

An analysis of lunar occultations in the years 1943–1974 for corrections to the constants in Brown's theory, the right ascension system of the *FK4*, and Watts' lunar-profile datum

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Summary. The times of 62 000 occultations of stars by the Moon observed during 1943–74 are analysed to determine corrections to the lunar ephemeris $j = 2$ (based on Brown's theory), the right ascension system of the fundamental star catalogue *FK4*, and the datum of the lunar-profile in Watts' charts. The method of reduction and analysis of the occultations is given in some detail. The value of $-13'' T^2$ is adopted for the tidal term in the Moon's mean longitude and no clear evidence is found for any divergence between the dynamical and atomic time-scales. After removing all the corrections found in this analysis, quasi-periodic fluctuations – amounting to $0''.2$ – still remain in the residuals: these are ascribed to the truncation of planetary terms in Brown's theory.

1 Introduction

Sixty years ago E. W. Brown (1919) published his celebrated work *Tables of the Motion of the Moon*. Brown based these *Tables* on his lunar theory which he had developed over a period of 30 years, beginning from a study of G. W. Hill's papers made at the suggestion of his former teacher, G. H. Darwin. The accuracy of Brown's lunar theory is such that it remained unsurpassed until the electronic computer made feasible the direct numerical integration of the equations of motion.

Improvements in the precision of the lunar ephemeris were brought about by more precise transformations of Brown's basic solution of the main problem made by Eckert, Jones & Clark (1954) and Eckert, Walker & Eckert (1966). However, in these later improvements the values of some of the fundamental constants and arguments in Brown's *Tables*, which had been determined by him mainly from Greenwich meridian-circle observations made in the nineteenth century, were unchanged. Inaccuracies in the determination of these fundamental quantities set a limitation on the accuracy of the final ephemeris and therefore work has been undertaken to compare the ephemeris with observations in order to determine corrections to the fundamental quantities. Van Flandern (1969a,b) derived corrections from a comparison of the ephemeris with meridian-circle observations made in the period 1952–67 and with occultations in the period 1950–68. Morrison & McBain

Sadler (1969) also derived corrections from a preliminary analysis of lunar occultations in the period 1960–66.

The prime objective of the present work is to compare the version of Brown's analytical ephemeris currently used in *The Astronomical Ephemeris* with 62 000 timings of occultations of stars made in the period 1943–74 in order to derive corrections to Brown's value for the fundamental constants. Some of the constants have already been revised (IAU 1966) these include the flattening of the Earth, the ratio of the masses of the Earth and Moon, the parallaxes of the Sun and Moon, and the parallactic inequality. These cannot be further improved using occultation data, so they are not investigated in this analysis.

In association with the derivation of corrections to the fundamental arguments we carry through to the lunar ephemeris the effect of the recent recommendation of the 1977 General Assembly of the IAU concerning the revised definition of the time-argument of the ephemerides (IAU 1977). The IAU resolution recommends that the unit and origin of the time-argument should be specified in relation to the international atomic time-scale (TAI) and that this should replace the ephemeris time-scale (ET) which specifies the unit and origin in relation to the ephemeris of the Sun. I shall denote this revised time-scale by dynamic time (TD).

Another objective of this analysis is to determine the position of the origin of right ascension in the *Fourth Fundamental Catalogue, FK4* (1963) with respect to the dynamical equinox. The principle of the method is to use the orbital plane of the Moon as an intermediary in defining the plane of the ecliptic and hence the location of the point (here called the dynamical equinox) where the ecliptic intersects the equator. Occultation observations directly relate the positions of the stars and Moon and so the method affords one of the best ways of determining the position of the *FK4* equinox relative to the dynamical equinox.

In reducing the times of occultations, the irregularity of the lunar profile is removed by applying limb-profile corrections taken from the charts of the marginal zone published by Watts (1963). It was Watts' intention to give the limb-profile heights relative to a mean circular datum whose centre was the position in the Moon to which the lunar ephemeris (as published in *The Astronomical Ephemeris*) referred. However, he found difficulty in achieving this aim. During the preparation of the charts, measurements indicated that the preliminary datum was elliptical and that its centre was displaced from the ephemeris centre. Watts sought to remove these defects by applying corrections to the preliminary datum. The effectiveness of these corrections is investigated in this analysis of the occultation data.

The main conclusions of this analysis are collected in Section 6.

2 Observations

2.1 NUMBER AND DISTRIBUTION OF OCCULTATION OBSERVATIONS

About 62 000 times of occultations made in the years 1943–74 (inclusive) are analysed in this paper. This total contains only timings of stars included in the *Catalog of 3539 Zodiacal Stars for the Equinox 1950.0* (Robertson 1940) – hereinafter referred to as the *ZC* – and so nearly all the occultations are of stars brighter than magnitude 7.5. A further 18 000 timings of stars, mostly fainter than 7.5, contained in the *Smithsonian Astrophysical Observatory Star Catalog* (1966) – the *SAOC* – were also collected; but these are not considered in the present analysis for two main reasons. Firstly, it is important to use a homogeneous system of star places as possible. The *FK4* would be the best in this respect.

but there are only about 400 stars in the occultation zone and there are too few occultation observations of these stars to give reliable solutions for the various parameters to be considered in this analysis. Secondly, the majority of observations of the *SAOC* stars have been made since 1969 and so they are very unevenly distributed over the period of this analysis.

The observations of all the occultations observed in the years 1943–71, together with the positions and names of the observers are being prepared for publication on microfiche (Morrison 1978). It is intended that the observations for the years 1972 onwards should also be published on microfiche.

Of the 62 000 timings in the years 1943–74, 12 000 are rejected in the reduction and analysis stages for reasons that are given later. The distribution by lunation of the remaining 50 000 used in this analysis is shown in Fig. 1. The two groups of high peaks in the periods 1949–54 and 1967–72 are nearly all due to the large numbers of observations when the Moon occulted the bright stars of the Pleiades. The other high peaks are usually due to occultations of one or other of the four first magnitude stars in the occultation zone. An annual cycle in the numbers of observations is evident, the maxima occurring in the northern-winter months because most observations are made during the longer nights in the temperature zone of the northern hemisphere. There is an underlying trend for the number of observations to increase over the period.

Accurate timings of the instant of occultation are only possible at the dark limb so, apart from occultations which occur near the poles of the Moon, all the timings used in this analysis are of disappearances at the dark limb (DD) during the first half of the lunation and reappearances at the dark limb (RD) during the second half. It is to be expected that the distribution of observations in each lunation is highly correlated with the phase of the Moon and this is evident in Fig. 2 which shows a histogram of the observations subdivided into 12° phase-groups of the mean elongation of the Moon (D). The high peak at phase 60° is due to the large number of observations of the Pleiades stars during the occultations of 1969 March and 1972 March: taken together, these account for about 2500 observations. (Their contribution is shown by the shaded area.) The distribution of observations shown in Fig. 2 demonstrates clearly the problem of reliably determining corrections to any of the lunar parameters which have periods close to the argument D or its higher harmonics.

2.2 METHODS OF TIMING OCCULTATIONS

2.2.1 Timing of visual observations

About 95 per cent of the observations were made visually using telescopes of comparatively small aperture. Of these, about 10 per cent were made by professional astronomers and the remainder by amateurs. There are many methods of timing visual observations, but in essence they fall into two categories. The most widely used method is to start a clock (e.g. stopwatch or chronometer) at the instant of disappearance or reappearance of the star. A few minutes later the clock is stopped at a precisely known instant, usually on the minute-marker of a broadcast time-signal, and the elapsed time is subtracted from the time at which it is stopped. The second most popular method of timing is the so-called eye-and-ear method in which the observer estimates when the star disappeared or reappeared relative to the audible second-markers of a time-signal. In both these methods a precision of 0.1s can be achieved: reported timings with less than this precision are rejected in this analysis. Events which are noted as not being instantaneous are also rejected (but are to be studied separately later).

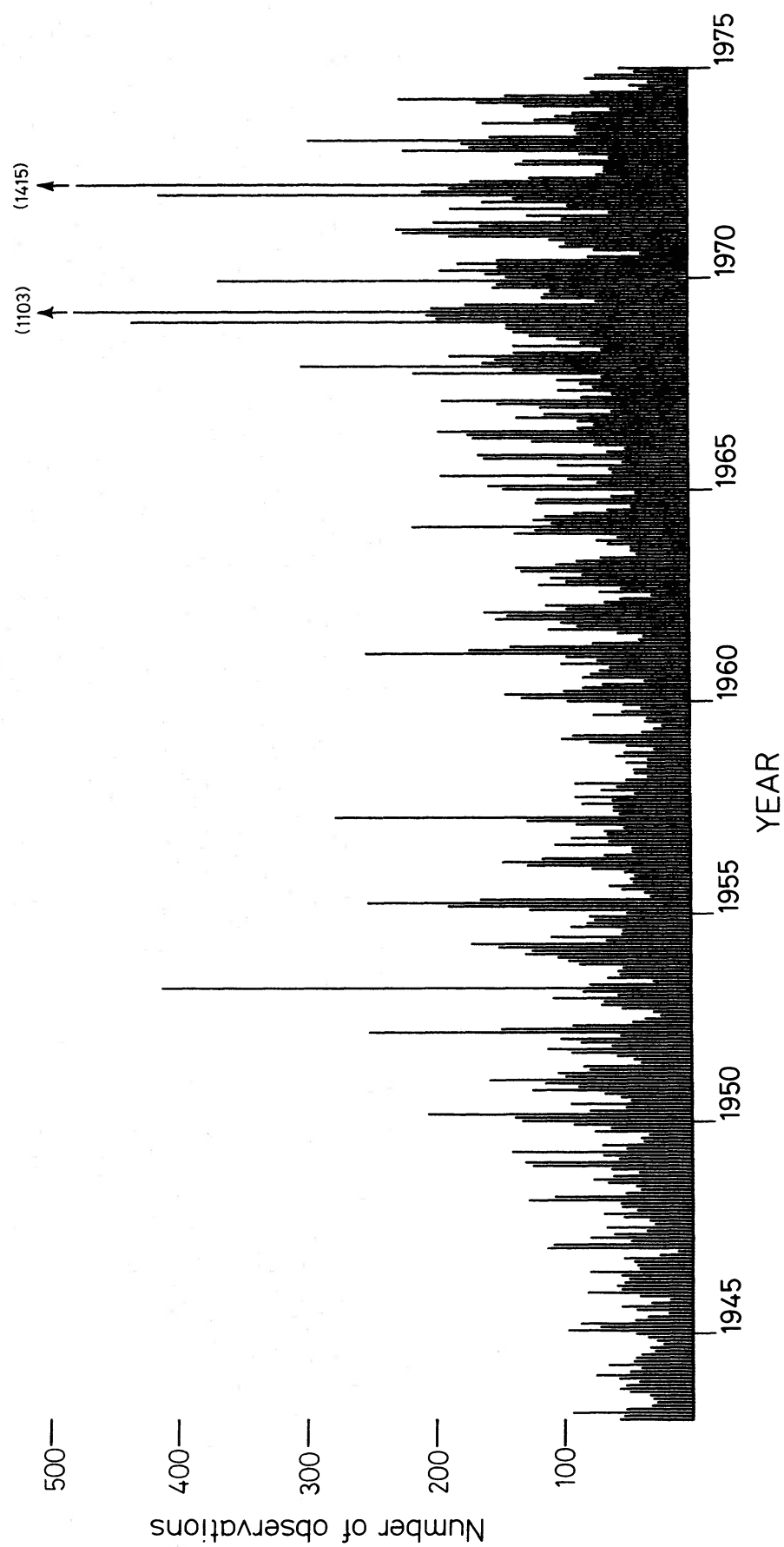


Figure 1. Distribution of occultation observations by lunation.

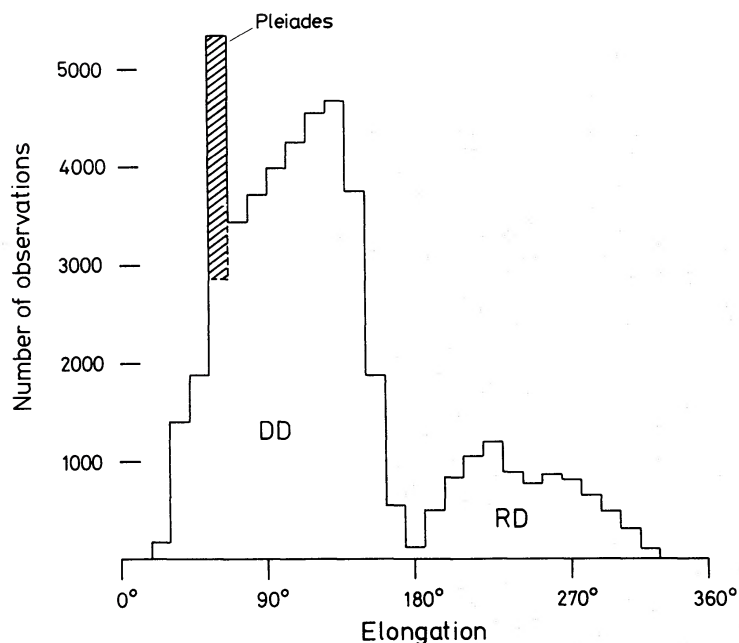


Figure 2. Distribution of occultation observations within lunations.

Personal equation. The first method described above requires a physical reaction on the part of the observer. The observer's reaction time, or personal equation, varies according to the circumstances, such as the magnitude of the star and it is sometimes estimated by the observer for each observation. Where no estimate is given an average value of -0.3 s is assumed for disappearances and -0.5 s for reappearances. These are considered to be the averages of all the observer's estimates under various observing conditions.

By comparing the time-residuals of several thousand observations made using the eye-and-ear method with those using the stop-watch method we found that the eye-and-ear timings were earlier systematically by 0.3 s. We concluded that no correction for personal equation was necessary for disappearances using this method, and that a correction of -0.2 s should be applied to reappearances. However, it was not clear that the reappearances timed by the eye-and-ear method did require any correction. This correction is regarded as being provisional and will be amended if necessary as a result of this analysis. The mean values of the personal equations that were adopted in reducing the observations when no value was specified by the observer are summarized in Table 1.

Accuracy. Observers were requested to estimate the accuracy of their observations, either numerically or on a descriptive scale ranging from 'excellent' to 'very poor'. The translation of these descriptive phrases into numerical estimates of uncertainty is set out in Table 2. The uncertainties in Table 2 are used in the determination of the weights to be assigned to each observation (see Section 4.1). Although the personal equation for the eye-and-ear method

Table 1. Adopted mean values of personal equations.

Phenomenon Method	<i>D</i> (s)	<i>R</i> (s)
Stop-watch etc.	-0.3	-0.5
Eye-and-ear	0.0	-0.2

Table 2. Accuracy of visual observations.

Description	Accuracy (s)
Excellent, very good	± 0.2
Good, certain	± 0.3
Satisfactory	± 0.4
Uncertain	± 0.7
Very doubtful	± 1.0
No description given	± 0.4

is zero for disappearances, the standard deviation of the residuals is nearly equal to that of other methods, once they have been corrected for personal equation.

2.2.2 Timing of photoelectric observations

Approximately 5 per cent of all the observations are made using photoelectric equipment with which the time of occultation is generally recorded with a precision of 0.001 s and comparable accuracy. These observations therefore have increased weight in this analysis, but only marginally so, because of the considerable contribution to the errors from the catalogue positions of the stars and the corrections for the outline of the Moon (see Morrison 1971). However, the photoelectric observations have the added usefulness in providing a standard against which to measure the observers' reaction times. Provided that visual and photoelectric observations are made from the same location, the error due to the star position and limb outline will be common to both. Sinzi & Suzuki (1967) have analysed such observations and conclude that the general value of the personal equation lies between 0.3 and 0.4 s, but they found a fairly wide dispersion in the results and this increased with the faintness of the stars.

2.3 SYSTEMS OF UNIVERSAL TIME SINCE 1943

Apart from the occasional timings made in local mean sidereal time, all observations were timed on the universal time-scale using the time-signal broadcasts of various national and international time services. The basis of the transmitted time-signals has changed over the period of the observations under discussion. As far as possible it has been the intention in this analysis to reduce all the occultation timings to the UT1 scale since this is the time-scale required in computing the local hour-angle of the Moon. The period of the observations can be divided into three ranges.

1943–1955. Before 1956 the broadcast time-signals were usually based upon quartz-crystal clocks. Their rates were adjusted as necessary to give the observed rate of the universal time scale deduced from astronomical observations smoothed over a few nights. The broadcast time-signals therefore closely followed UT0. Beginning with 1947 some observatories removed the effect of polar motion, thus giving UT1; but this change was only introduced by a few observatories at this time, so most national time-signals continued to be based on UT0 until 1956 when an internationally agreed procedure was introduced for removing polar motion. Therefore, timings in the period 1943 to 1955 were mostly made on UT0, but some were made on UT1. The task of identifying, then removing the effects of polar motion from the uncorrected time-signals is daunting and it is doubtful whether any significant improvement would accrue to the results of this analysis. The maximum change in time due to polar

motion is $0^s.03 \tan \phi$, where ϕ is the latitude of the observer. This is an order of magnitude less than other errors involved in occultation measurements, so it can be safely neglected.

1956–1960. Around 1956 primary atomic standards were introduced to monitor the rates of the quartz clocks. Since the stability in rate of the atomic standards could be relied upon over long periods, it was found to be advantageous to fit the rate to the average observed rate of rotation of the Earth over periods longer than the seasonal fluctuations. The epoch and rate of the broadcast time-signals were adjusted therefore so that the transmitted signals followed UT2 (i.e. UT1 amended for the estimated seasonal variation) as determined by the national observatories. The closeness with which the various time-signals followed UT2 varied from country to country, so, in reducing the timings to UT1, the simplifying assumption was made that the correction to be applied for the difference UT1 – UT2 was that published by the Bureau International de l'Heure (BIH).

1961–1974. In 1961 several of the national time-services decided to coordinate their time-signals so that the emitted time-scale (UTC) was maintained within 0.1 s of UT2; to achieve this they simultaneously made step adjustments in the unit and epoch of UTC. In this analysis, it has been assumed that unless otherwise stated the timings have been recorded on UTC since 1961 January 1 and that the corrections to derive UT1 were those published by the BIH. When observations were reported on an independent UT2 scale, the corrections to give UTC published by the relevant national timekeeping agency were applied and they were then reduced to UT1 in the normal way. Since 1972 January 1 the unit of the UTC scale has been held equal to that of the international atomic time-scale (TAI) and only step-adjustments of exactly 1 s have been made to keep it within 0.9 s of UT1.

3 Reduction

3.1 METHOD OF REDUCTION

The topocentric angular separation between the star and the centre of the Moon was computed for the time of each observation. Fig. 3 shows schematically the computed position C of the centre of the Moon and the computed position S of the star for the

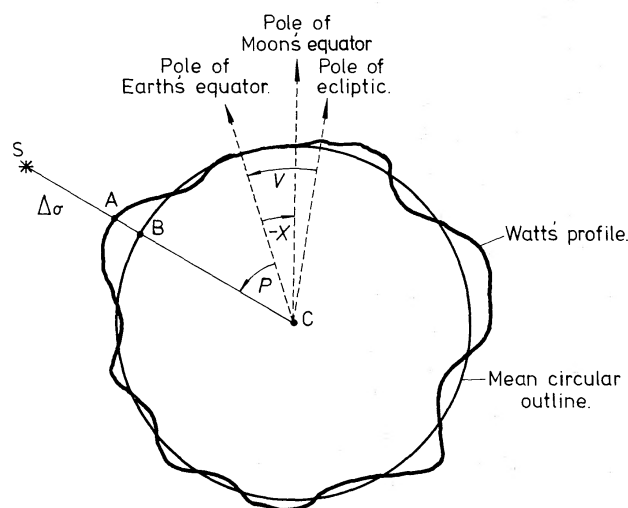


Figure 3. Schematic representation of computed residual ($\Delta\sigma$) and position angles required in the reduction and analysis of occultation observations.

observed time of occultation. The separation between the star and the point B on the near circular outline of the Moon is given by

$$SB = [255(\alpha' - \alpha)^2 \cos \delta' \cos \delta + (\delta' - \delta)^2]^{1/2} - \rho$$

where $\alpha = \alpha_0 + \Delta\alpha$ (seconds of time), $\delta = \delta_0 + \Delta\delta$ (seconds of arc) and α, δ, ρ = computed topocentric position and semi-diameter of the Moon, α_0, δ_0 = apparent geocentric right ascension and declination of the Moon, $\Delta\alpha, \Delta\delta$ = parallax corrections due to the displacement of the observer from the centre of the Earth, α', δ' = computed apparent right ascension and declination of the star.

The geocentric ephemeris of the Moon used in this analysis is the lunar ephemeris denoted by $j = 2$ (IAU 1968). A description of the basis of this ephemeris is given in Section 3.2. The geocentric positions were computed at half-daily intervals from the analytical theory and the values were represented by Chebyshev polynomials which facilitated the interpolation to the time of observation. The ephemeris was interpolated to the dynamical time (TD) of each observation using the relation

$$TD = UT + \Delta T.$$

The preliminary values of ΔT before 1955 July 1 were taken from a linear approximation to the values of ET-UT in *The Astronomical Ephemeris* for 1971 and the values after that date were taken from the relation,

$$\Delta T = (TAI - UT1) + 32.184 \text{ s.}$$

The procedure is explained more fully in Section 3.3.

The usual expressions given in *Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac* (1961) (hereinafter denoted by *ES*) were used to compute the parallax corrections $\Delta\alpha$ and $\Delta\delta$. An account of the method by which the observers' positions were referred to the centre of mass of the Earth is given in Section 3.4. The geocentric semi-diameter, ρ_0 , was computed from the semi-diameter at mean distance (932.58 arcsec) using the relation (*ES*, p. 109),

$$\rho_0 = 0.0799 + 0.272453 \pi_0,$$

where π_0 is the geocentric equatorial horizontal parallax. The topocentric semi-diameter was derived from the geocentric value by multiplying it by the ratio of the geocentric to the topocentric distance of the Moon.

The apparent positions of the stars were computed from their catalogue places and proper motions given in the *ZC*. Their parallaxes were taken (where available) from the *General Catalogue of Trigonometric Stellar Parallaxes* (Jenkins 1952). The places and proper motion were adjusted to the *FK4* system as explained in Section 3.5. The method of calculation of the apparent places is that given by Emerson (1973) using the same values for the constant of aberration, nutation and precession as in the lunar ephemeris $j = 2$.

The computed separation SB between the star and the mean circular outline of the Moon was corrected for the height of the outline at the point of occultation given in Watts' (1963) charts of the marginal zone. The arguments for entering his charts include the topocentric librations (l, b) and the angle X between the two great circles through C on the topocentric celestial sphere containing respectively the projections of the north poles of the Earth' (true) equator and the Moon's mean equator. These topocentric angles were derived by adding the following parallactic corrections to the geocentric librations (l_0, b_0, X_0),

$$\Delta l = (\Delta\alpha \cos \delta \cos X_0 - \Delta\delta \sin X_0) \sec b_0$$

$$\Delta b = -\Delta\alpha \cos \delta \sin X_0 - \Delta\delta \cos X_0$$

$$\Delta X = \Delta l \sin b_0 + \Delta\alpha \sin \delta.$$

The geocentric librations were calculated from the expressions for the optical and physical librations given in *ES*, p. 319. In the computation of the optical librations, the value of $1^\circ.564$ was used for the inclination of the mean lunar equator to the ecliptic because this was the value that Watts (1955) used in constructing his charts.

The position angle of the point of occultation, measured positive through east from the projection of the pole of the Moon's equator, which is required to enter Watts' charts, is given by

$$Q = P - X,$$

where P is the position angle of the occultation measured from the projection of the pole of the Earth's equator and calculated from the expression

$$P = \tan^{-1} \{ \cos \delta' \sin (\alpha' - \alpha) / [\sin \delta' \cos \delta - \cos \delta' \sin \delta \cos (\alpha' - \alpha)] \}.$$

The position angle N from the projected pole of the ecliptic was calculated from

$$N = P + V$$

where

$$V = \tan^{-1} \{ \sin \epsilon \cos \alpha / [\cos \epsilon \cos \delta + \sin \epsilon \sin \delta \sin \alpha] \}.$$

Details of the application of Watts' profile corrections are given in Section 3.6.

A small correction for the effect of refraction at low altitudes was applied to the computed separation, SB; the details are given in Section 3.7.

The calculated topocentric angular separation between the star and the limb of the Moon for the time of occultation was scaled to correspond to the geocentric distance by multiplication by the ratio of the topocentric to the geocentric distance. If this computed separation at geocentric distance is denoted by σ_c and the observed separation by σ_o , the residual (in the sense Observed – Computed, O – C) of the observational equation is given by $\sigma_o - \sigma_c$, which is conventionally denoted by $-\Delta\sigma$. Since the observed separation is zero,

$$O - C = -\Delta\sigma = -\sigma_c.$$

The subscript c will henceforth be omitted.

The partial derivatives with respect to universal time $\partial\sigma/\partial t$ and ephemeris time $\partial\sigma/\partial T_E$ were calculated by numerical differentiation. Other partial derivatives required in the analysis were calculated as follows:

$$\frac{\partial\sigma}{\partial\lambda} = -\sin N, \quad \frac{\partial\sigma}{\partial\beta} = -\cos N, \quad \frac{\partial\sigma}{\partial\pi} = -0.2725,$$

$$\frac{\partial\sigma}{\partial\alpha'} = +\sin P, \quad \frac{\partial\sigma}{\partial\delta'} = +\cos P, \quad \frac{\partial\sigma}{\partial\rho} = -\frac{\pi}{\pi_0}.$$

3.2 LUNAR EPHEMERIS $j = 2$

In this analysis the observations are compared with the lunar ephemeris designed by $j = 2$. It is based on Brown's theory with the following correction applied to the expression for mean longitude:

$$-8''.72 - 26''.74 T_E - 11''.22 T_E^2,$$

where T_E is measured in Julian centuries of 36 525 ephemeris days from the epoch 1900 January 0 Greenwich mean noon. Clemence (1948) first proposed that this empirical correction, which was derived from the work of Spencer Jones (1939), be applied to Brown's purely gravitational theory (i.e. the *Tables* less the empirical term) in order to introduce to the motion of the Moon the epoch and unit of ephemeris time as defined by the motion of the Sun in Newcomb's *Tables of the Sun*. The direct evaluation of the harmonic series on which Brown's *Tables* are based, together with the introduction of ephemeris time as the time-argument, formed the basis of the *Improved Lunar Ephemeris* (Eckert *et al.* 1954), hereinafter denoted by *ILE*. The *Tables* already contained a quadratic term in mean longitude ($+7''.14 T_E^2$) arising from the secular change in the eccentricity of the Earth's orbit and therefore the addition of the term ($-11''.22 T_E^2$) for the tidal acceleration of the Moon resulted in the quadratic term $-4''.08 T_E^2$ in the *ILE*.

The *ILE* was later revised for the introduction of the IAU 1964 system of astronomical constants (IAU 1966). The direct effect on the lunar ephemeris of these changes in the constants is given in a supplement to the *AE* 1968: they include changes to the constant of sine parallax, the constant of parallactic inequality and the ellipticity of the Earth's figure. The values of the following basic constants, which were adopted by Brown after the construction of his *Tables* and were used in the *ILE*, were not changed in subsequent revisions of the astronomical constants:

- (a) in the Moon's orbit – the eccentricity, the inclination and the longitudes of perigee and node,
- (b) in the Sun's orbit – the eccentricity and the longitude of perigee.

The accuracy of the *ILE* was later improved by incorporating the full precision of Brown's solution for the solar terms evaluated by Eckert *et al.* (1966). So, in summary, the lunar ephemeris $j = 2$ is essentially Brown's gravitational theory, with ephemeris time as the argument of time, corrected for the IAU 1964 system of astronomical constants.

The origin of longitude in the ephemeris $j = 2$ is unchanged from that in the *ILE* which was derived from the results of Spencer Jones (1939), as explained by Aoki (1960). In his comparison of Brown's *Tables* with the observed position of the Moon deduced from occultations, Spencer Jones used the star places in Hedrick's (1905) *Zodiacal Catalog*. Therefore, his derived correction to the mean longitude in Brown's *Tables* is dependent on the origin of right ascension in Hedrick's catalogue, which is the same as that in Newcomb's (1882) *Catalogue of 1098 Standard Clock and Zodiacal Stars*: it is commonly referred to as Newcomb's equinox. Therefore, the origin of longitude in $j = 2$ is expected to be close to Newcomb's equinox, rather than the equinox of *FK4*.

3.3 DYNAMICAL TIME-SCALE

The argument of time in the lunar ephemeris $j = 2$ is ephemeris time which is defined in relation to the motion of the Sun (see *ES*, p. 69). It has been recommended (IAU 1977) that a new time-scale specified in relation to the international atomic time-scale TAI should be used in the dynamical equations of motion. This new time-scale is here called dynamical time and is denoted by TD. Its origin and unit are specified by the requirements that:

- (a) its measure is 1977 January 01^d 00^h 00^m 32^s.184 exactly at the instant 1977 January 01^d 00^h 00^m 00^s TAI, and
- (b) that at this instant, the second of this new time-scale is the SI second at sea-level.

In order to carry through these recommendations, it will be necessary to adjust the existing values of the mean longitudes in the orbital theories of solar system objects so that

the dynamical time-scale, realized by inverse interpolation in the ephemerides, will accord with the definition above. In order to satisfy this requirement the following procedure has been adopted in the reduction of the occultation data.

Dynamical time-scale after 1955 July 1. Subsequent to the introduction of the atomic time-scale on 1955 July 1, the dynamical time-scale was taken to be

$$TD = TAI + 32^s.184.$$

The time on the TAI scale corresponding to the observed time of occultation on the UT1 scale was derived from the expression,

$$TAI = UT1 + (TAI - UT1),$$

where the difference $TAI - UT1$ was taken from the published data of the Bureau International de l'Heure. By this procedure the lunar ephemeris was entered for the time-argument $TAI + 32^s.184$ and any systematic departures with time in the residuals are to be treated as corrections to the constant and linear term in the expression for mean longitude in the ephemeris $j = 2$. The treatment of the quadratic term is described later in Section 4.2.

Preliminary dynamical time-scale before 1955 July 1. Before the existence of the atomic time-scale the ephemeris was entered for the dynamical time given by the linear approximation

$$TD = UT + 31^s.24 + 0^s.44 (Y - 1955.5),$$

where Y is the year (and fraction). The constant, $31^s.24$, is the value of $32^s.184 + (TAI - UT1)$ on 1955 July 1. By choosing the constant in this way the dynamical time derived from the expressions before and after 1955 July 1 give the same value at that epoch. This linear relation is good enough to ensure that the ephemeris may be interpolated with sufficient accuracy to compute the position angle with an error of less than $\pm 0''.01$: this accuracy is required to obtain the correct point on the limb-profile in Watts' charts.

3.4 POSITIONS OF OBSERVERS

Only observations made from locations whose coordinates were reported to a precision of $1''$ in longitude and latitude and 30 m in height were used in this analysis. This precision is such that the parallax corrections $\Delta\alpha$ and $\Delta\delta$, due to the displacement of the observer from the centre of mass of the Earth, may be computed to a precision of about $0''.01$, which is about an order of magnitude better than the accuracy of an observation.

Nearly all the positions used are geodetic coordinates, but they are referred to several different national datums which use various reference ellipsoids. The centres of these ellipsoids may be several hundred metres apart, and therefore it is desirable to reduce all the observers' geodetic coordinates to one standard ellipsoid whose centre is the centre of mass of the Earth. In so far as it was possible, each position was obtained with respect to one of the following three major datums; 1927 North American Datum, European Datum and Australian Geodetic Datum. The positions of observers in Japan, Africa and South America were referred to one of these major datums using the interdatum displacements given by Fischer (1968). The geocentric displacements (in rectangular coordinates) of these major datums with respect to the centre of mass of the Earth are known to within a few metres and are given, for example, by Fischer (1968). The conversion of the geocentric displacements to changes in longitude, latitude and height were computed using the standard formulae given in text-books on geodesy (e.g. Heiskanen & Moritz 1967), care having been taken with the sign in longitude which is conventionally measured positive to the west of

Greenwich in astronomical usage. The reference ellipsoid was taken to have an equatorial radius of 6 378 160 m and a flattening of 1/298.25 in order that it would be consistent with the IAU 1964 system of constants.

When this work was started it was not possible to refer the positions of all the observers to one of the afore-mentioned datums, especially in the European countries. For some observatories we have only the astronomically determined coordinates. Some of these in Europe have been corrected to the European Datum by applying the results for the deviation of the vertical published by the Association Internationale de Géodésie (1967). Failing the acquisition of station positions with respect to the major datums, the positions on the various national systems were used, since the displacements of their coordinates from the Earth-centred system should be more or less random and should not lead to any significant systematic effects in the residuals.

3.5 SYSTEM OF STAR PLACES

The star places and proper motions in the *ZC* were systematically adjusted to the *FK4* as follows. Differences were formed for the 432 stars common to both catalogues. Averages were taken in hourly bands of right ascension and these were smoothed by forming the simple moving average $(f_{-1} + 2f_0 + f_1)/4$. The resulting systematic differences are given in Table 3. These differences were applied to the *ZC* places and proper motions. It is noted that the difference between the zero points of right ascension (RA) in the *FK4* and Newcomb's catalogue is such that on the average

$$\text{RA}(\text{FK4}) - \text{RA}(\text{Newcomb}) = -0^{\text{s}}.050.$$

Table 3. Systematic differences: *FK4* – *ZC*.

RA (hr)	$\Delta\alpha$ (0 ^s .001)	$\Delta\mu$ 0 ^s .00001/yr	$\Delta\delta$ (0 ^{''} .01)	$\Delta\mu'$ (0 ^{''} .0001/yr)
0	+7	+16	–5	–5
1	+2	+6	–4	+5
2	–3	–5	–6	+7
3	–4	–4	–4	+3
4	–2	+1	+1	+8
5	0	+4	+6	+7
6	+5	+14	+7	+19
7	+7	+25	+4	+12
8	+5	+31	–1	–2
9	+1	+25	–3	–9
10	–3	+11	–3	–14
11	–3	+3	–5	–20
12	0	–2	–3	–18
13	–2	–13	0	–13
14	–1	–16	+2	–9
15	+3	–8	+7	+2
16	+3	–5	+11	+12
17	+3	–4	+9	+9
18	+5	–2	+6	+1
19	+3	–5	+3	–7
20	+2	–5	–1	–13
21	+4	+2	–1	–10
22	+5	+7	0	–6
23	+7	+13	–4	–8

Since the origin of longitude in the lunar ephemeris $j = 2$ is close to the origin of right ascension in Newcomb's catalogue, we expect this difference to be evident in the occultation residuals.

3.6 LIMB-PROFILE CORRECTIONS

The tabular arguments for entry to Watts' charts are the position angle Q from the axis of the Moon and the topocentric librations (l, b). Morrison (1970) has shown that the limb profiles taken from Watts' charts accord better with the results of grazing occultations if the tabular position angles of the charts are amended systematically by $-0^\circ.25$. Therefore, the heights of the profile taken from the two charts adjacent to the position angle $Q + 0^\circ.25$, which were read from a computerized version of Watts' charts (Morrison & Martin 1971), were interpolated to the height at position angle $Q + 0^\circ.25$. This height was scaled to the true distance of the Moon and subtracted from the calculated separation between the star and the mean circular outline of the Moon (SB in Fig. 3).

By this procedure the tacit assumption has been made that the surface to which Watt's profiles are referred is circular and that its centre is coincident with the centre of mass of the Moon. Both assumptions are probably wrong and this is borne in mind when forming the equation of condition in Section 4.5.

A code number from 1 to 4 was assigned to each limb correction as an indication of its uncertainty. Codes 1 and 2 were assigned for an uncertainty of $\pm 0''.2$, and code 3 for an uncertainty of $\pm 0''.5$ which occurred in regions where Watts' profiles had to be extrapolated. Where no limb correction could be obtained code 4 was assigned. These codes were used later in Section 4.1 when selecting and weighting the observations.

In computing the position angle Q from the axis it is important that the computed position angle P , measured from the north point, should be within $\pm 0^\circ.01$ of the correct angle in order to obtain the correct point on Watts' profile. The steepest lunar slopes have an inclination of about 13° , so the greatest change in height along the profile is about $0''.04$ in $0^\circ.01$ of position angle. This precision in the computation of the position angle is therefore adequate when it is considered that the standard error of the heights taken from Watts' charts is about $\pm 0''.15$. There are three main causes of the error in computing the position angle P : (1) the uncertainty of the catalogue position of the star, (2) the uncertainty of the Moon's position in the ephemeris, and (3) the difference between the origin of right ascension in the ephemeris and the *FK4*. The standard errors for the first two are about $\pm 0''.3$ and $\pm 0''.2$, respectively, and the systematic contribution from the third is about $0^s.050$ in right ascension. This difference would contribute a systematic displacement of $0^\circ.04 \cos P$ in computing the position angle so, to avoid this, $+0^s.050$ was added to the right ascension of stars when computing P . The uncertainties in the star position and linear ephemeris will produce an accidental error of about $\pm 0^\circ.02$: this has been ignored.

3.7 REFRACTION AT LOW ALTITUDE

It can be shown (*ES*, p. 55) that in order to compensate for the change in altitude due to refraction, R , in the computation of the circumstances of an occultation occurring at a true altitude, z , the geocentric radius of the observer, h , should be increased by an amount δh , where,

$$\frac{\delta h}{h} = \frac{\mu_0(z - R)}{\sin z} - 1.$$

The mean index of refraction of the atmosphere at the place of observation is denoted by μ_0 . This expression has been evaluated for mean values of R and a range of z and they are tabulated in *ES*, p. 55. The effect becomes significant in reducing occultations below an altitude of 15° where the value of δh is 30 m. With sufficient accuracy, the change in the computed separation between the star and the limb of the Moon, σ , due to δh is given by,

$$\delta\sigma = -\frac{\delta h}{h}(\Delta\alpha \sin P + \Delta\delta \cos P),$$

where, as before, $\Delta\alpha$ and $\Delta\delta$ are the components of the parallactic displacements of the Moon. The size of this correction varies from $0''.02$ at an altitude of 15° to about $0''.9$ on the horizon. However, the refraction at very low altitudes can change considerably with temperature and pressure and therefore alter $\delta\sigma$. Measurements of pressure and temperature are not usually recorded with occultation observations, so mean values were assumed in the few cases where this was necessary, rather than reject the observations.

4 Analysis

Outline and scope of the analysis. The objective of this work is to analyse the calculated residuals ($\Delta\sigma$) for the following effects: corrections of a linear and periodic character in the lunar ephemeris $j=2$; corrections to the preliminary dynamical time-scale adopted in the reduction of observations before 1955.5; systematic corrections to the *FK4* catalogue; corrections to the basic datum of Watts' charts.

4.1 SELECTION AND WEIGHTING OF THE OBSERVATIONS

The following categories of occultation observations were rejected in this analysis: observations having timing errors of 1 min or greater, or the wrong star identification (about 1 per cent of the total); observations with poor or no limb profile corrections in Watts' charts and observations of the components of double stars not having separate positions in the *ZC* (about 9 per cent). So, of the 62 000 reduced observations, about 56 000 remained after this stage. A histogram of their residuals is plotted in Fig. 4 together with a normal frequency curve with zero mean and standard deviation of $0''.49$. The systematic bias in the residuals ($\Delta\sigma$) due to the equinox displacement discussed in Sections 3.2 and 3.5 was removed by applying $+0^s.050$ ($\partial\sigma/\partial\alpha'$) to each residual. This correction was only applied during this stage of selection of observations and was *not* applied to $\Delta\sigma$ in Section 4.2 or subsequently.

It was decided to reject residuals with an absolute value greater than $1''.3$, which is 2.6 times the standard deviation. If the residuals were distributed normally, only 1 per cent of them should have been rejected by this limit: in fact, about 10 per cent were rejected! It will be seen in Fig. 4 that the wings of the distribution lie well above the normal frequency curve in both directions. The spread of residuals in the wings is believed to be due to mistakes made by observers in identifying or recording the correct second of the events. For all the reasons above, 12 000 of the 62 000 reduced observations were rejected.

The weight assigned to each observation was determined from the following contributory factors:

(1) *Timing accuracy.* If the estimate of the timing uncertainty taken from Table 2 is denoted by ϵ_t , then the contributory error in the residual, $\Delta\sigma$, is given by,

$$\epsilon_1^2 = \left(\epsilon_t \frac{\partial\sigma}{\partial t} \right)^2.$$

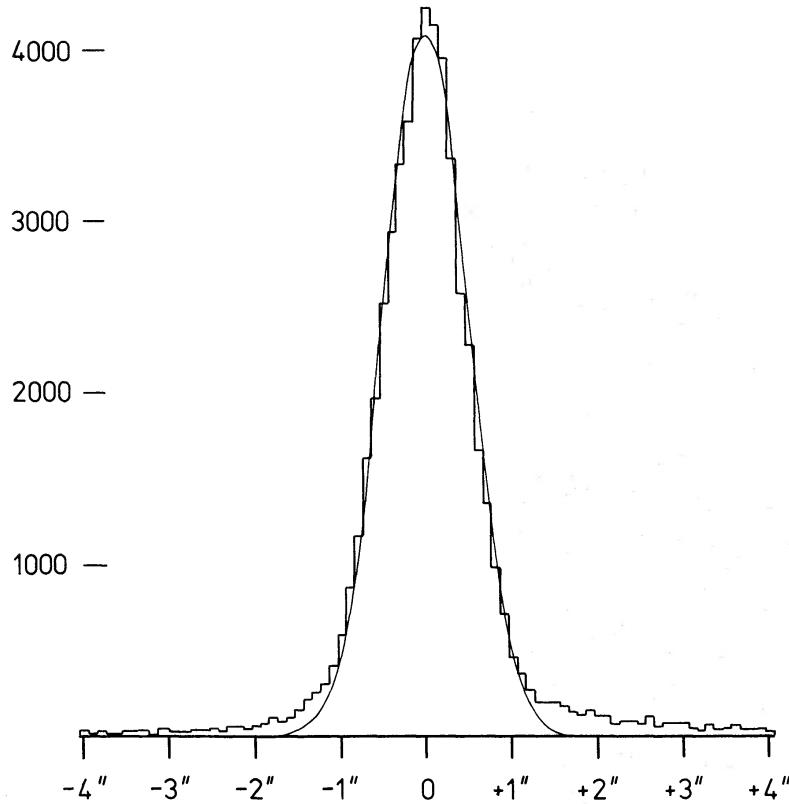


Figure 4. Histogram of computed angular residuals ($\Delta\sigma$) with fitted normal frequency curve having zero mean and standard deviation of $0''.49$.

(2) *Star positions.* The errors in right ascension and declination ($\epsilon_\alpha, \epsilon_\delta$) taken from the ZC give the following error in $\Delta\sigma$,

$$\epsilon_2^2 = \left(\epsilon_\alpha \frac{\partial \sigma}{\partial \alpha'} \right)^2 + \left(\epsilon_\delta \frac{\partial \sigma}{\partial \delta'} \right)^2.$$

(3) *Watts' chart.* The errors (ϵ_3) in the limb-profile heights were taken to be $\pm 0''.2$ or $\pm 0''.5$, depending on the amount of extrapolation of the contours that was required, as described in Section 3.6.

(4) *Lunar ephemeris $j = 2$.* A standard error of $\pm 0''.16$ was adopted for all observations, irrespective of the position angle.

(5) *Pleiades.* It is desirable to have an even distribution of stars around the Moon's orbit in order to separate the systematic distortions due to the star field from the corrections to various lunar parameters. Apart from the Pleiades, the stars in the ZC have a distribution of one star per square degree. In the Pleiades there are 12 stars per square degree and therefore the weight of each observation of these stars was reduced by the factor $1/12$.

The weight, W , of each observation was computed from,

$$W = f(0.47)^2 / (\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 + (0.16)^2),$$

where f has the value 0.083 for occultations of stars in the Pleiades and 1.0 otherwise.

4.2 CORRECTION TO THE EXPRESSION FOR THE MEAN LONGITUDE IN $j = 2$

As explained in Section 3.3 the observations made after 1955 July 1 were reduced by entering the lunar ephemeris with the time-argument $\text{TAI} + 32^s.184$. At this stage in the

analysis we shall regard this time-argument as being identical with dynamical time and therefore, to bring the lunar ephemeris into agreement with observations, we adjust the values of the constant and linear terms in the expression for the mean longitude. However, the most crucial decision which has to be made at this stage is which value of the quadratic term should be used.

The value for the quadratic term, $-4''.08 T_E^2$, in the lunar ephemeris $j = 2$ is the sum of two contributions: the first, $+7''.14 T_E^2$, is the theoretical value derived from point-mass attraction in Newtonian gravitational theory; the second, $-11''.22 T_E^2$, is the empirical value for the tidal acceleration based on the results of Spencer-Jones (1939). Whilst the theoretical part is well determined, the tidal part is not. From an analysis of ancient astronomical observations (mainly total solar eclipses), Muller (1975) found the tidal acceleration to be $(-14'' \pm 1'') T_E^2$; and from an analysis of the transits of Mercury in the period 1677–1973, Morrison & Ward (1975) found it to be $(-13'' \pm 1'') T_E^2$. But from analyses of occultations timed since 1955 July on the atomic time-scale TAI, Morrison (1973) found the tidal acceleration to be $(-21'' \pm 3'') T_A^2$, and Van Flandern (1975) found it to be $(-18'' \pm 5'') T_A^2$. Van Flandern has argued that the apparent difference between the tidal accelerations measured on the atomic and ephemeris time-scales is real and can be explained on the hypothesis that the universal constant of gravitation is decreasing with respect to TAI but not ET (or TD). I am of the opinion that the difference between the two sets of results is not significant and therefore T_E and T_A denote equivalent intervals of time. Accordingly, the subscripts will henceforth be omitted and T regarded as the time in centuries of 36 525 ephemeris or atomic days since the epoch 1900 January 0 Greenwich mean noon. In this analysis the value $-13''.T^2$ has been adopted for the tidal acceleration, but since the observations were reduced with a value of $-11''.22 T^2$, a differential correction of $-1''.78 (T - 0.77)^2 \partial\sigma/\partial\lambda$ was applied to the computed separations. This constraint imposed on the T^2 term is removed later in the analysis (see Section 5.1) in order to test whether there is indeed another accelerative component, besides the tidal one, when the Moon's motion is measured with respect to TAI.

The 36 000 observations made after 1955 July 1 were then analysed for corrections to the constant and linear terms which would be consistent with the observations and the requirement that the time-argument of the ephemeris should be TAI + 32^s.184. The following weighted observational equation of condition in three unknowns, a , b and c , which allows for a constant correction in latitude, was solved by the method of least-squares:

$$[a + b(T - 0.77)] \frac{\partial\sigma}{\partial\lambda} + c \frac{\partial\sigma}{\partial\beta} = -\Delta\sigma.$$

The solution gave:

$$a = -0''.806 \pm 0''.006,$$

$$b = -0''.411 \pm 0''.060,$$

$$(c = -0''.095 \pm 0''.004).$$

These corrections are to be applied in the sense of correcting the computed ephemeris position of the Moon to the observed position which is measured relative to the star positions. As explained in Sections 3.2 and 3.5, the origin of right ascension in the lunar ephemeris is referred approximately to Newcomb's equinox, whilst the positions of the stars in the *ZC* are referred to the *FK4* equinox. The average correction to be applied to the right ascension in the *FK4* is $-0^s.050$, which is equivalent to $-0''.69$ in ecliptic longitude. Thus, as

expected, a large part of the correction a arises from this difference in the positions of the equinox.

The correction to be applied to the expression for the mean longitude in the lunar ephemeris $j = 2$ in order that the argument of time should be TAI + 32^s.184 and that the origin of longitude should be the zero point of right ascension in the *FK4* is therefore,

$$\delta L = -0''.806 - 0''.411(T - 0.77) - 1''.78(T - 0.77)^2;$$

or, rearranging this to the epoch 1900.0,

$$\delta L = -1''.544 + 2''.330 T - 1''.78 T^2.$$

The expression for the mean longitude in $j = 2$, amended by the addition of this correction, is used throughout the rest of this analysis.

4.3 CORRECTION OF THE PRELIMINARY DYNAMICAL TIME-SCALE BEFORE 1955 JULY 1

In the reduction of the data before the introduction of the atomic time-scale in 1955 July 1 we formed an approximation to the dynamical time-scale by adding a linear correction in time to the universal time-scale;

$$TD = UT + 31^s.24 + 0^s.44 (Y - 1955.5),$$

where Y is the calendar date expressed in years. It is not possible to separate simultaneously the contributions in the residuals, $\Delta\sigma$, due to fluctuations in UT, from the contributions due to errors in the lunar ephemeris; so some simplifying assumption must be made about the behaviour of UT.

Regarding the behaviour of UT relative to TAI in the 20-year period since 1955.5 as being not uncharacteristic of its behaviour in the previous 20 years it was decided to analyse the occultation residuals in the period 1943.0–1955.5 for a cubic expression in time in order to relate TD to UT. This cubic expression, added to the linear approximation, should give a representation of the dynamical time-scale which nowhere departs from the true scale by more than 0.2 s, and is generally less than 0.1 s (see O'Hara 1975, fig. 3): this is adequate for the purposes of this analysis.

The following equation of condition for the unknowns b, c, d and e was set up, the computed residuals having been amended for the quadratic expression δL in Section 4.2:

$$\{b(Y - 1955.5) + c(Y - 1955.5)^2 + d(Y - 1955.5)^3\} \frac{\partial \sigma}{\partial T} + e \frac{\partial \sigma}{\partial \beta} = -\Delta\sigma.$$

No unknown for the constant is included in this equation because it was required to be held fixed at the value of +31^s.24 so that there is no discontinuity in dynamical time at the epoch 1955.5 (see Section 3.3).

The solution of the 14 000 equations of condition by the method of least-squares gave;

$$b = -0^s.109 \pm 0^s.011,$$

$$c = -0^s.009 \pm 0^s.003,$$

$$d = +0^s.0002 \pm 0^s.0002,$$

$$(e = -0''.125 \pm 0''.006).$$

Applying these corrections to the preliminary expression for TD given above and neglecting the small cubic term, we find;

$$TD = UT + 31^s.24 + 0^s.331(Y - 1955.5) - 0^s.009(Y - 1955.5)^2.$$

This was adopted as the definitive expression for dynamical time in the period 1943.0–1955.5 and all the computed residuals, $\Delta\sigma$, were adjusted to conform to this expression.

4.4 LUNATION SOLUTIONS FOR $\delta\lambda$ AND $\delta\beta$ AND PERIODOGRAM ANALYSES OF THESE SOLUTIONS

It is evident from the distribution of occultation observations shown in Fig. 2 that most of the disappearances are observed on several days around first quarter. This poor distribution of observations with respect to the lunar argument D raises problems in analysing the residuals for the presence of periods close to the frequency of D . However, if we accept that we are unlikely to separate terms with frequency D or its multiples from spurious ones due to the distribution of the data we can greatly simplify the periodogram analysis by regarding the residuals in each lunation as appertaining to the same phase of D . For the disappearances in each lunation the weighted means of the observations nearly always lie within a range of 24° about the phase $D = 100^\circ$. The loss of information in assuming that all the weighted means lie exactly at the phase angle 100° is not serious in relation to the scatter in the data. Therefore, to a good approximation, the weighted solutions for single corrections $\delta\lambda$ and $\delta\beta$ in each lunation can be regarded as a series of equally spaced 'normal points' at an interval of one synodic month.

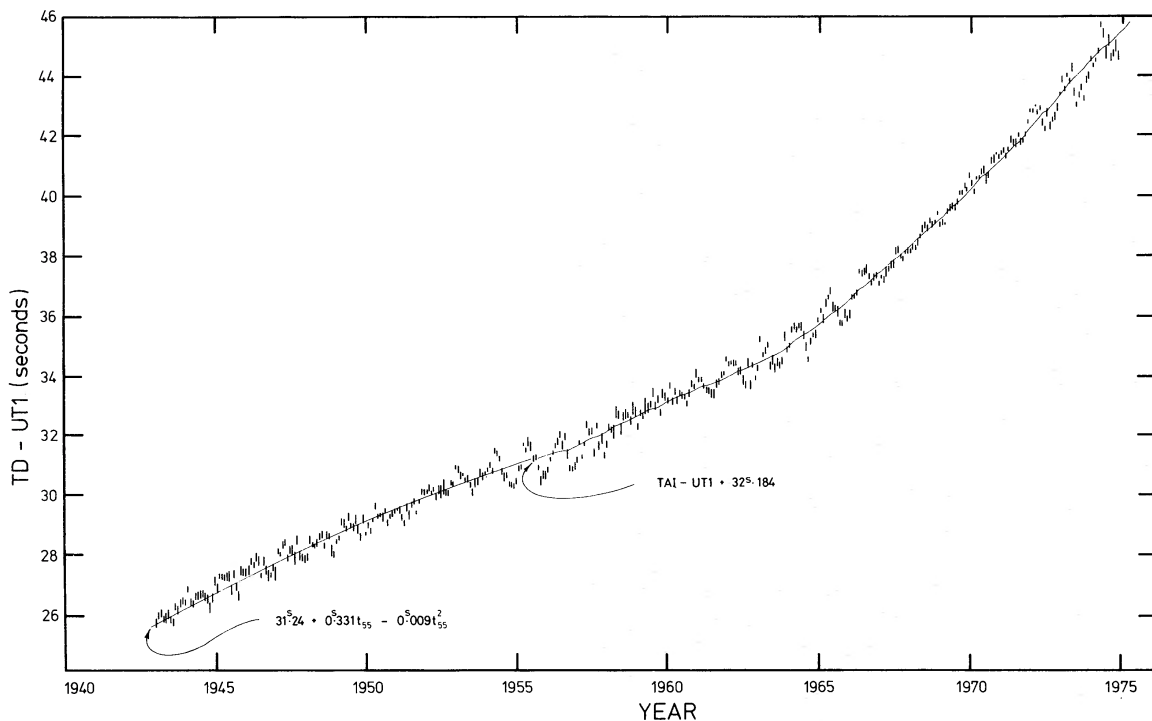


Figure 5. Difference between the dynamical time-scale (TD) and UT1. The vertical bars are the solutions obtained by grouping observations into lunations: half-lengths are one standard error. The quadratic expression in t , where t is measured in years from 1955.5, is fitted to the solutions in the period 1943.0 to 1955.5. The expression for the mean longitude of the Moon in this analysis is chosen to give the best agreement in the period 1955.5 to 1974.0 between TD–UT1 and TAI–UT1 + $32^s.184$. The fluctuations are mainly due to errors in the longitude of the Moon computed from the ephemeris $j = 2$ (see text).

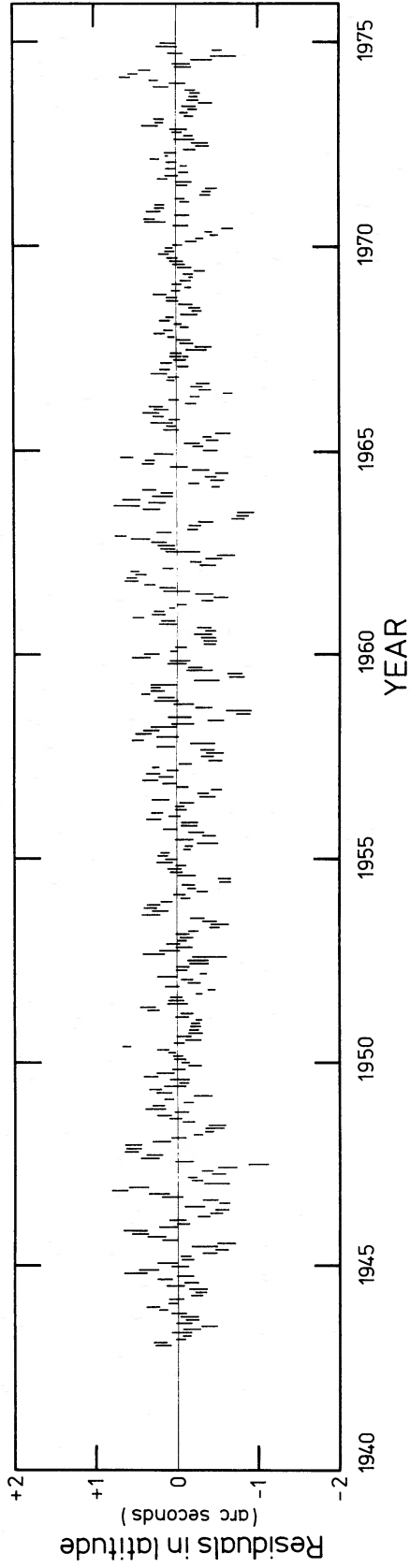


Figure 6. Solutions for observed minus computed latitude obtained by grouping observations into lunations: half-bar lengths are one standard error.

The following weighted observational equation of condition was solved for corrections to the ecliptic longitude ($\delta\lambda$) and latitude ($\delta\beta$) in each lunation:

$$\frac{\partial\sigma}{\partial\lambda}\delta\lambda + \frac{\partial\sigma}{\partial\beta}\delta\beta = -\Delta\sigma.$$

The residual, $-\Delta\sigma$, was adjusted for the finally adopted expressions for the mean longitude and the dynamical time-scale given in Sections 4.2 and 4.3. The solutions for the 396 lunations are plotted in Figs 5 and 6 where the half-bar length is one standard error. For convenience of presentation the solutions for $\delta\lambda$ have been multiplied by the inverse of the Moon's mean motion ($1.821\text{s}''$) to convert them to time and plotted relative to the finally adopted dynamical time-scale which is shown as a continuous line in Fig. 5.

The presence of periodicities in the lunation solutions is evident. These solutions were subjected to periodogram analysis and the resulting periodograms are shown in Figs 7 and 8,

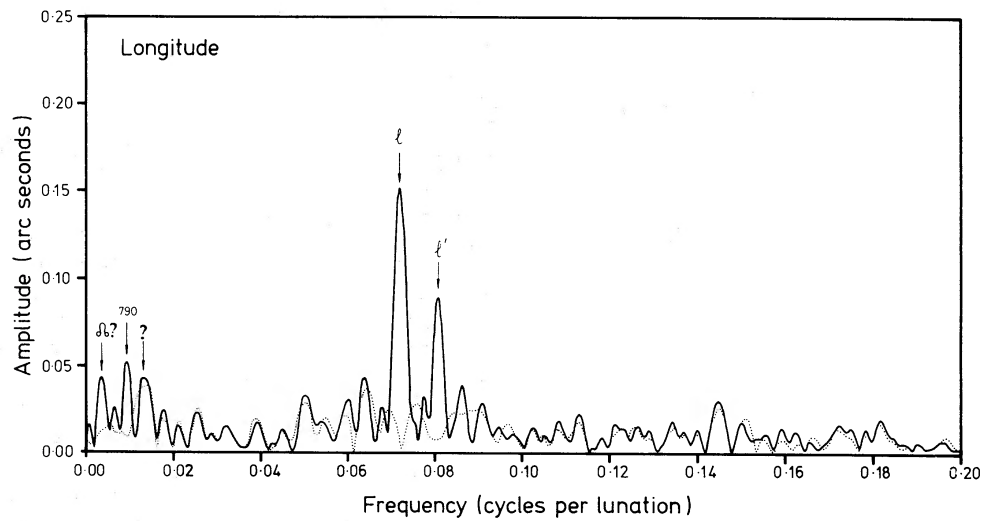


Figure 7. Periodogram of the fluctuations in longitude shown in Fig. 5. The dotted curve is the periodogram obtained after removing the main solution in Table 6 from the data.

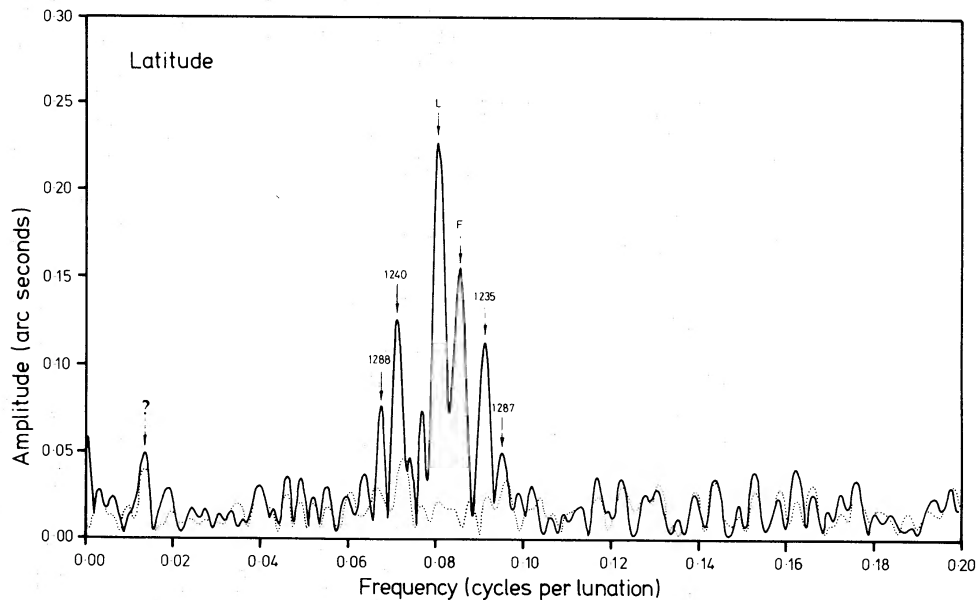


Figure 8. Periodogram of the fluctuations in latitude shown in Fig. 6. The dotted curve is the periodogram obtained after removing the main solution in Table 6 from the data.

where the amplitude is in seconds of arc and the frequency in cycles per lunation. The figures have been terminated arbitrarily at a frequency of 0.2 cycle/lunation as there are no significant peaks beyond these points.

If ν denotes the frequency in cycle/lunation of a peak in Figs 7 or 8, then it can be interpreted as one, or a combination of the basic causal frequencies, $D(\nu \pm n)$ cycle/day where $n = 0, 1, 2, \dots$, and D is in units of cycle/day. In order to select the most likely frequencies we need to consider the possible sources of periodic error in the lunar theory and in the stellar reference frame.

Periodic terms arising from the lunar theory. The coefficients and arguments (or frequencies) of the terms in the harmonic series for the longitude and latitude in the theory depend upon the observationally determined values of the following parameters:

Constant	Primary argument
e , eccentricity of the Moon's orbit ϖ , longitude of the Moon's perigee	l
e' , eccentricity of the Sun's orbit ϖ' , longitude of the Sun's perigee	l'
γ , sine half inclination of Moon's orbit Ω , longitude of Moon's node	F
α_1 , constant of parallactic inequality L' , longitude of the Sun	D

Corrections to the values of some of these parameters are clearly indicated by the presence of peaks at l and l' in Fig. 7 and at F in Fig. 8. These are the only peaks arising from corrections to these basic parameters that we would expect to detect above the noise level in Figs 7 and 8 because the frequencies of other related terms are either compounded with a multiple of the frequency D , or have coefficients in the theory which are an order of magnitude less than those of the primary frequencies.

The highest peak in the latitude periodogram is coincident with the frequency of L which is the argument of mean longitude. There is a term with this frequency in the lunar theory due to the figure of the Earth; but there could not be an error of this magnitude in that term. Further discussion of this peak is deferred until the next section which is concerned with the stellar reference frame. Besides the peaks at F and L , there are four other significant peaks at frequencies $\nu = 0.0711, 0.0912, 0.0670$ and 0.0950 cycle/lunation. If these are regarded as beating frequencies of the basic frequencies $(\nu + 1)D$, where $D = 0.03386$ cycle/day, we find that the basic frequencies have the values 0.0362, 0.0369, 0.0361 and 0.0371. Inspection of the table of planetary terms in the *ILE* reveals the following pairs of terms shown in Table 4. In the arguments of the terms, T , V and J denote the longitudes of the

Table 4. Suspect planetary terms in latitude.

<i>ILE</i> reference no.	Coefficient (")	Argument	<i>ILE</i> frequency (cycle/day)	Periodogram frequency
1240	+0.704	$L_0 + 51^\circ.6 - 5T + 3V$	0.03626	0.0362
1235	+0.077	$L_0 + 215.6 + 5T - 3V$	0.03694	0.0369
1288	+0.018	$L_0 + 24 - 2J$	0.03614	0.0361
1287	+0.035	$L_0 + 168 + 2J$	0.03706	0.0371

Earth, Venus and Jupiter, respectively. It is very improbable that the close agreement between the theoretical and observed frequencies could be accidental, and, as found later (in Section 5.6), that the phases of three of the terms should also be in such good agreement.

Of the remaining peaks in the latitude periodogram that at frequency 0.0135 cycle/lunation appears to be significant, but we can find no likely term in the lunar theory to account for it. The amplitude of 0".06 at zero frequency will later be attributed to a combination of the displacement of the centre of figure in Watts' charts from the centre of mass and the ellipticity of his datum surface.

As well as the obvious peaks at l and l' in the longitude periodogram, there appear to be several other significant ones. The peak near 0.0040 cycle/lunation = 0.00014 cycle/day, is close to the frequency of revolution of the node (-0.000147 cycle/day), and we regard this as the most likely cause. The peak near 0.0096 cycle/lunation = 0.00033 cycle/day is close to the frequency of the planetary term, *ILE* Reference No. 790 with argument $199^\circ.0 \cdot 5T - 3V$ and frequency 0.000338 cycle/day. This is the same commensurability as already noted in latitude, so it seems probable that there is some error in the theory associated with this particular commensurability. Assuming that this hypothesis is correct and that the errors in terms containing this commensurability are roughly proportional to the size of the coefficients, the next largest correction would produce a peak only one-quarter the size of that of *ILE* No. 790 and hence would not be detectable in Fig. 8. As in the longitude periodogram, no explanation is found for the peak near 0.0138 cycle/lunation.

Periodic terms arising from the stellar reference frame. Corrections to the equinox and the obliquity are determined from observations of the position of the Moon using the same principles as for the Sun. The mean orbital plane of the Moon over the period of revolution of the nodes defines the plane of the ecliptic and its intersection with the equator determines the actual position of the equinox. An error in the location of the equinox will contribute to the longitude of the node and hence produce an error-term associated with the argument Ω ; but it cannot produce any other periodic effects in the theory of the motion in latitude. Therefore, a term with the frequency of L in the latitude residuals must be attributed to errors in the stellar reference frame. Differentiating the following relation with respect to α ,

$$\sin \beta = -\cos \delta \sin \alpha \sin \epsilon + \sin \delta \cos \epsilon,$$

we find

$$\cos \beta \frac{\partial \beta}{\partial \alpha} = -\cos \delta \cos \alpha \sin \epsilon.$$

Substituting,

$$\cos \delta \cos \alpha = \cos \beta \cos \lambda,$$

and introducing incremental changes, we find,

$$\delta \beta = -\cos L \sin \epsilon \delta \alpha,$$

where the mean longitude, L , has been substituted as a sufficiently good approximation to the true longitude, λ . Since the occultation residuals are formed from the separation between the stars and the Moon, a positive correction $\delta \alpha$ to the Moon's position corresponds to a negative correction $\delta \alpha'$ to the star's position; therefore, in the residuals

$$\delta \beta = +\cos L \sin \epsilon \delta \alpha'.$$

It can also be shown that a correction $\delta\epsilon$ to the assumed value for the obliquity will produce a term $+\sin L \delta\epsilon$ in the latitude residuals. Therefore the large peak at L in Fig. 8 is due to a combination of corrections to the equinox and obliquity. These will be distinguished by their phases in the subsequent analysis.

It is also likely that there are systematic errors in right ascension and declination in the star places which behave approximately as $a \sin(\alpha + b)$, where a and b are unknown constants. Such systematic corrections in declination cannot be satisfactorily separated from a correction in latitude of the form $c \sin L + d \cos L$ which, as has already been shown, can arise from corrections to the obliquity and equinox. The contribution from these systematic errors will have to be assessed when considering the solutions for the obliquity and equinox. This correlation between unknowns in the stellar reference frame and the lunar theory is not so troublesome in longitude. Systematic corrections in right ascension are most likely to be confused with simple elliptic corrections to the Sun's orbit of the form,

$$\delta\lambda' = 2\delta e' \sin l' - 2e'\delta\varpi' \cos l'.$$

However, in the lunar theory there are other significant first-order terms due to the Sun's orbit which help to reduce this correlation between systematic corrections to the right ascension of the stars and to the Sun's orbit. Indeed, the correlation is reduced so much that lunar occultations may provide more reliable corrections to the Sun's orbit than those that can be obtained directly from observations of the Sun!

4.5 FORMULATION OF THE EQUATION OF CONDITION IN 40 UNKNOWNNS

Basic equation of condition. Having identified as many as possible of the periodic terms present in the residuals, an observational equation of condition was formed which included these terms and also corrections to the assumed diameter, shape and location of the centre of Watts' datum, and corrections to the preliminary values of the personal equations. The equation of condition took the basic form:

$$\frac{\partial\sigma}{\partial\lambda}\delta\lambda + \frac{\partial\sigma}{\partial\beta}\delta\beta + \frac{\partial\sigma}{\partial\pi}\delta\pi + \frac{\partial\sigma}{\partial\alpha'}\delta\alpha' + \frac{\partial\sigma}{\partial\delta'}\delta\delta' + \frac{\partial\sigma}{\partial\rho_0}\delta\rho_0 + \frac{\partial\sigma}{\partial t}\delta t = -\Delta\sigma.$$

Inserting the expressions for the partial derivatives from Section 3.1 we find,

$$\sin N \delta\lambda + \cos N \delta\beta + 0.2725 \delta\pi - \sin P \delta\alpha' - \cos P \delta\delta' + \frac{\pi}{\pi_0} \delta\rho_0 - \frac{\partial\sigma}{\partial t} \delta t = \Delta\sigma.$$

Expansion of the incremental changes, $\delta\lambda$ etc. The incremental change in true longitude $\delta\lambda$ was expanded as the linear sum of corrections to the mean longitude and the basic constants and arguments of the lunar theory:

$$\begin{aligned} \delta\lambda = & \delta L + \sum_{i=1,8} \frac{\partial\lambda}{\partial\mu_i} \delta\mu_i \\ & + \text{correction to coefficient of } \sin \Omega \\ & + \text{correction to planetary term } ILE \text{ No. 790,} \end{aligned}$$

where δL is a quadratic expression in time and μ_i represents the primary arguments l, l', F, D and the constants e, e', γ, α_1 .

The incremental corrections in latitude and parallax were expanded as follows:

$$\delta\beta = \sum_{i=1,8} \frac{\partial\beta}{\partial\mu_i} \delta\mu_i$$

+ corrections to planetary terms *ILE* Nos 1235, 1240, 1287, 1288

$$+ \cos L \sin \epsilon \delta\alpha'_0$$

$$+ \sin L \delta\epsilon;$$

$$\delta\pi = \sum_{i=1,8} \frac{\partial\pi}{\partial\mu_i} \delta\mu_i.$$

The values of the partial derivatives with respect to the basic constants and arguments were calculated to three significant figures by differentiating and summing the component parts of the terms in the theory listed by Brown (1908). The values were checked subsequently against a computer listing (kindly provided by Dr Van Flandern) in which the derivatives had been computed from the new solution of the Main Problem by Deprit, Henrard & Roa (1971).

The incremental correction $\delta\alpha'$ was regarded as a correction for a systematic periodic error in the right ascension of the *FK4* of the form,

$$\delta\alpha' = a \sin(\alpha' + b),$$

where a and b are constants. The equinox correction $\delta\alpha'_0$ has been included in the expansion of $\delta\beta$ where it can be determined more reliably. No unknown was included for $\delta\delta'$ because it was regarded as being inseparable from the possible displacement in latitude between the centre of Watts' datum for the profile and the centre of the figure.

The incremental correction $\delta\rho_0$ to the lunar semi-diameter at mean distance was expanded as follows:

$$\delta\rho_0 = \delta r_0 + \cos(Q - R_1) \delta r_1 + \cos 2(Q - R_2) \delta r_2,$$

where, δr_0 = correction to the adopted semi-diameter of $932''.58$ at mean distance, corresponding to a value of $3422''.608$ for the equatorial horizontal parallax; δr_1 = shift required along position angle R_1 to bring the centre of the mean profile through Watts' data into coincidence with the centre of figure defined by occultation observations; δr_2 = correction to the ellipticity of Watts' reference datum along position angle R_2 in order to reduce it to a circle.

The incremental change in time, δt , was expanded as a correction to one of the four categories of personal equation: (1) stop-watch etc., disappearances; (2) eye-and-ear, disappearances; (3) stop-watch etc., reappearances; (4) eye-and-ear, reappearances.

Final equation of condition. The incremental expansions were substituted in the basic equation to give the final form of the equation of condition. The 40 unknowns in the equation are listed in Table 5.

The correction to the coefficient of the term in $\sin \Omega$ in longitude was introduced as an unknown because of doubts expressed by Van Flandern (1976) about the revision of Brown's theory for the change in the value of the ellipticity of the Earth's figure from $1/294$ to the IAU 1964 value of $1/298.25$. The largest corrections to the ephemeris $j = 2$ which Van

Table 5. The 40 unknowns in the main solution.

Unknown		Symbol
1 Constant in Moon's mean longitude	} Only for observations after 1955 July 1	L
2 Linear term in Moon's mean longitude		$\dot{L}$
3 Quadratic term in Moon's mean longitude		L
4 Constant part of Moon's perigee		$\bar{\omega}$
5 Linear part of Moon's perigee		$\dot{\bar{\omega}}$
6 Constant part of Moon's node		Ω
7 Linear part of Moon's node		$\dot{\Omega}$
8 Constant part of Sun's mean longitude		L'
9 Constant part of Sun's perigee		$\bar{\omega}'$
10 Constant of eccentricity of Moon's orbit		e
11 Constant of inclination of Moon's orbit		γ
12 Constant of parallactic inequality		α_1
13 Constant of eccentricity of Sun's orbit		e'
14 Coefficient of term $\sin \Omega$ in Moon's longitude		$s\Omega$
15 Coefficient of term $\cos \Omega$ in Moon's longitude		$c\Omega$
16 Coefficient of sine of argument of planetary term <i>ILE</i> No. 790		$s790$
17 Coefficient of cosine of argument of planetary term <i>ILE</i> No. 790		$c790$
18 Coefficient of sine of argument of planetary term <i>ILE</i> No. 1235		$s1235$
19 Coefficient of cosine of argument of planetary term <i>ILE</i> No. 1235		$c1235$
20 Coefficient of sine of argument of planetary term <i>ILE</i> No. 1240		$s1240$
21 Coefficient of cosine of argument of planetary term <i>ILE</i> No. 1240		$c1240$
22 Coefficient of sine of argument of planetary term <i>ILE</i> No. 1287		$s1287$
23 Coefficient of cosine of argument of planetary term <i>ILE</i> No. 1287		$c1287$
24 Coefficient of sine of argument of planetary term <i>ILE</i> No. 1288		$s1288$
25 Coefficient of cosine of argument of planetary term <i>ILE</i> No. 1288		$c1288$
26 Constant correction in right ascension of <i>FK4</i>		α
27 Linear correction in right ascension of <i>FK4</i>		$\dot{\alpha}$
28 Constant correction to obliquity		ϵ
29 Linear correction to obliquity		$\dot{\epsilon}$
30 Systematic correction in right ascension of <i>FK4</i> varying as $\sin \alpha$		$s\alpha$
31 Systematic correction in right ascension of <i>FK4</i> varying as $\cos \alpha$		$c\alpha$
32 Correction to semi-diameter of Moon		r_0
33 Longitude component of shift of centre of Watts' datum		$r_1 sR_1$
34 Latitude component of shift of centre of Watts' datum		$r_1 cR_1$
35 Longitude component of correction to ellipticity of Watts' datum		$r_2 sR_2$
36 Latitude component of correction to ellipticity of Watts' datum		$r_2 cR_2$
37 Correction to adopted personal equation: disappearance, stop-watch etc.		$pe1$
38 Correction to adopted personal equation: disappearance, eye-and-ear		$pe2$
39 Correction to adopted personal equation: reappearance, stop-watch etc.		$pe3$
40 Correction to adopted personal equation: reappearance, eye-and-ear		$pe4$

Flandern proposes are,

$$\delta\lambda = -0''.02 \sin \Omega,$$

and

$$\delta\beta = +0''.02 \sin L.$$

These should be detectable in the occultation data, but the term in latitude will be confused with the term having the same argument arising from an error in the obliquity of the ecliptic.

The unknowns in the quadratic expression in mean longitude were introduced in order to test whether the acceleration of the Moon measured with respect to TAI departed significantly from the adopted value of $-13'' T^2$ which had been measured with respect to ET.

Table 6. Main solution.

Unknowns (see Table 5)	Solution (")	se (")	Increase in sum of squares	Correlation coefficients of 0.5 and greater
L	-0.04	± 0.01	2.1	L with $\dot{L}$ -0.8
$\dot{L}$	+1.7	± 0.3	5.6	L with $\ddot{L}$ +0.7
$\ddot{L}$	-11.0	± 2.0	6.0	L with $s\Omega$ +0.5
ϖ	-0.27	± 0.05	6.5	L with $c\Omega$ +0.7
$\dot{\varpi}$	-5.1	± 0.3	58.7	L with $r_1 sR_1$ -0.7
Ω	+1.65	± 0.09	71.9	$\dot{L}$ with $\ddot{L}$ -1.0
$\dot{\Omega}$	+3.4	± 0.6	6.0	$\dot{L}$ with $s\Omega$ -0.8
L'	+0.68	± 0.08	15.3	$\dot{L}$ with $c\Omega$ -0.6
ϖ'	-10.2	± 1.3	13.3	$\ddot{L}$ with $s\Omega$ +0.8
e	+0.057	± 0.002	229.3	$\ddot{L}$ with $c\Omega$ +0.5
γ	-0.053	± 0.003	84.2	ϖ with $\dot{\varpi}$ -0.7
α_1	+0.46	± 0.03	47.0	Ω with $\dot{\Omega}$ -0.8
e'	-0.17	± 0.02	16.1	α_1 with r_0 +0.7
$s\Omega$	+0.005	± 0.006	0.2	α with $\dot{\alpha}$ -0.8
$c\Omega$	-0.009	± 0.005	0.6	e with $\dot{e}$ -0.8
$s790$	-0.032	± 0.004	14.3	r_0 with $pe2$ +0.7
$c790$	-0.008	± 0.004	1.1	r_0 with $pe4$ +0.6
$s1235$	-0.092	± 0.005	67.8	$r_1 sR_1$ with $pe2$ -0.7
$c1235$	-0.018	± 0.005	2.5	$r_1 sR_1$ with $pe4$ -0.6
$s1240$	-0.098	± 0.005	74.1	$r_1 cR_1$ with $r_2 sR_2$ -0.6
$c1240$	0.000	± 0.005	0.0	$r_2 cR_2$ with $pe2$ -0.5
$s1287$	-0.037	± 0.005	10.9	$pe1$ with $pe3$ +0.8
$c1287$	+0.002	± 0.005	0.1	$pe2$ with $pe4$ +0.7
$s1288$	-0.026	± 0.005	5.7	
$c1288$	-0.050	± 0.005	19.1	
α	+0.71	± 0.02	191.3	
$\dot{\alpha}$	+1.5	± 0.2	11.8	
ϵ	+0.014	± 0.008	0.8	
$\dot{\epsilon}$	-0.10	± 0.05	0.8	
$s\alpha$	-0.030	± 0.004	10.5	
$c\alpha$	+0.041	± 0.005	16.4	
r_0	+0.152	± 0.006	136.5	
$r_1 sR_1$	-0.015	± 0.010	0.5	
$r_1 cR_1$	-0.191	± 0.004	407.2	
$r_2 sR_2$	+0.084	± 0.003	136.8	
$r_2 cR_2$	-0.035	± 0.004	14.1	
	(s)	(s)		
$pe1$	-0.25	± 0.02	23.2	
$pe2$	+0.07	± 0.04	0.7	
$pe3$	-0.16	± 0.03	7.8	
$pe4$	+0.26	± 0.04	7.1	
Sum of squares of reduced residuals			10 325.6	
Number of observations			49 986	
Variance			0.207	

It was realized that strong correlations might exist between some of the 40 unknowns in Table 5 and that some of the linear changes might hardly be detectable over the 32-year time-span of the data. However, they were included for the completeness of the discussion.

4.6 SOLUTION FOR 40 UNKNOWNNS

The 50 000 weighted equations of condition in 40 unknowns were solved by the method of least-squares. The solution, together with the standard errors and correlation coefficients

between the unknowns, which were calculated in the usual way from the variance–covariance matrix, are displayed in Table 6. The correlation coefficient is given where it is greater or equal to 0.5. The variance of the solution is 0.207, giving a standard error of $\pm 0''.46$ for an observation of unit weight. Solutions were also performed by removing each unknown in turn and computing the reduced sum of squares of the residuals. The increase in the sum of squares due to the exclusion of each unknown from the solution is shown in column 4 of Table 6. The result for the simultaneous exclusion of the three unknowns in mean longitude is also given.

5 Discussion

5.1 SOLUTION FOR THE QUADRATIC CORRECTION IN MEAN LONGITUDE

The solution given in Table 6 for the correction to the adopted motion in mean longitude measured with respect to the time-scale TAI + 32^s.184 is

$$-(0''.04 \pm 0''.01) + (1''.7 \pm 0''.3) (T - 0.58) - (11'' \pm 2'') (T - 0.58)^2.$$

The correction $-11'' T^2$ is to be added to the value of the quadratic term in the ephemeris which already contains the tidal term of $-13'' T^2$ adopted in Section 4.2. We recall that the residuals were adjusted in Section 4.2 for the revised expression in longitude obtained by introducing the time-scale TAI + 32^s.184 as the time-argument of the ephemeris whilst constraining the value of the tidal term to be $-13'' T^2$. If the tidal term measured with respect to TAI (as distinct from TD) were indeed $-13'' T^2$, then we would expect to find no significant correction, rather than the result given above.

The correction $-11'' T^2$ can be interpreted in one, or possibly a combination of the following ways: (1) the adopted value $-13'' T^2$ for the tidal term is substantially in error, (2) the lengths of the time-units on the dynamical and atomic time-scales vary with respect to one another, or (3) there is some cosmological explanation, such as the variability of the universal constant of gravitation. Before considering these possibilities it is advisable to enquire whether the standard error of $\pm 2''$ reflects the true uncertainty of the result.

The degree of correlation between the unknowns for the constant, linear and quadratic terms is such that when the quadratic term is omitted from the solution the constant changes to $(+0''.01 \pm 0''.01)$ and the linear term to $-(0''.02 \pm 0''.07) T$, with a consequential increase of only 6.0 in the sum of squares of the residuals. Apart from this difficulty of correlations between the unknowns, it is possible that the result for the quadratic term is distorted by the presence in the residuals of small long-period terms due to deficiencies in the planetary perturbations in Brown's lunar theory. When the unknown for an eight-year period term in longitude (*ILE* reference No. 790) is omitted, the solution for the quadratic term in mean longitude changes to $-(19'' \pm 2'') T^2$, and yet the correlation coefficient is only -0.4 . When the unknowns in $\sin \Omega$ and $\cos \Omega$ are omitted, the result becomes $-(12'' \pm 1'') T^2$ and $-(9'' \pm 2'') T^2$ respectively. From the variability of these solutions and the slowness of their effect on the residuals, I conclude that there is no firm evidence in this analysis for a quadratic term which is greater negatively than $-13'' T^2$.

The problems associated with the planetary perturbations in the theory can be avoided by comparing the observations with an ephemeris generated from the equations of motion by numerical integration. Van Flandern (private communication) has done this and he finds a correction of $-(5'' \pm 2'') T^2$ to the tidal term of $-13'' T^2$. He has interpreted this result as evidence for the variability of the gravitational constant, but it could also be regarded as setting an upper limit of $-18'' T^2$ on the tidal term or alternatively, as indicating the degree

of agreement between the dynamical and atomic time-scales. If we accept that the tidal term with respect to TAI is $-13''T^2$ and that there is no other contribution, apart from the term $+7''.14T^2$ due to planetary attractions, then a further correction of $-5''T^2$ to the mean longitude of the Moon would arise from a divergence between the atomic and dynamical time-scales of 9 s century^{-2} ; i.e. the lengths of the seconds on the two scales would diverge by 6 parts in $10^{11}/\text{yr}$. This result must be regarded as very tentative. I will hold the value of the tidal term fixed at $-13''T^2$ and neglect any other contribution in the rest of this analysis.

5.2 CORRECTIONS TO THE FRAME OF REFERENCE OF THE FK4

FK4 equinox. The correction to be added to the right ascension of the stars in the *FK4* catalogue in order that they should be referred to the actual intersection of the equator and the ecliptic (herein called the dynamical equinox) is found to be,

$$\delta\alpha'_0 = +0''.71 \pm 0''.02 + (1''.5 \pm 0''.2)(T - 0.5).$$

From the coefficient in the equation of condition in Section 4.5 it will be seen that the equinox correction is determined entirely from the latitude component of the O-C, thus avoiding the difficulty of separating a constant correction in right ascension from a constant correction in longitude. The constant correction of $+0''.71$ to the right ascension of the *FK4* is the result of the interpretation in this analysis of an apparent correction of $+0''.28 \cos L$ to the latitude of the Moon. This must surely be the only possible interpretation because such a term could not be explained by some deficiency in the lunar theory or by a bias in the observations. Because of the direct link in the observations between the positions of the stars and the Moon, whose orbit is an intermediary in establishing the position of the ecliptic, occultations provide an accurate method for the determination of the position of the equinox. It does not suffer from the severe observational problems associated with meridian observations of the Sun and inner planets. Even meridian observations of the outer planets and minor planets do not at present provide very reliable results for the equinox, usually because of limited coverage of their orbits which causes difficulty in separating the equinox correction from a correction in mean longitude (see, e.g. Adams, Bestul & Scott 1964).

The secular part of the equinox correction is not very reliably determined in our solution. It is closely correlated with the constant part and when the unknown $\dot{\Omega}$ (for which the solution is only of marginal significance) is excluded the solution for $\delta\alpha'_0$ reduces to $+0''.90/\text{cy}$. However, this analysis does indicate that a correction in the region of $+1''/\text{cy}$ should be applied to the proper motions in right ascension in the *FK4*. This result from the dynamics of the lunar orbit agrees with Fricke's (1967) result of $+1''.20 \pm 0''.11/\text{cy}$ from the kinematics of fundamental proper motions. Fricke's result for the motion is much more reliable, so we prefer to adopt our solution for the constant part at the mean epoch of the observations (around 1959.0) and his for the motion: the preferred solution is then,

$$\delta\alpha'_0 = +0''.82 \pm 0''.02 + (1''.20 \pm 0''.11)(T - 0.59).$$

In ecliptic coordinates this corresponds to the following corrections to the star positions:

$$\begin{aligned} \cos \beta \delta\lambda' &= [0''.75 \pm 0''.02 + (1''.10 \pm 0''.10)(T - 0.59)] \cos \beta \\ &\quad - [0''.33 \pm 0''.01 + (0''.44 \pm 0''.04)(T - 0.59)] \sin \beta \sin \lambda \\ \delta\beta' &= -[0''.33 \pm 0''.01 + (0''.44 \pm 0''.04)(T - 0.59)] \cos \lambda. \end{aligned}$$

Alternatively, these corrections with the opposite signs may be regarded as the corrections to the lunar ephemeris required to refer it to the *FK4* frame of reference. The correction in longitude has already been absorbed in the total correction to longitude deduced in Section 4.2, so no further adjustment is needed, but the correction in latitude has yet to be applied. Therefore, in order to refer the latitude of the Moon, taken from the ephemeris $j = 2$, to the *FK4* frame of reference, the following correction should be added:

$$\delta\beta = + [0''.33 \pm 0''.01 + (0''.44 \pm 0''.04)(T - 0.59)] \cos \lambda.$$

Reducing this to the epoch 1900.0 we find,

$$\delta\beta = + [0''.07 \pm 0''.01 + (0''.44 \pm 0''.04) T] \cos \lambda.$$

If, on the other hand, it is desired to refer the Moon's position to the dynamical equinox, then in this respect no correction is required to the latitude in $j = 2$, and the correction to the longitude is the sum of the correction found in Section 4.2 and the correction given above; that is:

$$\text{correction to } j = 2 \text{ to refer to } FK4 \text{ equinox} = -1''.54 + 2''.33 T - 1''.78 T^2$$

$$\text{correction to } FK4 \text{ to refer to dynamical equinox} = +0''.10 + 1''.10 T$$

$$\therefore \text{correction to } j = 2 \text{ to refer to dynamical equinox} = -1''.44 + 3''.43 T - 1''.78 T^2,$$

where the correction to the *FK4* equinox has been reduced to the epoch 1900.0 and the small term varying as $\sin \lambda \sin \beta$ has been ignored.

Precession. No attempt has been made in this analysis to derive a correction to Newcomb's value for the constant of luni-solar precession. In principle, the correction could be derived from a comparison of the observed and theoretical motions of the node and perigee, but the observed motions are too uncertain to make this practicable.

Obliquity. No significant correction to Newcomb's expression for the obliquity of the ecliptic is found in this analysis. In particular, we find no support for a correction to the secular change as great as $-0''.3/\text{cy}$ which has been found by some previous investigators. Duncombe & Van Flandern (1976) have explained how such a spurious correction might have arisen in these earlier investigations.

Systematic corrections in right ascension. The correction to the right ascension of the *FK4* system was found to be,

$$\delta\alpha' = 0''.03 \sin \alpha + 0''.04 \cos \alpha,$$

i.e.

$$\delta\alpha' \approx -0^s.003 \sin (\alpha - 4^h).$$

This correction only applies to the narrow belt about 13° wide centred on the ecliptic, so it cannot be compared directly with those from meridian observations which are usually tabulated for zones of right ascension and declination. The main reason for the inclusion of systematic errors of this type was to judge their effect on the solution for the solar elements: this is exhibited later in Table 9. However, the semi-amplitude of $0^s.003$ found here is consistent with the range of systematic corrections to the *FK4* found, for example, by Thomas & Wallis (1971) in their analysis and discussion of Danjon astrolabe observations in the period 1959–63 and by Afanas'eva & Gorshkov (1974) in their comparison of the USSR Time Services catalogue with the *FK4*.

5.3 SOLUTION FOR THE FUNDAMENTAL ARGUMENTS Ω , ϖ AND L'

The corrections to the fundamental arguments Ω , ϖ and L' in Table 6 are

$$\delta\Omega = + (1''.65 \pm 0''.09) + (3''.4 \pm 0''.6)(T - 0.5)$$

$$\delta\varpi = - (0''.27 \pm 0''.05) - (5''.1 \pm 0''.3)(T - 0.5)$$

$$\delta L' = + 0''.68 \pm 0''.08 \quad (\text{epoch } 1959.0).$$

The relationships between the primary and fundamental arguments are

$$F = L - \Omega$$

$$l = L - \varpi$$

$$D = L - L'.$$

With a time-span of only 32 years in the data and the degree of correlation that exists between some of the unknowns it is necessary to be cautious about the significance of the solutions for the secular terms. When the unknown $\dot{\alpha}$ is omitted from the solution, the result for $\dot{\Omega}$ reduces to $+(1''.6 \pm 0''.6)(T - 0.5)$, and from Table 6 it is seen that the sum of squares of the residuals is only increased by 6.0 when $\dot{\Omega}$ is omitted. This evidence suggests that there is not a strong case here for changing Brown's empirical value for the motion of the node and therefore I will adopt the correction to the constant part of $\delta\Omega$ found when the secular term is omitted: this solution gives

$$\delta\Omega = + 2''.02 \pm 0''.06 \quad (\text{epoch } 1959.0).$$

The correction to the secular part of $\delta\varpi$, on the other hand, appears to be more reliable but its correlation with the constant part is troublesome. When ϖ is omitted from the solution, $\dot{\varpi}$ alters to $-6''.3 \pm 0''.2$ and the sum of squares of the residuals only increases by 6.5. If $\dot{\varpi}$ is omitted, ϖ changes to $-0''.86 \pm 0''.03$ and the sum of squares increases by 58.7. So there is a fairly strong indication that Brown's value for the secular motion of the perigee should be reduced by a few seconds of arc per century. Until a firmer result is obtained from an analysis of a longer time-span of data I prefer to quote only the correction to the constant part for the mean epoch of observation; thus

$$\delta\varpi = - 0''.86 \pm 0''.03 \quad (\text{epoch } 1959.0).$$

The correction to the mean longitude found in Section 5.2 which refers $j = 2$ to the dynamical equinox is given by

$$\delta L = - 1''.44 + 3''.43 T - 1''.78 T^2.$$

Evaluating this expression for the epoch 1959.0 we find

$$\delta L = - 0''.04.$$

Thus, from the relations between the primary and fundamental arguments given above we find

$$\delta F = - 2''.06 \pm 0''.06$$

$$\delta l = + 0''.82 \pm 0''.03$$

$$\delta D = - 0''.72 \pm 0''.08.$$

The correction to the mean elongation, δD , can be compared with the result obtained by Duncombe, Haupt & Duncombe (1971) from the analysis of times of contact during the

total solar eclipses of 1966 November 12 and 1970 March 7. They found that the lunar ephemeris $j = 2$ had to be interpolated for a value of ephemeris time $0^s.911 \pm 0^s.034$ earlier than that used in interpolating in Newcomb's theory of the Sun. Alternatively, this result could be regarded as a correction to the mean elongation of the Sun from the Moon of $-0^s.911 \times 0.508$, where the factor 0.508 is the rate of change in the mean elongation in seconds of arc per second:

$$\therefore \delta D = -0''.46 \pm 0''.02 \quad (\text{eclipses, epoch 1968.0}).$$

This is to be compared with the occultation result:

$$\delta D = -0''.72 \pm 0''.08 \quad (\text{occultations, epoch 1959.0}).$$

The quoted error of $\pm 0''.02$ for the eclipse result only reflects the internal scatter of the results: errors in the lunar ephemeris and the shape of the profile taken from Watts' charts are liable to have contributed a systematic error of at least $\pm 0''.1$. The result from lunar occultations is likely to be affected by systematic errors in timing. The principal terms in the partial derivative $\partial\lambda/\partial D$ are $+0.044 \cos(l - 2D)$ and $+0.023 \cos 2D$. As shown in Fig. 2 the occultation observations are poorly distributed with respect to the mean elongation, D , and it is possible that there are systematic departures in visual timings which depend on the level of illumination and hence the phase of the Moon. Fortunately, the larger of the two partial derivatives is also coupled with the argument l , therefore this will help to reduce the degree of correlation. If the result from eclipses is taken to be $-0''.5 \pm 0''.1$, then the result from the occultations would appear to be in tolerably good agreement with it.

5.4 SOLUTION FOR THE DELAUNAY PARAMETERS e , γ AND α_1

The corrections to the Delaunay parameters of eccentricity and inclination ($\gamma = \sin i/2$) used in $j = 2$ are

$$\delta e (\sin 1'')^{-1} = +0''.057 \pm 0''.002$$

$$\delta \gamma (\sin 1'')^{-1} = -0''.053 \pm 0''.003;$$

i.e.

$$\delta e = +(28 \pm 1) \times 10^{-8}$$

$$\delta \gamma = -(26 \pm 1) \times 10^{-8}.$$

Both values are very stable under variation of the set of unknowns in the solution.

The correction to the Delaunay parameter α_1 ($= 0.002\,509\,35$) in the ephemeris $j = 2$ is

$$\delta \alpha_1 (\sin 1'')^{-1} = +0''.46 \pm 0''.03;$$

i.e.

$$\delta \alpha_1 = +(22 \pm 2) \times 10^{-7}.$$

The parameter α_1 is related to the ratio of the masses of the Moon and Earth (denoted by μ) and the ratio of the lunar and solar parallaxes:

$$\alpha_1 = \frac{1 - \mu \pi'}{1 + \mu \pi}.$$

The values of μ , π and π' are now so well known that it is very unlikely that the correction to α_1 could be greater than 1×10^{-7} , so we must regard the correction above as being spurious and investigate the possible cause of this.

The principal term in the partial derivative $\partial\lambda/\partial\alpha_1$ is $-0.24 \sin D$, and therefore taking $\delta\alpha_1$ as $+0''.46$, the implied correction to the computed longitude of the Moon is $-0''.11 \sin D$. The required correction of $-0''.11$ around first quarter could be absorbed by increasing the personal equation for disappearances from $-0^s.3$ to about $-0^s.6$ and the correction of $+0''.11$ at last quarter by decreasing the personal equation for reappearances from $-0^s.5$ to $-0^s.2$. Whilst it is quite possible that the average personal equation for disappearances should be increased, it is very unlikely that that for reappearances should be reduced. An alternative explanation might be that the semi-diameter should be reduced and the correction for the ellipticity of Watts' datum altered. To test the feasibility of these interpretations we have inspected the effects on the correlated unknowns of removing each one in turn from the solution: the results are presented in Table 7. The standard errors are given only for the main solution since they are almost the same for the others.

Table 7. Solutions for unknowns correlated with the constant in parallactic inequality α_1 .

Solution	Main	1	2	3	4	5	6	7	8
Unknown									
α_1	$+0''.46 \pm 0''.03$	—	$+0.40$	$+0.44$	$+0.60$	$+0.48$	$+0.53$	$+0.51$	$+0.61$
r_0	$+0''.152 \pm 0''.006$	$+0.144$	—	$+0.130$	$+0.131$	$+0.145$	$+0.141$	$+0.131$	$+0.123$
r_2cR_2	$-0''.035 \pm 0''.004$	-0.008	$+0.013$	—	-0.026	-0.032	-0.031	-0.025	-0.022
$pe1$	$-0^s.25 \pm 0^s.02$	-0.40	-0.04	-0.21	—	-0.24	-0.13	-0.24	—
$pe2$	$+0^s.07 \pm 0^s.04$	$+0.25$	-0.60	-0.08	$+0.03$	—	$+0.05$	-0.10	—
$pe3$	$-0^s.16 \pm 0^s.03$	-0.32	$+0.04$	-0.12	$+0.06$	-0.16	—	-0.16	—
$pe4$	$+0^s.26 \pm 0^s.04$	$+0.44$	-0.41	$+0.11$	$+0.22$	$+0.20$	$+0.24$	—	—
Increase in sum of squares		47.0	136.5	14.1	23.2	0.7	7.8	7.1	30.7

The exclusion of unknowns in either α_1 or r_0 leads to unacceptable results for the personal equations as well as substantially increasing the sum of squares of the residuals. The application of the corrections for $pe3$ and $pe4$ in the main solution to the adopted values in Table 1 leads to values of $-0^s.16$ and $+0^s.06$ for the disappearances and reappearances by the eye-and-ear method. It is very unlikely that the personal equation for reappearances could be less than that for disappearances, so this solution is rejected too. Some constraint must be applied to the unknowns in order to reach an acceptable solution. If it is accepted that no personal equation should be applied to disappearances timed by the eye-and-ear method, then solution 6 of Table 7 would apply. This solution also indicates that no personal equation should be applied to the reappearances using the eye-and-ear method ($pe4$), since the adopted value in the reduction was $-0^s.2$. This is regarded as being the most satisfactory solution and it will be adopted as the definitive one for the remainder of the analysis.

The effects of personal equation can be eliminated by using only photoelectric observations. The solution for α_1 , r_0 and r_2cR_2 is given in Table 8. Unfortunately, there are only about 1400 observations, and, although the timing error is negligible, the contribution from other sources of error leads to quite large standard errors for the values of the unknowns. The differences between the solutions using the photoelectric data and all the data indicate that there may still be appreciable systematic errors in visual timings which give spurious results for α_1 and the other correlated unknowns, but the evidence is not conclusive. More photoelectric observations need to be analysed in order to resolve this problem. Meanwhile

Table 8. Solution for α_1 and correlated unknowns using only photoelectric observations.

Unknown	Solution
α_1	$-0''.03 \pm 0''.22$
r_0	$+0''.05 \pm 0''.03$
$r_2 c R_2$	$+0''.03 \pm 0''.02$

we will adopt the results in solution 6 of Table 7, except for the result for α_1 which we will regard as arising from a spurious and unexplained bias in the visual observations.

5.5 SOLUTION FOR THE SOLAR ELEMENTS e' AND ϖ'

The corrections to Newcomb's values of the solar elements e' and ϖ' , which were adopted by Brown after the construction of his *Tables* and subsequently carried through to the *ILE* and $j = 2$, are

$$\delta e'(\sin 1'')^{-1} = -0''.17 \pm 0''.02$$

i.e.

$$\delta e' = -(82 \pm 10) \times 10^{-8},$$

and

$$\delta \varpi' = -10''.2 \pm 1''.3,$$

or

$$e' \delta \varpi' = -0''.17 \pm 0''.02.$$

These results are in good agreement with those found by Adams & Scott (1968) from an analysis of transit-circle observations of the Sun in the period 1956–62:

$$\begin{aligned} \delta e'(\sin 1'')^{-1} &= -0''.15 \pm 0''.01 \\ e' \delta \varpi' &= -0''.19 \pm 0''.01. \end{aligned} \quad (\text{transit circle})$$

The unknowns for the solar elements are correlated with the systematic corrections to the right ascension system of the *FK4*. The effect of the interaction on the solution can be judged from Table 9. The distortion in the results for the solar elements is not too severe; however, the real uncertainty of the results would appear to be about three times the standard error.

Table 9. Solutions for solar elements and correlated unknowns.

Unknown	Solution				
	Main	1	2	3	4
	(")	(")	(")	(")	(")
e'	-0.17 ± 0.2	—	-0.17	-0.23	-0.24
ϖ'	-10.2 ± 1.3	-9.9	—	-7.1	-14.3
$\sin \alpha$	-0.03 ± 0.004	-0.04	-0.02	—	-0.03
$\cos \alpha$	$+0.04 \pm 0.005$	$+0.06$	$+0.05$	$+0.04$	—
Increase in sum of squares		16.1	13.3	10.5	16.4

5.6 CORRECTION TO THE COEFFICIENT OF $\sin \Omega$ IN LONGITUDE

No significant correction is found in this analysis to the coefficient of the term in $\sin \Omega$ in the ephemeris $j = 2$. As noted in Section 4.4, Van Flandern (1976) has predicted a correction of $-0''.02 \sin \Omega$; however, it may be that this long-period correction has been absorbed in the corrections to the preliminary dynamical time-scale deduced and applied in Section 4.3. It might have been wiser to have used only data after 1955.5 when the dynamical time-scale could be directly related to the atomic time-scale.

5.7 CORRECTIONS TO PLANETARY TERMS

The corrections in Table 6 to four of the terms in the lunar theory with planetary argument supports the conclusions reached in Section 4.4 following the periodogram analysis of the lunation solutions. Only in the case of term *ILE* No. 1288 does the solution for the cosin part strongly indicate that either the phase of the argument requires revision or the identification of the correction with this term is wrong. The application of the correction to the terms in the *ILE* is set out in Table 10. These terms are amongst the largest planetary terms and the inference is that there must be serious problems associated with the planetary part of Brown's lunar theory. However, the periodogram analyses shown in Figs 7 and 8 indicate that there are no other errors in the coefficients of the planetary terms greater than $0''.04$.

Table 10. Corrections to planetary terms.

<i>ILE</i> ref. no.	Argument angle	<i>ILE</i> coefficient (")	Revised coefficient (")
790	$199^\circ.0 + 5T - 3V$	+0.092	$+0.060 \pm 0.004$
1235	$L_0 + 215^\circ.6 + 5T - 3V$	+0.077	-0.015 ± 0.005
1240	$L_0 + 51^\circ.6 - 5T + 3V$	+0.074	-0.024 ± 0.005
1287	$L_0 + 168^\circ + 2J$	+0.035	-0.002 ± 0.005
1288	$L_0 + 24^\circ - 2J$	+0.018	-0.051 ± 0.005

(Revised angle 105°)

5.8 CORRECTIONS TO THE REFERENCE DATUM IN WATTS' CHARTS

Watts intended that the reference datum in his charts of the marginal zone should be a sphere whose centre is coincident with the centre of mass. Thus the outline of the Moon against the stellar background for any particular libration could in principle be found by taking the circle defined by the position and semi-diameter in the ephemeris and adding the profile from the charts. But this assumes that the ephemeris position refers to the centre of mass.

The ephemeris position in longitude refers to the centre of *figure* defined by the occultation observations which were reduced without the application of profile corrections as used by Spencer Jones in the derivation of his expression for the mean longitude. This expression was subsequently introduced into the *ILE* and later $j = 2$. So, this centre of figure will not necessarily coincide with the centres of the best-fitting ellipses or circles drawn through the maria or highlands in the marginal zone; it will lie somewhere between the two if they are indeed separate from one another. Sjogren & Wollenhaupt (1976) have obtained results using laser altimetry data from the *Apollo 15*, *16* and *17* missions which show that the centre of figure is about 1 km ($\sim 0''.5$) east of the centre of mass on the geocentric sphere, irrespective of whether the reference points are taken on the maria or highland

However, Bills & Ferrari (1977), by combining laser altimetry data with orbital and Earth-based photogrammetry obtain a result which is about half that of Sjogren & Wollenhaupt. From the disparity of these results we conclude that, whilst there is good agreement that the centre of figure is 2 km closer to the Earth than the centre of mass, there is still uncertainty about the size of the displacement along the direction of the lunar orbit. For the present we will adopt the result of Sjogren & Wollenhaupt, and it will be assumed that the longitude in the ephemeris refers to the centre of figure which leads the centre of mass in orbital longitude by $\sim 0''.5$.

The ephemeris position in latitude, on the other hand, refers to the centre of mass and not the centre of figure, because the orbital plane must contain the centre of mass. It has been known for many years that the centre of figure is approximately 1 km south of the centre of mass and Watts sought to compensate for this displacement in the construction of his charts by moving the centre of his reference circle (which was originally located close to the centre of figure) $0''.3$ along position angle 333° , in an attempt to bring it nearer to the ephemeris centre which is at the same latitude as the centre of mass.

In Fig. 9 M is the centre of mass; C is the ephemeris centre at the same latitude, but $\sim 0''.5$ east of M on the geocentric sphere; and F is the centre of figure defined by the occultation observations which is assumed to be close to C in longitude for the reason given above, but $\sim 0''.5$ south in latitude. In the application of Watts' limb profile, the centre of his reference circle is superimposed on the ephemeris centre at C; so the centre of the mean profile in Watts' charts is at some point W which is closer to F than C. The position angles are measured positively through east from the projection of the Moon's principal axes of inertia which strictly passes through M and not C, but the displacement between them is negligible in computing the angles.

The analysis gives the difference δr_1 between W and F along the position angle R_1 or, equivalently, the shift required to Watts' reference circle along the position angle $R_1 + 180^\circ$

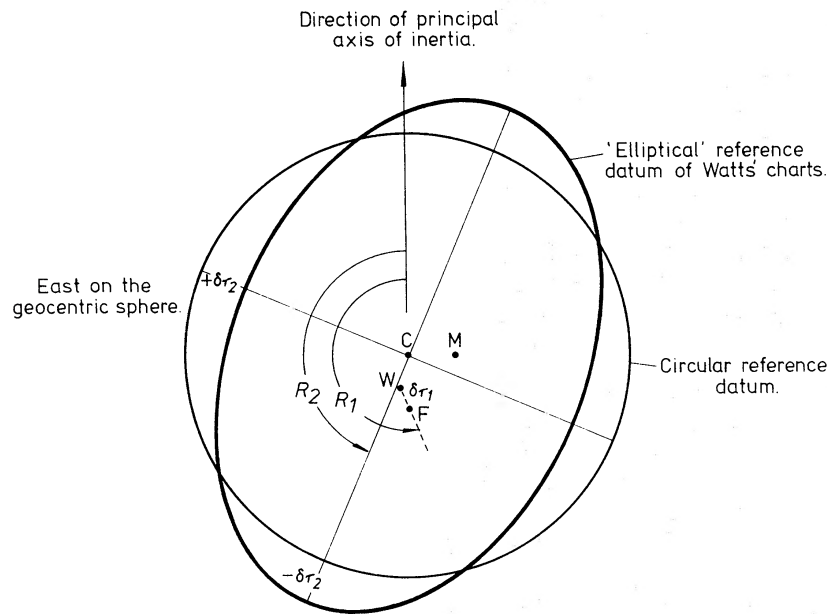


Figure 9. Diagram showing the corrections δr_1 and δr_2 (defined in the text) to the reference datum in Watts' charts. The centre of Watts' 'elliptical' reference datum is placed at the ephemeris centre C. W is the centre of the mean outline of Watts' limb-profile and F is the centre of figure defined by occultation observations. M is the centre of mass.

so that when it is superimposed on the ephemeris centre the profile will agree better with the observations.

It has been assumed that Watts' reference datum is indeed circular; however, Watts suspected that it might be elliptical. He determined and applied the following corrective term to the limb corrections to try to remove the ellipticity:

$$\delta r_2 = + (0''.15 \pm 0''.08) \cos 2(Q - 153^\circ \pm 15^\circ).$$

In this analysis we also allow for a correction of this form in order to test his result. Fig. 9 shows the ellipse with its semi-major axis in position angle R_2 and the correction δr_2 required to reduce it to a circle. The unknown δr_0 is the correction to the adopted semi-diameter of $932''.58$ for the radius of Watts' reference datum at mean distance (corresponding to a value of $3422''.608$ for the equatorial horizontal parallax).

The main solution in Table 6 gives:

$$\delta r_0 = + 0''.152 \pm 0''.006$$

$$\delta r_1 = + 0''.19 \pm 0''.01, \quad R_1 = 184^\circ \pm 3^\circ$$

$$\delta r_2 = - 0''.09 \pm 0''.01, \quad R_2 = 146^\circ \pm 1^\circ.$$

The difficulties in the interpretation of these results arising from the correlations between the unknowns for the semi-diameter, the constant of parallactic inequality and the persona equations have been discussed already in Section 5.4. It was decided there to adopt solution 6 of Table 7 where the correction to the semi-diameter is

$$\delta r_0 = + 0''.14 \pm 0''.01 \quad (\text{preferred solution}).$$

The result for δr_1 indicates that the centre of the mean limb-profile in Watts' charts lie $0''.19$ above the centre of figure defined by occultation observations; thus Watts should have moved the centre of his reference datum further in the direction of position angle $\sim 0^\circ$ than he did. The uncertainty of the position angle is greater than the standard error of $\pm 3^\circ$ would lead one to believe because of the correlations between the displacement of the reference centre at right angles to the axis and the corrections to the mean longitude and the persona equations. The occultation data alone are insufficient to determine the position of the centre of Watts' reference datum in longitude with respect to the mean physical outline. The most that can be concluded from this analysis is that in longitude the centre of Watts' reference datum is compatible with the position of the centre of the Moon in the ephemeris $j = 2$ amended by the expression in mean longitude given in Section 4.2. Where this centre is in relation to the centre of mass cannot be determined from these data and in this respect I am sceptical of the significance of the displacement in longitude obtained by Mulholland (1977) from a combination of occultation and laser ranging data. However, this displacement could be determined using altimeter measurements from lunar orbiters to features in the margin zone which are identifiable on Watts' charts.

From the results of $-0''.09$ for δr_2 and 146° for R_2 it would appear that Watts overestimated the correction required to remove the ellipticity from his reference datum: he should have applied $+0''.06 \cos 2(Q - 153^\circ)$.

The statements above regarding the location of the ephemeris centre of the Moon can be summarized as follow.

(a) The ephemeris $j = 2$, amended in mean longitude (Section 4.2) and entered for the time-argument TAI + 32^s.184, refers to a point (C in Fig. 9) which is at the same latitude as the centre of mass, but $\sim 0''.5$ east of it on the geocentric sphere. In order, therefore, to refer

the mean longitude to the centre of mass, $-0''.5$ should be added to the expression found in Section 4.2; thus

$$\delta L = -2''.04 + 2''.33 T - 1''.78 T^2.$$

(b) Superposition of Watts' profile data on a circle centred at C of semi-diameter $932''.72$ at mean distance causes the centre of figure W in Watts' charts to be situated $0''.19$ above the centre of figure defined by occultation observations.

(c) Application of the correction $+0''.19 \cos(Q - 184^\circ)$ to Watts' profile heights effectively lowers the centre of figure W to the point F, which is the centre of figure defined by occultation observations.

(d) Application of the correction $-0''.09 \cos 2(Q - 146^\circ)$ removes the spurious ellipticity from Watts' reference datum.

The origin of longitude in the ephemeris $j = 2$, amended by the expression in mean longitude given in Section 4.2, is the zero point of right ascension in the *FK4*; so by following the procedures in (a)–(d) above the centre of mass of the Moon is located in longitude with respect to the *FK4* frame of reference. In order to refer the origin of longitude to the dynamical equinox, the correction to the *FK4* equinox given in Section 5.2 should be applied; thus, finally

$$\delta L = -1''.94 + 3''.43 T - 1''.78 T^2.$$

5.9 VALUES OF PERSONAL EQUATIONS

The results for the personal equations have already been discussed at some length in Section 5.4 in connection with the apparently anomalous solution for the constant in parallactic inequality (α_1). It was decided there to adopt the corrections in solution 6 of Table 7; thus the adopted value of $-0^s.3$ for the average delay in timing disappearances using stop-watches etc. should be increased to $-0^s.4$. This increased value of the personal equation for disappearances is in broad agreement with the results of an investigation by Sinzi & Suzuki (1967) who compared visual and photoelectric timings of the same occultations.

The adopted value of $-0^s.5$ for reappearances is unchanged, but the analysis indicates that the reaction time for *both* disappearances and reappearances using the eye-and-ear method is less than $0^s.1$. The corrected values of the average personal equations are given in Table 11.

Table 11. Corrected values of average personal equations.

Phenomenon Method	D (s)	R (s)
Stop-watch etc.	-0.4	-0.5
Eye-and-ear	0.0	0.0

6 Conclusions

The principal results of this analysis are summarized in this section.

6.1 ZERO POINT OF RIGHT ASCENSION IN *FK4*

The correction to be applied to the right ascension of the stars in the *FK4* in order that the origin of right ascension should be the position of the dynamical equinox is

$$\delta \alpha'_0 = +0''.82 \pm 0''.02 + (1''.20 \pm 0''.11)(T - 0.59),$$

where T is measured in centuries from 1900.0. Fricke's (1967) value for the secular motion has been adopted.

6.2 DYNAMICAL TIME-SCALE

The dynamical time-scale (TD) used in this analysis is related to the atomic time-scale (TAI) after 1955 July 1.0 by the relation

$$TD = TAI + 32^s.184.$$

In the period 1943.0–1955.5 the dynamical time-scale may be approximated by

$$TD = UT + 31^s.24 + (0^s.331 \pm 0^s.011) (Y - 1955.5) - (0^s.009 \pm 0^s.003) (Y - 1955.5)^2,$$

where Y is the calendar date expressed in years.

6.3 CORRECTIONS TO THE LUNAR EPHEMERIS $j = 2$

The correction to the expression for the mean longitude in the ephemeris $j = 2$ is found to be

$$\delta L = -1''.94 \pm 0''.02 + (3''.43 \pm 0''.10) T - (1''.78 \pm 1'') T^2,$$

where T is measured in Julian centuries from the epoch 1900 January 0.5 TD. Application of this correction has the following consequences:

- (a) the time-argument of the ephemeris is $TAI + 32^s.184$,
- (b) the origin of longitude is the dynamical equinox,
- (c) the point in the Moon to which the ephemeris refers is the centre of mass,
- (d) the contribution due to tidal friction is $-13'' T^2$.

If it is required to refer the position to the *FK4* equinox, the correction in longitude becomes

$$\delta L = -2''.04 \pm 0''.01 + (2''.33 \pm 0''.06) T - (1''.78 \pm 1'') T^2,$$

and a correction has to be introduced into latitude of the amount

$$\delta\beta = [+ (0''.07 \pm 0''.01) + (0''.44 \pm 0''.4) T] \cos L.$$

In referring the position in longitude to the centre of mass I have adopted the rather arbitrary value of $-0''.50$ for the difference in longitude, centre of mass *minus* centre of figure (see Section 5.8).

The corrections to the fundamental arguments ϖ , Ω , L' and ϖ' for the mean epoch of the solution, 1959.0, are:

Referred to the dynamical equinox of the solution	Referred to the <i>FK4</i> equinox
$\delta\varpi = -0''.86 \pm 0''.03$	$-1''.61 \pm 0''.05 - (1''.10 \pm 0''.10) (T - 0.59)$
$\delta\Omega = +2''.02 \pm 0''.06$	$+1''.27 \pm 0''.06 - (1''.10 \pm 0''.10) (T - 0.59)$
$\delta L' = +0''.68 \pm 0''.08$	$-0''.07 \pm 0''.08 - (1''.10 \pm 0''.10) (T - 0.59)$
$\delta\varpi' = -10''.2 \pm 1''.3$	$-10''.9 \pm 1''.3 - (1''.10 \pm 0''.10) (T - 0.59).$

Combining these corrections with the correction to δL , we find the consequential changes to the arguments l, F, D and l' as given in the *ILE* and listed in section 4C of the *ES* to be:

$$\delta l = -1''.08 \pm 0''.03 + (3''.43 \pm 0''.12) T - (1''.78 \pm 1'') T^2$$

$$\delta F = -3''.96 \pm 0''.06 + (3''.43 \pm 0''.12) T - (1''.78 \pm 1'') T^2$$

$$\delta D = -2''.62 \pm 0''.08 + (3''.48 \pm 0''.12) T - (1''.78 \pm 1'') T^2$$

$$\delta l' = +10''.9 \pm 1''.3.$$

The revised arguments of the lunar ephemeris $j = 2$ are therefore:

$$L = 270^\circ 26' 01''.05 + 1336^\circ 307' 53''.02 T - 5''.86 T^2 + 0''.0068 T^3$$

$$l = 296^\circ 06' 15''.51 + 1325^\circ 198' 51''.00 T + 31''.31 T^2 + 0''.0518 T^3$$

$$F = 11^\circ 14' 59''.24 + 1342^\circ 82' 01''.33 T - 13''.34 T^2 - 0''.0012 T^3$$

$$D = 350^\circ 44' 12''.33 + 1236^\circ 307' 06''.54 T - 6''.95 T^2 + 0''.0068 T^3$$

$$l' = 358^\circ 28' 43''.27 + 99^\circ 359' 02''.59 T - 0''.54 T^2 - 0''.012 T^3,$$

where L is measured from the dynamical equinox and all the arguments refer to the centre of mass. In deriving these expressions, no corrections have been applied to Brown's values for the secular motions of the fundamental arguments ϖ, Ω, L' and ϖ' , and the corrections to the constant parts derived for the mean epoch of the observations (1959.0) have been taken to apply at 1900.0. However, this analysis indicates that the secular motions of the perigee and node, in particular, require correction. Until a longer time-span of data has been analysed, it will not be possible to derive reliable corrections for these. So the arguments l and F given above are likely to be in error by a few seconds of arc when evaluated a century from the mean epoch of this solution.

The corrections to the constants e, γ and e' are

$$\delta e = + (28 \pm 1) 10^{-8} = (0''.057 \pm 0''.002) (\sin 1'')$$

$$\delta \gamma = - (26 \pm 1) 10^{-8} = (0''.053 \pm 0''.003) (\sin 1'')$$

$$\delta e' = - (82 \pm 10) 10^{-8} = (0''.17 \pm 0''.02) (\sin 1'').$$

The revised values of these constants in the ephemeris $j = 2$ are therefore:

$$e = 0.05490\,077$$

$$\gamma = 0.04488\,671$$

$$e' = 0.01675\,022.$$

The following constants are unchanged by this analysis:

$$\alpha_1 = 0.00250\,935$$

$$\pi' = 8''.794$$

$$\pi = 3422''.608$$

$$E/M = 81.30$$

$$f = 1/298.25.$$

The corrections to the fundamental arguments and constants listed above contribute the following corrections greater than $0''.01$ to the lunar ephemeris $j = 2$:

$$\begin{aligned}\delta\lambda = & \delta L + \delta l [+0.110 \cos l - 0.022 \cos (l - 2D) + 0.007 \cos 2l] \\ & + \delta l' [-0.003 \cos l' - 0.001 \cos (l' + l - 2D)] \\ & + \delta D [+0.044 \cos (l - 2D) + 0.023 \cos 2D] \\ & + \delta e [+2.00 \sin l - 0.40 \sin (l - 2D)] \\ & + \delta e' [-0.19 \sin l' - 0.06 \sin (l' + l - 2D)] \\ \delta\beta = & \delta F (+0.090 \cos F) + \delta\gamma (+2.00 \sin F).\end{aligned}$$

6.4 LUNAR FIGURE IN WATTS' CHARTS

The correction to the limb-profile heights in Watts' charts, which follows from the requirement that the reference datum should be circular and coincident with the centre of mass, is found to be

$$\begin{aligned}\delta\rho_0 = & +0''.14 \pm 0''.01 + 0''.50 \cos (Q - 90^\circ) + (0''.19 \pm 0''.01) \cos (Q - 184^\circ \pm 3^\circ) \\ & - (0''.09 \pm 0''.01) \cos 2(Q - 146^\circ \pm 1^\circ),\end{aligned}$$

where Q is the position angle measured through east on the geocentric sphere from the axis of the Moon.

The first term is the correction to the provisional value of $932''.58$ for the radius of Watts' reference datum at a mean distance corresponding to an equatorial horizontal parallax of $3422''.608$. The second and third terms arise from the fact that the centre of Watts' reference datum is $0''.50$ east and $0''.19$ south of the centre of mass on the geocentric sphere. The value of the displacement in longitude is rather arbitrary (see Section 5.8), but that in latitude is very reliably determined. If Watts' limb-profile corrections are to be used in conjunction with the uncorrected ephemeris $j = 2$, where the longitude (but *not* the latitude) refers to the centre of figure, the second term should not be applied. The fourth term removes the effect of the spurious ellipticity in Watts' reference datum.

6.5 CONCLUDING REMARKS

The computed residuals, reduced by the main solution in Table 6, were re-solved by lunation for corrections $\delta\lambda$ and $\delta\beta$, as described in Section 4.4. The amplitudes of the periodic fluctuations in Figs 5 and 6 were greatly reduced, but some quasi-periodic fluctuations, amounting at times to a semi-amplitude of $0''.2$, still remained. The solutions for $\delta\lambda$ and $\delta\beta$ were analysed again by periodogram analysis and the resulting periodograms are shown by dotted lines in Figs 7 and 8. There are no peaks remaining in the periodograms greater than $0''.04$.

The main contributory cause of the fluctuations still remaining in the lunation solutions is probably the truncation of the planetary terms in Brown's theory. Garthwaite, Holdridge & Mulholland (1970) compared $j = 2$ with an ephemeris generated by numerical integration and identified several features in their spectral analysis of the differences which corresponded to known planetary perturbations. The present analysis identifies at least four planetary terms which require substantial corrections – two of them being about $0''.09$. The periodograms show that there are no other planetary terms with semi-amplitudes greater than $0''.04$, besides those identified in this paper. However, in aggregate, several such corrections will produce errors of $\sim 0''.2$ at times.

Further progress in the development of a more accurate analytical lunar ephemeris would appear to depend on two points. The first is improved values for the secular changes in the fundamental arguments and the second is a new solution for the planetary perturbations: both are currently under way.

Acknowledgments

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