Astron. Astrophys. 80, 104–109 (1979)

ASTRONOMY AND ASTROPHYSICS

Primordial Black Holes

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Received January 11, 1979

Summary. The processes of primordial black hole formation and accretion of matter onto the primordial black holes already formed are investigated. We give the limits on the possible number of primordial black holes of various masses inferred from astrophysical observations.

Key words: cosmology – primordial black hole – relativistic hydrodynamics – evaporation of black hole

1. Introduction

It seems probable that at the very first stages of the expansion of the Universe primordial black holes (PBH) were formed.

In fact, today the Universe is very homogeneous and isotropic on the scale of the order of the cosmological horizon. At the same time the Universe has a well-developed structure on the scale of galaxies and smaller formations. Galaxies seem to have been formed by the growth of small inhomogeneities as a result of gravitational instability. In such a case the theory requires that at the beginning of the expansion of the Universe near the cosmological singularity, there should have existed small but finite perturbations of the Friedmann metric with amplitude of the order of 0.01 containing the same amount of baryons as contemporary galaxies do. It would be natural to assume the amplitude of perturbations to be even larger on smaller scales. On a scale where the amplitude of perturbations was of the order of unity, the gravitational collapse of primordial matter can occur when the cosmological horizon becomes of the order of the perturbation scale.

This is the way primordial black holes can be formed with masses from Planck mass and higher[1].

The possibility of primordial black hole formation was mentioned in 1966 by Zeldovich and Novikov (1966) and in 1971 by Hawking (1971). Later the PBH problem was treated in numerous papers.

Primordial black holes have become a subject of great interest since the quantum evaporation of low mass black holes was discovered by Hawking (1974, 1975a), because only primordial black holes can have such small masses. The Hawking process is essential for the physics of the early stages of the expansion of the Universe on the one hand, and as a possible way to detect primordial black holes in the Universe on the other hand.

This paper deals with the problem of PBH formation, with matter accretion onto them at the early stages of the Universe and the related process of their mass growth; emphasis will be given to the limits on the possible number of primordial black holes of various masses inferred from astrophysical observations.

2. Formation of the Primordial Black Holes

Here, the following two problems are basic:

1. What are the deviations from the Friedmann cosmological model at the beginning of the expansion that result in PBH formation?
2. How does the accretion of surrounding material proceed onto the PBH already formed?

Both problems had been formulated and analyzed in the first papers on PBH (Zeldovich and Novikov, 1966, 1967). It turned out that only numerical computations can give an exhaustive answer to the above questions[2].

Appropriate calculations carried out by Nadejin et al. (1977, 1978) show the hydrodynamic picture of PBH formation and the subsequent non-steady gas accretion, under the simplest assumption of the spherical symmetry of the processes under consideration.

The dependence of the process of PBH formation on the amplitude of deviation from the flat Friedmann model near the singularity (the beginning of the expansion) and on the profile of that deviation (albeit within the assumption of spherical symmetry) is also discussed in this paper. The metric perturbation near the singularity is assigned as a spherical region with a comoving 3-space of constant positive curvature, i.e. the perturbed region corresponds to some part of the closed Friedmann model. The deviation amplitude can be characterized by a number measuring the fraction of the closed space with the constant positive curvature cut out. This part of the closed Universe mat-

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1 It should be mentioned that there exist so called entropy theories of galaxy formation (Doroshkevich et al., 1967; Dicke and Peebles, 1968) in which only fluctuations of the matter composition – but not metric perturbations – are required at the early stages of the Universe. Even under the assumption that there were no substantial metric perturbations, fluctuations of the order of unity due to which PBHs with $m \approx 10^{-5}$ g can form seem inevitable on the scale of 10^{-33} cm

2 Besides PBH, white holes are also discussed in the literature. It was shown that quantum effects near the singularity (Zeldovich et al., 1974) as well as the accretion (Eardley, 1974) are important for white holes due to which they rapidly become a sort of black holes

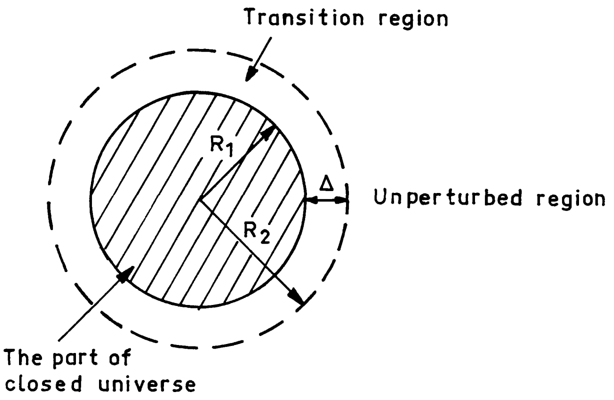

Fig. 1. The spherical perturbation and the transition region

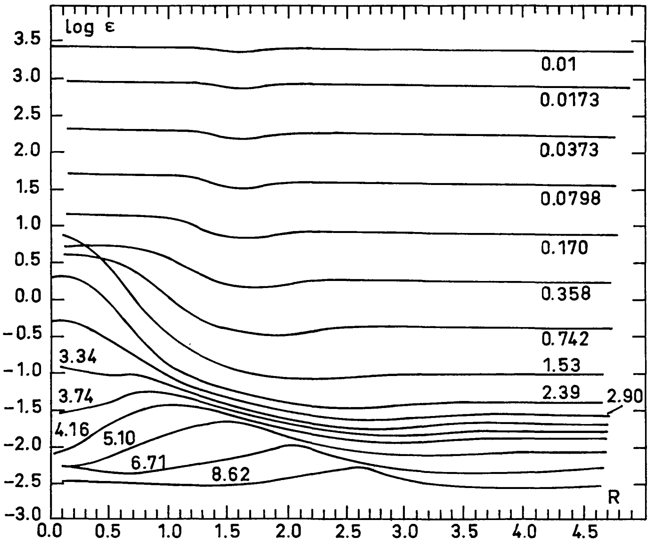

Fig. 2. The development of the density perturbation with time. Different curves correspond to the different moments of time. ε-energy density, R is the Lagrange radius. This figure corresponds to $R_1=0.75\ R_{\max}$, $\Delta=0.5\ R_1$ (see text). The black hole does not form

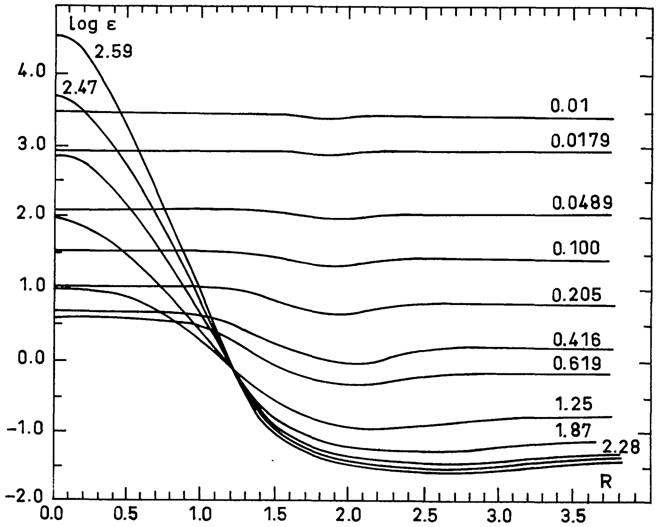

Fig. 3. The same as Fig. 2, but $R_1=0.9\ R_{\max}$. The black hole forms

ches the flat Friedmann model via a transition region. The width of the latter, according to our calculations, is the second important parameter of the problem.

It is assumed also that outside the perturbed region the solution is exactly that of flat Friedmann model. Hence, the perturbation is such that the total mass of material inside the perturbed region is exactly the same as it would have been in this region in the case of unperturbed Friedmann model.

The space-time metric is given by

$$ds^2=c^2 e^{\sigma} dt^2 - e^{\omega} dR^2 - r^2 (d\theta^2 + \sin^2\theta d^2). \tag{1}$$

The comoving system of reference is chosen, R is the Lagrange radius of particles. The matter has the equation of state $P=\varepsilon/3$. The non-perturbed metric is characterized by the following formula

$$ds^2=c^2dt^2-a^2(t)[dR^2+R^2(d\theta^2+\sin^2\theta d^2)]. \tag{2}$$

Deviations from this formula characterize perturbations of the metric. Near the singularity in the perturbed region we assume the following metric:

$$ds^2=c^2dt^2-a^2(t)\ [dR^2+\sin^2 R(d\theta^2+\sin^2\theta d^2)]. \tag{3}$$

Hence, the perturbation amplitude can be described by R_1 – the value of R on the boundary of the perturbed region of the 3-space with positive constant curvature (see Fig. 1). Further the transition region extends over the range:

$$R_1<R<R_2, \quad R_2-R_1=\Delta,$$

where the solution gradually matches the external non-perturbed region. The development of the process strongly depends on the width of this transition region. In fact, if Δ is small enough, steep pressure gradients and violent hydrodynamic phenomena develop as the density perturbations $\delta\rho$ grow. If Δ is large, pressure gradients are small.

Figures 2–4 show the results of calculations (Nadejin et al., 1978).

If the perturbation of the metric is small (small $R_1=0.75\ R_{\max}$, where $R_{\max}=\pi/2$, Fig. 2) the initial density perturbation turns into an acoustic wave propagating to infinity and the black hole does not form.

For larger $R_1=0.80\ R_{\max}$ density perturbations are large, but still no primordial black holes form, the perturbation spreads out as a wave-package.

For $R_1=0.9\ R_{\max}$, $\Delta=0.5\ R_1$ (Fig. 3, 4) a PBH forms. In Fig. 5 the curve is depicted that shows for which R_1 and Δ a primordial black hole forms, and for which it does not and the initial perturbations become acoustic waves.

The following conclusions can be made. PBH's can form only for very large deviations from Friedmann model which correspond to $R_1\approx 0.85$–$0.9\ R_{\max}$. The width of the transition region has a strong effect on the PBH formation. The narrower Δ, the greater is the role of pressure gradients that hinder PBH formation.

Before numerical calculations had been performed, attempts were made to estimate the importance of pressure in PBH-formation by developing steady-state or self-similar solutions. An assumption was made that pressure could contribute to gas accretion by PBH's in the process of the formation and significantly enlarge their masses. Carr and Hawking (1974) showed that there is no self-similar solution that results in a catastrophic accretion of matter by a PBH, with its size growing as fast as the cosmological horizon. Our calculations show, that in fact pressure strongly hinders PBH formation, making their masses smaller than they

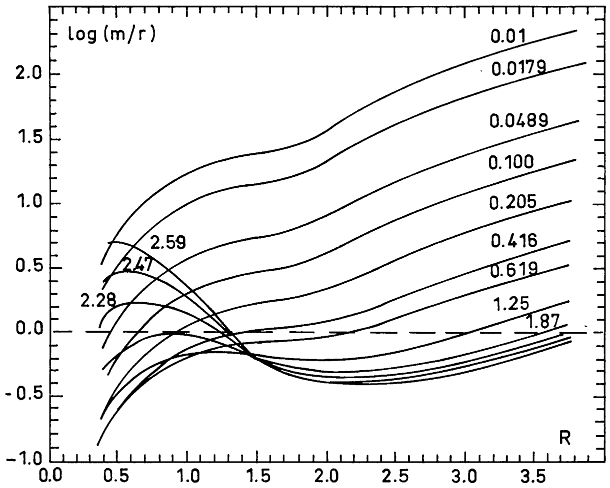

Fig. 4. The criterion of a black hole formation. Different curves correspond to the different moments of time. The moment of time, when the black hole arises, corresponds to the curve which is tangent to the line $\log(m/r)=0$

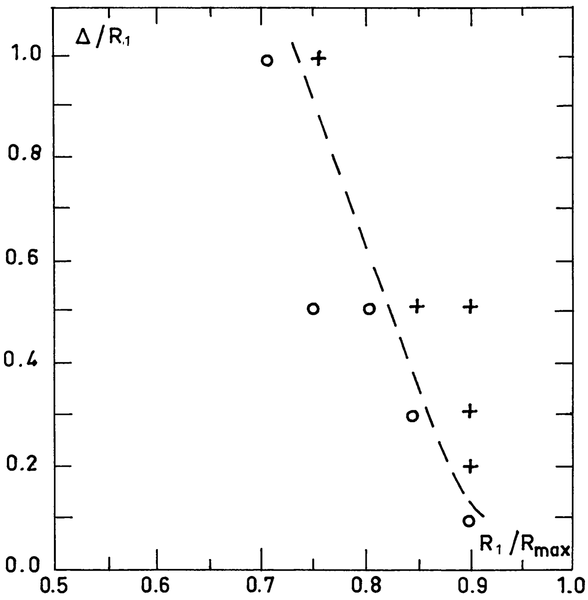

Fig. 5. The dependence of the PBH formation on the amplitude R_1 of the perturbation and the transition region. The crosses correspond to the formation of PBH, the circles correspond to the transmutation of the perturbation into the acoustic wave

would have been with the same initial perturbation but with no pressure, $P=0$. Indeed, near the singularity within the spatial cross-section $t=\text{const}$, the energy density in the perturbed region $R<R_1$ is higher than far from the center in the flat Friedmann model, and the outward pressure gradient at R_1 tends to throw away the matter. In the transition region Δ, the density ρ is minimal, and on the outer boundary of this region R_2 the inward pressure-gradient gives rise to accretion. However, this effect is less significant in PBH formation than the above mentioned gradient at R_1 which results in the outflow of matter from the perturbed region. As a result the mass of the PBH which actually forms is 0.2–0.3 that of the PBH which would have formed with no outflow at all, i.e. in case $P=0$. It should be emphasized that the size of the PBH just after its formation is much smaller than the cosmological horizon. When a PBH forms, its mass is about 0.01–0.06 of that trapped within a sphere with a radius equal to the cosmological horizon.

Under such conditions the accretion onto a PBH is slow and it only slightly increases the mass of the PBH in the course of the subsequent evolution. Our calculations show this clearly.

This conclusion was proved for self-similar solutions by Carr and Hawking (1974) [and was mentioned as one of the possibilities by Zeldovich and Novikov (1968)]. Many other interesting hydrodynamical phenomena arise in the course of PBH formation. For example if we take a very narrow transient region, then shock waves arise (see Nadejin et al., 1978). In case of low pressure the tidal interactions destroy spherical symmetry and prevent PBH formation in case of small perturbation.

3. Upper Limits on the Mass Spectrum of PBH's

Another approach to the problem of primordial black holes consists in the determination of upper limits on the mass spectrum of PBH's on the basis of various observational evidences or cosmological consequences of their existence. The PBH mass spectrum is of great interest not only by itself; it is also unambiguously related to the initial spectrum of adiabatic perturbations at $t=t_{\text{pl}}$, where $t_{\text{pl}}\approx 10^{-43}$ s (see more detailed formulas in Carr, 1975).

How could primordial black holes be discovered? Those with $M>10^{15}$ g, once formed, have not effectively changed up to the present stage of the Universe. Their mass loss due to quantum effects is small, and the mass growth due to accretion could hardly be considerable as has been shown above. Such PBH's could be discovered only due to their gravitational attraction. At present the contribution of PBH to the total cosmological energy density cannot appreciably exceed the critical density, $\rho_{\text{crit}}\approx 5\ 10^{-30}$ g cm^{-3} $(H/50\ \text{km s}^{-1}\ \text{Mpc}^{-1})^2$.

As was first shown in Zeldovich and Novikov (1966), this condition places a very stringent limitation on ε_{PBH} at the instant of their formation. This moment t_0 depends on the mass of PBH:

$$t_0(s)\approx GM/c^3\approx 2\ 10^{-39}\ M\ (g).$$

The reason for this is that at $t<10^{12}\div 10^{13}$ s when the Universe expands according to $a(t)\sim\sqrt{t}$, the energy density of radiation decreases as $a^{-4}\sim t^{-2}$, while $\varepsilon_{\text{PBH}}\sim a^{-3}\sim t^{-3/2}$ (because of the cosmological expansion PBH velocity rapidly decreases, hence the effective equation of state for PBH is $P=0$).

Let us characterize the PBH spectrum at the moment of its formation by the quantity $\beta(M)=(\varepsilon_{\text{PBH}}/\varepsilon_m)_{t=t_0}$, where $\varepsilon_{\text{PBH}}(M)$ is the PBH energy density with the typical mass of an individual PBH of the order of M, ε_m is the total radiation and matter energy density for $\Omega=1$ (if $\beta(M)\ll 1$, then $\varepsilon_m\sim 1/Gt^2$). Here we assume that the main part of PBH's is concentrated near one value of mass M. In the case of an extended spectrum of PBH's its magnitude should be characterized by the derivative $d\beta(M)/dM$. The quantity $\beta(M)$ represents the total fraction of matter that has collapsed into primordial black holes with the mass M at the moment of their formation. The solid curve in Fig. 6 represents the upper limit on β(M) for the case $\Omega_m=0.1$.

The upper limit on $\beta(M)$ in the range 10^{16} g$<M<10^{48}$ g follows from the above argument (Zeldovich and Novikov, 1966) and is $\beta(M)\lesssim 10^{-25}\ M^{1/2}$ (g).

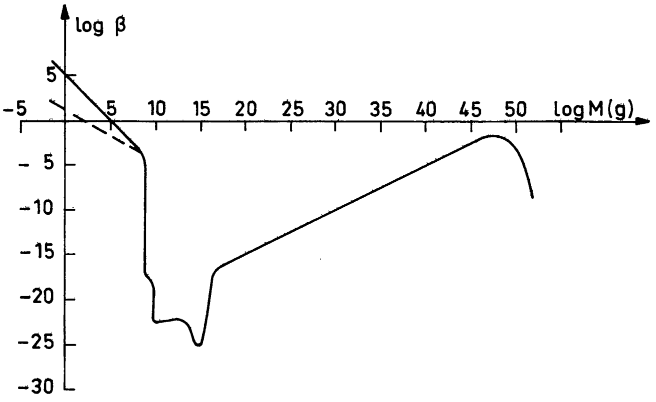

Fig. 6. The upper limit on mass fraction β of matter that has collapsed into the PBH's with the mass M at the moment of their formation (continuous line). The dotted line on the left side of this Figure corresponds to improved upper limit on $\beta(M)$, obtained from entropy argument, if PBH's have a smooth broad mass spectrum

For $M > 10^{15}\ M_\odot \approx 10^{48}$ g the upper bound on $\beta(M)$ and $\varepsilon_{\rm PBH}$ can be significantly improved on the basis of the absence of 24 h large-scale anisotropy in the relict background radiation[3].

Other ways of setting an upper limit on β for $M > 10^{15}$ g are discussed in (Carr, 1975) (see also the recent review Carr, 1978) but they do not strongly change the above stated results.

The situation changes drastically in case of PBH's with the masses less than 10^{15} g because, as a result of the effect of black hole evaporation discovered by Hawking (1975), such PBH's have already evaporated by the present time. When evaporating, a PBH emits particles and antiparticles with characteristic energies $E \approx 4-6\ T_{\rm BH}$, where $T_{\rm BH} = \hbar c^3/8\pi G M \approx 10^{13}/M(g)$ GeV (for a Schwarzschild black hole). The lifetime t_1 of a PBH depends on its mass and is of the order of $10^{-27}\ M^3\ (g)$ s. Here we assume the Elementary Particle picture (but not the Composite Particle picture) with the number of different types of the "truly elementary" particles approaching $20-30$; this implies that the equation of state in the early Universe is of the form $P_m = \varepsilon_m/3$. Let us introduce the quantity $\alpha(M) = (\varepsilon_{\rm PBH}/\varepsilon_m)_{t=t_1(M), \Omega=1}$ (i.e. at the moment of the complete evaporation of PBH with the mass M). If $\alpha(M) < 1$ then β is related to α through

$$\beta(M) = \alpha(M)\ M_{\rm pl}/M,$$

where $M_{\rm pl} = 2\ 10^{-5}$ g. After the completion of PBH evaporation the density of emitted particles is

$$n_e = \alpha(10^{-7}/t_1^{1/6})\, n_m \approx \frac{20\,\alpha\, n_B}{\Omega t_1^{1/6}},$$

where n_m is the total particle-number density and n_B is the baryon number density at the moment t_1 (t_1 in seconds)[4]. At $t \gtrsim 1$ s, the main contribution to n_m is made by massless particles and (if $t < 10$ s) by e^-e^+-pairs. As calculated by Page (1976), 45% of the energy emitted by a PBH with $5\ 10^{14}\ {\rm g} < M < 10^{17}$ g is in ultrarelativistic electrons and positrons, 45% is in neutrinos, 9% is in photons and 1% is in gravitons. PBH's with smaller masses emit also π-mesons, nucleons and other strongly interacting particles.

Upper limits on $\varepsilon_{\rm PBH}$ with $M \sim 10^{14} - 10^{15}$ g, which evaporate after recombination, were found by Chapline (1975), Page and Hawking (1976), and Carr (1976). The most stringent limitation was obtained for PBH's with $M \sim 10^{15}$ g from γ-ray observations (if there is no PBH's clustering within galactic halos). The total γ-ray density emitted by these PBH's must not exceed the observed γ-ray cosmic background density. This limit is: $\alpha(10^{15}\ {\rm g}) \lesssim 10^{-8}$, $\beta(10^{15}\ {\rm g}) \lesssim 10^{-25}$.

A very promising method of the search for PBH of this mass was recently proposed by Rees (1977). He emphasizes that collective interaction of electrons and positrons ejected in the final explosion of a PBH with the interstellar magnetic field can generate an instantaneous radio-burst (see also the detailed calculation of this process in Blanford, 1977). However, at present this mechanism can be used to improve the above mentioned limit only under additional assumptions of the PBH clustering within galactic halos and of the value of the interstellar magnetic field in the vicinity of a PBH (see the results and further perspectives in Meikle, 1977). Besides, a strong limit on the PBH spectrum over the mass range $10^{13}\ {\rm g} < M < 10^{14}$ g follows from the consideration of the effect of the PBH high-temperature radiation on the kinetics of the recombination (Naselskii, 1978).

PBH's in the mass range $10^{11}\ {\rm g} < M < 10^{13}$ g evaporate before recombination, but the emitted radiation does not attain the equilibrium distribution because the baryon and electron number densities are low and the bremsstrahlung cannot provide sufficient number of photons in the Rayleigh-Jeans region of spectrum (for details see Sunyaev and Zeldovich, 1970; Illarionov and Sunyaev, 1974, where the case of an arbitrary energy source is investigated). Thus, the emitted photons must distort the spectrum of the background radiation. Comparison with the observed spectrum of the background electromagnetic radiation shows that $\alpha(M) < 10^{-2} - 1$, $\beta(M) M/M_{\rm pl} < 10^{-2} - 1$ over the mass range under discussion (Zeldovich and Starobinsky, 1976).

Several upper limits on $\varepsilon_{\rm PBH}$ in the mass range $10^9\ {\rm g} < M < 10^{12}$ g can be obtained by taking into account the effect of high-energy hadrons (Zeldovich et al., 1977) and neutrinos (Vainer and Naselskii, 1977) emitted by PBH's on the nucleosynthesis of helium (Zeldovich et al., 1977; Vainer and Naselskii, 1977) and deuterium (Zeldovich et al., 1977) in the early Universe.

It is not difficult to verify that in the adopted model with the limited number of "truly elementary" particles the interaction between the particles emitted by a single black hole with a sufficiently small mass is not essential even in the case when the emitted particles are hadrons.

Really, if $T_{\rm BH} \gg m_p$ (or the quark mass), then all emitted particles are ultrarelativistic. On the other hand, particles are emitted one after another at characteristic intervals $\Delta t = \delta^{-1}\, r_g/c$ with δ of about $(2-4)\ 10^{-2}$, if hadrons constitute the major part of the emitted particles and the number of elementary hadrons is some $10-20$. Due to the discreteness of the emission process, at the moment of creation the particle is effectively surrounded by a vacuum cavity with the size of about $\delta^{-1}\, r_g$, and at $r > \delta^{-1}\, r_g$ the number density of previously emitted particles is $n(r) \approx \delta r_g^{-1}\, r^{-2}\, (4\pi)^{-1}$. The inner boundary of the cavity expands with an ultrarelativistic velocity. Taking into account that the relative velocities of particles in the BH rest frame are small (of about cm_p^2/E^2, where E is the characteristic particles energy, $E \sim (4-6)\ T_{\rm BH} \gg m_p$) and assuming the cross-section of strong interaction to approach $(\hbar/m_\pi c)^2$, the number of collisions of any emitted particle with all other particles emitted by the same BH can be calculated. It turns out to be $\nu = 4\pi\delta^2 (m_p/m_\pi)^2 (m_p T_{\rm BH}/E^2) < 1$, if $T_{\rm BH} > m_p$.

3 If there were such large black holes, we should fall down in the gravitational field of the nearest of them and should see the anisotropy in the relict background radiation

4 The assumed value of the Hubble constant is $50\ {\rm km\ s^{-1}\ Mpc^{-1}}$

Therefore small PBH's inject high-energy nucleons and antinucleons into the surrounding space. Antinucleons emitted by PBH's with $M \sim 10^9 - 10^{10}$ g ($t_1 \sim 1 - 10^3$ s) annihilate with the background nucleons and increase neutron to proton ratio in the background matter without changing the whole baryon number density $n_p + n_n$, because BH radiation is symmetric with respect to the baryonic charge to an accuracy of inessential statistical fluctuations [if no special mechanism connected with the *CP*-violation (Hawking, 1975b; Zeldovich, 1976) is present]. This results in an increase of the primordial He^4 abundance. Then the following limit can be derived from the comparison with the observed He^4 abundance (the mass fraction $Y_{He^4} < 30\%$):

$$\alpha(M) < 10^{-2}\, t_1^{1/6} \Omega,$$

for 10^9 g $< M < 10^{10}$ g (t_1 here and below is in seconds). Limitations on the PBH's spectrum obtained from considering the effect of high-energy ν_e and $\bar{\nu}_{\bar{e}}$ emitted by PBH's on n/p-ratio and He^4 nucleosynthesis are approximately 10^3 times weaker if $\Omega = 0.1$.

A strong limit on ε_{PBH} over the range 10^{10} g $< M < 10^{13}$ g can be obtained from the deuterium abundance. In this case, the most interesting process for us is the spallation of primordial He^4 nuclei by ultrarelativistic nucleons and antinucleons emitted by PBH's. Neutrons born by this process (and those initially emitted by PBH's) get rapidly captured by the background protons forming D nuclei. The neutron mean free path h before D-formation is smaller than $c \times$ [neutron life time ($\sim 10^3$ s)], if $t_1 < 10^5\, \Omega^{2/3}$ s. Deuterium does not burn out in reactions $D + D \begin{smallmatrix} \nearrow T + p \\ \searrow He^4 + n \end{smallmatrix}$ in significant amounts, if $t_1 > 3\; 10^3$ s ($\Omega = 1$) or $t_1 > 10^3$ s ($\Omega = 0.1$) and $Y_D = 5\; 10^{-5}$. Since Y_D cannot exceed $5\; 10^{-5}$, we obtain the following upper limit on α over the range 10^{10} g $< M < 5\; 10^{10}$ g (Zeldovich et al., 1977):

$$\alpha \lesssim 10^{-6}\, t_1^{1/6} \Omega.$$

Specifically, if $M \sim 10^{10}$ g and $\Omega = 0.1$, then $\beta < 5\; 10^{-22}$. When $t_1 > 10^5$ s, the main contribution to the effect in question is from the reaction of direct deuterium production in He^4 spallation. It is known that the branching ratio for this reaction is approximately 25% at $E \gtrsim 1$ GeV. The characteristic collision time between nucleons and antinucleons emitted by PBH's and background He^4 nuclei is smaller than the horizon even if $M \sim 10^{13}$ g ($t_1 \sim 10^{12}$ s). This leads to a limit on α over the range 10^{11} g $< M < 10^{13}$ g:

$$\alpha < 5\; 10^{-6}\, t_1^{1/6} \Omega.$$

Thus, the primordial deuterium abundance is a very sensitive indicator of PBH's.

It is important to emphasize another aspect of this result. It is well known that the observed D-abundance is difficult to explain within the ordinary nucleosynthesis scheme, if one adopts a high density of matter in the Universe [$\rho > 6\; 10^{-31}$ g/cm^{-3}, $\Omega > 0.1$, or $\Omega > 0.5$ in case of the strong, specially adjusted inhomogeneity (Wagoner, 1973; Zeldovich, 1975)]. From the results obtained above it follows that the presence of a small number ($\beta \sim 10^{-20} \div 10^{-21}$) of PBH's with $M \sim 10^{10} - 10^{12}$ g gives us a unique possibility to explain the observed deuterium abundance even if $\Omega = 1$. And by this practically no other property of the Universe changes. In particular, at $\Omega > 0.1$ the proposed mechanism gives a small increase in He^3 abundance: $Y_{He^3} \sim (1-3)\; 10^{-5}$, which does not contradict the observational data (with the possibility of He^3 stellar destruction taken into account). He^3 is formed both directly in the process of He^4 spallation and from D by the reaction $D + D \rightarrow He^3 + n$. The abundance of He^3 produced in this way never exceeds the deuterium abundance. The proposed mechanism does not result in an increase of Li^7 abundance. It should be noted here, that, according to Wagoner's calculations, the abundance of Li^7 formed in the early Friedmannian Universe (without PBH's) with $\Omega > 0.1 - 0.2$ agrees with the observations.

It is essential that the suggested mechanism does not encounter two main difficulties characteristic of all previously proposed mechanisms of deuterium production. These difficulties are (see, e.g. Epstein et al., 1976; Epstein, 1977):

a) the overproduction of Li^6 and Li^7, and also B^{11};

b) the overproduction of γ-rays.

The first difficulty does not arise because, on one hand, particles emitted by PBH's have very high characteristic energies $E \sim 50$–5000 GeV over the PBH mass range 10^{10} g $\lesssim M \lesssim 10^{12}$ g (so the cross-sections of Li^7 and Li^6 formation are very small) and, on the other hand, the fraction of α (and $\bar{\alpha}$) particles in the PBH radiation is exceedingly small (it is just in $\alpha + He^4$ collisions that Li nuclei form). Even lower is the rate of B^{11} formation in this model. The second difficulty does not arise because γ-rays produced in He^4 spallation have enough time to be absorbed by matter in the course of the subsequent evolution of the Universe.

If $M_{PBH} < 10^9$ g ($t_1 < 1$ s), then PBH radiation quickly thermalizes and the specific entropy (per baryon) S of the Universe increases. Therefore, upper limits on $\alpha(M)$ for $M < 10^9$ g can be obtained by comparing the accumulated amount of entropy with the presently measured value $S \approx 10^9/\Omega$ (H/50 km s^{-1} Mpc^{-1})^2, where the contribution from relic thermal $\nu_e, \bar{\nu}_e$ and $\nu_\mu, \bar{\nu}_\mu$ is taken into account. It turns out that the entropy is mainly produced not at the moment of particle creation by PBH's but during the subsequent relaxation and thermalization of these particles. Calculations show (Zeldovich and Starobinsky, 1976) that if the most PBH's have the typical mass M, then the specific entropy after relaxation is

$$S = (1 + S_0)\, \beta M / M_{pl},$$

where S_0 is the initial specific entropy before PBH evaporation.

From this it follows that $\beta \lesssim (10^9/\Omega)\,(M_{pl}/M)$. In particular, all the presently observed entropy may result from the PBH evaporation; for this to occur it is sufficient, for example, that one-half of matter in the initially cold Universe ($S_0 \ll 1$) collapses into PBH's with $M \sim 10^4/\Omega$ (g). Note that even if $\beta(M) < 1$ in this case, later there will be an evolutionary stage with $\varepsilon_{PBH} \gg \varepsilon_m$ and the law of expansion $a(t) \sim t^{2/3}$. If $M < 10^4/\Omega$ (g), then $\beta(M) > 1$ is possible.

If PBH's do not concentrate near some one mass value but rather have a smooth broad mass spectrum, then the upper limit on $\beta(M)$ obtained from the entropy argument can be improved (see the dotted line on the left side of Fig. 6). Assume, e.g., that the PBH mass spectrum is of the form $d\beta(M)/dM = \gamma/M$ over the range $M_{pl} < M < M_{max}$, where $\gamma = \text{const} < 1$. This spectrum corresponds to the flat spectrum of initial adiabatic metric perturbations at $t = t_{pl}$. Then the upper limit on γ turns out to be $\gamma^2 (M/M_{pl})^{4/3} < 10^9/\Omega$, if 10^4 g $< M_{max} < 10^9$ g. The value of $\gamma \sim 1$ is possible only when $M_{max} < 100$ g. In this case, the evolution of the Universe passes through a sequence of power-law stages with $a(t) \sim t^{q_n}$, where $q_n = 2/3 - 1/6 \cdot 3^n$, $n = 0, 1, 2, \ldots$, so that $q_0 = 1/2$, $q_1 = 11/18$, $q_2 = 35/54$ and so on. In the real Universe at 10^4 g $< M_{max} < 10^9$ g, only three first stages can take place and the last stage with $q \neq 1/2$ must end before $t = 1$ s.

When obtaining the foregoing limitation, the essential assumption was that BH's leave no remnants, though the theory of pair creation in the BH gravitational field treated as classical is applicable only if $M > M_{pl} \approx 2\; 10^{-5}$ g. If we assumed that BH's evaporate only down to the mass M_{pl} and after that the evaporation somehow stops, we should obtain very strong upper limits on $\beta(M)$ at $M<10^9$ g: $\beta<10^{-20}\,M^{3/2}$ (g). This limit follows from the arguments about the age of the Universe and it is the same as the argument for PBH with M in the range 10^{16} g$<M<10^{48}$ g. In particular, $\beta(M)<10^{-27,5}$ at $M\sim10^{-5}$ g corresponds to the meansquare amplitude of metric perturbations $h<0.03$ on the scale 10^{-33} cm (if the gaussian distribution for perturbations and equation of state $P=\varepsilon/3$ are assumed). Therefore, the cosmological arguments present an evidence that either BH's evaporate completely or metric fluctuations on the scale 10^{-33} cm are significantly less than unity.

4. Conclusion

As a result, it should be concluded that the initial singularity was smooth enough in any scales.

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