Flicker Noises in Astronomy and Elsewhere

1. Introduction

Astronomical sources which vary by order unity on a variety of different timescales are always interesting, and a part of the exotic attractiveness of quasars must surely by due to the fact that they fall into this category. Figure 1 shows the light curve of 3C273 from the data of Kunkel,[11] here reproduced from Fahlmann and Ulrych.[6]

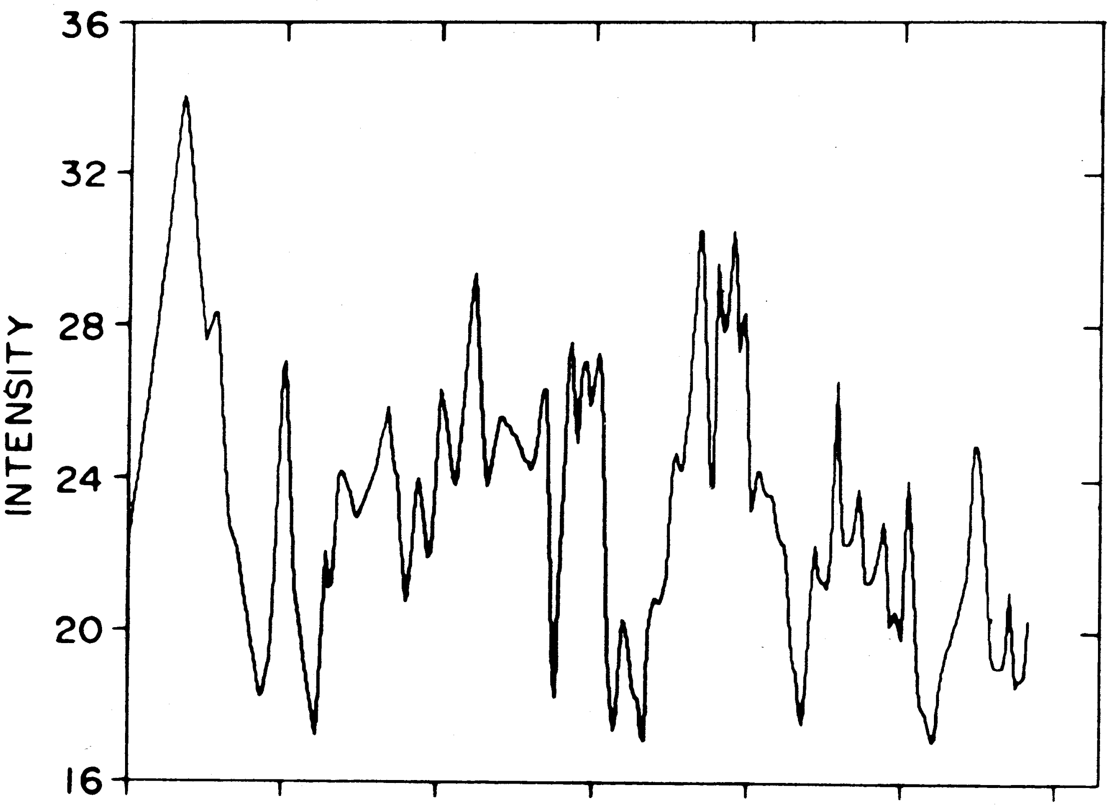

FIGURE 1 Light curve of the quasar 3C273 over a period of 80 years, from 1887 to 1967. The data of Kunkel[11] is here plotted by hundred-day averages; the ordinate is in arbitrary intensity units. (Reproduced from Fahlmann and Ulrych[6]).

Comments Astrophys.
1978, Vol. 7, No. 4, pp. 103–119

The data extend over nearly a century and are plotted by hundred-day averages. What I plan to review in this paper is just the fact that this stochastic property of variation on all timescales is not something unique to Figure 1 or even to modern astronomy, but rather is something seen all the time in a wide variety of physical phenomena. Even when these phenomena can be studied under laboratory conditions, their explanation has been exceedingly difficult; in most cases they are presently unexplained – thus similar to quasars. The main difference between the astronomer's outlook and that of people in other fields seems to be that astronomers refer to these stochastic fluctuations as 'signal', while in other fields the most common terminology is 'noise'. We will adopt this un-astronomical point of view here and look at some properties of various types of noise.

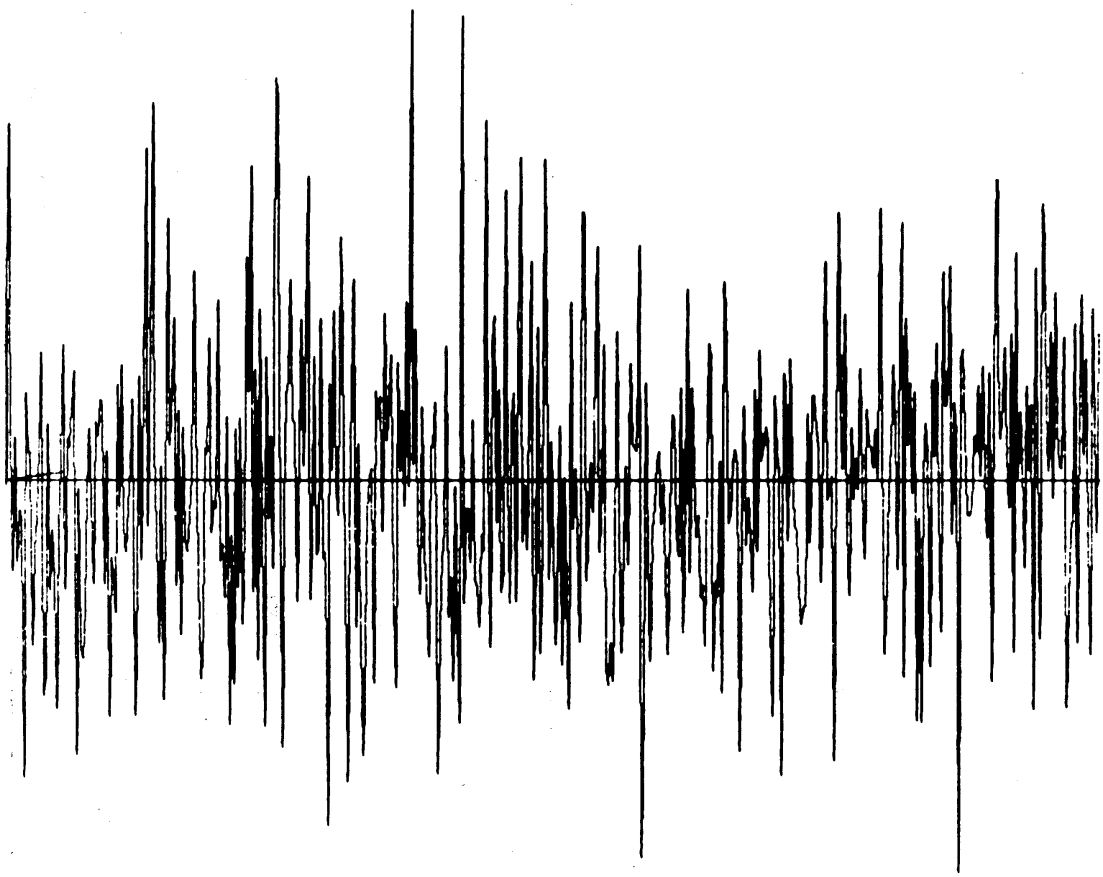

FIGURE 2 White noise, here sampled at 500 equally spaced points. The 500 amplitudes are independently random values with a normal distribution. This noise has a flat power spectrum.

Figure 2 shows a type of noise that almost everyone has some familiarity with, namely 'white noise'. White noise has equal power in every unit of bandwidth (i.e. per one Hertz). If we integrate this power from some finite frequency down to zero frequency—a finite bandwidth—we get some finite total power at the low frequency end of the spectrum; the power spectrum is convergent at low frequencies. Things are not so nice if we integrate up from some finite frequency towards infinity. Here we get a divergence: there is an infinite amount of power at high frequencies. The way that this shows itself in Figure 2 is that the function shown is infinitely choppy, no matter how fine a horizontal scale one chooses to look at. The value of the function does not converge to a limiting value at a point. (Of course my computer cannot plot this infinitely discontinuous function, so I had to limit the bandwidth in some way. The figure actually shows 'sampled' white noise, with 500 sampling points across the graph. The algorithm for generating sampled white noise is just to pick independently random numbers at each of the 500 points, with a normal probability distribution for each number.)

Since white noise converges at low frequencies but diverges at high frequencies, the *mean* value of the noise converges as we average it over longer and longer time intervals, but the instantaneous value of the noise is undefined, Suppose we now generate a noise not by laying down random numbers, but rather by incrementing a running sum by random steps, normally distributed with zero mean. This gives a 'random walk' function, and an example is shown in Figure 3. What is the power spectrum of this noise? The random walk is, by

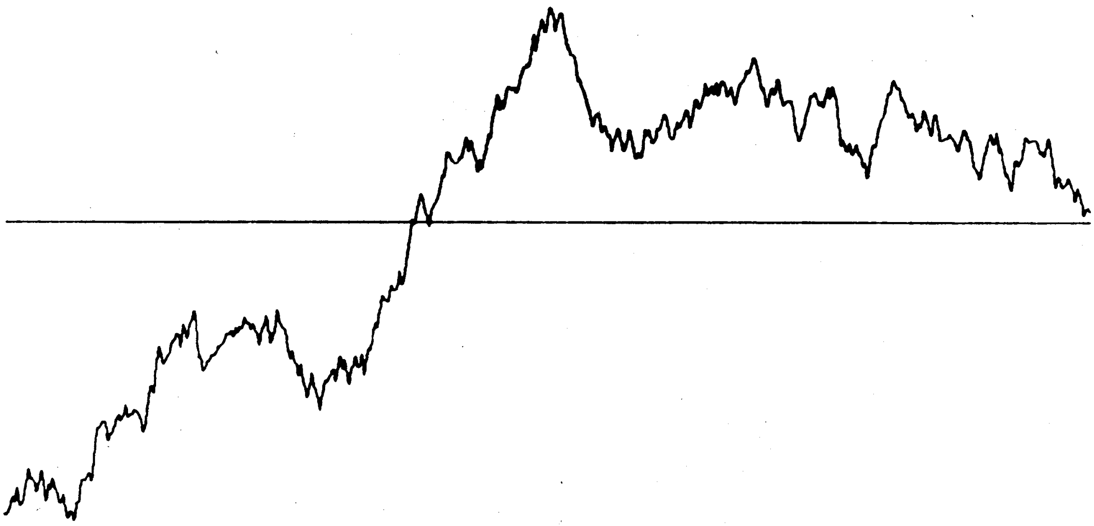

FIGURE 3 Random-walk noise. The derivative of this noise is white noise as in Figure 2. Random-walk noise has a power spectrum $\propto 1/f^2$.

definition, an integral of white noise. Doing this integration brings in a factor of $1/f$ in the Fourier transform, which is a factor of $1/f^2$ in its square, the power spectrum. Since white noise has a flat spectrum $\propto f^0$ (constant), random walk has a spectrum $1/f^2$. Notice that this spectrum is *convergent* when we integrate in frequency from some constant to infinity. There is only a finite amount of power at high frequencies. In Figure 3 we see that the function does have a well-defined value at each point. But the $1/f^2$ spectrum diverges when integrated down to zero frequency. This is just a statement of the fact that if we look over longer and longer timescales, the value of a random walk function wanders farther and farther away from its initial value. This noise has no well-defined *mean* value over long times.

My superficial impression is that neither Figure 2 nor Figure 3 looks very much like Figure 1, the quasar light curve. Figure 1 looks somehow 'in between' the other two in its variability. A more general comment would be that neither Figure 2 nor Figure 3 would excite astrophysicists very much, no matter what they purported to be observations of. One is too rough, the other is too smooth.

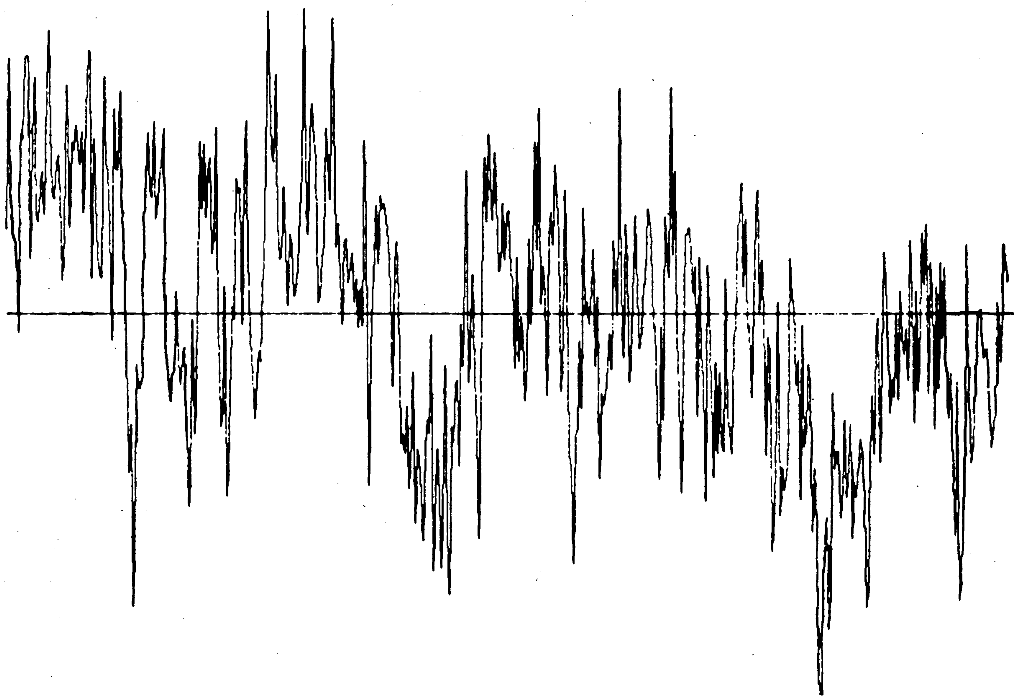

FIGURE 4 Flicker noise or $1/f$ noise. The power spectrum of this noise ($\propto 1/f$) has a divergent integral at both high and low frequencies.

But let us look at Figure 4. Now this is exciting! Notice all that rapid fluctuation down to the 'instrumental resolution limit'! Look at that general downward trend across the whole graph! And if you look closely you can see two (and maybe three) dips in the curve, spaced almost evenly across the graph. What theorist could resist the temptation to interpret those?

The stochastic function shown in Figure 4 is something called 'flicker noise' or $1/f$ noise. Its power spectrum varies as f^{-1}, so it is exactly halfway between white noise and random walk noise. A way of generating flicker noise, which we will discuss in more detail below, is to take 'half an integral' of a white noise process. This, when squared, then gives the $1/f$ spectrum. Flicker noise is divergent when integrated to *either* zero frequency or infinite frequency. It has neither a well-defined long-term mean, nor a well-defined value at a single point. But these divergences are both only logarithmic – so slow that we could extend frequency cutoffs to absurdly high or low values of frequency and hardly change the appearance of the noise at all.

Below, I will return to discussing some of the mathematical properties of flicker noise (or low-frequency divergent noises in general). But first I would like to summarize a series of examples of physical phenomena (mostly non-astronomical) in which flicker noise appears very persistently—usually defying explanation. The best-documented examples are of electrical flicker noise. This noise was first noticed some decades ago as an effect in vacuum tubes. Figure 5 (taken from Brophy[4]) shows the spectrum of noise actually measured in several current-carrying devices. Of course any electrical device has Johnson or Nyquist noise—white, thermal noise—independent of whether or not there is a current flowing in it. Flicker noise generally requires a current flow. If a component of $1/f$ noise is superposed on a thermal noise spectrum, the former will dominate at sufficiently low frequencies. In Figure 11, the 'vacuum tube' curve shows a change from white to flicker noise at around 1000 Hz. The white noise here

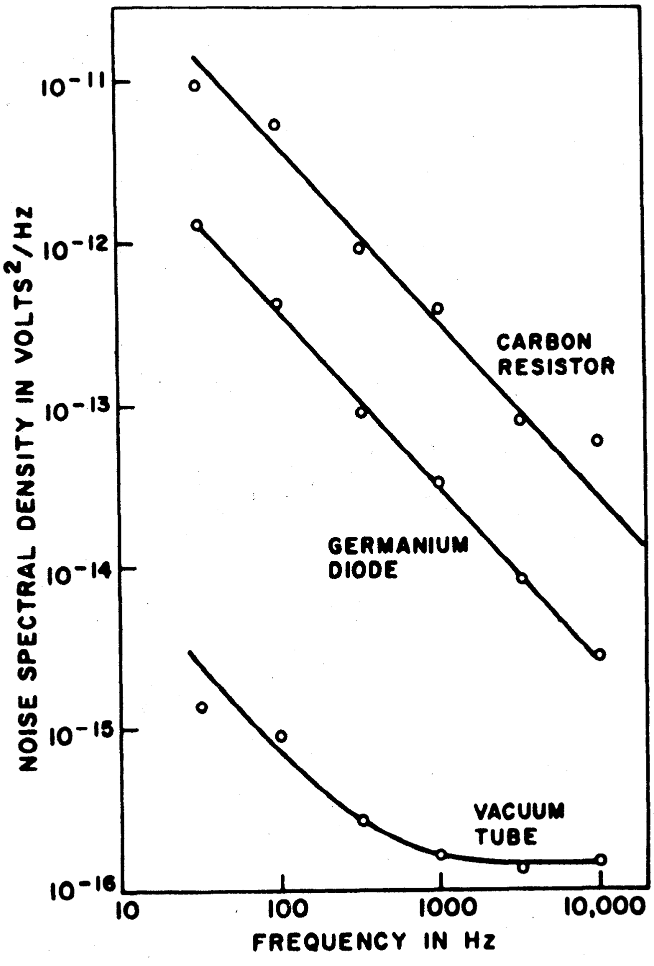

FIGURE 5 Typical electrical noise spectra for some current-carrying devices: 50 KΩ carbon resistor, 2N2000 germanium diode-connected transistor, and 12AX7 vacuum tube. (Reproduced from Brophy).[4]

is presumably not actually Nyquist noise but rather the 'shot' noise of the individual electrons arriving at the anode. Flicker noise, when it comes to dominate, represents long timescale fluctuations in the number of electrons, and therefore in the current. There is a common denominator among devices which show a strong flicker noise spectrum: they typically have currents flowing through very small or thin physical regions, so that small fluctuations in the number of current carriers can be important. In the carbon resistor, whose spectrum is also in Figure 5, the bottlenecks are the sharp points where the carbon grains touch each other. Wirewound resistors, supposedly, do not show flicker noise at this level. Among solid-state devices, those with very thin contact junctions show flicker noise preferentially: MOS transistors as opposed to junction field-effect transistors, for example (see Radeka[18] for references). Thin metallic films are also a source of electrical flicker noise (Ref. 23 and references therein).

The frightening thing about Figure 5 is that the noise spectra shown are still rising along a beautiful power law at the smallest frequencies measured. We might well ask: how far down in frequency does this divergence extend? Some evidence on this comes from the makers of precision clocks and frequency standards (cesium, quartz, hydrogen maser, and so on). A simplified form of the standard measure for a clock's long-term stability is: With what fractional accuracy can a given clock measure an interval of length t? It is not entirely obvious *a priori*

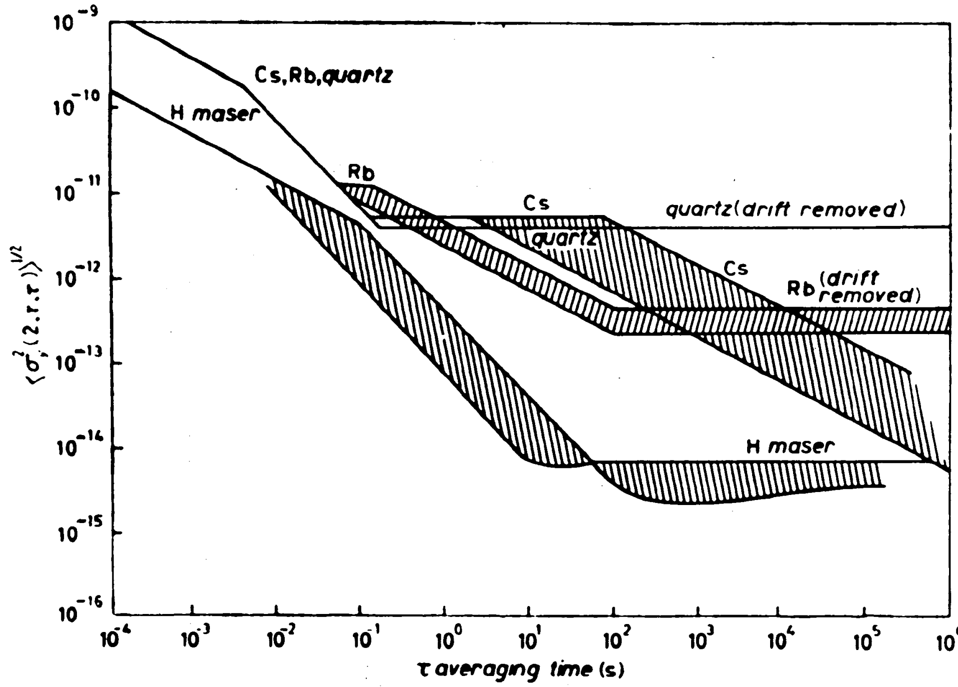

FIGURE 6 Long-term stability of cesium, rubidium, quartz, and hydrogen frequency standards. The ordinate shows essentially the fractional accuracy to which an interval (the abcissa) can be timed. The tendency of the curves to become horizontal for long times indicates that flicker noise dominates these clocks' frequency stability. (Reproduced from Vessot).[22]

just how this accuracy should scale with increasing *t*. Consider one possibility: suppose we have a clock with a certain average 'ticking' rate, but each tick is of slightly variable length, with the errors independent from tick to tick. In this case the fractional accuracy with which an interval can be measured will get better as the interval is lengthened, as the square root of the interval. The point here is that we have added white noise, uncorrelated in time, to our clock's frequency. In real life this behavior – increasing fractional accuracy with increasing interval–is virtually never seen for long intervals. Figure 6, from a review by Vessot[22] shows this fractional accuracy parameter for a variety of clocks. As averaging time is increased, accuracy gets better for a while, but then it levels off; the fractional accuracy with which an interval can be measured becomes independent of the interval. (This is why one can speak of a clock of accuracy 10^{-12} or 10^{-14}, with no dimensional units specified.) Evidently the ticking rates of these clocks are subject not to independent perturbations, but rather to perturbations which are correlated on all timescales–in fact they turn out to be flicker noise. A generic property of physical measurements subject to flicker noise error processes is that the accuracy does not improve beyond a certain point even as more and more measurements are averaged together. Where does the clock flicker noise come from? According to Vessot, it can usually be traced to electrical flicker noise in the electronic components of the clocks–in other words to the extension of Figure 5 towards very low frequencies. So Figure 6 gives us this indirect evidence that the flicker noise in a carbon resistor, say, gives rise to fluctuations coherent over times as long as 10^6 seconds. This is quite fantastic! How does the resistor remember over a period of weeks or months that it is in an 'up' fluctuation? There are no generally-accepted answers to this question, although there have been various proposals in the literature. One can start with recent papers by Weissman[27] and by Handel[8] and references therein to get an idea of the range of mechanisms proposed. My opinion is that the explanation of electrical flicker noise is in about the same state of certainty as the explanation of quasar energy sources.

Let me turn from electrical noises to a completely different area. Figure 7 shows a plot, from work by Taft *et al.*[20] of the spectrum of undersea ocean currents at a depth of 3100 meters in the central Pacific. This data is gathered essentially by mooring a buoy at this depth, leaving it for a month to record, then recovering the data and Fourier analysing it (cf. Ref. 26). The oceanographic interest in Figure 7 lies partly in features like the sharp peaks which occur around 12 and 24 hours; these are the tidal components of the current. The oceanographic people understand less well the fact that this tidal structure should be superposed on a general power-law trend which is rising towards low frequencies and keeps a well-defined slope over (here) three orders of magnitude. Most oceanographers have faith that the spectrum will stop rising at low enough frequencies, but at present the turnover has not been unambiguously seen.

Temperature variations measured by Wunsch and his group at MIT also show this power-law behavior, as do measurements of low frequency seismic noise at quiet sites (this may in fact be due to distant ocean noise). I have not yet

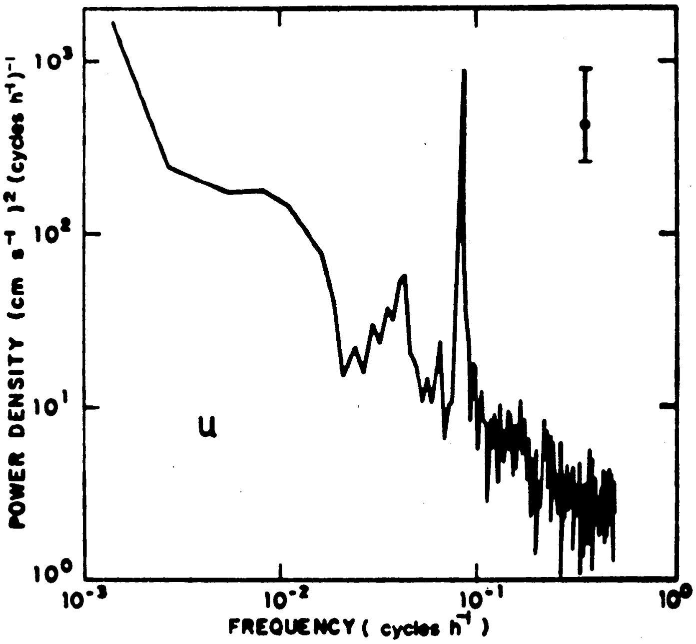

FIGURE 7 Power spectrum of the east-west component of ocean current velocities at a depth of 3100 m at 1°02.3′S, 149°50.7′W. The length of the bar at upper right shows 95% confidence limits. (Reproduced from Taft *et al.*)[20]

mentioned the *slope* of the power law in Figure 7. One can see by eye that it is close to -1, i.e. to flicker noise. It is claimed in the literature[17] that a whole variety of meteorological and climatic time series have flicker noise power spectrum: One example is the flow rate of the Nile over the last 2000 years. Another is the annual thickness of glacial varves (clay deposits formed from the yearly runoff from a receding glacier) which presumably tell something about spring temperatures of the last few thousand years. One should emphasize, however, that the value of the slope is not all that well determined in some of these data, so that a statement that something or other is 'flicker noise' should be taken with a grain of salt. A more conservative statement would typically be that some power spectrum is rising to low frequencies, and with slope in the range of -1.0 ± 0.3, say. Figure 8, taken from Wunsch,[25] will demonstrate that not *everything* is flicker noise. It shows the power spectrum of variations of sea level in Bermuda, for periods extending up to of order a year. The sharp spikes again are the ocean tides. If you draw a line through the power law trend, you will find its slope to be very nearly -2. It seems as though the sea level at Bermuda is changing by a random walk process, with deviations increasing, typically as $t^{1/2}$. If you postpone your Bermuda vacation for too long, the island may be underwater!

An interesting and totally unexpected occurence of flicker noise was found recently by Voss and Clarke[23] at Berkeley. They took the audio output

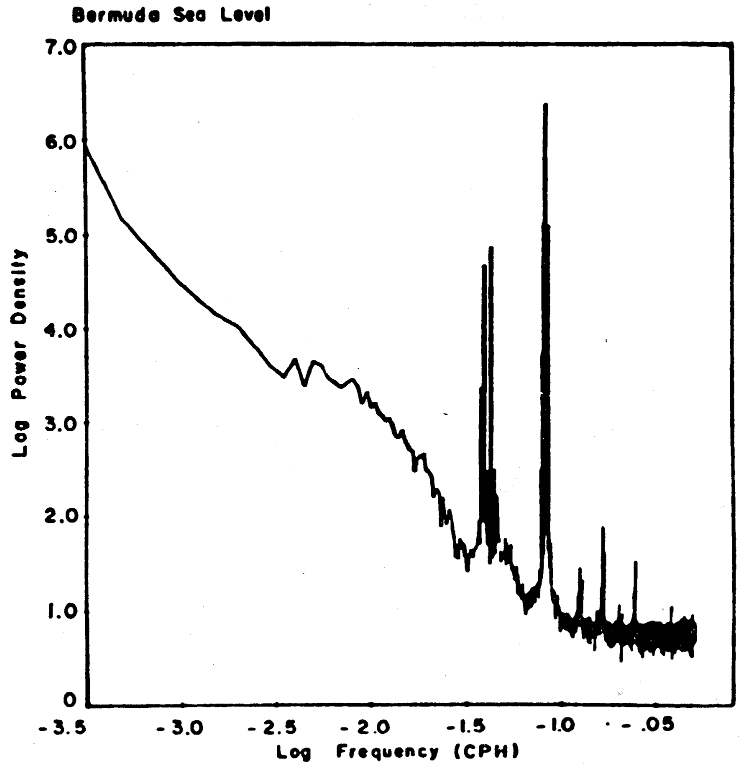

FIGURE 8 Power spectrum of sea level at Bermuda. Note the strong tidal components at 12 hours and 24 hours. The rise of power at low frequencies is here steeper than $1/f$ (and is close to $1/f^2$). (Reproduced from Wunsch.[25])

of a radio, squared it, low pass filtered it at 20 Hz, and measured the power spectrum over a period of many hours. Their measured quantity is thus the instantaneous 'loudness' of the music or talk on the radio. The top half of Figure 9 shows the amazingly good fit to $1/f$ to everything from Scott Joplin piano rags to news-and-talk shows. Voss and Clarke then modified their apparatus to measure the instantaneous number for zero crossings of the audio signal – roughly equivalent to the instantaneous pitch of the signal – and got the results shown in the bottom half of Figure 9. Here the news-and-talk station shows structure, presumably due to systematic patterns in sentence intonation and such things. But the classical music shows a $1/f$ law over almost five decades in frequency, up to corresponding periods of more than an hour. What does this mean? Music certainly does have structure on all different timescales. It would appear that these scales are distributed something like logarithmically in time and have comparable loudness or pitch variations at each level of structure. There are three notes to a phrase, say, and three phrases to a bar, and three bars to a theme, and three repetitions of a theme in a development, and three developments in a movement, and three movements in a concerto, and perhaps three concertos in a radio broadcast. I do not mean this really literally, but I think the idea is clear enough. This type of argument helps to explain the general trend of the Voss and Clarke data, but I think there is still the real mystery of why the agreement with $1/f$ looks so *precise.*

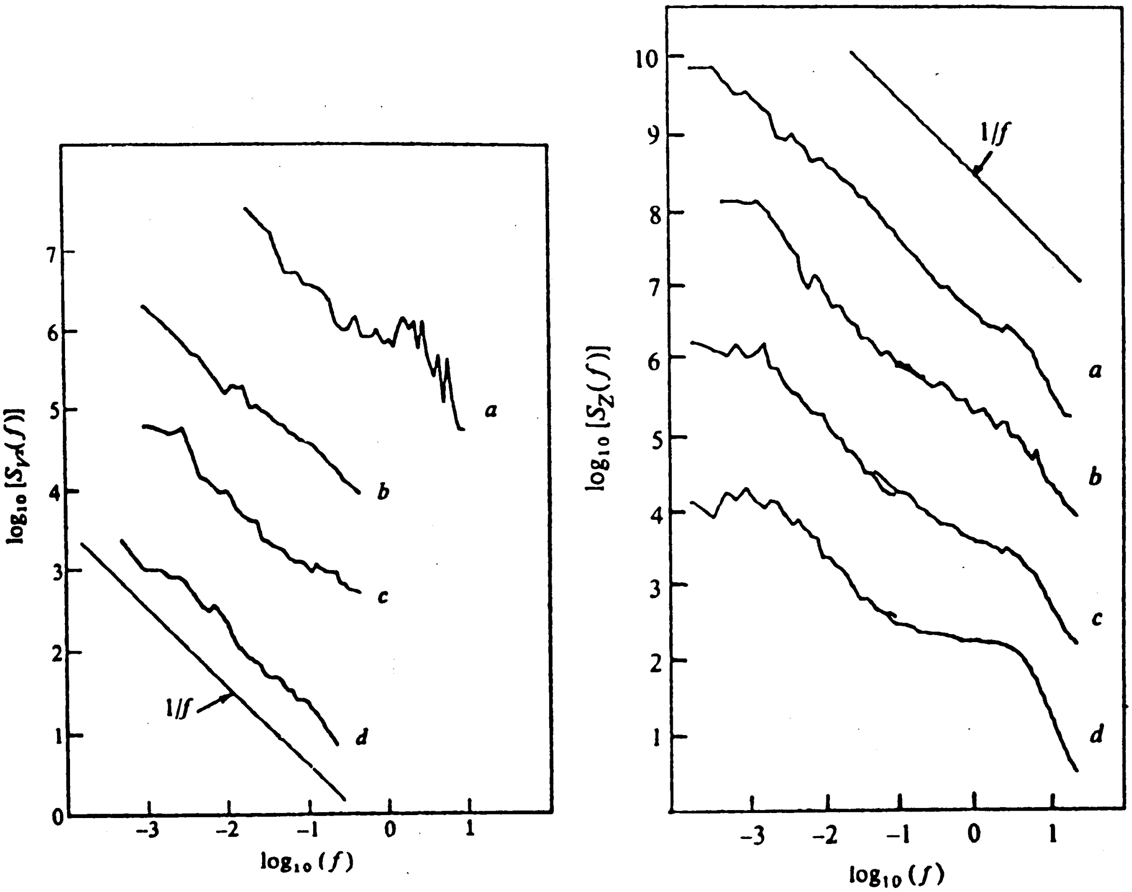

FIGURE 9 (*left*) Loudness fluctuation spectra as a function of frequency f (in Hz) for: *a,* Scott Joplin Piano Rags; *b,* classical radio station; *c,* rock station; *d,* news-and-talk station. (Reproduced from Voss and Clarke.[23]) (*right*) Power spectra of pitch fluctuations for four radio stations: *a,* classical; *b,* jazz and blues; *c,* rock; *d,* news-and-talk. (Reproduced from Voss and Clarke (1975).)

I should certainly not slight astronomical phenomena. Sunspot number has been measured for quite a long time. The raw time series of sunspot number shows clearly the 11 or 22 year period of the solar cycle. One might expect that this cycle would show up as an obvious bump in the power spectrum of the data. But in fact[17] there is no clear bump if the raw data is treated without sophistication. The solar cycle does not maintain good phase coherence over more than a small number of cycles, and thus the bump is washed out in a strong power-law trend which is rising towards low frequencies with slope (can you guess?) approximately –1. One wonders about the Maunder minimum (the period of 50 years when there may have been *no* sunspots[5]): was some *new* physical phenomenon coming into play, or was this just another fluctuation due to some continuous, power-law spectrum, to below some critical threshold?

There has been a lot of interest recently in the possible identification of solar normal modes with periods around one hour[10] or several hours (e.g. Ref. 3). People who have looked at these data tell me that although the resonance peaks themselves are still somewhat controversial, a striking feature of the data is the general trend-rising towards the low frequency end of the spectrum. Perhaps flicker noise phenomena in the sun extend over the six orders of magnitude from hours to centuries (the electrical and musical flicker noises can extend this far or farther).

Power spectra which rise to low frequencies as apparent power laws have also been seen in careful photometry of white dwarfs (Ref. 9; see especially Figure 5) and central stars of planetary nebulae.[12] Spectral indices around -1 (flicker noise) are not uncommon, although they may often be somewhat closer to zero.

A few more examples seem worth mentioning briefly. I do not vouch for any of these other than to say that they can be found in the literature: fluctuations in the potential of nerve membranes;[2,21] flow of sand in an hourglass;[19](this rules out the otherwise clever idea of building a huge hourglass which is more accurate than the best hydrogen maser over *sufficiently long time intervals*); stock market prices and other econometric indicators (cf. Ref. 16).

III. Generation of Flicker Noise

Let us turn now to some elementary mathematical questions which arise when one considers low-frequency divergent noise. Of course, the real goal should be to understand how nature manages to generate flicker noise under such general circumstances. A first step in that direction is to understand how *we* might generate flicker noise mathematically (e.g., how I actually did generate the example of Figure 4). Above, I pointed out that a flicker noise signal was the 'half-integral' of a white noise signal. The way to make this precise is by Fourier (or Laplace) transforms (cf. Ref. 1): let $f(t)$ be a function whose half-integral we wish to take and let $F(t)$ be the half-integral. We proceed by taking a Fourier transform, then dividing by $\omega^{1/2}$, then taking an inverse Fourier transform, as follows:

$$\begin{aligned} F(t_0) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega\, e^{i\omega t_0}\, \omega^{1/2}\left[\int_{-\infty}^{+\infty} dt\, f(t) e^{i\omega t}\right] \\ &= \int_{-\infty}^{+\infty} dt\, f(t)\left[\frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega\, e^{i\omega(t-t_0)}\, \omega^{-1/2}\right] \end{aligned} \qquad (1)$$

$$\equiv \int_{-\infty}^{+\infty} \mathrm{d}t\, f(t)\, G(t - t_0)$$

In the second equality the order of integration was reversed to show explicitly that the half-integral consists of linear convolution with a kernel (or Green's function) $G(t - t_0)$. The difficulty with this kernel is that it has a branch cut in the denominator, so that there is more than one way to evaluate it in the complex plane, depending on just where we choose to put the branch cut. It is not difficult to show that the kernel can be written as a general linear combination of two different functions, $G_+(t - t_0)$, and $G_-(t - t_0)$

$$G_+(t - t_0) = \begin{cases} \text{const.} \times (t - t_0)^{-1/2} & t > t_0 \\ 0 & t \leqslant t_0 \end{cases} \qquad (2)$$

$$G_-(t - t_0) = \begin{cases} 0 & t \geqslant t_0 \\ \text{const.} \times (t_0 - t)^{-1/2} & t < t_0 \end{cases}$$

You can see that one of these represents an 'advanced' response of the half-integral to the function $f(t)$; the other represents a 'retarded' response; and we can have any linear combination of advanced and retarded responses.

The way, now, to generate a flicker noise from a white noise (e.g. a Poisson-random sequence of impulses) is (i) to decide how much advanced and retarded response you want and construct the appropriate $G(t, t_0)$, and (ii) convolve this 'impulse response function' (as it would be called in the language of filters) with your sample of white noise, It is evident that flicker noise generated in this way is not symmetric under time reversal (except in the special case of half-advanced plus half-retarded). Gaussian white noise does not define an arrow of time; nor does its integral, random walk; but its half-integral, flicker noise, *does*. Now we can also see just how flicker noise can maintain a 'memory' of its fluctuation state over very long timescales: In the 'retarded' case, a delta-function impulse in $f(t)$ produces a filter response which rises at once to a cusp, then decays away only as $t^{-1/2}$. The superposition of these slowly dying tails from all previous impulses gives the very long time correlations in the noise.

How might *nature* generically produce systems whose response to stimulii decays away as $t^{-1/2}$? If we knew the answer to this, we would, I think, know much of the explanation of flicker noise. Unfortunately, there are no very compelling mechanisms known. Let me give you an example of one which is, however, somewhat artificial. Suppose we have a box containing some ionizable substance, and suppose that it is subjected to a Poisson random flux of cosmic ray showers. Each shower, hypothetically, produces a large number of 'ions' in the box, and these ions then decay away by recombination. The differential equation for the number of ions is

$$\frac{dN}{dt} = -\text{const.} \times N^n \quad (3)$$

Here n tells how-many-body recombination is taking place. Normal, two-body recombination goes as the square of the density, so that $n = 2$; but some sort of hypothetical three-body recombination would have $n = 3$. The solution to Eq. (3) for $n > 1$ is

$$N = \text{const.} \times (t - t_0)^{-1/(n-1)} \quad (4)$$

So an impulse response function which generates flicker noise will result when $n = 3$. (The value $n = 2$ gives a white noise; the value $n = \infty$ gives a random walk – there is in this case no recombination at all and the number of ions just accumulates in a running sum.) If three-body processes were the rule in nature, we might have here the germ of a general explanation; but it is hard to think of any three-body processes at all! Perhaps electrons, holes, and lattice defects or something like that, but one would not want to press the point. I have heard that P. Anderson devised some kind of model involving two-electron traps in metals, but I have not seen this in print.

There is another mathematical way of generating power-law noise that has been discussed in the literature by Halford[7] among others. This one is a generalization of a shot noise model. The idea is to choose some canonical 'pulse shape' for a single event. If we superimpose these pulses randomly in time, we get a noise which is typically white at sufficiently low frequencies, and which has the spectrum of the individual events at high frequencies. To get flicker noise (or any other desired spectrum, for that matter) we superimpose random events, but we also stretch the pulses by a randomly varying factor. If the probability of a certain stretching factor varies as some power of the factor (over some large range), then the composite spectrum will also be pure power law: this follows the well-known maxim that anything convolved with a power-law gives a power-law.

I do not find scale superimposition a very convincing mechanism for explaining the naturally occurring flicker noises, because I find it hard to conceive of physical mechanisms which contribute stretched pulses with just the right frequency of occurrence over, say, six orders of magnitude. Scale superimposition just transfers the mystery to the random 'stretching factor' process. The scale superimposition idea does however bring to the fore a fact that we might have emphasized from the stars: not all flicker noises are the same, nor must they even *look* the same by eye. In Figure 10 one sees three examples of flicker noise generated by scale superimposition of exponentially decaying pulses. These all have the same power spectrum (as verified by direct numerical computation). However, their 3-point (and higher) correlations are different. The first example obtains its high frequency components by superimposing very frequent, but small, exponential pulses. The third example generates the same high-frequency spectrum by means of very rare, but huge, spikes (which are visible in the figure). The middle example is in-between the two extremes. Although these examples all have the same power spectrum (and therefore the

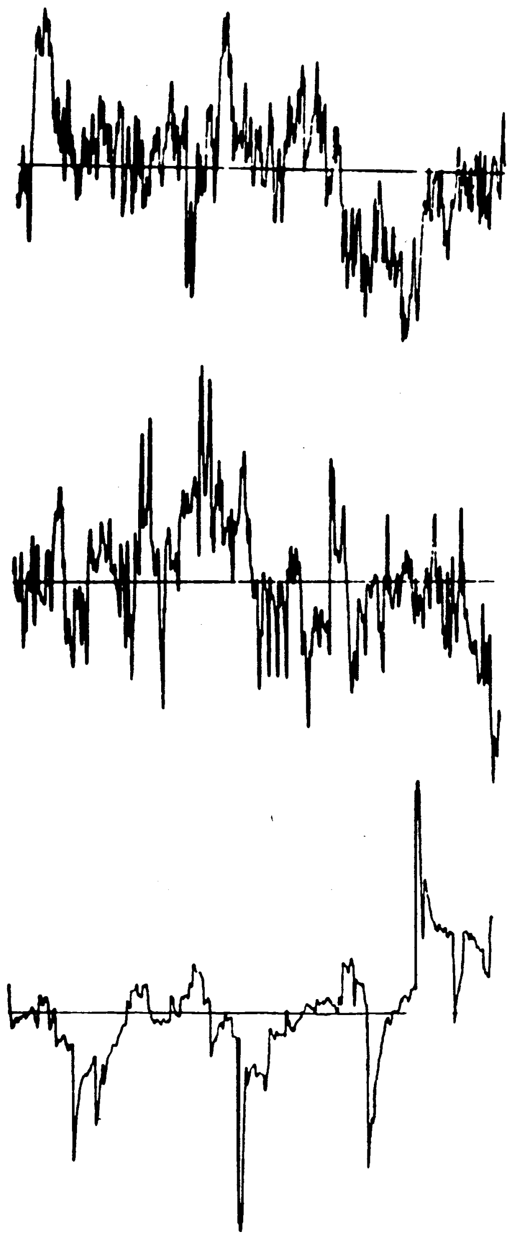

FIGURE 10 $1/f$ noises produced by superposition of exponential pulses with a power-law distribution of widths τ. The probability per time of a pulse of scale τ is proportional to τ^{-2} (top figure), τ^{-1} (middle figure), and τ^{0} (lower figure).

same 2-point autocorrelation), they can easily be told apart by a measurement of 3- and 4-point correlations. Such correlation measurements can also disambiguate the flicker noise generated by a causal filter, from that of an acausal one.

IV. Final Examples and Conclusions

You might wonder whether you can see by eye the difference between flicker noise generated by advanced and retarded filters. Figure 11 may answer this; I find the two halves very hard to tell apart.

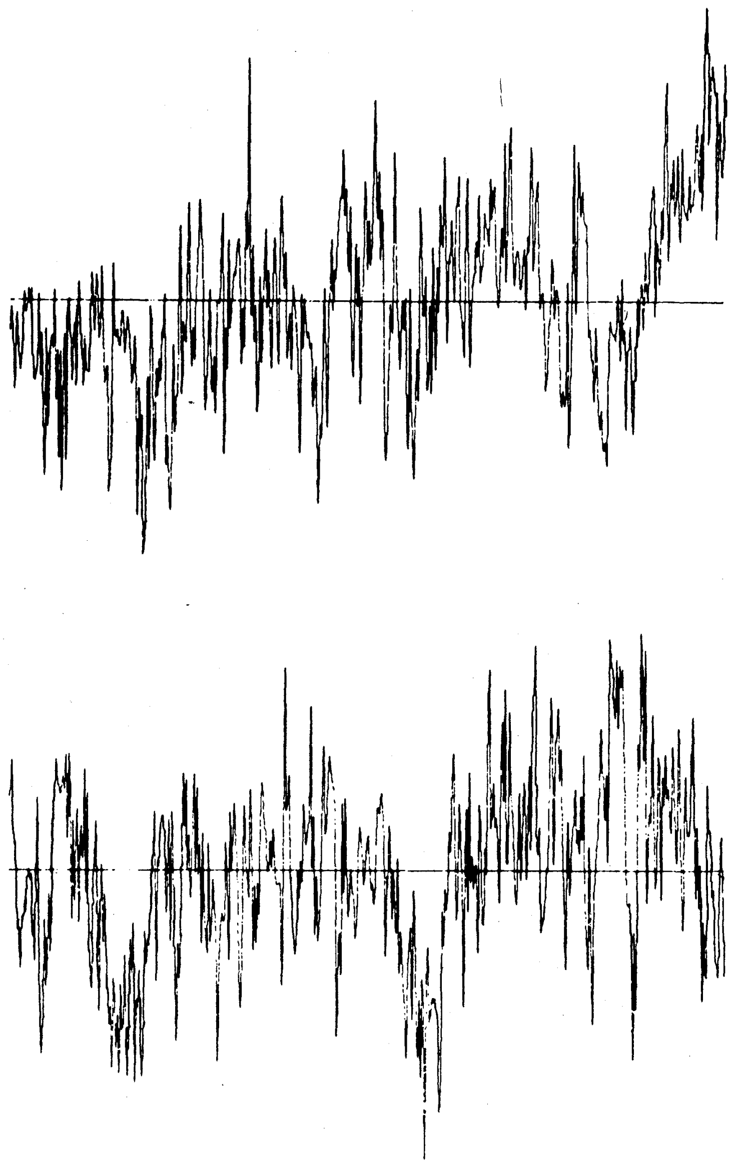

FIGURE 11 (*top*) $1/f$ noise produced by filtering a sample of Gaussian white noise. The impulse response of the filter is $G_-(t - t_0)$ of Eq. (2), the fully retarded (causal) response. (*bottom*) Same as top figure, except the response is here $G_+(t - t_0)$, the fully advanced (acausal) response. Figure 4 also shows filtered white noise; its response function is $1/2\,(G_+ + G_-)$.

Let me conclude with only some speculations and questions. First, I speculate that 4-point autocorrelation measurements on electrically generated flicker noise should show it to be a completely casual, filtered Poisson process. This is because I expect the *underlying* stochastic process, whatever it is, to have no long memory; therefore I expect the flicker spectrum to come entirely from a decaying power-law response such as was discussed above. Second, regarding the musical flicker spectrum, I expect that one can distinguish good composers from bad composers on the basis of whether the loudness spectrum of their music is time-reversible (as seen in 4-point correlations). A bad composer writes each bar on the basis of the music that he has already set down – therefore his impulse response function to a sudden musical idea should be highly time unsymmetric. A good composer has all his good ideas worked out in advance, so the advanced and retarded responses should be about equal. One might compare Bach to Telemann. I would like to hedge my third speculation by giving two possibilities:

a) One might speculate that the sun has flicker noise processes on all timescales and that these induce the climatic and meteorological noise processes that are seen on Earth. The trouble with this is that it is very hard to see how the sun couples so directly to, say, deep ocean currents on timescales of hours. So one is led to

b), the speculation that $1/f$ noise properties ought to be a general calculable feature of highly nonlinear hydrodynamic systems, like a turbulent atmospheres or stellar envelopes. Here one is swimming against the current of present interest in nonlinear systems. In a number of areas, the very powerful 'soliton' picture is emerging, that nonlinear systems can generically have *non*dispersive (and very un-noise-like) wave solutions, even in regimes where all intuition (about weakly nonlinear coupled-oscillator systems) would predict total chaos and equipartition. So I should really phrase this speculation: After all the soliton-like behavior of a nonlinear system has been accounted for, there should be a 'bunch of junk' left over, and there should be some very general reason why this likes to take on a $1/f$ fluctuation spectrum, at least in hydrodynamic systems.

As for quasars:

1) Are their fluctuations low-frequency divergent? If not, where does the power spectrum really turn over at low frequencies? (Ozernoi and Chertoprud,[13] among others have claimed that the turnover is at least as short as ten years, but this has been contested by Terrell and Olson,[20a] and others).

2) How time-asymmetric is a quasar light-curve? Is it really well established that rise times are short and decay times long, or is it just a theoretical prejudice?

3) If the output of a quasar is a convolution of random, impulsive events with some response function of long decay time, then what is the shape of this response function?[6,20a] Of particular interest, I think, is the late-time shape of the pulse – where it is poorest determined by the data. This probably tells us most about the large-scale environment in which the quasar energy source

(whatever its nature) is embedded.

One final remark about 'quasiperiodicity' seems not out of place: Mandelbrot[14] and others have noted that there is a tendency for the eye to pick out sinusoidal variations even when they are not statistically real. For low-frequency divergent noises, most power is on scales comparable to the length of the entire record. But the eye is also efficient at removing a 'linear' trend (because of perspective?), which eliminates the very longest pseudo-sinusoids. A rule of thumb is that the strongest eye-apparent period in (actually non-periodic) data will be about one-third the length of the date sample. In astronomy, one might do well to take 'three-cycle' quasiperiods with more than a grain of salt.

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