

The Obliquity of the Ecliptic

A. Wittmann

Universitäts-Sternwarte, Geismarlandstr. 11, D-3400 Göttingen, Federal Republic of Germany

Received July 25, 1978

Summary. In view of the discrepancy between observed and calculated secular rates of change $d\varepsilon/dT$, a reassessment of observed values of the obliquity of the ecliptic ε is made. Least-squares adjusted values of the obliquity are derived for a number of epochs (1000 B.C. to AD 2000). A simple numerical expression for the obliquity is given which supersedes the standard expression and amply approximates the expected cyclic behaviour for a time span of at least 10^6 years.

Key words: ecliptic – obliquity – precession

1. Introduction

The mean apparent obliquity of the ecliptic is presently decreasing by approximately $47''$ per century. At the standard epoch 1900.0 (JD 2415020.0) the observed rate is $d\varepsilon/dT = -47''.11 \pm 0''.05$ (cf. Table 1 of Lieske, 1970), whereas the theoretical rate calculated from planetary perturbations is $d\varepsilon/dT = -46''.83 \pm 0''.01$ (Newcomb, 1906; Lieske, 1970; Laubscher, 1972).

As Lieske (1970) has pointed out, the discrepancy could be due to observational or computational errors, a residual motion of the earth's mantle (cf. Aoki, 1969), or a combination of both. As the discrepancy has not yet been resolved in a satisfactory way, it will be of interest to review the main observational evidence and to compare it with the generally adopted expression for the obliquity, which was derived by Newcomb from the theoretical rate of change (e.g. Newcomb, 1906). If T is reckoned in centuries from 1850.0 (JD 2396758.0), Newcomb's polynomial fit reads:

$$\varepsilon = 23^\circ 27' 31''.68 - 46''.837 T - 0''.0085 T^2 + 0''.0017 T^3. \quad (1)$$

Equation (1) was originally derived from the motion of the ecliptic between $T = -2.5$ and $T = +2.5$ with T in tropical centuries. If T is reckoned in julian centuries of 36,525 days, this will have a negligible effect. For instance, at the epoch 2000 the difference amounts to only $0''.0015$. Therefore, Eq. (1) may be treated as if it were derived with T in julian centuries, and the coefficients may readily be transformed to more recent standard epochs $T = 0$. For instance, with T reckoned from 1900.0 we have from Eq. (1):

$$\varepsilon = 23^\circ 27' 08''.26 - 46''.844 T - 0''.00595 T^2 + 0''.0017 T^3. \quad (2)$$

For the present investigation we shall make use of the standard expression adopted by the Astronomical Ephemeris (Wilkins, 1960):

$$\varepsilon = 23^\circ 45' 22.94 - 1^\circ 30' 12.5 \cdot 10^{-2} T - 1^\circ 64 \cdot 10^{-6} T^2 + 5^\circ 03 \cdot 10^{-7} T^3. \quad (3)$$

Table 1. Observed obliquity ε

epoch	author/source	T	$\varepsilon(^{\circ})$
-1100	Chou Li	-30.0	23.881
-350	Pytheas	-22.50	23.819
-250	Erathostenes	-21.50	23.856
-150	Hipparchus	-20.50	23.858
-114	Greece (Newton, 1974)	-20.1397	23.7183
-30	Liu Hsiang	-19.2997	23.745
89	Chia Khuei	-18.1097	23.66
173	Liu Hung & Tshai Yung	-17.2697	23.837
173	Liu Hung & Tshai Yung	-17.2697	23.689
178	Liu Hung & Tshai Yung	-17.2197	23.631
450	Tsu Chhung-Chih	-14.4997	23.636
630	Li Shun-P'eng	-12.6997	23.661
800	al-Ma'mūn	-10.9997	23.562
820	Hsü Ang	-10.7997	23.541
845	Benū Mūsā	-10.5497	23.583
885	al-Battāni	-10.1497	23.583
900	Pien Kang	-9.9997	23.556
911	Ibn Corrah	-9.8897	23.558
918	Abdel ben Amājūr	-9.8197	23.583
965	al-Sūfi	-9.3497	23.6525
970	Abu Jaafar al-Chazzan	-9.2997	23.5833
994	al-Chojendi	-9.0597	23.5392
999	al-Buziani & Abu Hamed	-9.0097	23.5833
1003	Ibn Junis	-8.9697	23.5833
1007	Abul Rihan	-8.9297	23.5833
1019	al-Biruni	-8.8097	23.5833
1061	al-Zarkālī	-8.3897	23.5583
1140	Liu Hsiao-Jung	-7.5997	23.5245
1174	Moses ben-Maimon	-7.2597	23.5000
1240	al-Marrākusi	-6.5997	23.5625
1278	Kuo Shou-Ching	-6.2197	23.5367
1290	Nassir Odin	-6.0997	23.5000
1363	Ibn al-Shātir	-5.3697	23.5167
1436	Mirza Ulugh Beg	-4.6397	23.5047
1490	Walther / Lacaille	-4.0997	23.4964
1587	Tycho Brahe / Riccioli	-3.1297	23.5067
1590	Tycho Brahe / Lalande	-3.0997	23.4978
1646	Riccioli	-2.5397	23.5056
1650	Boulliaud	-2.4999	23.5333
1661	Hevelius / Lalande	-2.3899	23.4861
1672	Cassini & Cassini	-2.2799	23.4825
1690	Flamsteed	-2.0999	23.4835
1703	Bianchini	-1.9700	23.4764
1709	Rømer / Horrebow	-1.9100	23.4797
1737	de la Condamine	-1.6300	23.4733
1743	de Thury	-1.5700	23.4764
1743	le Monnier	-1.5700	23.4761
1750	Bradley	-1.5000	23.4717
1750	Lacaille	-1.5000	23.4719
1756	Mayer	-1.43997	23.47111

Table 1. (Continued)

1800	Bessel	-1.00000	23.465222
1800	Peters	-1.00000	23.465061
1850	Leverrier	-0.49999	23.458842
1850	Hansen & Olufsen	-0.49999	23.458728
1850	Newcomb	-0.49999	23.458800
1870	van de Sande Bakhuyzen	-0.29999	23.456111
1900	---	0.00000	23.452174

Table 2. Least-squares adjusted obliquity ε

Epoch	T	$\varepsilon(^{\circ})$
-1000	-29.0	23.89 ± 0.040
0	-18.9997	23.75 ± 0.060
900	-9.9997	23.581 ± 0.007
1500	-3.9997	23.507 ± 0.010
1600	-2.9999	23.495 ± 0.010
1700	-2.0000	23.482 ± 0.009
1750	-1.49996	23.471837 ± 0.000049
1800	-1.00000	23.465291 ± 0.000028
1850	-0.49999	23.458746 ± 0.000010
1900	0.00000	23.452195 ± 0.000011

Here and in what follows T is reckoned in julian centuries from 1900.0.

According to the way it has been derived, Eq. (3) will strictly be applicable for small $|T|$ only. For large $|T|$ the cubic term will cause a runaway of the polynomial beyond any finite limit, while it is known that the obliquity actually oscillates in a quasi-periodic fashion between approximately $22^{\circ}5$ and $24^{\circ}5$. Recent numerical integrations lead to a mean period of the order of 41,000 yr (Berger, 1976). For example, in 47,000 B.C. (i.e. $T = -489$) the obliquity was approximately $24^{\circ}4$, whereas Eq. (3) yields $\varepsilon = -29^{\circ}4$.

For quite a range of applications (such as long-term ephemerides, astroarcheology, paleoclimatic investigations, etc.) it is necessary to have for $\varepsilon(T)$ a numerical expression with the following properties: (a) it shall reproduce the most reliably determined values of ε (i.e. from AD 1700 onwards) with negligible error; (b) the accuracy for $T \geq -100$ shall be better or at least as good as that of Eq. (3); and (c) it shall approximately reproduce the expected cyclic behaviour with a mean period of the order of 41 centuries. In the present paper most of the observationally determined values of the obliquity – including the value adopted by Newcomb for the epoch 1850.0 – are numerically analyzed and compared with Newcomb's theoretical expression. Finally, an empirical expression is derived which has the aforementioned properties and which largely supersedes Eq. (3).

2. Analysis of Observations

In ancient times the obliquity was determined by measuring the length of the shadow cast by a gnomon (a vertical pole) at the time of the summer and winter solstices. Most of these measurements were made in China and have been discussed in detail by Needham (1959). For an assessment of the accuracy involved see Newton (1973). More recent and more accurate determinations from positional observations have been summarized by e.g. Flamsteed (1735), Lalande (1771), Ambronn (1921), Zinner (1931), and

Table 3. Calculated obliquity and residuals

T	ε	Eq. (4)	Eq. (3)	ε -Eq. (4)	ε -Eq. (3)
-9935.	24.05	24.34	-493264.26	-0.29	+493288.31
-707.	22.36	22.66	-145.92	-0.30	+168.28
-489.	24.40	24.28	-29.39	+0.12	+53.79
-285.	22.20	22.70	15.38	-0.50	+6.82
-178.	23.50	23.88	22.88	-0.38	+0.62
-100.	24.26	24.35	24.23	-0.09	+0.03
-29.0	23.89	23.825637	23.816010	+0.064363	+0.073990
-18.9997	23.75	23.700510	23.695486	+0.049490	+0.054514
-9.9997	23.581	23.583736	23.581749	-0.002736	-0.000749
-3.9997	23.507	23.504833	23.504282	+0.002167	+0.002718
-2.9999	23.495	23.491661	23.491302	+0.003339	+0.003698
-2.0000	23.482	23.478488	23.478309	+0.003512	+0.003691
-1.49996	23.471837	23.471902	23.471807	-0.000065	+0.000030
-1.00000	23.465291	23.465319	23.465305	-0.000028	-0.000014
-0.49999	23.458746	23.458736	23.458800	+0.000010	-0.000054
0.00000	23.452195	23.452157	23.452294	+0.000038	-0.000099
+104.	22.60	22.64	22.65	-0.04	-0.05
+220.	23.50	23.74	25.87	-0.24	-2.37
+311.	24.00	24.35	34.38	-0.35	-10.38
+518.	22.50	22.64	86.18	-0.14	-65.68
+715.	24.30	24.36	197.17	-0.06	-172.87
+9751.	24.20	24.34	166094.36	-0.14	-466070.16

Wilkins (1960). For a discussion of a few misleading quotations in the literature see Needham (1959) and Newton (1974).

The 57 determinations of the obliquity that will be analyzed below are summarized in Table 1 which has been compiled from the sources quoted above. The value for 1900 has been inferred from the secular rate of change determined by several authors (see Wilkins, 1960).

The data of Table 1 may conveniently be split into three parts: (i) data for epochs later than 1749; from these ε can be inferred with an accuracy of $\pm 0''.18$ or better. (ii) data for epochs between 800 and 1749; from these ε can be inferred with an accuracy of approximately $\pm 36''$. (iii) data before 800; in this case the accuracy is limited to approximately $\pm 0''.06$.

By going backward in time, a series of least-squares fits involving the items of Table 1 in reversed order gave the results listed in Table 2. If, for example, the last N rows of Table 1 are used, one obtains for, say, $N=10$: $\varepsilon = (23.452203 \pm 0.000064) - (0.013061 \pm 0.000066) \cdot T$. At $T = -1.49996$ this leads to $\varepsilon = 23.471794$. Then $\varepsilon(-1.49996)$ is obtained as the mean from all those fits [$N=2, 3, \dots, 10$ in case of group (i)]. The least-squares adjusted obliquity obtained in this way will represent a weighted mean with the larger weight attached to the more recent data.

3. Results and Conclusions

The data of Table 2 were approximated by the expression

$$\varepsilon = \varepsilon_0 + \varepsilon_1 \sin(\varepsilon_2(T + \varepsilon_3)). \quad (4a)$$

Here again an unweighted least-squares fit would not be adequate because the accuracy of the data decreases with decreasing T . However, a weighted fit fulfilling conditions (a)–(c) was obtained as follows:

$$\varepsilon_0 = 23^{\circ}49'69.32 \pm 0''.001200 \quad (4b)$$

$$\varepsilon_1 = -0''.860 \pm 0''.005 \quad (4c)$$

$$\varepsilon_2 = 0.01532 \pm 0.0009 \text{ (rad/century)} \quad (4d)$$

$$\varepsilon_3 = 3.40 \pm 0.10 \text{ (centuries)} \quad (4e)$$

Note that 8 decimal places of ε_0 must be retained in order to meet condition (a).

In Eq. (4) the period is $200\pi/\varepsilon_2 = 41013$ yr, which is essentially identical with the mean period calculated by Berger (1976). From Eq. (4) the secular rate of change at the epoch 1900.0 is

$$\left(\frac{d\varepsilon}{dT}\right)_{T=0} = \varepsilon_1 \varepsilon_2 \cos(\varepsilon_2 \varepsilon_3) = -47''.37 \pm 0''.39, \quad (5)$$

while from the least-squares fits of group (i) alone we have:

$$\left(\frac{d\varepsilon}{dT}\right)_{T=0} = -47''.13 \pm 0''.15. \quad (6)$$

As the theoretical rate seems to be fairly well established by independent calculations, this result provides additional evidence in favour of a significant discrepancy ($\approx -0''.4$) between observations and theory, which – if positively confirmed by new observations – may have important implications upon precession (cf. Lieske et al., 1977), proper motions, rotation of the galaxy, rotation of the earth, continental drift, etc.

On the other hand it is seen from Eq. (5) and (6) that the discrepancy becomes smaller for the more recent observations, suggesting that the discrepancy may be entirely observational in nature. An analysis kindly carried out by J. H. Lieske has shown that two other reasonable ways of assigning weights to the data of Table 1 lead to $d\varepsilon/dT = -46''.98$, so that the most probable observed rate becomes $(d\varepsilon/dT)_{T=0} = -47''.11$. In this way the discrepancy could nearly be “weighted away”, but any more definite conclusion must be based on new observational data.

The obliquity calculated from the two representations (3) and (4) is compared in Table 3, where the first and last rows correspond to the approximate epochs of minimum, mean or maximum obliquity from Berger (1976).

For group (i) the sums of the squared deviations are $s_3 = 1.38 \cdot 10^{-8}$ for Eq. (3) and $s_4 = 6.55 \cdot 10^{-9}$ for Eq. (4). For groups (ii) and (iii) taken together we have $s_3 = 0.00848$ and $s_4 = 0.00414$.

From the foregoing analysis the predicted obliquity at the forthcoming standard epoch 2000.0 (JD 2451545.0) is $\varepsilon = 23^\circ 43' 10.7'' \pm 0''.000054$. For comparison, Lieske et al. (1977) have adopted $\varepsilon = 23^\circ 43' 29.1''$, while Eqs. (3) and (4) yield $\varepsilon = 23^\circ 43' 28.0''$ and $\varepsilon = 23^\circ 43' 00.5''$, respectively.

Acknowledgement. The computations were made with the Univac 1108 MPS of the Gesellschaft für wissenschaftliche Datenverarbeitung mbH, Göttingen.

References

- Ambrohn, L.: 1921, *Astronomie*, p. 142, J. Hartmann (Ed.), Teubner, Leipzig
- Aoki, S.: 1969, *Astron. J.* **74**, 284
- Berger, A. L.: 1976, *Astron. Astrophys.* **51**, 127
- Flamsteed, J.: 1735, *Historia Coelestis Britannica*, Vol. 3, p. 26; Meere, London
- Lalande, J. J. F.: 1771, *Astronomie*, Vol. 3, p. 139; Desaint, Paris
- Laubscher, R. E.: 1972, *Astron. Astrophys.* **20**, 407
- Lieske, J. H.: 1970, *Astron. Astrophys.* **5**, 90
- Lieske, J. H., Lederle, T., Fricke, W., Morando, B.: 1977, *Astron. Astrophys.* **58**, 1
- Needham, J.: 1959, *Science and Civilization in China*, Vol. 3, p. 284; Cambridge University Press, Cambridge
- Newcomb, S.: 1906, *A Compendium of Spherical Astronomy*, p. 237; Macmillan, New York
- Newton, R. R.: 1973, *Quart. J. Roy. Astron. Soc.* **14**, 367
- Newton, R. R.: 1974, *Monthly Notices Roy. Astron. Soc.* **169**, 331
- Wilkins, G. A. (Ed.): 1960, *Explanatory Supplement to the Astronomical Ephemeris*, pp. 180, 187; H. M. Stationery Office, London
- Zinner, E.: 1931, *Geschichte der Sternkunde*, p. 288; Springer, Berlin