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VIKING RELATIVITY EXPERIMENT: VERIFICATION OF SIGNAL RETARDATION BY SOLAR GRAVITY

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ABSTRACT

Analysis of 14 months of data obtained from radio ranging to the *Viking* spacecraft verified, to an estimated accuracy of 0.1%, the prediction of the general theory of relativity that the round-trip times of light signals traveling between the Earth and Mars are increased by the direct effect of solar gravity. The corresponding value for the metric parameter γ is 1.000 ± 0.002, where the quoted uncertainty, twice the formal standard deviation, allows for possible systematic errors.
Subject headings: gravitation — relativity — solar system: general

I. INTRODUCTION

General relativity predicts that the round-trip, or echo, delays of light signals traveling between the Earth and another planet will have an "extra" contribution, $\Delta\tau$, due to the direct effect of solar gravity on spacetime. This extra delay, for the generalized Schwarzschild metric (see, e.g., Will 1973), is given by

$$\Delta\tau = \frac{2r_0}{c}\frac{1+\gamma}{2}\ln\frac{r_e + r_p + R}{r_e + r_p - R}\,, \qquad (1)$$

where $r_0 = 2GM_\odot/c^2 \approx 3$ km is the gravitational radius of the Sun; $M_\odot$, the mass of the Sun; c is the speed of light; r_e and r_p are the distances from the Sun to the Earth and the target planet, respectively; and R is the distance between the Earth and the target planet. The metric parameter γ is identically unity in the general theory of relativity. For signals that just graze the Sun, $\Delta\tau$ is about 250 μs for $\gamma = 1$.

Prior measurements of this relativistic delay yielded values for γ that differed from unity by less than the estimated standard errors; the smallest uncertainty, 2%, stemmed from analysis of the echo delays obtained from ranging to the *Mariner 9* spacecraft. (See Shapiro [1979] for a review.)

The four *Viking* spacecraft arrived at Mars in the summer of 1976 and passed through superior conjunction on November 25 of that year. Ranging to the *Viking* spacecraft, the concern of this *Letter*, offered the possibility of substantial improvement over the *Mariner 9* relativity experiment for two reasons: (i) the accuracy of measurement of the echo delays was somewhat higher; and, more importantly, (ii) the accuracy of interpretation of these delays was substantially higher. The latter reason, in turn, stems from two factors. First, the two *Viking* landers were implanted on the surface of Mars so that their trajectories could be determined accurately. Second, the two *Viking* orbiters transmitted S-band ($\sim$2.3 GHz) and X-band ($\sim$8.4 GHz) signals that were coherent with the single S-band signal received by the spacecraft. This dual-band, one-way ranging allowed estimation of the contribution of the solar-corona plasma to the echo delays obtained from ranging to the landers, which could be carried out only at the S-band frequency.

In the remainder of this *Letter* we describe the *Viking* observations that we used (§ II), our data-analysis technique (§ III), and the results we obtained (§ IV) from this relativity experiment.

II. OBSERVATIONS

The observations we used consisted of the measurements of the (ambiguous) echo delays of the signals transponded by the *Viking* spacecraft. The techniques of observation and of ambiguity removal were discussed by Shapiro *et al.* (1977). The dual-band echo delays from ranging to the orbiters were, for the most part, obtained either simultaneously, or nearly simultaneously, with the measurements of echo delays from ranging to the landers. Simple temporal interpolation schemes, coupled with a thin-screen model, were used to determine plasma corrections from the dual-band echo delays (Eubanks *et al.* 1980). Corrections for the delays of the signals through the transponder and the ground equipment were determined, respectively, from preflight calibration and from direct measurement at the time of observation. All of the corrections were applied to the measurements of echo delays from ranging to the landers; the uncertainties in these corrections dominated the standard errors assigned to the corresponding corrected echo delays, especially for the measurements made near superior conjunction.

III. ANALYSIS

The echo delays obtained during the 14 months after the first *Viking* landing on Mars were analyzed with the Planetary Ephemeris Program (Ash 1972), which contains a parametric model of the solar system. The orbits of the Earth and Mars, based on the generalized Schwarzschild metric, were each numerically integrated with the perturbing orbits of the Moon and planets having been based on prior laser, radar, and classical optical observations of these bodies (Ash, Shapiro, and Smith 1971; King, Counselman, and Shapiro 1976). The model for the Earth's rotation was based on standard formulae and constants. The model used for the rotation of Mars was similar to that developed by Reasenberg and King (1979).

The coordinates of the radio tracking stations on the Earth were based on the analysis of data from an ensemble of past spacecraft missions (Ellis 1978). Other relevant parameters of our dynamical model of the solar system, such as the masses of the planets, were obtained from prior analyses (see, e.g., Ash *et al.* 1971).

This model of the solar system was used to calculate the theoretical value of each corrected echo delay; the partial derivatives of these values with respect to each relevant parameter were evaluated either solely from an analytical formulation or in part from the numerical integration of the variational equations (Ash 1972).

The following 25 parameters were estimated via a standard linearized weighted-least-squares algorithm: the six initial conditions each for the orbits of the Earth and Mars, the three scalars defining the period of rotation and the direction (at epoch) in inertial space of the axis of rotation of Mars, the three aerocentric coordinates of each of the *Viking* landers, the rotation phase of the Earth at a given epoch (included to account for a possible rotation difference between our coordinate system and that in which the locations of the tracking stations on the Earth were determined), the Earth-Moon mass ratio, the light-second equivalent of the astronomical unit, and the metric parameter γ. The data set for this solution contained all of the echo delays from ranging to the landers for the stated 14 month period, save for $\sim$10% that were deleted because of their very large, up to 20 μs, residuals; most of these deleted delays were obtained near superior conjunction and those obtained on different days were markedly inconsistent with each other as well as with the rest of the delay data. No definitive explanation has been obtained for these anomalies (see Shapiro *et al.* [1977] for further discussion).

To test for possible systematic effects on the parameter estimates, we repeated the solution with the data and parameter sets altered. For example, the data from successively larger intervals, up to 100 days, centered on superior conjunction, were unweighted and γ was set to unity. The postfit residuals of these unweighted data showed no systematic trends about superior conjunction. In other tests, the full data set was used and the parameter set altered, both by increasing and by decreasing the number of members. The set was increased primarily by the addition of the masses of planets, for example, Jupiter, Saturn, and Uranus. The estimates of these masses were the more poorly determined the farther the planet was from the orbits of the Earth and Mars. Although some of these estimates differed by many standard deviations from the nominal values (Ash *et al.* 1971) of these masses, the estimate of γ differed from unity by at most 2 standard deviations. The parameter set was decreased primarily by the maintenance of various subsets of the orbital elements of the Earth and Mars at their *a priori* values. These solutions also yielded values for γ that were consistent with the one obtained from the 25 parameter solution. In all cases, the formal standard deviations of the parameter estimates were obtained by uniform scaling of the measurement standard deviations such that the mean weighted square of the postfit residuals was unity.

Figure 1 displays postfit residuals from the 25 parameter solution. The root-mean-square (rms) of these residuals is about 50 ns, equivalent to an rms in one-way range of about 7.5 m. The residuals show some signs of short-period systematic trends that probably indicate remaining imperfections in our plasma calibrations. Nonetheless, the histogram of the residuals approximates a Gaussian curve rather closely.

IV. RESULTS

The result we obtained for γ from the basic solution is

$$\gamma = 1.000 \pm 0.002\,. \tag{2}$$

The uncertainty given, about twice the formal standard deviation, is based on the spread obtained in the estimates of γ from the many test solutions and on a (subjective) judgment of the reliability of the myriad procedures used in the collection and analysis of the data.

The stability, or consistency, of the estimates of γ obtained from the different solutions is due partly to the rather weak correlations between each estimate of γ and those of the other parameters. The cumulative effect of these correlations can be quantified in terms of a masking factor, $\mu(\gamma) \equiv \sigma(\gamma)/\sigma^*(\gamma) \geq 1$, where σ and σ^* are, respectively, the standard deviations of the estimates of γ when all parameters are estimated simultaneously and when only γ is estimated. For our basic solution, which involves 25 parameters, μ takes on the rather modest value of 13. By contrast, for the estimate (Reasenberg and Shapiro 1976) of the time rate of change of the gravitational constant, μ exceeds 80.

This estimate of γ from the *Viking* experiment (see also Cain *et al.* 1978) appears to be an order of magnitude more accurate than any value obtained without *Viking* data (Shapiro 1979), and is in excellent accord with the predictions of general relativity. Although our result for γ does not rule out any alternate theories of gravitation that agree with the general theory of

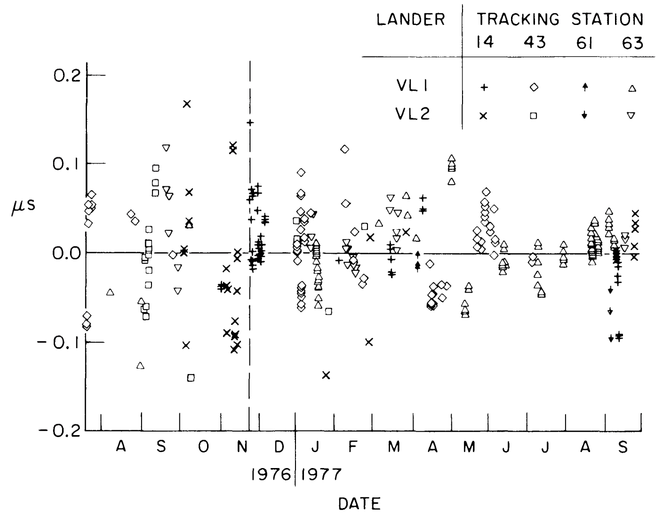

FIG. 1.—Postfit residuals for measurements of echo delays from ranging to the *Viking* landers (see text). The standard deviations of the measurements were omitted to avoid cluttering; their values ranged from 50 to 100 ns. The dashed vertical line denotes the epoch of superior conjunction, 1976 November 25. VL1 and 2 denote, respectively, *Viking* landers 1 and 2, and 14, 43, 61, and 63 denote, respectively, Deep Space Network tracking stations in Goldstone, California (64 m diameter antenna), Canberra, Australia (64 m), and Madrid, Spain (26 m and 64 m).

relativity in the post-Newtonian limit, it does serve to severely restrict the coupling constant, ω, of the Brans-Dicke (1961) theory. This theory reduces to general relativity in the limit as ω approaches infinity. Our result implies that $\omega \geq 500$.

The continued collection and analysis of *Viking* data appear likely to provide an improvement in the estimate of γ by a factor of at least 2. Although not yet analyzed, some apparently useful echo-delay data were obtained near the time of the recent superior conjunction of Mars in 1979 January. In addition, augmentation of the data set with the measurements of echo delays obtained from other parts of the orbits of the Earth and Mars, and with the measurements of the Doppler shifts of the echoes, will allow the masking factor to be reduced significantly. Finally, further development of the techniques for plasma calibration may allow a significant reduction to be made in the uncertainties accompanying these calibrations.

Beyond *Viking*, appreciable improvement in this test of general relativity will probably require another planetary lander with the capability for better plasma calibration through, for example, dual-band ***round-trip*** measurements of delay. This additional capability could allow at least another order-of-magnitude decrease in the uncertainty in the estimate of γ.

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