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STAR FORMATION AND GALACTIC EVOLUTION. I. GENERAL EXPRESSIONS AND APPLICATIONS TO OUR GALAXY

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ABSTRACT

This study of galactic evolution involves three mechanisms for triggering star formation in interstellar clouds: (i) star formation triggered by a galactic spiral density wave, (ii) star formation triggered by shock waves from supernovae, and (iii) star formation triggered by an expanding H II region. Useful analytic approximations to the birthrate per unit mass are obtained by treating the efficiencies of these various mechanisms as time independent. In situations where shock waves from high-mass stars (either expanding H II regions or supernova explosions) are the only important star-forming mechanisms, the birthrate is exponential in time. This case is appropriate for the past evolution of an elliptical galaxy, nuclear bulge, or galactic halo. In the disk of a spiral galaxy where all three mechanisms operate, the birthrate consists of an exponential term plus a time-independent term. In both situations, the value of the time constant T in the exponential term is directly related to the efficiency of the shock waves from massive stars in initiating star formation.

For our Galaxy, this simplified model is used to compute the radial distributions of young objects and low-mass stars in the disk, and the past and present birthrates in the solar-neighborhood shell.

Subject headings: galaxies: evolution — galaxies: stellar content — stars: formation — stars: stellar statistics — stars: supernovae

I. INTRODUCTION

The rate of star formation is one of the important factors governing the evolution of galaxies. In constructing models for galactic evolution, one should connect the functional form adopted for the stellar birthrate to one or more mechanisms for producing stars. Various star-formation mechanisms have been suggested in the literature, and it is probable that several different types of star-forming mechanisms operate within galaxies. In this paper, we investigate simplified galactic-evolution models which incorporate three star-forming mechanisms for which there is both observational and theoretical evidence. These are (i) star formation triggered by passage of a galactic spiral density wave through an interstellar cloud, (ii) star formation triggered by passage of a shock wave from a supernova through an interstellar cloud, and (iii) star formation triggered by passage of the shock front from an expanding H II region through an interstellar cloud. Young stars and H II regions are often found near the edges of dense molecular clouds (Woodward 1976). This suggests that some triggering mechanism is usually involved in star formation.

The above mechanisms for triggering star formation have received much attention in recent literature. Toomre (1977) summarizes the main observational evidence for spiral density shock waves in certain spiral galaxies, e.g., the presence of dust lanes coinciding with the peak radio continuum flux on the concave side of the visible spiral arms and the agreement between the optical and H I arms. Hydrodynamic calculations of Woodward (1976) for plane-parallel shocks indicate that spiral density shock waves can trigger nonuniform gravitational collapse of a suitable interstellar cloud. Bash, Green, and Peters (1977) interpret CO line profiles in terms of star clusters born in spiral density shock waves. Jensen, Strom, and Strom (1976) show that star formation by spiral density waves can account qualitatively for the metal-abundance gradients observed in various disk galaxies. On the other hand, a growing number of observations link expanding H II regions to the formation of protostars in neighboring molecular clouds (see Elmegreen and Lada 1977; Loren 1977). Elmegreen and Lada show that when the shock and ionization fronts of an expanding H II region propagate into a molecular cloud, a dense neutral layer forms between the two fronts. If the initial cloud density is sufficiently large, this neutral layer becomes gravitationally unstable within a few million years. Observations also connect the shock waves from supernovae with new star formation: Ögelman and Maran (1976) and Herbst and Assousa (1977, 1978) find very young stars near the edges of old, expanding supernova remnants. Furthermore, the simple explanation which Cameron and Truran (1977) give for the observed excess of ^{26}Mg in the Allende meteorite (Lee, Papanastassiou, and Wasserburg 1976) associates the origin of the Sun with the explosion of a nearby supernova. Chevalier and Theys (1975) show that if a supernova shock wave which is optically thin and radiating impinges on a local inhomogeneity in the interstellar medium, significant density enhancement can result. While supernova shock waves are not plane parallel, Woodward's results can probably be applied, in a qualitative sense, to supernova shocks.

My object is to show how the general character of galactic evolution is governed by these postulated star-forming mechanisms. We shall make simplifying assumptions that enable us to obtain analytic solutions. Quantities associated with star formation triggered by the spiral density shock waves, supernova shock waves, and expanding H II regions will be denoted by the subscripts "sp," "SN," and "H II," respectively. We often lump together star formation induced by expanding H II regions and star formation induced by shock waves from supernovae and call the process "star formation by high-mass stars," since, aside from uncertainties about the progenitors of Type I supernovae, both mechanisms depend on the existing number of high-mass stars. Quantities associated with star formation by high-mass stars will be denoted by the subscript "HM."

After setting forth basic terminology in § II, we consider in § III the simple case where only star formation by high-mass stars operates. Under simple assumptions, we show that the birthrate, as a function of time, is a decaying exponential. Now, decaying exponential birthrates are popular in studies of galactic evolution and are assumed in the models of Truran and Cameron (1971), Searle, Sargent, and Bagnuolo (1973), Ostriker and Thuan (1975), Tinsley (1976), etc. Also, in a closed system, taking a birthrate proportional to the mass of interstellar gas (as in Salpeter 1959) is approximately equivalent to adopting a decaying exponential birthrate. The derivation in § III provides a physical justification for using a decaying exponential birthrate where spiral structure is unimportant, and relates the time constant to the efficiency of the shockwaves in initiating star formation.

In § IV we obtain an expression for the stellar birthrate in a galactic disk where star formation by the spiral density wave and by high-mass stars both operate. The resulting birthrate consists of an exponential term plus a time-independent term. In Kaufman (1979, hereafter Paper II), we use the observed distribution of high-excitation H II regions to estimate the relative importance of the two star-formation mechanisms in our Galaxy and solve for the value of the time constant in the exponential term. In § V we solve for the values of the other birthrate parameters appropriate to our Galaxy and compute (i) the radial distribution of young objects in the disk, (ii) the birthrate and the stellar age distribution in the solar neighborhood, and (iii) the radial distribution of low-mass stars and the metal-abundance gradient in the disk. These predictions are compared with observations. We summarize our results in § VI.

II. BASIC TERMINOLOGY

Divide the portion of the galaxy to be modeled into a series of coaxial, similar shells, concentric with the center of the galaxy. Each shell has mass dM_r, where r is a measure of the distance to the galactic center. For the disk of a spiral galaxy, each shell is a cylindrical shell with radius (dr, r) and height equal to the thickness of the disk. For an elliptical galaxy, nuclear bulge, or galactic halo, each shell is a spheroidal shell which has major axis (dr, r) and excludes the disk (in a spiral galaxy). We use the notation (dr, r) to mean within an interval dr centered on r.

Now focus on a particular shell with (dr, r). Let $B(m, t, r)dmdtdM_r$ equal the number of stars of mass (dm, m) born at model age (dt, t) in this shell. We assume that the birthrate $B(m, t, r)$ is a separable function of stellar mass and time. Thus

$$B(m, t, r) = B(t, r)\psi(m)\,, \tag{1}$$

where $\psi(m)$, the initial mass function (IMF), is normalized so that

$$\int_{m_L}^{m_U} \psi(m)dm = 1\,, \tag{2}$$

and $\psi(m)$ is zero outside this mass range. To save on notation we write the same $\psi(m)$ for all three star-forming mechanisms. Now, observations indicate that star formation by high-mass stars and star formation by the spiral shock wave can both produce massive stars. In Paper II we use the specific assumption that the upper IMF is the same in all cases. However, unless otherwise noted, our arguments are independent of whether the lower IMF is the same for all mechanisms. For numerical estimates, we adopt the usual approximation $\psi(m) \propto m^{-(1+x)}$ and take $x = 1.3$ for $m = m_L$ to $0.4\,m_\odot$, $x = 0.25$ for $m/m_\odot = 0.4$–1, and $x = 1.35$ for $m/m_\odot = 1$–3 from Tinsley and Ostriker (1977), and $x = 1.9$ for $m = 3\,m_\odot$ to $m_U = 60\,m_\odot$ from Kaufman (1975*a*). For m_L, we use either $0.1\,m_\odot$ or $0.2\,m_\odot$. We let $S(t, r)dM_r$ represent the total number of stars born in the shell up to time t.

It is useful to divide the evolution of spiral galaxies into two galactic eras: a short, transient, initial era (Collapse Phase) and a long subsequent era (Disk Phase). For the disk models, we take $t = 0$ when the spiral density wave first turns on. This is presumably near the beginning of the Disk Phase and has been preceded by star formation during the Collapse Phase. Similarly, the formulation in § III, where only star formation by high-mass stars operates, assumes some prior star-formation mechanism, e.g., collisions of gas clouds as the galaxy contracts (see Larson 1969) or contraction of massive clouds as the galaxy forms. Here the definition of $t = 0$ is less natural but may be taken as near the end of the Collapse Phase.

Finally, t_1 refers to the present time and $r_\odot$ to the solar-neighborhood shell. For the disk of our Galaxy, we take t_1 equal to 8×10^9 yr, unless otherwise noted, and $r_\odot$ equal to 10 kpc.

III. CONSEQUENCES OF STAR FORMATION BY HIGH-MASS STARS

Let us study the simple case where star formation by high-mass stars is the only mechanism for producing stars. This case could apply to a halo surrounding a disk galaxy, to an elliptical galaxy, or to those portions of a spiral galaxy interior to the inner Linblad resonance or exterior to the outer Linblad resonance.

Consider a particular galactic shell with (dr, r), and let $B_{\mathrm{HM}}(t, r)$ equal the stellar birthrate in this shell. Now $B_{\mathrm{HM}}(t, r)$ has two contributions:

$$B_{\mathrm{HM}}(t, r) = B_{\mathrm{SN}}(t, r) + B_{\mathrm{H\,II}}(t, r)\,, \tag{3}$$

where B_{SN} is the birthrate associated with star formation by shock waves from supernovae and $B_{\mathrm{H\,II}}$ is the birthrate for star formation by expanding H II regions. Since $B_{\mathrm{SN}}(t, r)$ is determined by the supernova rate in the shell at time $t - \tau_f$, where τ_f is the average time interval between supernova explosion and new star formation, we have

$$B_{\mathrm{SN}}(t, r) = A_{\mathrm{SN}} \int_{m_s}^{m_U'} B(t - \tau_m^* - \tau_f, r)\psi(m)dm\,. \tag{4}$$

Here A_{SN} is the average number of stars born for each supernova, τ_m^* is the stellar lifetime, the mass range is the stars that become the appropriate type of supernova, and the integral represents the supernova rate at time $t - \tau_f$. We choose $m_U' \geq 40\, m_\odot$ and m_s equal to either $5\, m_\odot$ or $9\, m_\odot$.

Observations by Herbst and Assousa (1977), Herbst, Racine, and Warner (1978), and Ögelman and Maran (1976) indicate values of $\tau_f \gtrsim 10^6$ yr. However, it is difficult to detect the supernova remnant if τ_f is appreciably larger than 10^6 yr. Because of this observational selection effect, it is not clear whether time scales as short as 10^6 yr are typical. An approximate upper limit to τ_f can be deduced as follows: As the interior pressure in an expanding supernova remnant (SNR) approaches interstellar values, the ability of the SNR to trigger star formation decreases. For spherically symmetric models, the SNR reaches pressure equilibrium with the interstellar medium $1\text{–}2 \times 10^6$ yr after the explosion (Smith 1977; Ikeuchi 1978). If the SNR encounters a cloud with initial hydrogen (proton) density $n > 10^2\ \mathrm{cm}^{-3}$, then the protostars formed require at most 2×10^6 yr to reach the main sequence, and hence, $\tau_f \gtrsim 4 \times 10^6$ yr.

We expect the birthrate $B_{\mathrm{H\,II}}$ to be proportional to the number of OB subgroups (and OB field stars) in the galactic shell. Since the number of OB stars in a subgroup is observed to be nearly constant (Blaauw 1964), we take

$$B_{\mathrm{H\,II}}(t, r) = A_{\mathrm{H\,II}} B(t - \tau_{\mathrm{H\,II}}, r) \int_{m_{\mathrm{OB}}}^{m_U} \psi(m)dm\,, \tag{5}$$

where $A_{\mathrm{H\,II}}$ is the average number of stars born for each OB star via the expanding H II region mechanism, the mass range describes stars or groups of stars producing a sufficient number of Lyman-continuum photons, and $\tau_{\mathrm{H\,II}}$ is the average time interval between the birth of an OB star and the birth of the stars formed by its expanding H II region.

The theory of Elmegreen and Lada (1977) and the observations of Blaauw (1964) indicate that $\tau_{\mathrm{H\,II}}$ can be as small as $2\text{–}3 \times 10^6$ yr. In the observed examples, the shock fronts encounter rather dense clouds with n equal to $10^3\text{–}10^4\ \mathrm{cm}^{-3}$. Consequently, gravitational instability and contraction to the main sequence occur rapidly. Again, observational selection effects favor finding dense molecular clouds and short time scales $\tau_{\mathrm{H\,II}}$. Larger values of $\tau_{\mathrm{H\,II}}$ would apply if the shock-ionization front has to travel some distance before encountering a sufficiently dense cloud or if the OB stars trigger gravitational instability in adjacent clouds which have initial densities $10^2 < n < 10^3\ \mathrm{cm}^{-3}$. An approximate upper limit to $\tau_{\mathrm{H\,II}}$ is the lifetime of the OB stars which supply the Lyman-continuum photons necessary to drive the shock-ionization front.

We look for the solution at model age $t \gg \tau_m^*$, i.e., $t \gg 9 \times 10^7$ yr if m_s is as small as $5\, m_\odot$. Then we can use the linearization,

$$B(t - \tau, r) = B(t, r) - \tau \frac{\partial B}{\partial t}\,. \tag{6}$$

Since we are combining two star-formation rates, we simplify notation by introducing (1) the net efficiency of star formation by high-mass stars,

$$A_{\mathrm{HM}} = A_{\mathrm{SN}} + A_{\mathrm{H\,II}}\,; \tag{7}$$

(2) the effective fraction of the initial mass function involved in star formation by high-mass stars,

$$f_{\mathrm{HM}} = \frac{A_{\mathrm{SN}}}{A_{\mathrm{HM}}} \int_{m_s}^{m_U'} \psi(m)dm + \frac{A_{\mathrm{H\,II}}}{A_{\mathrm{HM}}} \int_{m_{\mathrm{OB}}}^{m_U} \psi(m)dm\,; \tag{8}$$

(3) the time interval τ_{HM} between the birth of a given high-mass star and the formation of the new stars triggered by shock waves from this high-mass star; and (4) the average value of τ_{HM} as given by the expression

$$\bar{\tau}_{\mathrm{HM}} = \frac{A_{\mathrm{SN}}}{A_{\mathrm{HM}}} f_{\mathrm{HM}}^{-1} \int_{m_s}^{m_U'} (\tau_m^* + \tau_f)\psi(m)dm + \frac{A_{\mathrm{H\,II}}}{A_{\mathrm{HM}}} f_{\mathrm{HM}}^{-1} \tau_{\mathrm{H\,II}} \int_{m_{\mathrm{OB}}}^{m_U} \psi(m)dm\,. \tag{9}$$

With this notation, the above equations give

$$B(t, r) = B_{\mathrm{HM}}(t, r) = A_{\mathrm{HM}} f_{\mathrm{HM}} \left[B(t, r) - \bar{\tau}_{\mathrm{HM}} \frac{\partial B}{\partial t} \right]\,. \tag{10}$$

The value of $\bar{\tau}_{\mathrm{HM}}$ lies in the range 2×10^6 yr to 1.6×10^7 yr if $m_s = 9\, m_\odot$, and in the range 2×10^6 to 4×10^7 yr if $m_s = 5\, m_\odot$. The lower portions of these ranges pertain if A_{SN} is negligible in comparison with $A_{\mathrm{H\,II}}$ and the values assumed by Elmegreen and Lada for cloud parameters apply.

As an approximation, we assume that the global values of f_{HM} and the efficiency A_{HM} are time independent. The distribution of interstellar clouds is subject to statistical fluctuations which determine whether a particular shock wave is successful in triggering star formation. In taking A_{HM} as globally time independent we are assuming that interstellar clouds remain sufficiently numerous in the galaxy (or in the portion of the galaxy modeled) so that the rate of star formation is governed by the triggering mechanisms. Then the solution to equation (10) is

$$B(t, r) = B(0, r)e^{-t/T}, \tag{11}$$

with time constant

$$T = \frac{A_{HM} f_{HM} \bar{\tau}_{HM}}{1 - A_{HM} f_{HM}}. \tag{12}$$

Note that this birthrate has the same functional form as the birthrates assumed in the models of Searle, Sargent, and Bagnuolo (1973), Tinsley (1976), etc. We now have a physical interpretation: this type of birthrate applies to star formation by massive stars with approximately constant efficiency.

The exponential-type solution could have been anticipated, since the process (each massive star giving birth to A_{HM} new stars) is mathematically identical to population growth. The derivation is presented to show the relation between the time constant T in the birthrate and the star-formation efficiency A_{HM}. If interstellar conditions are such that $A_{HM} f_{HM} < 1$, then the birthrate is the popular decaying exponential; otherwise the birthrate is a growing exponential (which might apply during the Collapse Phase).

Since this birthrate is unstable against either exponential growth or decay, it seems unlikely that the efficiency A_{HM} would adjust to yield a time-independent birthrate, i.e., if star formation by high-mass stars is the only mechanism operating, we do not expect a constant birthrate. Thus in galactic models, such as those of Mueller and Arnett (1976) and Gerola and Seiden (1978), which attempt to explain spiral structure in terms of star formation by high-mass stars in a differentially rotating galaxy, one must either assume that a spiral galaxy somehow knows the correct value of A_{HM} to maintain zero population growth via some negative feedback mechanism or that some other type of star-forming mechanism is also in operation.

Once T is specified, the value of the efficiency A_{HM} follows from equation (12), i.e., the average number of new high-mass stars created by each high mass star is

$$y = A_{HM} f_{HM} = \left[1 + \frac{\bar{\tau}_{HM}}{T}\right]^{-1}. \tag{13}$$

We define y as the ratio of the number of high-mass stars formed in each generation to the number of high-mass stars in the previous generation. Thus y is not the same as the probability P_{st} in Gerola and Seiden (1978). To obtain the value of y corresponding to P_{st} in the models of Gerola and Seiden, one must sum appropriately over all their geometric cells since P_{st} is the probability that one "star" will trigger star formation in a particular geometric cell. Their requirement that "a propagating structure have a probability of unity to survive to the next time step" indicates that $y \approx 1$ in their models.

According to Tinsley (1975), a decaying exponential birthrate with time constant $T \leq 10^9$ yr and age $t_1 = 12 \times 10^9$ yr yields *UBV* colors in agreement with the observed mean colors of elliptical galaxies. This upper limit on T implies that the average past efficiency of star formation by high-mass stars is small enough to explain the usual absence of young stars in present elliptical galaxies. The most active star formation in an elliptical galaxy occurs when the elliptical galaxy is young, when both expanding H II regions and supernova shock waves trigger star formation. For values of T in the range 10^8–10^9 yr (this is the range of values for the time constant in the massive halo models of Ostriker and Thuan 1975) and $\bar{\tau}_{HM}$ in the range 2×10^6 to 4×10^7 yr, the value of $A_{HM} f_{HM}$ lies between 0.71 and 0.998. Note that most of the high-mass stars have two opportunities to trigger star formation (first by developing an H II region and later by becoming a supernova). The value of $A_{HM} f_{HM}$ represents the sum of both opportunities. If (i) $m_{OB} = m_s$, or (ii) A_{SN} is negligible, or (iii) $A_{H\,II}$ is negligible, we obtain the values of A_{HM} listed in Table 1 for the above values of T and $\bar{\tau}_{HM}$. In this table, m_{HM} is the relevant lower mass limit (i.e., either m_s or m_{OB}) in equation (8).

The assumption in equation (11) that A_{HM} is time independent is obviously an approximation. If the interstellar gas becomes so depleted in an elliptical galaxy (e.g., by the action of galactic winds) or in a galactic halo that the number of interstellar clouds falls below some critical value, then the values of A_{HM} and T would decrease. Now, observational limits on the present star formation rate in normal elliptical galaxies cannot distinguish between a present birthrate of zero and a birthrate that is merely down by a factor of e^{-12} (for $T = 10^9$ yr) from its initial value. Thus a lack of young stars could be attributed either to the effects of galactic winds or to a low value for the efficiency A_{HM}. Equation (11) should provide a useful description for elliptical galaxies

TABLE 1

AVERAGE NUMBER OF STAR BIRTHS TRIGGERED BY EACH MASSIVE STAR IF $T = 10^8$–10^9 YEARS

m_L ($m_\odot$)	m_{HM} ($m_\odot$)	f_{HM}	$A_{HM} f_{HM}$	A_{HM}
0.2..........	9	4.6×10^{-3}	0.86	190
			1.00	220
0.1..........	9	2.2×10^{-3}	0.86	400
			1.00	460
0.2..........	5	1.4×10^{-2}	0.71	50
			1.00	70
0.1..........	5	6.8×10^{-3}	0.71	100
			1.00	150

until $e^{-t/T}$ becomes very small; thereafter the precise form of the birthrate is not of much interest.

IV. THE BIRTH RATE IN DISK MODELS

We consider star-forming mechanisms appropriate to the annulus in a galactic disk where star formation by the spiral density wave operates, i.e., from the inner Linblad resonance at r_1 to the outer Linblad resonance at r_2. The frequency of star-forming events associated with the spiral density shock wave is $\pi^{-1}[\Omega(r) - \Omega_p]$, where Ω_p is the angular speed of the spiral pattern and $\Omega(r)$ is the rotational speed of the matter. The combined stellar birthrate for a cylindrical shell with radius (dr, r) is

$$B(t, r) = B_{\rm HM} + B_{\rm sp}\,, \qquad (14)$$

with

$$B_{\rm sp} = A_{\rm sp}[\Omega(r) - \Omega_p]\,, \qquad (15)$$

and $B_{\rm HM}$ given by equations (3), (4), and (5). The efficiency $A_{\rm sp}$ is π^{-1} times the number of stars born per unit shell mass when the spiral shock passes.

Let us make the simplifying assumptions that $A_{\rm HM}$ and $A_{\rm sp}$ are time independent and again use the linearization given by equation (6). Then from equations (4), (5), (14), and (15), we obtain

$$B(t, r) = B(0, r)e^{-t/T} + \gamma(r)(1 - e^{-t/T})\,, \qquad (16)$$

where T is given by equation (12) and

$$\gamma(r) = \frac{A_{\rm sp}[\Omega(r) - \Omega_p]}{A_{\rm HM}f_{\rm HM}}\left(\frac{T}{\bar{\tau}_{\rm HM}}\right)\,. \qquad (17)$$

Thus the birthrate consists of an exponential term, $[B(0, r) - \gamma(r)]e^{-t/T}$, and a "constant" term, $\gamma(r)$.

Note that for applications involving only young stars, as in § V*b* and Paper II, we do not need to assume that $A_{\rm HM}$ and $A_{\rm sp}$ are time independent for all t. In such cases, we require only the weaker assumption that the efficiencies remain constant for a time interval long compared to either $\bar{\tau}_{\rm HM}$ or T.

In § V we solve for values of the parameters $B(0, r)$ and $\gamma(r)$ appropriate to our Galaxy. To avoid problems with shells that are not closed systems, we work in terms of averages (angular brackets) over the relevant mass shells in the galactic disk:

$$\langle B(t, r)\rangle = \frac{1}{M_D}\int B(t, r)dM_r = \frac{1}{M_D}\int_{r_1}^{r_2} B(t, r)2\pi\sigma(r)rdr \qquad (18)$$

and

$$\langle S(t, r)\rangle = \int_0^t \langle B(t', r)\rangle dt'\,, \qquad (19)$$

where $\sigma(r)$ is the projected surface density and

$$M_D = \int_{r_1}^{r_2} 2\pi\sigma(r)rdr\,. \qquad (20)$$

The physical requirement that both $\langle B(0, r)\rangle$ and $\langle\gamma(r)\rangle$ be positive constrains the value of

$$h(t_1) = \frac{\langle S(t_1, r)\rangle}{\langle B(t_1, r)\rangle}\,, \qquad (21)$$

i.e., $\langle B(0, r)\rangle$ is positive for

$$h(t_1) > \frac{t_1 \exp(t_1/T)}{\exp(t_1/T) - 1} - T\,, \qquad (22)$$

while $\langle\gamma(r)\rangle$ is positive for $h(t_1) < T[\exp(t_1/T) - 1]$. Although the second inequality can always be satisfied for a small enough value of T, no value of T will satisfy the first inequality unless $h(t_1) > \frac{1}{2}t_1$. Thus we require $h(t_1) > 4 \times 10^9$ yr if the disk age $t_1 \geq 8 \times 10^9$ yr.

The value $h(t_1) = t_1$ is a critical value for the behavior of the birthrate $\langle B(t, r)\rangle$. If $h(t_1) > t_1$, then $[\langle B(0, r)\rangle - \langle\gamma(r)\rangle]$ is positive, so $\langle B(t, r)\rangle$ will decrease monotonically with time to a constant value $\langle\gamma(r)\rangle$. However, if $h(t_1) < t_1$, then $\langle B(t, r)\rangle$ will increase monotonically with time to a constant value $\langle\gamma(r)\rangle$. We conclude in § V*a* that for the Disk Phase in our Galaxy, the monotonically *decreasing* case is preferred. This agrees with the choice usually made in the literature.

Using equations (10) and (17), we find that the relative importance of star formation by the spiral density wave is

$$\frac{B_{\rm sp}}{B_{\rm HM}} = \frac{\bar{\tau}_{\rm HM}}{T}\frac{\gamma(r)}{B(t - \bar{\tau}_{\rm HM}, r)} \approx \frac{\bar{\tau}_{\rm HM}}{T}\frac{\gamma(r)}{B(t, r)}\,. \qquad (23)$$

If the birthrate is a decreasing function of time, then $B_{\rm sp}/B_{\rm HM}$ increases with time and approaches a constant value $\bar{\tau}_{\rm HM}/T$ for large t. In this case, as a young spiral galaxy ages, the spiral arms increase in prominence until the "constant" term $\gamma(r)$ dominates in the birthrate.

V. SOLUTIONS APPROPRIATE TO THE DISK OF OUR GALAXY

a) Values for the Parameters in the Birthrate

In Paper II, we find that $T = (2.4 \pm 1.6) \times 10^7$ yr for the present disk of our Galaxy. Using this value, we can calculate values for the unknown parameters $\langle\gamma(r)\rangle$ and $\langle B(0, r)\rangle$ if, for example, we have estimates for $\dot{n}_{\rm SN}(t_1)$, the present total supernova rate in the disk, and $\mu(t)$, the mass fraction of the disk locked up in long-lived stars (and remnants of stars) born during the Disk Phase. If we let

$$f_{\rm SN} = \int_{m_s}^{m_U'} \psi(m)dm \qquad (24)$$

and

$$\bar{\tau}_{\rm SN}f_{\rm SN} = \int_{m_s}^{m_U'} (\tau_m{}^* + \tau_f)\psi(m)dm \qquad (25)$$

(analogous to the definitions of $f_{\rm HM}$ and $\bar{\tau}_{\rm HM}$), then equations (4), (16), and (18) yield the following expression for the supernova rate:

$$\frac{\dot{n}_{SN}(t_1)}{f_{SN}M_D(t_1)} = \left(1 + \frac{\bar{\tau}_{SN}}{T}\right)[\langle B(0, r)\rangle - \langle\gamma(r)\rangle] \times \exp(-t_1/T) + \langle\gamma(r)\rangle . \quad (26)$$

Let α equal the average mass locked up per star ($\alpha = 0.3$–$0.5\ m_\odot$). Then

$$\mu(t_1) = \alpha\langle S(t_1, r)\rangle , \quad (27)$$

provides a second relation between the unknown parameters. If no mass flows into the disk, then $\mu(t) = G(0) - G(t)$, where $G(t)$ is the mass fraction of the disk in the form of interstellar matter. More generally,

$$\mu(t) = G(0) - G(t) + [1 - G(0)]\left[1 - \frac{M_D(0)}{M_D(t)}\right] , \quad (28)$$

so μ can exceed $G(0) - G(t)$ if inflow has increased the mass of the disk.

In Paper II we find that $T/t_1 \ll 1$ for our Galaxy. Hence we can use the following approximations: (*a*) the present birthrate $B(t_1, r) \approx \gamma(r)$; (*b*) equation (26) reduces to $M_D\langle\gamma(r)\rangle \approx \dot{n}_{SN}/f_{SN}$; (*c*) the observed value of $h(t_1)$ can be obtained from

$$h(t_1) \approx \frac{\mu(t_1)}{\alpha}\frac{M_D f_{SN}}{\dot{n}_{SN}} ; \quad (29)$$

and (*d*) the ratio of the initial birthrate in the disk to the present birthrate in the disk is

$$\frac{\langle B(0, r)\rangle}{\langle\gamma(r)\rangle} = \frac{h(t_1)}{T}\left[1 - \frac{t_1 - T}{h(t_1)}\right] . \quad (30)$$

Note, for small values of T/t_1, that there are no physically allowable solutions if $h(t_1) < t_1 - T$. This eliminates all models for the Disk Phase of our Galaxy in which the birthrate increases with time to a constant value, except for the small range of models with $t_1 - T < h(t_1) < t_1$.

To compute numerical values for $\langle\gamma(r)\rangle$ and $\langle B(0, r)\rangle$, we specify values for the input parameters m_s, $M_D(t_1)$, etc. Initially we try the following values. We take $\dot{n}_{SN}(t_1)$ as either 0.02 yr^{-1}, from counts of SNRs (Ilovaisky and Lequeux 1972), or 0.05 yr^{-1}, from observations of other Sbc galaxies (Tammann 1970, 1974, 1977). Allowing for star formation during the Collapse Phase and assuming $G(t_1)$ in the range 0.1–0.2, we set $\mu(t_1)$ equal to either 0.7 or 0.5. For the remaining parameters, we choose $m_s = 5$ or $9\ m_\odot$, $m_L = 0.1$ or $0.2\ m_\odot$, $M_D(t_1) = 6 \times 10^{10}$ or $10^{11}\ m_\odot$, $t_1 \geq 8 \times 10^9$ yr, and $T = 2 \times 10^7$ yr. We then rule out combinations of input values that yield $h(t_1) < 8 \times 10^9$ yr. Furthermore, constraints presented in § Vc eliminate values of $h(t_1) > 19 \times 10^9$ yr. So choosing combinations that give $h(t_1)$ in the range 8–19 $\times 10^9$ yr, we find that the present total birthrate in the disk $M_D\langle\gamma(r)\rangle$ is in the range 3.5–11 yr^{-1}. If t_1 is 8×10^9 yr, then values for the ratio $\langle B(0, r)\rangle/\langle\gamma(r)\rangle$ range from 1 [when $h(t_1) = t_1$] to 51 [when $h(t_1) = 9 \times 10^9$ yr] to 550 [when $h(t_1) = 19 \times 10^9$ yr]. Large values for the birthrate at the beginning of the Disk Phase would be consistent with the predictions of Larson (1974) and Kaufman (1975*b*) for the birthrate near the end of the Collapse Phase.

b) *The Radial Distribution of Young Objects*

We compare our model with the observed radial distribution of young objects such as giant H II regions, CO, OH, and near-infrared radiation. Since $T/t_1 \ll 1$ in our Galaxy, the present birthrate $B(t_1, r)$ has the same radial dependence as $\gamma(r)$, i.e., *the radial gradient is governed by the spiral density wave.* For $\gamma(r)$, we treat two possible cases: in case (I), the efficiency A_{sp} is proportional to the compression in the spiral shock wave, with the expression for the compression and values of the angular drift speed $\Omega(r) - \Omega_p$ from Shu, Milione, and Roberts (1973); in Case II, A_{sp} is independent of the spiral shock compression. In both cases, the birthrate decreases as r increases from r_1 to r_2, since $\gamma(r)$ behaves either like $r^2[\Omega(r) - \Omega_p]^3$ or like $\Omega(r) - \Omega_p$.

Let $b(r)$ represent the number of young objects per pc^2 projected onto the galactic plane and averaged over a shell with radius (dr, r). For our model, $b(r)$ equals $\sigma(r)B(t, r)$, and we use values of the projected mass surface density $\sigma(r)$ from Innanen (1973). In Table 2, we compare predicted and measured values for $b(r)$ in the 5 kpc ring and in the solar-neighborhood shell. For the measured values of $b(r)$, we choose various types of radiation thought to be associated with young objects. We see that the various measured values cover a sufficiently wide range to include both of our predicted values.

Bash, Green, and Peters (1977) construct a galactic model based on star formation by the spiral density wave, alone. Their predicted radial gradient in $b(r)$ is not as steep as we obtain because we defined $B(t, r)$ as the birthrate per unit mass rather than the birthrate per unit volume. Their model does not account for star formation at either $r < r_1$ or $r > r_2$. Our model has the advantage that star formation by high-mass stars can operate throughout the galaxy.

TABLE 2

Radial Distribution of Young Stars in Our Galaxy

Object	$b(5.5\ \text{kpc})/b(10\ \text{kpc})$	Reference
Predicted for case I.......	16	...
Predicted for case II......	8	...
Young OH/IR stars......	11	Oort and Baud 1978
2.4 μm surface brightness	18	Okuda *et al.* 1978
CO emission.............	5.5	Gordon and Burton 1976
H109α emission..........	8	Mezger 1972
H109α emission..........	4.7	Burton 1976
H166α emission..........	21	Burton 1976

c) *The Local Birthrate: Past and Present*

Let $H_1(r)$ equal $S(t_1, r)/B(t_1, r)$, the ratio of the time-integrated birthrate to the present birthrate for a shell with radius (dr, r). The observed value of $H_1(r_\odot)$ for the solar-neighborhood shell provides a constraint on the values for model parameters. From limits on the local IMF near $1\ m_\odot$, Tinsley (1976) estimates $8 \gtrsim H_1(r_\odot)/10^9$ yr $\gtrsim 20$. The local IMF is based on Gliese's (1957, 1969) catalog, and there are some indications that this catalog is incomplete for nearby GV stars (see Wei-Hwan 1978). Keeping this in mind as a source of uncertainty, we shall proceed to use Tinsley's range of values for $H_1(r_\odot)$.

For $T/t_1 \ll 1$, equation (16) gives

$$H_1(r) \approx \frac{B(0, r)}{\gamma(r)} T + t_1 - T . \tag{31}$$

Note that $B(0, r)$ is defined as the initial birthrate per unit shell mass. As an approximation, we now assume that $B(0, r)$ is nearly uniform and set $B(0, r_\odot) = \langle B(0, r)\rangle$. Then letting $\varphi(r) = \gamma(r)/\langle\gamma(r)\rangle$ [where $\langle\gamma(r)\rangle$ depends on the present galactic mass distribution], we have

$$H_1(r_\odot) = \frac{h(t_1)}{\varphi(r_\odot)} + (t_1 - T)\left[1 - \frac{1}{\varphi(r_\odot)}\right] . \tag{32}$$

Thus $H_1(r)$ is determined largely by the value adopted for $h(t_1)$. With the surface-density distribution $\sigma(r)$ of Innanen (1973), case I of § V*b* gives $\varphi(r_\odot) = 0.5$, while case II gives $\varphi(r_\odot) = 0.9$. If t_1 equals 8×10^9 yr, then the above limits on $H_1(r_\odot)$ imply that $8 \leq h(t_1)/10^9$ yr ≤ 14 in case I and $8 \leq h(t_1)/10^9$ yr ≤ 19 in case II. Thus we rule out certain combinations of values for the input parameters listed in § V*a*. The condition $8 \leq h(t_1)/10^9$ yr ≤ 14 can still be satisfied by choosing (*a*) $M_D = 6 \times 10^{10}\ m_\odot$, $\mu = 0.5$, $m_s = 9\ m_\odot$, and $\dot{n}_{\rm SN} = 0.02$ yr^{-1}; or (*b*) $M_D = 10^{11}\ m_\odot$, $m_s = 9\ m_\odot$, $m_L = 0.2\ m_\odot$, and $\dot{n}_{\rm SN} = 0.05$ yr^{-1}; or (*c*) $M_D = 6 \times 10^{10}\ m_\odot$, $\mu = 0.5$, $m_s = 5\ m_\odot$, $m_L = 0.1\ m_\odot$, and $\dot{n}_{\rm SN} = 0.05$ yr^{-1}. (Unspecified parameters are allowed to take any of the values listed in § V*a*.) So it is not difficult to obtain a suitable value for $H_1(r_\odot)$ with plausible values for the input parameters.

Tinsley (1976) estimates that the present birthrate per pc^2 in the solar neighborhood is 3.2–$13 \times 10^{-9}\ m_\odot$ pc^{-2} yr^{-1}. If we adopt a value of $80\ m_\odot$ pc^{-2} for $\sigma(r_\odot)$ and the above limits on $h(t_1)$, then $\gamma(r_\odot)\sigma(r_\odot)$, the mean present birthrate per pc^2 in the solar-neighborhood shell, is 3–$6 \times 10^{-9}\ m_\odot$ pc^{-2} yr^{-1} in case I and 4–$11 \times 10^{-9}\ m_\odot$ pc^{-2} yr^{-1} in case II. Thus within the context of our disk models, the global values assumed for the present supernova rate and the total number of stars born up to the present time in our Galaxy are consistent with the observed values of the present local birthrate per pc^2 and the total number of stars born up to the present time in the solar neighborhood.

d) *Low-Mass Stars*

We now discuss the radial variation in the number of stars born up to the present time. In numerical examples, we set t_1 equal to 8×10^9 yr.

In the disk, $S(t_1, r) = H_1(r)\gamma(r)$. If we assume that $B(0, r)$ is independent of r, then we can use equation (32) for $H_1(r)$ and obtain the following relation between the radial gradient in $S(t_1, r)$ and the radial gradient in young stars:

$$\begin{aligned}\frac{\partial}{\partial r} S(t_1, r) &= \langle\gamma(r)\rangle(t_1 - T)\frac{d\varphi}{dr} \\ &= (t_1 - T)\frac{\partial}{\partial r} B(t_1, r) . \end{aligned} \tag{33}$$

For r close to $r_\odot$, $d\varphi/dr = 0.27$ kpc^{-1} (case I) or 0.23 kpc^{-1} (case II) and $\partial S/\partial r$ lies in the range 0.11–0.33 kpc^{-1}. In Table 3 we show the decrease in $S(t_1, r)$ between the 5.5 kpc ring and the solar-neighborhood shell for the range of values of $h(t_1)$ found appropriate in § V*c*. Let $N_{\rm LM}(t, r)$ equal the average number of low-mass stars per pc^2 in a shell with radius (dr, r): $N_{\rm LM}(t, r) = \sigma(r)S(t, r)$. With $\sigma(r)$ from Innanen (1973), we have $N_{\rm LM}(t_1, 5.5\ {\rm kpc})/N_{\rm LM}(t_1, r_\odot) = 3S(t_1, 5.5\ {\rm kpc})/S(t_1, r_\odot)$.

The radial color gradient in the disk depends on the gradient in the ratio of old-to-young stars,

$$\frac{dH_1}{dr} = -\frac{h(t_1) + T - t_1}{\varphi^2}\frac{d\varphi}{dr} , \tag{34}$$

on the gradient in the metal abundance Z, and on the gradient in reddening by dust. Table 3 lists the increase in $H_1(r)$ between the 5.5 kpc ring and the solar-neighborhood shell. Using the relation between $B - V$ color and H_1 from Larson and Tinsley (1978), we find that the gradient in H_1 alone would make the 5.5 kpc ring bluer than the solar neighborhood by $\Delta(B - V) \leq 0.03$ if $H_1(r_\odot) \leq 10 \times 10^9$ yr, or by $\Delta(B - V) = 0.08$–0.1 if $H_1(r_\odot)$ is 20×10^9 yr. On the other hand,

TABLE 3

NUMBER OF STARS BORN UP TO THE PRESENT TIME AS A FUNCTION OF RADIUS

$h(t_1)$ (10^9 yr)	$\dfrac{H_1(5.5\ {\rm kpc})}{H_1(r_\odot)}$	$\dfrac{S(t_1, 5.5\ {\rm kpc})}{S(t_1, r_\odot)}$	$\dfrac{S(t_1, r_\odot)}{S_{\rm nuc}(t_1)}$
Case I: $A_{\rm sp} \propto$ Compression			
8............	1.0	5.4	200.0
9............	0.84	4.5	4.9
10............	0.73	3.9	3.0
14............	0.51	2.7	1.7
Case II: $A_{\rm sp}$ Independent of Compression			
8............	1.0	2.6	360.0
9............	0.93	2.4	8.0
10............	0.87	2.3	4.6
14............	0.72	1.9	2.2
19............	0.63	1.6	1.7

the observed gradient in metal abundance would act to make the 5.5 kpc ring redder than the solar neighborhood. If the observed abundance gradients derived by Peimbert (1978) for $r = 8$–14 kpc apply also to the range $r = 5$–10 kpc, then the average value of O/H is about 3 times higher at 5.5 kpc than at 10 kpc. According to Larson and Tinsley (1978), this increase in Z would make $B - V$ larger by roughly 0.08. Thus, if $H_1(r_\odot) \gtrsim 10^{10}$ yr, the outward blueing associated with the metal-abundance gradient would dominate the outward reddening associated with the gradient in the ratio of old-to-young stars.

If the same IMF applies both to star formation by the spiral density wave and to star formation by high-mass stars, then we can compare the number of low-mass stars per pc^3 in the disk with the number of low-mass stars per pc^3 in the nuclear bulge. For the nuclear bulge, we adopt the model of § III and let $S_{\rm nuc}(t, r)$ represent the time-integrated birthrate for a mass shell in the nucleus. Since $T/t_1 \ll 1$, $S_{\rm nuc}(t_1, r) \approx B(0, r)T$. Again assuming $B(0, r)$ independent of r, we obtain

$$\frac{S(t_1, r_\odot)}{S_{\rm nuc}(t_1, r)} = 1 + \frac{t_1 - T}{h(t_1) - t_1 + T}\varphi(r_\odot) . \quad (35)$$

Values for this ratio are listed in Table 3. With Innanen's (1973) mass-density distribution for our Galaxy, we find that there are 0.07–15 times as many low-mass stars per pc^3 at $r = 1$ kpc as in the solar-neighborhood shell. With the mass-density distribution of Sanders and Lowinger (1972), we find that the number of low-mass stars per pc^3 at $r = 100$ pc is 3–70 times the number of low-mass stars per pc^3 in the solar-neighborhood shell. Equation (35) does not apply to the high-density region in the central pc of our Galaxy, since collisions between interstellar clouds would be important in triggering star formation there.

e) Comments on the Metal-Abundance Gradient

The rate at which Z increases in some region of a galaxy depends on the yield p, the star formation rate, and the amount of interstellar gas with which the supernova ejecta mix before new stars are formed. The "simple model" for chemical evolution devised by Searle and Sargent (1972) postulates that p is constant and that each zone in the galaxy can be treated as isolated (see Pagel and Patchett 1975 for a more detailed description). This means that the mass fraction in the form of interstellar gas $G(t, r) = G(0, r) - \alpha S(t, r)$. Dividing our Galaxy into a series of isolated mass shells (as in Talbot and Arnett 1975), we find that the radial gradient in the interstellar metal abundance is

$$\frac{\partial Z}{\partial r} \approx \frac{\alpha p}{G(t, r)}\frac{\partial S}{\partial r} , \quad (36)$$

provided we can neglect the radial dependence of $Z(0, r)$ and the radial dependence of $G(0, r)$. If $p \approx 0.01$ (Talbot and Arnett 1973) and $G(t, r_\odot) = 0.1$–0.2, then equations (33) and (36) give $-\partial Z/\partial r =$ 0.003–0.012 kpc^{-1} for the solar-neighborhood shell. With $Z(t, r_\odot) = 0.02$, Peimbert's (1978) observed gradient in O/H implies that $-\partial Z/\partial r = 0.005$ kpc^{-1} at $r_\odot$.

However, I caution the reader against accepting at face value the apparent agreement between the predicted and observed local abundance gradients. Peimbert (1978) finds a variety of values for O/H in H II regions with approximately the same percentage of interstellar gas, while the "simple model" predicts that, aside from variations in $Z(0, r)$ and $G(0, r)$, regions with the same value for $G(t, r)$ should all have the same metal abundance. Furthermore, in writing equation (36), we judiciously chose to use the theoretical gradient in $S(t_1, r)$ rather than the observed gradient in $G(t_1, r)$. The latter is much flatter (see Gordon and Burton 1976) and would yield a metal-abundance gradient much flatter than observed. The implication is that various mass shells in the disk are not isolated; gas flows such as differential infall or radial flows in the disk must play a significant role in determining $G(t, r)$.

The conclusion is that, indeed, the observed metal-abundance gradient does correlate with the predicted star-formation gradient, but further study of the gas-flow problem is required. This I postpone to a later paper.

As noted in § I, several authors have used models with decaying exponential birthrates to treat either the chemical evolution of our Galaxy or the colors of other galaxies. In our disk model, the birthrate consists of an exponential term plus a "constant" term: the exponential term is important during the early stages of the model but makes a negligible contribution to the current birthrate. To what extent does the exponential term influence the chemical evolution of the disk of our Galaxy? Let $\langle f_{\rm exp}(t)\rangle$ represent the fractional contribution of the exponential term to the total time-integrated birthrate for the disk. For $T/t_1 \ll 1$, we find that $\langle f_{\rm exp}(t_1)\rangle = [h(t_1) + T - t_1]/h(t_1)$. So the value of $\langle f_{\rm exp}(t_1)\rangle$ varies from 2.5×10^{-3} if $h(t_1) = t_1 = 8 \times 10^9$ yr to 0.5 if $h(t_1) = 19 \times 10^9$ yr.

f) Stellar Age Distribution in the Solar Neighborhood

For the solar-neighborhood shell, let $N(\tau_i)$ equal the number of stars with ages in an interval $d\tau$ about τ_i. For stellar ages $\tau_i \gtrsim 7.5 \times 10^9$ yr, our disk model has $(t_1 - \tau) \gg T$, and, consequently, the constant term dominates in the birthrate. This means that the age distribution in our disk model is the same as an age distribution calculated with a time-independent birthrate. Tinsley (1974) compares the age distribution for a time-independent birthrate with the observed age distributions given in Cayrel de Strobel (1973) and Clegg and Bell (1973). The two observed age distributions are not in good agreement with each other. This indicates problems with the stellar age determinations and/or the completeness of star counts. The age distribution predicted for our model (see Table 4) is a

TABLE 4

STELLAR AGE DISTRIBUTION

τ_1 (10^9 yr)	τ_2 (10^9 yr)	Observed $N(\tau_1)/N(\tau_2)$	Predicted $N(\tau_1)/N(\tau_2)$	Reference
3–5.......	1–3	0.55 ± 0.18	0.55	1
5–7.......	1–3	0.47 ± 0.17	0.21	1
6–8.......	3–4	0.40 ± 0.11	0.11	2

REFERENCES.—(1) Cayrel de Strobel 1973; (2) Clegg and Bell 1973.

bit steeper than the observed distribution in Cayrel de Strobel and considerably steeper than the observed distribution in Clegg and Bell. Unless $t_1 < 8 \times 10^9$ yr, Tinsley's (1976) simple exponential models which satisfy her limits on H_1 also have steeper age distributions than observed by Clegg and Bell. While the stellar age distribution is potentially a test of the model, a more accurate determination of the observed distribution is required.

VI. SUMMARY

The simplified galactic-evolution models in this paper are based on three star-formation mechanisms for which there is both observational and theoretical evidence. These mechanisms are star formation triggered by a galactic spiral density wave, star formation triggered by an expanding H II region, and star formation triggered by shock waves from supernovae. The latter two mechanisms are called star formation by high-mass stars.

With the assumption that the efficiencies of the above star-forming mechanisms remain approximately constant in time for the duration of the galactic model (which starts at the end of the initial Collapse Phase), we obtain the following results.

1. If star formation by high-mass stars is the only important star-forming mechanism, then the birthrate is exponential in time. This provides a physical interpretation for the exponential-type birthrates used by many authors to discuss the chemical evolution and colors of galaxies and indicates that exponential birthrates are appropriate for describing the past evolution of an elliptical galaxy, nuclear bulge, or galactic halo. The value of the time constant T in the exponential is determined by the efficiency with which shock waves from massive stars initiate star formation.

2. In the disk of a spiral galaxy, all three star-forming mechanisms operate, and the total birthrate contains contributions from B_{HM}, the birthrate for star formation by high-mass stars, and from B_{sp}, the birthrate for star formation by the spiral density wave. The birthrate then consists of an exponential term plus a time-independent term. The efficiency of star formation by high-mass stars determines the time constant T in the exponential term.

3. For our Galaxy, we have the following specific results. (i) Since the time constant $T \approx 2 \times 10^7$ yr (see Paper II) for the disk, the birthrate in the disk decreases with time during the early Disk Phase and then remains nearly constant in time for disk age $t \lesssim 10^9$ yr. (ii) In the disk, the radial gradient in the birthrate is governed by the spiral density wave and is in reasonably good agreement with the observed radial distributions of various types of young objects. (iii) Our model is consistent with the observed values for the present local birthrate per pc^2 and the total number of stars born up to the present time in the solar neighborhood. The present birthrate per unit mass in the solar neighborhood shell is smaller by a factor $\varphi(r_\odot) = 0.5$–0.9 than in the disk as a whole. (iv) We compute the radial variation in S, the total number of stars born up to the present time, and compare the disk with the nuclear bulge region. In the disk, the predicted gradient in S correlates with the observed metal-abundance gradient. Jensen, Strom, and Strom (1976) obtain a similar result for other disk galaxies. Considering only star formation by the spiral density wave, they find that metal-abundance gradients are correlated with star-formation gradients. (v) If $H_1(r_\odot) \gtrsim 10^{10}$ yr, then the outward blueing of the disk associated with the metal-abundance gradient dominates the outward reddening associated with the gradient in the ratio of old-to-young stars.

Thus we have a simplified theory of galactic evolution that applies to both elliptical and spiral galaxies and yields results in reasonably good agreement with observations.

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