Opportunities for detecting ultralong gravitational waves

M. V. Sazhin

Shternberg Astronomical Institute, Moscow

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The influence of ultralong gravitational waves on the propagation of electromagnetic pulses is examined. Conditions are set forth whereby it might be possible to detect gravitational waves arriving from binary stars. There are some prospects for detecting gravitational radiation from double superstars with masses $\mathfrak{M}_1 \approx \mathfrak{M}_2 \approx 10^{10} \mathfrak{M}_\odot$.

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Several methods have been proposed for detecting ultralong gravitational waves.[1] One possible technique would involve recording the change in the frequency of electromagnetic radiation in the gravitational wave field.[2] For pulses of electromagnetic radiation, this effect would be manifested as a change in the period between pulses. The time required for a pulse of electromagnetic radiation moving in a gravitational wave field to cover the path from the transmitter to the receiver will depend on the amplitude and the phase of the gravitational waves. If the pulses are radiated by the transmitter at equal time intervals Δ, the receiver will record them at time intervals $\Delta + \delta(t)$. The quantity $\delta(t)$ describes the shift in the pulses with respect to time.

Let us apply the geometrical optics approximation to find the change $\delta(t)$ in the interpulse period. Light rays move along trajectories for which $ds = 0$. We orient the coordinate system such that photons will travel along the x axis when gravitational waves are absent. The equation of motion will then have the form $c\,dt = dx$. If gravitational radiation comes into play, this equation will become

$$c\,dt = \left[1 + \frac{1}{2} h(t, x)\right] dx,$$

where $h(t, x)$ represents the corrections to the metric. Pulses radiated at different instants t_1, t_2 will traverse equal distances dx in different elapsed times:

$$c\,d\delta t = \frac{1}{2}\left[h(t_1, x) - h(t_2, x)\right] dx. \qquad (1)$$

The principal time shift between two pulses will build up while they traverse a distance L comparable to the wavelength. If L should be shorter than the wavelength, then $\delta(t) \approx h(t)\Delta \cdot L/\lambda$. If $L \gg \lambda$, then $\delta(t) \approx h(t)\Delta$, and the amplitude of the shift during the half-period of the gravitational waves will be $\theta \approx h\tau$, to order of magnitude.

We shall apply the theory outlined above to the pulse propagation scheme illustrated in Fig. 1. Suppose that the line joining the source of the electromagnetic pulses and the observer lies in the orbit plane of a binary star. Then the wave corrections to the metric will be

$$h(t, x) = -\frac{\alpha}{2} \frac{a^4 - a^2x^2}{(a^2+x^2)^{5/2}} \cos 2\omega \left(t - \frac{\sqrt{x^2+a^2}}{c}\right), \qquad (2)$$

where $\tau = 2\pi/\omega$ is the period of the binary star, a is the distance from the center of the binary system to the trajectory of the light rays, the coordinate x is measured along this trajectory from the point A (Fig. 1), and $\alpha = 6\pi^{2/3} c t_g^{5/3} \tau^{2/3}$. Here $t_g = (2G/c^3)(M_1M_2)^{3/5}/(M_1 + M_2)^{1/3}$; M_1 and M_2 are the masses of the two stars.

Substituting the expression (2) into Eq. (1) and integrating, we find that the interpulse period will change by

$$\delta(t) = \frac{\pi\alpha}{c} \frac{\Delta}{\tau} \int_{-b}^{s} dx \frac{a^4 - a^2x^2}{(a^2+x^2)^{5/2}} \sin\left[2\omega t + 2\frac{\omega}{c}\left(x - \sqrt{x^2+a^2}\right)\right],$$

where b is the distance from the source of the electromagnetic pulses to the point A (Fig. 1) and s is the distance from A to the observer. If the binary star is situated close to the path of the light ray, with $a^2 \ll c\tau b$ but $c\tau < a$, then after an elapsed time $\tau/4$ the shift will amount to

$$\theta = c^2 t_g^{5/3} \tau^{4/3}/a^2. \qquad (3)$$

However, if the binary star is far from the light ray, with $c\tau b^3 \ll a^4$ but $a < b$, then

$$\theta = c t_g^{5/3} \tau^{1/3} (a/b)^3/4a, \qquad (4)$$

while if $a \gg b$,

$$\theta = c t_g^{5/3} \tau^{1/3}/4a. \qquad (5)$$

One of the most appealing prospects in the search for ultralong gravitational waves would be to monitor the radiation of pulsars. The high stability of pulsar periods would give the method the requisite sensitivity. Compared to the use of drift-free satellites, pulsars would have two major advantages for detection of such signals: 1) The pulses will traverse a path $L \gg \lambda$, and the equation for

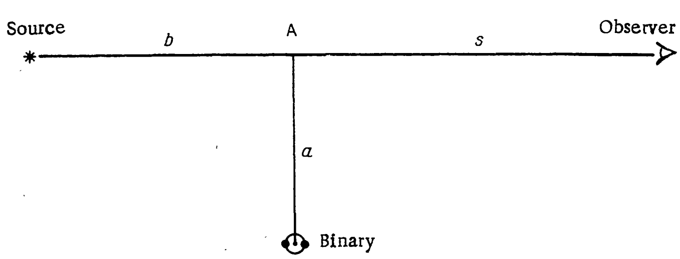

FIG. 1. Relative arrangement of the source of electromagnetic pulses, the observer, and the binary star.

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TABLE I. Binary Stars to Inspect for Adjacent Pulsars

α	δ	τ	$f\left(\frac{M}{M_\odot}\right)$	$\left(\frac{M_1}{M_\odot}\right)/\left(\frac{M_2}{M_\odot}\right)$	$\frac{\delta}{\Delta}\cdot 10^{18}$	φ°
$05^h11^m.6$	36°31′	4	–	19.7/19.7	3	1″
$06^h08^m.4$	−54°56′	1.7	–	12.7/6.7	3	0,3″
$07^h49^m.2$	22°16′	0.17	0.33	–	5.9	–
$07^h55^m.4$	−48°58′	1.45	6.77	–	25	1″
$08^h10^m.6$	−18°45′	0.07	0.6	–	70	–
$13^h32^m.7$	52°25′	0.2	0.3	–	3.9	–
$15^h35^m.3$	36°58′	12.6	–	9.4/9.9	1.5	180″
$17^h49^m.7$	−32°27′	12	8.8	–	1.5	–
$18^h46^m.4$	33°15′	12.9	8.5	–	1.5	–
$20^h15^m.0$	36°02′	1.9	–	37.3/32.7	22	0.3″
$20^h17^m.8$	36°36′	21.6	4.94	–	1.5	5″
$20^h48^m.1$	34°17′	3	–	17.6/17.3	3.4	1″
$22^h42^m.9$	57°33′	2.1	–	23.4/19.1	11	–
$22^h44^m.2$	64°34′	1.8	–	11.4/9.8	3.6	–

δ(t) will not contain the small factor L/λ that appears if satellites are used. 2) In the event that the ray from a pulsar toward the earth passes close to a binary star (with $a \approx c\tau$), the correction h to the metric will be considerably larger than at the earth, so that the shift in the pulses will be greater by a factor R/a, where R is the distance from the binary star to the earth. The action of gravitational waves on the pulses emitted by a pulsar will have the effect that sinusoidal variations will develop at the instant the pulses reach the observer.

Let us consider the possibilities for detecting gravitational waves from binary systems. Gravitational radiation would be perceptible under two conditions. First, the drift in a clock during the period of measurement must not exceed the shift in the pulses during the same time; that is, the quantity $4\,\theta/\tau \approx \delta/\Delta$ must exceed the relative accuracy of the time standard. Second, the time shift θ must be larger than the fluctuations $\sigma/\sqrt{N}$ in the arrival time of individual pulses from a given pulsar; here σ measures the size of the window for such pulses and N is the number of pulses received in time τ/4.

In the ideal case, the binary system would be so located that the distance to the ray trajectory between the pulsar and the earth is equal to the length of the gravitational waves, or $a \approx c\tau$; that is, the rays would pass adjacent to a nonwave zone of the source of gravitational radiation. Then if the binary star has parameters $M_1 \approx M_2 \approx 10^2\ M_\odot$ and $\tau \approx 10$ days, the gravitational wave effect will produce a quantity $\delta/\Delta \approx 10^{-14}$, which is at the level of the most precise time standards available, and a shift $\theta \approx 10^{-9}$ sec for a total number $N \approx 10^8$ pulses. Values of this order raise hopes that gravitational radiation might be detected.

The known short-period binaries described in Batten's catalog[3] are situated too far away from the rays joining known pulsars and the earth. Introducing the parameters of the binaries into Eqs. (3)-(5), we find that $\delta/\Delta \approx 4\theta/\tau$ is considerably smaller than the precision of existing time standards.

Two pulsar–binary star pairs turn out to have the largest shifts. The first pair consists of the Crab Nebula pulsar PSR 0531 + 22 and the binary star δ Geminorum. For this pair the shift $\theta \approx 10^{-13}$ sec and $\delta/\Delta \approx 10^{-20}$. The second pair includes the pulsar PSR 2148 + 63 and a binary star at $\alpha = 21^h54^m$, $\delta = 63°$; it has $\theta \approx 2\cdot 10^{-11}$ sec and $\delta/\Delta \approx 10^{-19}$. Clearly the detection of gravitational waves from sources confidently known to exist still lies well outside the realm of the possible. In order for gravitational waves to be detected the precision of time standards would have to be raised to roughly 10^{-19}.

Another possibility would be to search for binary stars at a distance $a \approx c\tau$ from pulsar–earth ray trajectories with parameters $t_g \approx 10^{-9}\,\tau$, or to seek pulsars in the neighborhood of binary stars. Evidently if we have different stars giving the same value of δ/Δ it will be more advantageous to seek gravitational radiation from stars with a long period, because the detection critierion $\sigma/\sqrt{N} \approx \theta$ should also be satisfied. Table I lists some stars near which a search for pulsars would be desirable. The first two columns give the right ascension and declination of each star; the third, the period in days; the fourth and fifth, the mass function and the ratio $(M_1 \sin^3 i)\cdot(M_2 \sin^3 i)^{-1}$; the sixth, the ratio δ/Δ; and the last column, the angular distance in seconds of arc at which one should look for a pulsar.

An appreciably more favorable situation will arise if one examines the likelihood of detecting gravitational waves from double superstars having masses $M \approx 10^8$–$10^{10}\ M_\odot$ and periods of several years. Suggestions have been made[4,5] that such objects ought to exist in the nuclei of certain galaxies and quasars.

If a double superstar with masses $M_1 \approx M_2 \approx 10^{10}\ M_\odot$ and a period $\tau \approx 10$ yr were located at a distance $a \approx 10^{28}$ cm, which is comparable with the observer's horizon, then in order to record its gravitational waves we would need a time standard whose relative precision is $\approx 10^{-15}$ or better, and we would have to measure a time shift of $\approx 0.1\ \mu$sec after accumulating $N \approx 10^8$–10^9 pulses in a single series of measurements. The implementation of such observations could either confirm the existence of double superstars with the mass indicated, or set an upper limit on their mass. Observations of this kind undoubtedly should be made simultaneously using several pulsars that have the most stable periods.

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Multiple scattering of pulsar signals by turbulent interstellar plasma

Dimitri S. Bugnolo*

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The stochastic transport theory originally developed by the author for the study of electromagnetic wave propagation in media characterized by a random-phase velocity is applied to the problem of the propagation of a pulsar signal through the turbulent interstellar plasma. The theoretical results are used to predict the amplitude fluctuations of pulsar radiation and the coherence of pulsar signals in the frequency domain. A comparison is made with a few experimental data. The mean scale of the turbulence is estimated. Analysis of this kind can not only elucidate the effects of the interstellar medium on pulsar characteristics but it can aid us in understanding the medium itself. Future extensions of the theory could be applied to improve the design of large-scale radio astronomy systems using adaptive receivers in searches for encoded signals generated by extraterrestrial civilizations.

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1. INTRODUCTION

As recently noted by Taylor et al.,[1] "Pulsar intensities are known to vary over at least five distinct time scales." In this paper I consider theoretically the variations that may safely be attributed to the effects of the interstellar plasma on the propagation of pulsar radiation. This analysis will not include the rapid pulse-to-pulse amplitude fluctuations that have also been reported in the literature. Such very rapid fluctuations, even after correction for dispersion, are probably due to variations within the neutron star itself, or to variations in whatever plasma may be in the immediate vicinity of the spinning neutron star.

While the statistics of the rapid pulse-to-pulse fluctuations have been amply reported in the literature (see, for example, Taylor et al.[1]), very little has been published in detail about the statistics of the effects of a pulsar signal attributable to interstellar plasma. An exception is a paper by Rankin and Counselman[2] reporting long-term variations (on the order of months) of the dispersion factor for NP 0532, the Crab Nebula pulsar. Huguenin et al.[3] have also studied the slow variations of pulsar intensities on a day-to-day basis. However, they attribute these changes to variations in the structure of the neutron star or in the excitation level of the emitting regions. This interpretation may be open to question.

Once a pulse of broad-spectrum electromagnetic radiation is emitted in our direction by a spinning pulsar, it must traverse a great deal of interstellar space. For example, the nearest and greatest estimated distances among the pulsars catalogued by Taylor and Manchester[4] are 100 pc for PSR 0950 + 08 and 17,000 pc for PSR 1924 + 19.

Since the interstellar medium is ionized, the average plasma density along the line of sight to any given pulsar may be calculated from the measured dispersion, using Maxwell's equations. This procedure yields the well-known result for the difference Δt in the arrival times of a pulse at two frequencies f_1, f_2:

$$\Delta t=4.1434\cdot 10^3\,\mathrm{DM}\,(1/f_2^2-1/f_1^2). \quad (1)$$

Here DM represents the dispersion measure [$cm^{-3}\cdot pc$]. It is in turn related to the pulsar distance d_{pc} expressed in parsecs and to the mean electron density $\langle N_e\rangle$ along the path by

$$\mathrm{DM}=d_{pc}\langle N_e\rangle. \quad (2)$$

Various suggestions for a galactic model for $\langle N_e\rangle$ may be found in the literature. Taylor and Manchester[4] suggest the form

$$N_e=0.03\exp(-z/1000)\ \mathrm{cm}^{-3}, \quad (3)$$

where z is the absolute value of the distance from the galactic plane in parsecs.

This paper examines how variations in the galactic

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