Long-wavelength perturbations of a Friedmann universe, and anisotropy of the microwave background radiation

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Observational evidence, which is necessarily confined to a region of the universe limited in space (within the observer's horizon), implies a high degree of homogeneity and isotropy for the large-scale structure of the universe. In principle, substantial deviations of the properties of the real universe from the parameters of an idealized Friedmann cosmological model could have prevailed on scales exceeding that of the horizon. Constraints on the amplitude of perturbations with such long wavelengths are imposed by the virtual isotropy ($\delta T/T < 10^{-4}$) of the observed background radiation. This information on $\delta T/T$ together with the natural hypothesis that the perturbations are statistically independent implies that on spatial scales exceeding the horizon there exist no significant (with amplitude of order greater than $\delta T/T$) perturbations in density. For certain types of perturbations in the metric (in the gravitational field), the amplitude could be appreciable without contradicting the empirical limits on $\delta T/T$.

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Studies of the 3°K microwave background radiation are a most valuable tool for probing the large-scale structure of the universe.[1] Empirical evidence concerning the spectrum and angular distribution of this radiation can set limits on the perturbations that may exist – the departures of the properties of the real world from the parameters of an idealized Friedmann cosmological model. But the observations are necessarily confined to a region restricted in space. If we nevertheless take advantage of the measurements that have been made and invoke a few general hypotheses, what can we say about the state of the universe beyond the region which is in principle accessible to observation today? To answer this question, we should at the outset make clear what is meant by the observable region, perturbations and their Fourier spectrum, and statistical independence.

We are presently observing photons emitted by the primordial plasma in the remote past, at an epoch when the plasma had become transparent as a result of recombination. Thus the photons of the background radiation have been traveling freely, without being scattered, for an extremely long although finite time. The time of free photon propagation determines the spatial scale that we call the horizon of recombination.

Small perturbations of the Friedmann model can be expanded in Fourier harmonics, each characterized by the corresponding wavelength λ_n. As time passes the wavelength changes in proportion to the scale factor of the isotropic universe.

Among the perturbations of various wavelengths, there are some for which λ_n exceeds not only the horizon of recombination but also the observer's horizon (or simply the horizon), as specified by the finite time of expansion from a superdense (singular) state to the present epoch. (Actually the relative difference between the recombination horizon and the observer's horizon is small, amounting to just a few percent.) Such waves are said to be long.

Now we should ascertain what quantitative constraints on the amplitude of long-wavelength perturbations can be imposed in view of the observed high degree of isotropy of the background radiation: $\delta T/T < 10^{-4}$. Might it not turn out that at our present epoch, on scales greater than the horizon, substantial perturbations in the density and in the metric exist (say with an amplitude $\delta\rho/\rho \approx 10^{-1}$, or with a dimensionless amplitude in the metric of the same order) which we do not even suspect, because we cannot yet observe them directly? Such effects would become accessible only to astronomers of the very remote future, $t \gg 2 \cdot 10^{10}$ yr, when the observer's horizon and the recombination horizon become comparable with the corresponding wavelength.

We would emphasize that we are here interested in smooth, long-wavelength perturbations that encompass the region of space inside the horizon as well, rather than perturbations that originate beyond the horizon, remaining always unchanged within it. In other words, we shall make the natural assumption that harmonic perturbations of differing wavelength are not specially correlated. If this were not the case, they could be selected in such a way that within the horizon the deviations from the Friedmann model would be particularly small (and hence $\delta T/T$ would be small), but beyond the horizon the perturbations would be large. But such a choice of different harmonics is most unlikely, for it would imply that an observer on the earth is in a singular position. Our starting assumption is, on the contrary, that all observers are equivalent and perceive approximately the same picture; thus for all observers, even for those causally unrelated, $\delta T/T < 10^{-4}$. The question nevertheless remains of whether small deviations, not noticeable with measurements of the present accuracy within each observer's horizon, might add up to a substantial large-scale perturbation. Graphically expressed, might not observers living on the slopes and humps and in the valleys of a long density wave or gravitational wave be able to recognize this fact by examining, with limited accuracy, only their immediate neighborhood?

The conclusion we shall reach may be stated as follows. Measurements of $\delta T/T$ indicate that the density perturbations cannot be appreciable; on any scale they are limited to an amplitude which is an infinitesimal of at least the same order as $\delta T/T$. As for the perturbations in the metric, for the same empirical constraints on $\delta T/T$ these can be larger than $\delta T/T$, and indeed in the range of long wavelengths the metric perturbations can reach values approaching unity, so that they are at the limit of applicability of the theory of small perturbations.

In this respect, with a restriction of order $\delta T/T$ imposed on the dimensionless amplitude of the perturbations, the result may appear paradoxical. At first glance it may seem obvious that for a fixed recombination horizon, a transition to increasingly long wavelengths would be equivalent to smoothing out the perturbations within the horizon, diminishing their contribution to $\delta T/T$, and thereby permitting a rise in the admissible amplitude of the perturbations without coming into conflict with the observations. Actually, however, it is not only the spatial dependence of a perturbation that is important, but also the speed of its time variation. A long-wavelength perturbation will manifest itself not as a wave but as an anisotropy in the deformation. It will make some contribution to $\delta T/T$ even as the wavelength characterizing the spatial periodicity increases without bound.

However, perturbations of the metric also exist which vary slowly with time and induce a particularly small deformation anisotropy. The constraints on the amplitude of such perturbations resulting from their connection with $\delta T/T$ will be relaxed. This is the reason why perturbations of the metric with a growing density mode, as well as metric perturbations such as the nonsingular mode of gravitational waves, may exceed $\delta T/T$ and reach substantial values for values of long wavelength.

We shall examine here a homogeneous and isotropic model with flat three-dimensional space, the density being equal to the critical density. Assume that "smeared out" matter has the equation of state $P = 0$, which is a good approximation for the postrecombination era. Then the metric of the universe (the gravitational field), including small perturbations, can be described by the line element

$$ds^2=a^2(\eta)(\eta_{ab}+h_{ab})dx^a dx^b=a^2(\eta)$$

$$(d\eta^2-dx^2-dy^2-dz^2)+a^2(\eta)h_{ab}dx^a dx^b.$$

The corrections h_{ab} are determined by solutions of the Einstein equations. As Lifshits demonstrated,[2] three independent types of corrections can be identified: 1) density perturbations; 2) eddy perturbations; 3) gravitational waves.

One should keep in mind that prior to recombination matter was hot, possessing entropy, so that density perturbations could have been either adiabatic or nonisentropic.[3] After recombination the adiabatic perturbations could have had a growing and decaying mode, whereas the entropy perturbations in the long-wavelength range would have been transformed only into the decaying mode of density perturbations. In the discussion below we shall consider the entropy perturbations separately.

The metric h_{ab} will be represented in the form proposed by Sachs and Wolfe,[4] except for trivial changes of notation. If $P = 0$, the equations given by Lifshits[2] and Sachs and Wolfe[4] coincide for all perturbations except for the eddy (rotational) perturbations.

Suppose that the photons become free instantaneously at the epoch η_E of emission. Let η_R denote the epoch of reception today. If the last scattering of photons occurred at the epoch of recombination, when $z \approx 1400$, then[1] $\eta_E/\eta_R \approx 1/40$. The direction of arrival of a light ray is defined by a vector e^α having the components $e^\alpha = \{\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta\}$. If at the epoch η_E of photon emission the temperature of the primordial background radiation was everywhere the same and equal to T_E, then at the time of photon arrival it would possess variations depending on the angle of arrival and on the observer's position. The temperature will take the value

$$T_R=T_E\frac{\eta_E^2}{\eta_R^2}\left(1+\frac{\delta T}{T}\right),$$

where[2]

$$\frac{\delta T}{T}=\frac{1}{2}\int\limits_0^{\eta_R-\eta_E}\left(\frac{\partial h_{\mu\beta}}{\partial\eta}e^\mu e^\beta-2\frac{\partial h_{0\beta}}{\partial\eta}e^\beta\right)dw. \tag{1}$$

The integration extends along the light geodesic $\eta = \eta_R - w$, $x^\alpha = e^\alpha w$, with a parameter w varying from zero to $\eta_R - \eta_E$.

We shall consider the three types of perturbations individually, representing each of them by a single plane wave.

1. <u>Density perturbations.</u> The growing mode of density perturbations is described by the equations

$$\frac{\delta\rho}{\rho}=\frac{1}{2}h(n\eta)^2e^{i(nx+\xi)},$$

$$h_{\mu\beta}=(10h\eta_{\mu\beta}+h\eta^2n_\mu n_\beta)e^{i(nx+\xi)},\qquad h_{0\mu}=0, \tag{2}$$

where h depends, in general, on n. The number n, the modulus of the wave vector, characterizes the spatial periodicity of the perturbation. For long waves the condition $n\eta_R < 1$ is satisfied. Here and subsequently we shall regard the z axis as directed along the vector $\mathbf{n}$, with the observer having coordinates $x = y = z = 0$.

Substituting the expressions (2) into Eq. (1) and integrating, we find that

$$\frac{\delta T}{T}=he^{i\xi}[in\eta_R\cos\theta-in\eta_E\cos\theta e^{in(\eta_R-\eta_E)\cos\theta}+1-e^{in(\eta_R-\eta_E)\cos\theta}].$$

In the long-wavelength limit, with $n\eta_R \ll 1$, we have

$$\frac{\delta T}{T}=\frac{1}{2}h(n\eta_R)^2\cos^2\theta\left\{\left(1-\frac{\eta_E^2}{\eta_R^2}\right)\cos\xi\right.$$

$$\left.-\frac{1}{3}n\eta_R\cos\theta\left(1-3\frac{\eta_E^2}{\eta_R^2}+2\frac{\eta_E^3}{\eta_R^3}\right)\sin\xi\right\}. \tag{3}$$

Since $\frac{1}{2}h(n\eta_R)^2 = (\delta\rho/\rho)_R$, it is evident from Eq. (3) that at the present epoch large $\delta\rho/\rho$ values are precluded in the long-wavelength range by the high degree of isotropy of the background radiation. As for the amplitude of the metric, these restrictions become considerably weaker because of the small value of the factor $(n\eta_R)^2$.

The parameter ξ describes the relative position of the observer. For $h > 0$ and $\xi = 0$ the observer will be at a maximum point of the density perturbation; for $\xi = \pi$, at a minimum. The integral of $\delta T/T$ over a sphere will be greater than zero for $\xi = 0$, so that at the density maximum the radiation will appear somewhat hotter than the average over all space. Correspondingly, if $\xi = \pi$ the radiation will seem just as much cooler. Only the angular variations in the temperature are significant for the observer, since he has no opportunity to compare the mean temperature he observes with the temperature averaged over all space.

If the observer is located near the inflection points, that is, at $\xi \approx \pm\pi/2$, then for fixed limits on $\delta T/T$ the quantity $(\delta\rho/\rho)_R$ can be increased because of the small factor $n\eta_R \cos\theta$. However, the probability that the observer will be situated in this range of ξ values is very small, since the departures from $\xi = \pm\pi/2$ should not exceed a small quantity of order $n\eta_R \cos\theta$. As very long wavelengths are approached ($n\eta_R \to 0$), this ξ interval will also tend to zero.

For the decaying mode of density perturbations we have

$$\frac{\delta\rho}{\rho}=\frac{1}{2}hn^2\frac{1}{\eta^3}e^{i(nx+\xi)}, \quad h_{\mu\beta}=\frac{1}{\eta^3}hn_\mu n_\beta e^{i(nx+\xi)}, \quad h_{0\mu}=0. \tag{4}$$

If $n\eta_R \ll 1$, an evaluation of $\delta T/T$ yields

$$\frac{\delta T}{T}=-\frac{1}{2}hn^2\cos^2\theta\frac{1}{\eta_E^3}\left\{\left(1-\frac{\eta_E^3}{\eta_R^3}\right)\cos\xi -n\eta_R\cos\theta\left(1-\frac{3\eta_E}{2\eta_R}+\frac{\eta_E^3}{2\eta_R^3}\right)\sin\xi\right\}=-\left(\frac{\delta\rho}{\rho}\right)_E\cos^2\theta\,\Phi(\xi,\theta), \tag{5}$$

where $\Phi(\xi, \theta)$ designates the expression in braces.

Since

$$\frac{1}{2}hn^2\frac{1}{\eta_E^3}=\left(\frac{\delta\rho}{\rho}\right)_E,$$

the density and metric perturbations at epoch η_E could not have exceeded $\delta T/T$, and a fortiori they should be small today.

After recombination, entropy perturbations in the long-wavelength range will have gone over primarily to the decaying mode of the density perturbations. In fact, the general equations for entropy perturbations[5] provide us with asymptotic relations in the long-wavelength limit which are applicable both before and after recombination. During the stage when radiation dominates (but after the annihilation of antibaryons has terminated, when $kT \ll m_pc^2$), an entropy perturbation of the total density will grow according to the slower of the two modes characteristic of adiabatic perturbations, that is, according to the law

$$\frac{\delta\varepsilon}{\varepsilon}\sim\left(\frac{\delta S}{S}\right)\frac{\eta}{\eta_e} \quad (\eta\ll\eta_e), \tag{6}$$

whereas the fast mode would give a law of the form $\delta\varepsilon/\varepsilon \sim \eta^2$ [In Eq. (6), η_e designates the epoch when the radiation density and the nonrelativistic plasma density are equal; $\delta S/S = \mathrm{const}\cdot e^{inx}$ represents the entropy perturbation.] During the stage when nonrelativistic plasma dominates (when $\eta \gg \eta_e$), the slower mode will be the decaying mode $\delta\rho/\rho \sim \eta^{-3}$ of density perturbations; by contrast, the growing mode will give $\delta\rho/\rho \sim \eta^2$. One finds that after recombination a density perturbation will in fact follow the $\delta\rho/\rho \sim \eta^{-3}$ law (at least until the wavelength of the perturbation exceeds the scale of the horizon). This statement is demonstrated by an actual calculation. If $\eta \gg \eta_e$, one can obtain for perturbations in the metric and in the density the approximate relations

$$\frac{\delta\varepsilon}{\varepsilon}=\frac{\delta\rho}{\rho}=\frac{1}{2}Cn^2\left(\frac{\eta_e}{\eta}\right)^3e^{i(nx+\xi)}, \quad h_{\mu\beta}=C\left(\frac{\eta_e}{\eta}\right)^3n_\mu n_\beta e^{i(nx+\xi)}, \tag{7}$$
$$h_{0\mu}=0 \quad (n\eta<1),$$

where C is a constant related to $\delta S/S$. The perturbations in the velocity are negligible. In the long-wavelength range, then, we obtain from entropy perturbations after recombination the decaying mode of density perturbations.

An intuitive explanation can also be given for this fact.[1] An entropy perturbation, by changing the state of primordial matter in various regions of space (within the wavelength of the perturbation) in the initial state, will insignificantly alter the total density and space geometry of the universe near the singularity. In every part of space, a three-dimensionally flat world will remain practically flat. After recombination, the density will be determined by nonrelativistic gas with a pressure $P = 0$ and, without regard to initial deviations in the entropy, it will fall off according to the law $\rho = [6\pi G(t+\tau)^2]^{-1}$. The quantity τ corrects for the fact that one cannot consider $P = 0$ at the beginning of the expansion. Entropy perturbations will cause τ to differ in different parts of space.

It is readily verified that the difference in density (that is, the perturbation) will diminish with increasing t as $\delta\rho/\rho = -2\delta\tau/t+\tau$, where the time t is measured from the singularity. Thus as time passes the density perturbation will behave in accordance with the decaying mode. This process will continue until the wavelength of the perturbation becomes comparable with the horizon. At that time very insignificant initial deviations in the geometry of various parts of space from a flat geometry and differences in the radiation density will become important. Various parts of the universe will now increasingly resemble a spatially open or a spatially closed world. The growing mode of density perturbations, negligibly small immediately after recombination, will surpass the decaying mode in amplitude a certain time after equality between the horizon and the wavelength has been achieved, and will thenceforth determine $\delta\rho/\rho$.

With regard to the question of the $\delta T/T$ produced by long-wavelength perturbations, we are interested in the solution (7). This solution coincides with Eq. (4) and leads to the same conclusions. We may infer from the limits $\delta T/T < 10^{-4}$ that density perturbations (originating as entropy perturbations) and the corresponding perturbations in the metric are bounded by a quantity of order 10^{-4} in the long-wavelength range.

2. Eddy perturbations. A rotational perturbation in the metric will have the form

$$\delta\rho=0,\quad h_{\mu\beta}=-\frac{2}{n}(n_\beta\varkappa_\mu+n_\mu\varkappa_\beta)\left(\frac{8}{\eta^3}+\frac{n^2}{\eta}\right)e^{i(nx+\xi)},$$

$$h_{0\mu}=-i\frac{2}{\eta^2}n\varkappa_\mu e^{i(nx+\xi)},\quad \varkappa^\mu n_\mu=0.$$

The last condition here means that $\varkappa^\mu$ has the components $\varkappa^\mu = (C_1, C_2, 0)$. The angular velocity vector Ω^α of matter lies in the (x, y) plane. If $C_1 = 0$ the vector Ω^α will be oriented along the x axis; if $C_2 = 0$, along the y axis. The observer is located at the maximum absolute value $\Omega_\alpha\Omega^\alpha$ for $\xi = 0$ and $\xi = \pi$, and the orientation of Ω^α is here distinguished by sign.

If $n\eta_R \ll 1$, we obtain for $\delta T/T$ the expression

$$\frac{\delta T}{T}=8\sin 2\theta(C_1\cos\varphi+C_2\sin\varphi)$$
$$\times\frac{1}{\eta_E^3}\left\{\Phi(\xi,\theta)+\left[\frac{n\eta_E}{8\cos\theta}\left(1-\frac{\eta_E^2}{\eta_R^2}\right)\right]\right\}. \tag{8}$$

The bracketed term owes its origin to the components $h_{0\mu}$. The integral of $\delta T/T$ over a sphere everywhere vanishes.

Equation (8) implies that with the constraints that can now be placed on the quantity $\delta T/T$, the amplitude of eddy-type perturbations in the metric cannot be significant for long wavelengths. It is worth recalling, incidentally, that near the singularity the eddy perturbations in the metric are large and are divergent.[6,7]

3. Gravitational waves. The perturbation in the metric is

$$\delta\rho=0,\quad h_{\mu\beta}=-\frac{d_{\mu\beta}}{\eta^3}(1-in\eta)e^{i(nx+n\eta+\xi)},$$
$$h_{0\mu}=0,\quad d_{\mu\beta}n^\beta=0,\quad d_\mu{}^\mu=0. \tag{9}$$

The nonvanishing components of the matrix $d_{\alpha\beta}$ will be written in the form $d_{11} = -d_{22} = C_1$, $d_{23} = d_{32} = C_2$, where C_1, C_2 correspond to the two possible polarizations of the wave.

In the form (9) given above, the real part of the solution describes the decreasing (singular) mode of the perturbations, divergent near the singularity; the imaginary part of the solution corresponds to the nonsingular mode, which, like the growing mode of density perturbations, is compatible with a quasi-isotropic (locally Friedmann) solution near the singularity.

If $n\eta_R \ll 1$, we have approximately for the decaying mode

$$h_{\mu\beta}=-\frac{d_{\mu\beta}}{\eta^3}\left(1+\frac{(n\eta)^2}{2}+\dots\right)e^{i(nx+\xi)},$$

which gives

$$\frac{\delta T}{T}=\frac{1}{2}\sin^2\theta(C_1\cos 2\varphi+C_2\sin 2\varphi)\frac{1}{\eta_E^3}\Phi(\xi,\theta). \tag{10}$$

Since the components C_1, C_2 directly yield the characteristic amplitude of the wave for the decaying mode (that is, the value of the metric at the epoch when the length of the perturbation becomes comparable with the horizon), Eq. (10) implies that the amplitude of the decaying mode cannot be greater than $\delta T/T$.

For the nonsingular mode we have approximately

$$h_{\mu\beta}\approx-\frac{1}{3}d_{\mu\beta}n^3\left(1-\frac{(n\eta)^2}{10}+\dots\right)e^{i(nx+\xi)},$$

which leads to the expression

$$\frac{\delta T}{T}=\frac{n^3}{60}\sin^2\theta(C_1\cos 2\varphi+C_2\sin 2\varphi)(n\eta_R)^2$$
$$\times\left\{\left(1-\frac{\eta_E^2}{\eta_R^2}\right)\cos\xi-n\eta_E\cos\theta\left(1-2\frac{\eta_E}{\eta_R}+\frac{\eta_E^2}{\eta_R^2}\right)\right\}. \tag{11}$$

In this case the characteristic amplitude of the wave is determined by the quantities C_1n^3, C_2n^3. In the main approximation the perturbation in the metric is constant and does not depend on time, so that the main terms make no contribution to $\delta T/T$. The relation between the quantities C_1n^3, C_2n^3, and $\delta T/T$ contains the small factor $(n\eta_R)^2$. Thus in the limit of long wavelengths the amplitude of the nonsingular mode of gravitational wave perturbations could have been comparatively high, of order $(\delta T/T)(n\eta_R)^{-2}$, without contradicting the observations.

For both modes the integral of $\delta T/T$ over a sphere vanishes. Both $\delta T/T$ itself and its gradient drop to zero for $\theta = 0$.

Equations (3), (5), (8), (10), and (11) give the exact relation between $\delta T/T$ and the amplitude of the perturbations in the long-wavelength limit. As ought to be the case, $\delta T/T \to 0$ as $\eta_E \to \eta_R$. For perturbations growing with time, the value of $\delta T/T$ will be determined by the amplitude of the perturbations at epoch η_R; for decaying perturbations, at epoch η_E. An approximate expression for $\delta T/T$ could have been obtained directly from Eq. (1) by replacing the integral by the difference in the values of the expression $h_{\alpha\beta}e^\alpha e^\beta - 2h_{0\alpha}e^\alpha$, taken at epochs η_R and η_E.

Thus empirical evidence on $\delta T/T$ in conjunction with the plausible hypothesis of statistically independent perturbations leads to the conclusion that on scales exceeding the size of the horizon there exist no significant (with an amplitude exceeding $\delta T/T$, in order of magnitude) density perturbations, rotational perturbations, or gravitational waves for the singular mode. As for gravitational waves representing the nonsingular mode, as well as perturbations in the metric associated with the growing mode of density perturbations, these could be appreciable without coming into conflict with the observational limits on $\delta T/T$.

1)This statement presuposes that secondary ionization occurred quite late, with the ionized gas having an optical depth less than unity.

2)Greek indices take the values 1, 2, 3.

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The hydrodynamics of primordial black hole formation

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The hydrodynamic behavior of primordial black hole (PBH) formation early in the expansion of the universe is examined, assuming that near the singularity the expansion was quasi-isotropic. The nonlinear, spherically symmetric problem of the development of initially strong perturbations relative to a Friedmann background model is solved numerically. The type of perturbations required for PBHs to form is ascertained. The role of pressure gradients is evaluated in detail. At the time of its formation a PBH will have a mass considerably smaller than the mass within the cosmological horizon; hence a catastrophic accretion process appears unlikely.

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1. INTRODUCTION

Zel'dovich and one of us[1–3] called attention in 1966, followed by Hawking[4] in 1971, to the possibility that black holes might have developed at the very beginning of the cosmological expansion from matter in its primordial state. Considerable work has subsequently been done (see, for example, Carr and Hawking[5,6] and Zel'dovich and Starobinskii[7]) on this problem of primordial black holes (PBHs).

Interest in the PBH question heightened after Hawking[8] discovered the principle of quantum evaporation of low-mass black holes, because PBHs would in fact have a low mass. Hawking's process is important not only for the physically early phases in the expansion of the universe, but also as a possible avenue for detecting PBHs in the universe today.[9–12]

Two problems are fundamental for the theory of PBHs: 1) What should the departures from a Friedmann cosmological model have been at the start of the expansion in order for PBHs to have formed? 2) How will surrounding matter be accreted onto a PBH that has formed?

Both these problems had already been stated and discussed in the original treatments of the PBH question.[1–3] Those and subsequent analyses have established that a full resolution of the two problems will require numerical calculations on a computer. Along with PBHs, white holes have also been considered in the literature. Quantum processes near the singularity and the accretion process (Refs. 1-3) have been shown to be important for white holes (Ref. 13), converting them into distinctive black holes.

In this paper we shall describe our method for calculating the hydrodynamics of these phenomena on a computer, adopting the simplest assumption, that the processes are spherically symmetric, and we shall give some of the results.

2. FORMULATION OF THE PROBLEM

PBHs could not have developed early in the expansion of the universe (when the equation of state was $P = \varepsilon/3$) if the deviations from a Friedmann model were small both in density and in the metric. In fact, as Lifshits demonstrated,[14] density perturbations $\delta\varepsilon/\varepsilon$ of small amplitude will grow only to the amplitude of the metric perturbations, which are presumed small. Afterward the conditions for growth of perturbations will be violated (the linear scale of the perturbations will become smaller than the horizon), and they will be transformed into acoustic vibrations.

On the other hand, deviations from a Friedmann model that would lead to the formation of PBHs are certainly possible. Suppose that near the singular state we consider a homogeneous semiclosed universe[15–17] connected by a narrow throat with a flat homogeneous cosmological model. As the semiclosed world evolves, a signal from the throat traveling at the acoustic velocity will be able to reach the interior regions of the semiclosed world when those regions are already in a state of contraction beneath their own gravitational radius, so that they necessarily constitute a black hole.

The question now arises: What would the critical deviation from the Friedmann model be in order that a black hole will form, whereas if the deviation is any smaller no hole will form? This is the problem to be solved in our present paper. It will be formulated more accurately below. In addition, we are interested in the hydrodynamic processes that should accompany the formation of PBHs.

Let us therefore consider strong departures from a Friedmann model in its initial state. In general there is a great deal of arbitrariness in the choice of initial conditions near the singularity.[18,19] It is important to recog-