in clusters and associations $\phi = -2.33 \pm 0.07$.

(3) Thirty-six cepheids with radii determinations $\phi = -2.42 \pm 0.046$ where the errors are internal ones.

Whilst the seven 'classical' calibrators give the most internally consistent value of ϕ they suffer from the disadvantage that most of them are at shorter periods than the LMC cepheids under consideration ($0.9 < \log P < 1.7$). Hence in using this value, we rely on the other cepheids to establish that there is no trend with period. The weighted mean value of ϕ in

(2) and (3) above is -2.39 ± 0.04 which happens to agree exactly with (1). We adopt thi: value. Hence we find

$$M_{\langle V\rangle} = -3.80 \log P + 2.70\,(\langle B_0\rangle - \langle V_0\rangle) - 2.39.$$

Since our coefficients α, β are from solution 4b of Table 4 we adopt also the value of γ foı this solution and hence

$$\text{Mod. LMC} = V_0 - M_V = 16.41 + 2.39 = 18.80.$$

This is a considerable revision from a recent LMC Modulus derived from a P–L–(relation, namely 19.39 (van den Bergh 1977a) although agreeing much more closely with th value (18.59) given by Sandage & Tammann (1971).

The value of the modulus given above, does not take into account any possible meta abundance difference between the LMC and the Galaxy. This is discussed in the nex section.

If we apply the relation for M_V given above to the 24 LMC cepheids in Table 3 witl $0.4 < \log P < 1.0$ and assume that on the average these cepheids have the same meaı reddening as those with measured *BVI*, we obtain a corrected distance modulus of 18.74 This modulus has the advantage that it is derived from cepheids in the same period range a the classical calibrators. However, it has the following disadvantages: (1) The reddening have not been measured for many of these stars. (2) The sample includes three stars assume to be pulsating in an overtone. (3) The photometry will have been more difficult for thes fainter cepheids. The comparison of photometry by different observers discussed abov (Table 2) refers to stars in a longer period range ($\log P = 0.9$–2.1 and with only one objec of period less than $\log P = 1.3$). (4) The discussion on abundance effects in the next sectioı is derived from observations in the range $0.9 < \log P < 1.7$. Clearly further work would b useful for the short-period cepheids since they may indicate that the distance modulus w have adopted is a slight overestimate.

7 Abundance effects

Various discussions have been given of metal abundances in the LMC. The most recent wor on H II regions (Pagel *et al.* 1978) suggests that any abundance deficiency is likely to be quit small. They find the LMC H II regions to be deficient by a factor of 1.38 compared to tl Orion nebula.

An abundance change is expected in principle to affect the P–L–C relation (Gascoign 1974). As Gascoigne shows, a lower metal abundance leads to a larger value of the coloı coefficient (our β). From the figures Gascoigne gives, it is clear that the change expected i β will be small (0.1 to 0.2) for a metal deficiency as small as 1.38. Our adopted value (β (2.70) is in fact different from the value of 2.55 derived semi-empirically by Gascoigr for normal composition, by about the right amount (and in the right sense). However, tl scatter in β values in Table 4 shows that our present determination is not of sufficieı accuracy to attach any great weight to this numerical coincidence.

If the LMC cepheids are metal deficient they are expected to be bluer at a given temper ture than those in the Galaxy. Bell & Parsons (1974) have computed values of $(B-V)$ fc metal-deficient yellow supergiants. From their results we estimate that for a met deficiency of 1.38 we expect $\Delta(B-V) \sim 0.04$ at $(B-V) = 0.80$ which is the reddest of the models. This $(B-V)$ is close to the mean $(\langle B_0\rangle - \langle V_0\rangle)$ of the cepheids used in solution 4 c Table 4 (= 0.78).

We noted in Section 5 that the mean $\log P$ versus $(\langle B\rangle - \langle V\rangle)$ relation lay to the blue of th galactic intrinsic line. We estimate from Fig. 5 that the displacement is about 0.025 mag. If $\Delta(B-V)$ is the intrinsic $(B-V)$ excess and E_{B-V} is the mean reddening for LMC cepheids, then:

$$\Delta(B-V) - E_{B-V} \simeq 0.025. \tag{1}$$

A metal deficiency will also affect the intrinsic line in the $(V-I)$, $\{(B-V)-(V-I)\}$ diagram of Fig. 3. The effect of lower abundances on $(V-I)$ has not been computed but it is expected to be smaller than on $(B-V)$ since the metal line blanketing is less in the red spectral region. For our present estimates we assume abundance effects on $(V-I)$ to be negligible. Thus a metal deficiency will just lower the intrinsic line in Fig. 3 by the appropriate $\Delta(B-V)$. This will result in lower estimates for E_{B-V}. The situation is illustrated in Fig. 7. In the case of the cepheids of Table 1 it is found that for $\Delta(B-V) = 0.05$, the value of E_{B-V} is decreased by 0.05. Hence approximately

$$\Delta(B-V) + E_{B-V} \simeq \text{constant} \simeq 0.086. \tag{2}$$

Equations (1) and (2) then give $\Delta(B-V) \simeq 0.056, E_{B-V} \simeq 0.030$.

These figures are of course rather uncertain but they show that it is possible to get consistency between the $\log P - \{\langle B\rangle - \langle V\rangle\}$ results and the *BVI* results by adopting a small metal deficiency.

We adopt $\Delta(B-V) = 0.05$ which would seem consistent with a mild metal deficiency for young LMC objects as estimated by Pagel *et al.* and based on an interpolation of Bell–Parsons models. It also gives $E_{B-V} \sim 0.04$ which is close to the value that has been generally adopted in recent times for the LMC ($E_{B-V} = 0.05$) although this latter value has not been determined very accurately.

As noted by Gascoigne (1974) a metal deficiency will affect the derived distance modulus of the LMC primarily through the effect on the $\log T_e-(B-V)$ relation. We can estimate the effect in the following way. The intrinsic colour of the (metal deficient) cepheid is given by point A in Fig. 7. We will obtain a modulus very close to the correct one by using in a P–L–C relation (calibrated as ours is from galactic cepheids) the $(B-V)$ of a normal star of the same temperature as that of the metal-deficient cepheid. This is given in our approximation by point C. The interstellar reddening is now given by AX rather than by BX as previously. It is easily found that under these conditions, with the coefficient of the colour term in the P–L–C relation, $\beta = 2.70$ and with $R = A_V/E_{B-V} = 3.3$, the necessary correction to the modulus derived in Section 6 above is

$$\Delta M = (3.3 - 2.70)(\Delta E) - 2.70\,\Delta(B-V)$$

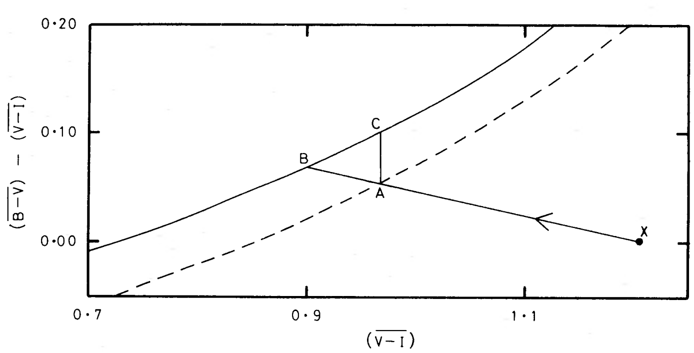

Figure 7. A $(\overline{V-I})$ against $\{(\overline{B-V})-(\overline{V-I})\}$ plot to illustrate the discussion of Section 7. Solid curve is the adopted (galactic) intrinsic relation. The dotted line shows the effect of a $B-V$ excess. $(B-V) = 0.05$ on this line.

where ΔE is the reddening corresponding to AB of Fig. 7. We find that if $\Delta(B-V) = 0.05$ then $\Delta E = 0.05$. Thus

$\Delta M = -0.11$ mag.

Hence the modulus of the LMC is 18.69. The following items contribute to the standar error in this modulus.

(1) From Section 2 (and Table 4) the scatter of a single LMC cepheid from our adopte P–L–C relation is 0.14 mag. Hence the standard error for the mean modulus derived fror the cepheids of solution 4b (Table 4) is $0.14/\sqrt{25} = 0.03$.

(2) Some uncertainty attached to ΔM derived above although it seems most unlikely tha the standard error of ΔM could be greater than 0.05 mag and it is probably smaller.

(3) The internal standard error of ϕ derived in Section 6 is 0.04 mag.

(4) The uncertainty in the absolute calibration of the galactic cepheid distances used t determine ϕ is difficult to estimate. Our adopted distance scale for cepheids in clusters an associations rests ultimately on the van Bueren (1952) distance modulus for the Hyade (3.03 mag). If we increase the cluster distance scale by 0.26 mag to agree with the Hyade modulus (3.29 mag) derived by Hanson (1975) then we would change our values of ϕ i Section 6 as follows:

(1) Seven classical cluster calibrators $\phi = -2.65 \pm 0.015$.
(2) Fourteen cluster cepheids $\phi = -2.59 \pm 0.07$.
(3) Thirty-six cepheids with radii determination $\phi = -2.42 \pm 0.045$ (unchanged).

Following our previous course of taking a weighted mean of (2) and (3) we should obtai $\phi = -2.47$ instead of our adopted -2.39. Thus our LMC modulus would be increased b 0.08. However, as can be seen the change in cluster distances has removed the previous goo agreement between the seven 'classical' calibrators and the 36 cepheids with radii. It i possible that the situation may be resolved along the lines suggested by van den Berg (1977b). That is, the increase in cluster distance scale may be at least partly offset by correction due to differences in metallicity between the clusters with cepheids and th Hyades. Unless one were prepared to abandon the value of ϕ from radius determinations i would seem unlikely that our adopted ϕ could be in error by more than 0.10–0.15. It i evident that this error dominates the others and we adopt 0.15 mag as a reasonable estimat of the standard error of the modulus.

Hence we find a true distance modulus of the LMC $= 18.69 \pm 0.15$ yielding a distance c 55 (± 4) kpc.

It should be borne in mind that in view of the larger values of the Hyades modulu obtained in recent times this is if anything likely to be an underestimate of the distance t the LMC.

8 Effect of a tilt of the LMC on the analysis

de Vaucouleurs (e.g. 1960) proposed that the LMC was a flattened system inclined at small angle to the plane of the sky. That this is so at least as far as very young objects aı concerned, is shown by the kinematics of H II regions and young stars (Feast 1964 Recently Gascoigne & Shobbrook (1978) have observed a number of cepheids on the easter and western extremities of the LMC in B and V and have detected a significant difference i distance modulus. Their results confirm the tilt of 27° deduced by de Vaucouleurs fror isophotes and show that the eastern edge of the LMC is the nearer edge to us. These ne cepheids have been included in the general solutions (Table 4, solutions 1 and 2) but onl one of them (HV 12700 which has *BVI* from SAAO work) is included in the final solutions (Table 4, solutions 3 and 4) which depend only on stars with *BVI.* Solutions with and without the new cepheids show that neither the coefficients of the P–L and P–L–C relations, nor the mean residuals are significantly affected by including these stars. Residuals for the new cepheids from solution 2b of Table 4 are:

	Δ(Mod)	X	N
East of axis	-0.096 ± 0.020	$-3^\circ.78$	8
West of axis	$+0.047 \pm 0.037$	$+2^\circ.78$	6

Here Δ(Mod) is the residual, N the number of stars and X is the mean angular distance from a major axis through the radio centre at an angle of 171°. This result confirms Gascoigne & Shobbrook's result. Further discussion of the tilt of the LMC will be given by Gascoigne & Shobbrook, here we simply consider whether it is likely to have significantly affected any of our discussions. The 26 cepheids with *BVI* and $\log P < 1.7$ were divided into two groups according to whether they lay east or west of the major axis. The mean residuals for these two groups are:

	Δ(Mod)	X	N
East of axis	-0.036 ± 0.041	$-2^\circ.07$	14
West of axis	$+0.043 \pm 0.026$	$+1^\circ.30$	12

The residuals are of the right sign to agree with Gascoigne & Shobbrook's result and they are also of about the expected amount. However, it is clear that the difference in residuals on the two sides of the major axis (0.079 ± 0.048) is barely significant for the area covered by these 26 cepheids with *BVI.* For such a small effect it seemed undesirable to add a position-dependent term in the P–L–C relation for these variables. As shown in Table 4 the standard error of one cepheid is 0.14. Allowing for the tilt of the LMC reduces this dispersion to 0.13, which is obviously only a marginal improvement. With the standard errors for $\langle V \rangle$ and $\langle B \rangle - \langle V \rangle$ adopted in Section 3, we would expect a value of $\sigma = 0.11$ from observational causes alone. It would appear therefore that the actual residuals are hardly greater than might be anticipated on observational grounds. In particular there is no evidence from the present data for a residual scatter due for example to a variation in the metal abundance amongst LMC cepheids. In principle the variation of the apparent distance modulus across the LMC could be due to an abundance gradient rather than a real change in distance but at the present time this seems a less attractive hypothesis than that proposed by Gascoigne & Shobbrook.

9 Comparison with recent work

Since this paper was drafted two papers by de Vaucouleurs (1978a and b = VI and VII) have appeared which deal amongst other things with the use of cepheids to determine the distance of the LMC. de Vaucouleurs (VII) obtains a true distance modulus uncorrected for abundance effects of 18.31. His value using cepheids alone is 0.04 mag less than this. This is strikingly different from the value of 18.80 (uncorrected for abundance effects) which we derive above. It is necessary to comment briefly on this discrepancy.

(1) *Zero Point.* The cepheid zero point in both the present work and that of de Vaucouleurs is partly based on absolute magnitudes for cepheids in clusters and associations.

There seems to be good agreement between us on the basic data used in this connection. For instance the values we have adopted for M_V from van den Bergh (1977a) for 14 cepheids in clusters and associations are also the values listed (M_V^c) in VI table 6. However our discussion rests ultimately on the van Bueren (1952) distance modulus for the Hyades (3.03 mag). de Vaucouleurs on the other hand has made these cepheid absolute magnitudes brighter by 0.26 mag to correspond to Hanson's (1975) distance to the Hyades (3.29 mag). This change is in the wrong sense to explain the discrepancy. In fact de Vaucouleurs combines a number of estimates of his zero point (VI table 7) and his finally adopted value is only 0.06 mag brighter than it would have been had he adopted 3.03 for the Hyades modulus. We conclude that the difference between the two LMC moduli is not due to the use of different galactic zero points.

(2) *P–L relation.* The very considerable scatter that exists in the P–L relation (see Section 2) makes it in our opinion much less satisfactory than a P–L–C relation in determining the LMC modulus. In particular with only limited numbers of LMC cepheids and (especially) galactic calibrating cepheids systematic errors will occur if the observed stars do not scatter uniformly about the true mean relation. It is possible that the P–L relations in VII suffer from this effect. In addition there are two effects which will decrease the difference between the two LMC moduli: (a) the P–L relation is probably fairly insensitive to abundance effects (Gascoigne 1974). Thus for the P–L relation the comparison should be with our P–L–C modulus corrected for abundance effects (e.g. 18.69). (b) The absorption corrections adopted by de Vaucouleurs seem too high. We have found (Section 7) a mean $E_{B-V} = 0.04$ for our LMC cepheids. With $A_V/E_{B-V} = 3.3$ for typical cepheids this gives $A_B = 0.17$. de Vaucouleurs adopts $A_B = 0.43$. Thus adopting our reddening increases the LMC modulus from de Vaucouleurs P–L relation by 0.26 mag.

These two effects together with the general uncertainty associated with a P–L relation are probably sufficient to explain the discrepancy (so far as it relates to P–L relations).

(3) *P–L–C relations.* The values of the 'apparent' moduli derived by using observed $\langle V\rangle$ or $\langle B\rangle$ and $(\langle B\rangle-\langle V\rangle)$ values in P–L–C relations (VII table 2) require even less correction for absorption than the P–L results just discussed. This follows since if $(m-M)_c$ is the modulus calculated in this way, then

$$(m-M)_c = \langle V\rangle - \alpha \log P - \beta(\langle B\rangle-\langle V\rangle) - \phi$$

since

$$(m-M)_0 = \langle V_0\rangle - \alpha \log P - \beta(\langle B_0\rangle-\langle V_0\rangle) - \phi$$

it follows that

$$(m-M)_0 = (m-M)_c - (R-\beta)E_{B-V}$$

where

$$R = A_V/E_{B-V}.$$

A similar relation holds if we work with $\langle B\rangle$ rather than $\langle V\rangle$. In either case $(R-\beta)$ small and the correction required to the P–L–C 'apparent' modulus is (in the LMC case very small (0.06 mag when the relations are used in $\langle B\rangle$ rather than $\langle V\rangle$). It is clear that these P–L–C moduli are corrected by $A_B = 0.43$ as is the net result in VII then too low distance modulus will result.

We regard any remaining (small) differences as due to the improved data with which we are working.

It should be noted that if the value of $E_{B-V} = 0.04$ which we have found for our sampl of LMC cepheids applies to other LMC objects then the distance moduli for a range of calibrators in VII will have been underestimated.

In conclusion it should be emphasized that we have considered it desirable in this analysis to confine our attention entirely to cepheids with photoelectric light curves and to deal especially with cepheids having *BVI* observations. There have been several extensive studies of LMC cepheids based on photographic magnitudes (e.g. most recently Butler 1978). These photographic studies will be discussed by one of us in connection with a new large SAAO photographic programme (Martin, in preparation).

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References

Balona, L. A., 1977. *Mon. Not. R. astr. Soc.*, **178**, 231.
Bell, R. A. & Parsons, S. B., 1974. *Mon. Not. R. astr. Soc.*, **169**, 71.
Butler, C. J., 1978. *Astr. Astrophys. Suppl.*, **32**, 83.
Cousins, A. W. J., 1976. *Mem. R. astr. Soc.*, **81**, 25.
Cousins, A. W. J., 1978. *Observatory*, **98**, 54.
Dean, J. F., Warren, P. R. & Cousins, A. W. J., 1978. *Mon. Not. R. astr. Soc.*, **183**, 569.
de Vaucouleurs, G., 1960. *Astrophys. J.*, **131**, 265.
de Vaucouleurs, G., 1978a. *Astrophys. J.*, **223**, 351.
de Vaucouleurs, G., 1978b. *Astrophys. J.*, **223**, 730.
Eggen, O. J., 1977. *Astrophys. J. Suppl.*, **34**, 1.
Feast, M. W., 1964. *Mon. Not. R. astr. Soc.*, **127**, 195.
Gascoigne, S. C. B., 1969. *Mon. Not. R. astr. Soc.*, **146**, 1.
Gascoigne, S. C. B., 1974. *Mon. Not. R. astr. Soc.*, **166**, 25P.
Gascoigne, S. C. B. & Kron, G. E., 1965. *Mon. Not. R. astr. Soc.*, **130**, 333.
Gascoigne, S. C. B. & Shobbrook, R. R., 1978. Preprint.
Hanson, R. B., 1975. *Astr. J.*, **80**, 379.
Iben, I. & Tuggle, R. S., 1975. *Astrophys. J.*, **197**, 39.
Kraft, R. P., 1961. *Astrophys. J.*, **134**, 616.
Madore, B. F., 1975. *Astrophys. J. Suppl.*, **29**, 219.
Martin, W. L. & Warren, P. R., 1979. *SAAO Circulars*, to be published.
Pagel, B. E. J., Edmunds, M. G., Fosbury, R. A. E. & Webster, B. L., 1978. *Mon. Not R. astr. Soc.*, **184**, 569.
Parsons, S. B., 1971. *Mon. Not. R. astr. Soc.*, **152**, 121.
Sandage, A. R., 1972. *Q. Jl R. astr. Soc.*, **13**, 202.
Sandage, A. & Tammann, G. A., 1969. *Astrophys. J.*, **157**, 683.
Sandage, A. R. & Tammann, G. A., 1971. *Astrophys. J.*, **167**, 293.
Schmidt-Kaler, Th., 1961. *Astr. Nachr.*, **286**, 113.
Tammann, G. A., 1970. In *The Spiral Structure of Our Galaxy*, p. 236, IAU Symp. 38, eds Becker & Contopoulos, D. Reidel, Dordrecht, Holland.
van den Bergh, S., 1977a. *IAU Coll. 37* = CNRS Coll. No. 263.
van den Bergh, S., 1977b. *Astrophys. J.*, **215**, L103.
van Bueren, H. G., 1952. *Bull. Astr. Insts Neth.*, **11**, 385.