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X-RAY AND RADIO EMISSION FROM CLUSTERS OF GALAXIES: THE HEATING OF INTRACLUSTER GAS BY RELATIVISTIC ELECTRONS

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ABSTRACT

We discuss the observed correlation between X-ray emission from clusters of galaxies and the presence of steep spectrum radio sources in the context of a thermal model for the X-ray emission. We calculate the rate at which energy is transferred from the relativistic electrons to the thermal gas, and find that under reasonable assumptions about the low-energy cutoff of the electron spectrum this rate is ample to power the X-ray source. We show that in general the temperature of the gas is independent of the heating mechanism, because of regulation by the cluster potential. A correlation between the steepness of the spectrum and strength of the radio source and the X-ray luminosity remains, however. We also discuss some constraints on inverse Compton models of X-ray production.

Subject headings: galaxies: clusters of — galaxies: intergalactic medium — radio sources: general — synchrotron radiation — X-rays: sources

I. INTRODUCTION

Many clusters of galaxies have now been observed to be sources of X-ray emission. Radio emission is also observed from these clusters, and considerable evidence exists for the correlation of X-ray and radio emission from clusters. Owen (1974) has shown that clusters which are strong X-ray sources tend to also contain strong sources of radio emission. A recent study by Erickson, Matthews, and Viner (1978) indicates that the low-frequency radio spectral index of a cluster is also correlated with its X-ray luminosity, with the steeper spectrum sources tending to have greater X-ray luminosities.

The radio emission from clusters is well understood to be synchrotron emission from relativistic electrons in a weak magnetic field. Models involving either bremsstrahlung emission from a hot, thermal gas or inverse Compton scattering by relativistic electrons of the 3 K microwave background radiation have been proposed to explain cluster X-ray properties. Thermal bremsstrahlung models involving only a hot gas, however, give no explanation for the observed correlations of cluster X-ray and radio properties. These correlations follow naturally from inverse Compton models, but these models run into difficulty with clusters such as Perseus and Coma which have steep, high-frequency X-ray spectra. Inverse Compton models also have difficulty explaining the iron line emission which is observed from many clusters (Mitchell *et al.* 1976; Serlemitsos *et al.* 1977; Mushotzky *et al.* 1978).

It has been suggested that the steep-spectrum radio sources are the result of confinement by the surrounding hot gas, increasing the source lifetime so that energy losses affect the spectrum. Such an explanation holds only for those sources having sizes much smaller than the size of the gas distribution. As we shall see (§ II*c*), such is not always the case for these sources. In addition, in many cases the spectrum of the cluster as a whole is flatter at high frequencies. This observation is inconsistent with the confinement hypothesis for a single source, requiring the existence of other, younger sources in the cluster. The low-frequency spectra can be explained, however, by an older electron population which has escaped the parent source or sources and now occupies a larger volume of space. This viewpoint is supported by the admittedly sparse data on source sizes at low frequencies. More confined steep-spectrum sources may also be produced by the back diffusion of these older electrons into expanded remnants of past radio source events (Christiansen and Holman 1978).

Inasmuch as observations indicate that both hot, thermal gas and relativistic electrons are present in these clusters and are interrelated, it is of interest to consider models which involve both and which can yield the observed correlations. In this paper we consider the interaction of nonthermal electrons with the thermal intergalactic medium. We shall assume that an intracluster medium of density $n \gtrsim 10^{-4}$ cm^{-3} is required, independent of the X-ray emission mechanism, to explain the radio tails which are observed in many clusters (e.g., Pacholczyk and Scott 1976). We find that under reasonable conditions the gas is significantly heated by the nonthermal particles and that the correlation between cluster X-ray and radio emission can be explained in terms of this process. We have omitted redshift dependences from our discussion since all sources so far observed have low z. These dependences are summarized in an Appendix for future reference.

We first discuss the global energetics of the cluster. Next we discuss the local heating and cooling of the gas, and the generation of cluster winds. Third, we consider a specific model in which the relativistic electrons are generated in the cluster center and then stream outward. In each case we shall be concerned with implications for both the thermal bremsstrahlung and inverse Compton models for the X-ray emission.

II. HEATING BY NONTHERMAL ELECTRONS

Sofia (1973) has pointed out that cosmic-ray heating may be important in clusters of galaxies. Here we develop the theory in detail, using the observed radio source parameters as a basis.

a) *The Electron Population*

Radio sources in clusters are often observed down to frequencies $\sim$26 MHz (e.g., Erickson, Matthews, and Viner 1978) and occasionally to 10 MHz (e.g., Clark 1967; Bridle and Purton 1968). The radio spectrum can usually be described by a power law of the form $F_\nu \propto \nu^{-\alpha}$. For those sources with curved spectra, we approximate the low-frequency spectrum over a small frequency band by this power-law form. The radio emission, interpreted as synchrotron emission, implies the existence of a population of electrons with energy spectrum $N_e(E) = KE^{-p}$ erg^{-1}, where $p = 2\alpha + 1$. The synchrotron emission from the source is

$$J_\nu = \left[\frac{a(\alpha)}{0.1}\right] 8 \times 10^{-5}(6.3 \times 10^{18})^{\alpha-1} K B^{\alpha+1} \nu^{-\alpha} \text{ ergs s}^{-1}\text{ Hz}^{-1}\text{ sr}^{-1} , \tag{1}$$

where B is the magnetic field strength in the source and $a(\alpha)$ varies slowly with α (see, e.g., Ginzburg and Syrovatskii 1964). We can therefore express K in terms of the flux F in janskys at frequency ν_F. For a source whose distance is $d_{100} \times 100$ Mpc,

$$K = 10^{34} F \nu_F{}^\alpha B^{-(\alpha+1)}(6.3 \times 10^{18})^{-(\alpha-1)}(0.1/a) d_{100}{}^2 . \tag{2}$$

We shall also need the electron density per unit volume, which depends on the source size. For a spherical source of radius $r_{50} \times 50$ kpc,

$$n_e(E) = kE^{-p} \text{ erg}^{-1}\text{ cm}^{-3} ; \qquad k = 8 \times 10^{-37} F \nu_F{}^\alpha B^{-(\alpha+1)}(6.3 \times 10^{18})^{-(\alpha-1)}(0.1/a) d_{100}{}^2 r_{50}{}^{-3} . \tag{3}$$

In terms of the synchrotron flux, the inverse Compton emissivity due to this population of electrons is

$$j_\nu = 5 \times 10^{-52} F \nu_F{}^\alpha (1.7 \times 10^4 T_{\rm BB})^{\alpha-1}(T_{\rm BB}/2.7\text{ K})^4 (0.1/a) B^{-(\alpha+1)} \nu^{-\alpha} d_{100}{}^2 r_{50}{}^{-3} , \tag{4}$$

where $T_{\rm BB}$ is the temperature of the scattered blackbody radiation. The total inverse Compton flux from the source is

$$F_{\rm ic} = 8 \times 10^{-35} F \nu_F{}^\alpha (1.7 \times 10^4 T_{\rm BB})^{\alpha-1}(T_{\rm BB}/2.7\text{ K})^4 (0.1/a) B^{-(\alpha+1)} \nu^{-\alpha} \text{ ergs cm}^{-2}\text{ s}^{-1}\text{ Hz}^{-1} , \tag{5}$$

independent of d and r. The inverse Compton flux increases as B decreases.

The frequency of synchrotron emission from an electron of energy E is

$$\nu_S \sim 10^{18} B E^2 \text{ Hz} , \tag{6}$$

while the frequency of inverse Compton emission is

$$\nu_{\rm ic} \sim 4 \times 10^{23} E^2 \text{ Hz} . \tag{7}$$

For 10 MHz radio emission, $E \sim 3 \times 10^{-6} B^{-1/2}$ ergs, while for a 1 keV X-ray, $E \sim 7.6 \times 10^{-4}$ ergs. For $B \leqslant 1$ microgauss, E(radio) is much greater than E(X-ray), so that the inverse Compton model requires an extrapolation of the electron spectrum to lower energies. We assume that the power-law energy distribution continues to a minimum electron energy $E_{\rm min}$, where $E_{\rm min} < 3 \times 10^{-3}\, B_{\mu\rm G}{}^{-1/2}$ ergs. Presumably $E_{\rm min}$ is determined by the electron acceleration mechanism.

b) *Heating of the Gas*

The minimum rate at which the relativistic electrons heat the gas is determined by Coulomb interactions between the relativistic electrons and the thermal electrons. Then the relativistic electron loses energy according to (e.g., Ginzburg and Syrovatskii 1964)

$$\frac{dE}{dt} = -10^{-18} n \left[\frac{\ln(\gamma) - \ln(n) + 73.4}{80}\right] \text{ ergs s}^{-1} . \tag{8}$$

The term in brackets changes only slowly for the parameters of interest here and can be neglected without significant error. The acutal heating rate may be somewhat greater than this due to nonlinear heating processes. Nonthermal electrons (and protons) which are streaming through a background plasma may be unstable to the generation of both electrostatic and hydromagnetic waves. These waves will be damped by, and hence will impart thermal energy to, the background plasma. These processes have not been sufficiently investigated for the conditions of interest here, and we will assume that the actual heating rate is not significantly greater than the collisional heating rate. The total heating rate due to the population of relativistic electrons is therefore

$$\Gamma = 10^{-18} n \int K E^{-(2\alpha+1)} dE . \tag{9}$$

Upon performing the integration, we have

$$\Gamma = 10^{-18} n K E_{\min}{}^{-2\alpha}/2\alpha \text{ ergs s}^{-1} \quad (\alpha > 0) , \tag{10}$$

where $E_{\min}$ is the low-energy cutoff in the electron spectrum. In a steady state, the cooling rate due to bremsstrahlung radiation from the gas is equal to the heating rate Γ, so we may set $L_x = \Gamma$. Thus, using (2),

$$L_x = 10^{16} n F \nu_F{}^{\alpha} B^{-(\alpha+1)} (6.3 \times 10^{18})^{-(\alpha-1)} (0.1/a) E_{\min}{}^{-2\alpha} d_{100}{}^2/(2\alpha) \text{ ergs s}^{-1} . \tag{11}$$

Taking ν_F to be 26.3 MHz and normalizing B to 10^{-6} gauss gives

$$L_x = 1.3 \times 10^{35} n F_{26} B_{\mu G}{}^{-2} E_{\min}{}^{-2} d_{100}{}^2 \alpha^{-1} (2.4 \times 10^5 B_{\mu G} E_{\min}{}^2)^{-(\alpha-1)} (0.1/a) . \tag{12}$$

For typical cluster parameters, $F_{26} \approx 100$, $n \approx 3 \times 10^{-3}$ cm^{-3} (in the thermal bremsstrahlung model) and $\alpha \approx 1$, we find $L_x \sim 10^{45}$ ergs s^{-1} if $E_{\min} = 6 \times 10^{-6} B_{\mu G}{}^{-1}$ ergs ($\gamma_{\min} \sim 8 B_{\mu G}{}^{-1}$). The steeper the radio spectrum, the more efficient is the heating. In Table 1 we show parameters for some of the cluster sources. Using these parameters, we have calculated the value of $E_{\min}$ required if $\Gamma = L_x$. We have used the 2–10 keV X-ray luminosity (Cooke *et al.* 1978), assuming that this number is not too different from the total luminosity if $T \sim 10^8$ K. Since $E_{\min} \sim L_x{}^{-1/2}$, this assumption will not have much effect on the value of $E_{\min}$. We conclude that the total energy requirements for the X-ray source are easily met by the observed radio sources if the electron spectrum extends to $E_{\min} \sim 10^{-4}$–10^{-5} ergs ($\gamma \sim 10$–100), even if the field is as high as 10^{-6} gauss ($E_{\min} \propto B^{-(\alpha+1)/2\alpha}$).

We have ignored the contribution of any possible cosmic-ray proton component in the above discussion. Since dE/dt is essentially independent of energy (eq. [8]), at least for $\gamma \gg 1$, the effect of including protons is to multiply the heating rate Γ and luminosity L_x by a factor of 2. If the protons are nonrelativistic but suprathermal, the effect is larger, since these particles heat more efficiently.

c) *Constraints on Inverse Compton Models*

Let us consider now the case in which the X-rays are produced by inverse Compton scattering. Making this assumption, we can calculate the value of the magnetic field in the cluster. We have done this in two ways: first, ignoring incompatibilities between the computed and observed X-ray spectra, we fitted the total X-ray luminosity in the 2–10 keV band to the observed value; second, we insisted that the electron spectrum break to $p = 3$ ($\alpha = 1$) at $\nu = 10$ MHz in order to provide a better fit to the observed X-ray spectrum. There is no *a priori* reason to assume

TABLE 1

CLUSTER RADIO AND X-RAY PARAMETERS

Cluster	F_{26}	α	z	d_{100}	F_x*	L_x†	B_{ic} (i)	B_{ic} (ii)
401	(65)	1.6	0.075	4.5	3.4	18.4×10^{44}	10^{-7}	4×10^{-8}
426 (Perseus)	751	1	0.018	1.1	47.4	13.6	4×10^{-8}	...
1367	141	1.2	0.02	1.4	3.6	0.6	9×10^{-8}	6×10^{-8}
1656 (Coma)	148‡	1.2§	0.023	1.4	14.8	7.8	5×10^{-8}	3×10^{-8}
2142	29	1.5	0.076	4.6	3.3	(25)	7×10^{-8}	5×10^{-8}
2256	69	2	0.06	3.6	3.6	9.6	10^{-7}	3×10^{-8}
Virgo	4341	0.79	0.0037	0.2	21.7	0.2	10^{-7}	...
Centaurus	184	1.14	0.017	0.7	4.8	2.4	9×10^{-8}	7×10^{-8}
Sc 0430–616	40‖	0.88‖	0.0601	3.6	2.3	...	4×10^{-8}	...
Sc 0627–544	332‖	1.06‖	0.0502	3.0	3.3	0.8	10^{-7}	...
Klemola 44	49‖	1.15‖	0.0274	1.6	1.8	1.2	7×10^{-8}	6×10^{-8}

NOTE.—Radio data from Erickson *et al.* 1977 except where noted. Numbers in parentheses are uncertain. The quantities d_{100} and L_x are derived for $H_0 = 50$ km s^{-1} Mpc^{-1}.

* X-ray flux in *Uhuru* counts s^{-1} from 4U catalog (Forman *et al.* 1978).

† 2–10 keV X-ray luminosity (Cooke *et al.* 1978).

‡ Viner and Erickson 1975.

§ Jaffe 1977.

‖ Data from Finlay and Jones 1973.

such a break, but it cannot, as yet, be ruled out by observation. These two procedures give maximum and minimum values of B if the inverse Compton X-ray flux is to equal the observed X-ray flux. As the energy of the break moves downward, B increases towards the "no-break" value. The computed values of the magnetic field strength are shown in Table 1. Taking $n \sim 10^{-4}$ cm^{-3}, we can ask what the lowest energy is to which the electron spectrum can extend in order that the thermal X-ray flux not exceed the observed flux. The result is $\sim 10^{-4}$ ergs, corresponding to inverse Compton photons at $E_{\rm ph} \sim 0.02$ keV.

We note also that the requirement that the inverse Compton X-ray flux not exceed the observed X-ray flux allows us to place a limit of $B \gtrsim (3\text{–}10) \times 10^{-8}$ gauss on the intergalactic magnetic field in clusters. Alternatively, the spectrum of particles must exhibit a turnover at E_T such that $3 \times 10^{-2} B_{-8}^{-1/2}$ ergs $> E_T > 2.4 \times 10^{-3}$ ergs and $p < 3$ below the break.

d) *Total Energy Requirements*

The total energy in relativistic electrons which is required to heat the intracluster gas, $E_{\rm Total} = KE_{\rm min}^{1-2\alpha}/(2\alpha - 1)$, can be obtained from the steady state condition $L_x = \Gamma$ (eqs. [10] and [11]), giving $E_{\rm Total} \sim 10^{60}\text{–}10^{62}$ ergs. On the other hand, the equipartition energies of discrete radio sources in clusters are found to be on the order of $10^{58}\text{–}10^{59}$ ergs (cf. Riley 1975). Hence, if a single galaxy supplies the relativistic particles, $10\text{–}10^4$ radio events are required. Studies of radio galaxy properties do in fact indicate that a single active galaxy undergoes $10\text{–}10^3$ radio events within its lifetime (Christiansen 1971). It is of course also possible that the heating particles are supplied by several of the cluster galaxies.

The total energy in relativistic electrons corresponds to $\sim 0.01\text{–}1\%$ of the total kinetic energy of a cluster and to less than 0.1% of the rest mass energy of a $10^{11}\ M_\odot$ galaxy. The total energy needed in the inverse Compton model to produce X-ray emission down to 0.1 keV is typically an order of magnitude greater than that needed in the thermal model.

e) *Predicted Temperatures*

The above discussion concerned the global energetics of the cluster. Here we consider the problem from a more local point of view. The heating rate per unit volume within the radio source is

$$\Gamma = 10^{-29} n E_{\rm min,\,-3}^{-2} F_{26} B_{\mu\rm G}^{-2} d_{100}^{2} r_{50}^{-3} (2 \times 10^{11} B E_{\rm min}^{2})^{-(\alpha-1)} \alpha^{-1} (0.1/a) \text{ ergs cm}^{-3}\text{ s}^{-1}\,, \tag{13}$$

where $E_{\rm min,\,-3}$ is in units of 10^{-3} ergs. The heating time to reach a temperature $T_8 \times 10^8$ K (ignoring cooling) is then

$$\tau_H = 4 \times 10^{13} B_{\mu\rm G}^{2} E_{\rm min,\,-3}^{2} F_{26}^{-1} T_8 d_{100}^{-2} r_{50}^{3} \alpha (2 \times 10^{11} B E_{\rm min}^{2})^{\alpha-1} \text{ yr}\,. \tag{14}$$

We shall denote $\tau_H(T_8 = 1)$ by τ_8. The time τ_8 is less than the Hubble time if $F_{26} \sim 100$, $B_{\mu\rm G} \sim 10^{-1}$, and $E_{\rm min} < 10^{-3}$ ergs. The parameter τ_8 is useful in describing the physics of the cluster. If $\tau_8 \sim 10^{10}$ years, then for a density $n \sim 3 \times 10^{-3}$ cm^{-3} the thermal X-ray source results. If $\tau_8 > 10^{10}$ years, the gas is too cool. If $\tau_8 < 10^{10}$ years, the gas will be overheated. We note also that τ_8 is independent of the gas density.

According to Raymond, Cox, and Smith (1976) the cooling rate $\Lambda \sim 5 \times 10^{-23} n^2$ ergs cm^{-3} s^{-1} for $T \sim (0.5\text{–}1) \times 10^7$ K. With $F_{26} = 100$, heating dominates cooling in this temperature range for $n \lesssim 2 \times 10^{-5} \times E_{\rm min,\,-3}^{-2} B_{\mu\rm G}^{-2}$ cm^{-3}. If $T > 10^7$ K, most of the cooling is due to bremsstrahlung so that $\Lambda \sim 10^{-27} T^{1/2} n^2$ ergs cm^{-3} s^{-1}. Using this cooling rate, we calculate an equilibrium temperature of a plasma heated by cosmic-ray electrons and cooled by bremsstrahlung:

$$T_{\rm eq} \sim 5 \times 10^{-5} n^{-2} F_{26}^{2} E_{\rm min,\,-3}^{-4} B_{\mu\rm G}^{-4} (0.1/a)^2 d_{100}^{4} r_{50}^{-6} (2 \times 10^{11} B E_{\rm min}^{2})^{-2(\alpha-1)} \alpha^{-2}\,, \tag{15}$$

or, writing $n_{-4} = n/(10^{-4}$ cm$^{-3})$,

$$T_{\rm eq} \sim 8 \times 10^{10} n_{-4}^{-2} (10^{10}/\tau_8)^2 \text{ K}\,. \tag{16}$$

If $n \sim 3 \times 10^{-3}$ cm^{-3} and $\tau_8 \sim 10^{10}$ yr, $T_{\rm eq} \sim 9 \times 10^7$ K. If $r_{50} \sim 10$, as is appropriate for the Coma cluster, then $\tau_8 \sim 10^{10}$ yr if $B_{\mu\rm G} = 1$ and $E_{\rm min} \sim$ few $\times 10^{-5}$ erg. These numbers agree with the calculations based on gross energetics in the previous section.

f) *Constraints on Inverse Compton Models from Local Energetics*

When $n \sim 10^{-4}$ cm^{-3}, as in the inverse Compton models, then $\tau_8 \sim 10^8$ yr leads to high values of $T_{\rm eq}$. For example, if we require that $E_{\rm min}$ be at least as small as that value which would give an inverse Compton photon at $\frac{1}{4}$ keV, then, taking $r_{50} = 10$, $T > 10^9$ K for most of the clusters, and $T > 10^{10}$ K for some (e.g., Coma). Such values are artificial, however, since the time required to heat the gas to such temperatures is greater than the Hubble time, and the gas is not bound by the cluster at these high temperatures. In addition, $T_{\rm eq}$ is very sensitive to the size of the radio source, which enters to the sixth power. Observational data on the radio source size are sparse. Viner and Erickson (1975) give the size of some sources at 20 MHz, corresponding to $E = 5 \times 10^{-2} B_{-8}^{-1/2}$

ergs. For the Coma cluster this size is 28′, which is consistent, within the errors, with the size at higher frequencies (and thus higher E). The implied value is $r_{50} = 11$. For 3C 84 in the Perseus cluster the source diameter is $\sim 7'$, again showing little change with frequency,[1] giving $r_{50} = 2.2$. For Perseus, then, $\tau_8 \sim 10^8$ years ($E_{\min, -3} = 0.38$) and $T_{\rm eq} \sim 6 \times 10^{14}$ K! For Coma, $\tau_8 \sim 10^{10}$ yr and $T_{\rm eq} \sim 8 \times 10^{10}$ K. In this case, the small value of τ_8 is telling us that energy is being pumped into the gas very rapidly. The result is that the gas flows out as a wind. Wind solutions (including the effects of momentum transfer) are derived in Appendix A. For the case in which $\Gamma \gg \Lambda$ (i.e., $T_{\rm eq} \gg 10^8$ K), the condition that the terminal velocity in the wind be greater than the escape velocity is

$$\tau_8 \ll 3.4 \times 10^7 (r_{50} a_{250})^{1/2} (\Delta V_8)^{-3} \text{ yr}, \tag{17}$$

where ΔV_8 is the cluster velocity dispersion in units of 1000 km s^{-1}, and a_{250} is the core radius in units of 250 kpc. For Perseus, this is $\tau_8 \ll 3.6 \times 10^6$ yr; for Coma, $\tau_8 \ll 1.6 \times 10^7$ yr. These values of τ_8 are attained for $E_{\min} \sim 7.2 \times 10^{-5}$ ergs and $E_{\min} \sim 3.2 \times 10^{-5}$ ergs, respectively. If the spectrum does extend this far, then the bulk gas motion would be sufficient to affect the structure of radio tail sources in these clusters, causing the tails to point outward. Statistically, the tails have no preferred direction (Lea 1976; Harris 1977) and specifically in the case of Perseus the smallest of the three tails in this cluster points inward (Miley *et al.* 1972). In addition, the mass loss rate implied by the wind (velocity $u_w \sim 2 \times 10^7 r_{50}^{1/2} (\tau_8/10^{10})^{-1/3}$ cm s^{-1}) is $\sim 6 \times 10^{-11}\, M_\odot$ yr^{-1} $M_\odot^{-1}$ for the Perseus cluster if $E_{\min} \sim 7 \times 10^{-5}$ ergs, a rather large mass loss rate for elliptical galaxies. Thus we can state that if the inverse Compton model for X-ray emission is to be compatible with the existence of radio tails in clusters, the electron spectrum *must* cut off at $E_{\min} \gtrsim 7 \times 10^{-5}$ ergs in the Perseus cluster, and $E_{\min} \gtrsim$ a few $\times\, 10^{-5}$ ergs ($\gamma \sim$ a few tens) in general.

In contrast, for the thermal model we find that the minimum γ is of the order of one, if the velocity in the wind is to be small with respect to the galaxy velocities. The minimum γ scales with B as $B^{-(\alpha+1)/2\alpha}$.

III. ORIGIN OF THE ELECTRONS

The previous discussion has centered on the observed electrons as inferred from the radio data. Here we consider a possible model for the radio source in which the electrons are continuously injected into the cluster by a central active galaxy (or galaxies). Since the magnetic field in the cluster is expected to be less than 4×10^{-6} gauss, the primary loss mechanism for the high-energy electrons is the inverse Compton mechanism (see, e.g., van der Laan and Perola 1969):

$$\tau_{\rm ic} \sim 2 \times 10^6 E^{-1} \text{ years}. \tag{18}$$

At the low energy end of the spectrum, the dominant loss mechanism is heating by Coulomb collisions:

$$\tau_{\rm cc} \sim 3 \times 10^{10} (E/n) \text{ years}. \tag{19}$$

Nonthermal bremsstrahlung losses are never important in this system. The two lifetimes are equal at an energy $E_{\rm eq} \sim 8 \times 10^{-3} n^{1/2}$ ergs.

The lifetime of the source can be estimated from source statistics. Of Abell clusters in distance classes 0 through 2, 88% are observed to be radio sources. Even including distance class 3, where selection effects start to become important, in the sense that the weaker sources drop below the sensitivity limit of the detectors, 65% of the clusters are observed. Restricting consideration to Bautz-Morgan types I, I–II, and II, the figures are 91% and 81%, respectively (Matthews 1977). If all clusters are sources at some time, then we conclude that the average source lifetime is $\gtrsim 65\%$ of the age of the cluster. Since the lifetime of the high-energy particles required to explain the radio emission is rather short ($\lesssim 10^8$ years at 100 MHz) for a magnetic field $\lesssim 10^{-7}$ gauss, continuous injection or reacceleration is implied.

Let the source inject particles with a spectrum QE^{-p}, and suppose that they diffuse outward. In the region where Compton losses dominate ($E > E_{\rm eq}$), $\dot{E} = -bE^2$, where $b \sim 1.6 \times 10^{-14}$ erg^{-1} s^{-1}. Then the distribution of electrons in a steady state is given by

$$f(r, E) = QE^{-p} (4\pi Dr)^{-1} \pi^{-1/2} \Gamma(p - 1) \exp(-bEr^2/4D) U(p - \tfrac{3}{2}, \tfrac{1}{2}, bEr^2/4D), \quad p > 1 \tag{20}$$

(e.g., Webster 1970), where U is the confluent hypergeometric function of the second kind and D is the diffusion coefficient. Since the size of the Perseus and Coma radio sources remains almost constant with frequency in the frequency range 26–600 MHz (Viner and Erickson 1975; Jaffe 1977) we require that

$$\frac{bEr^2}{4D} \ll 1. \tag{21}$$

Then the distribution becomes

$$f(r, E) = QE^{-p} (4\pi Dr)^{-1} \tag{22}$$

[1] We have not included here the extended halo 3C 84 B reported by Ryle and Windram (1968). The spectrum of 3C 84 B is flatter than that of 3C 84 A (halo) and therefore is expected to contribute little to the heating.

and we must have

$$D \gg 2 \times 10^{32} B_{\mu G}^{-1/2} \left(\frac{\nu_{max}}{600 \text{ MHz}}\right)^{1/2} \left(\frac{r}{500 \text{ kpc}}\right)^2 , \tag{23}$$

where ν_{max} is the maximum radio frequency at which the source still has an observed size r. In this model we can calculate the heating rate as a function of distance r from the particle source:

$$\Gamma(r) = \int \frac{QE^{-p}}{4\pi Dr} 10^{-18} n dE = \frac{10^{-18} nQ}{4\pi Dr} \frac{E_{min}^{-2\alpha}}{2\alpha} . \tag{24}$$

Q is related to K as determined from the radio data. Integrating over the whole source, we find

$$K = \frac{Q}{p - 1} \tau_{ic} . \tag{25}$$

However, if the source size is restricted to $r \ll (4D/bE)^{1/2}$, then

$$K = \frac{Q r_{max}^2}{2D} , \quad r_{max} < \left(\frac{4D}{bE_{max}}\right)^{1/2} , \quad \text{and} \quad k = \frac{3Q}{8\pi D r_{max}} . \tag{26}$$

Since the source size does not vary with frequency, this implies we are observing only regions having $r \ll (4D/bE)^{1/2}$, so this latter value is the one to use. The most probable reason for such a restriction is that the field decreases as density decreases in the cluster. Since $j_\nu \propto B^{\alpha+1} \sim B^2$, a decrease in B causes a rapid decrease in j_ν and hence restricts the source size. If this is in fact the case, then we must interpret B in our formulae as $\bar{B}$, where

$$\bar{B} = \left[\frac{1}{V} \int \frac{r_{max}}{r} B^{\alpha+1} dV\right]^{1/(\alpha+1)} , \tag{27}$$

where the integration is over the observed source volume.

An alternative is that the diffusion coefficient is not independent of energy but is roughly proportional to E. For scattering by magnetized clouds or in a turbulent magnetic field, an energy dependence is expected only if the characteristic size of the scattering regions is much less than the electron Larmor radius in these regions (Ginzburg and Syrovatskii 1964). Since the Larmor radius is on the order of a few astronomical units, this is not expected to be the case.

Then

$$\Gamma(r) = \tfrac{2}{3} 10^{-18} n \left(\frac{r_{max}}{r}\right) k \frac{E_{min}^{-2\alpha}}{2\alpha} , \tag{28}$$

independent of D. The heating rate gives an equilibrium temperature

$$T \sim \frac{25}{n_{-3}^2} \left(\frac{r_{max}}{r}\right)^2 \frac{E_{min,-3}^{-4}}{\alpha^2} F_{26}^2 (0.1/a)^2 B_{-6}^{-4} \frac{d_{100}^{+4}}{r_{50}^6} (2 \times 10^{11} B E_{min}^2)^{-2(\alpha-1)} . \tag{29}$$

Here n is also a function of r. This expression has a divergence at $r = 0$, which is an artifact of our choice of a point source of particles. Nevertheless, the temperature does increase rapidly as r decreases. Let us take the gas distribution to be a known function and write n as $n_0 f(r)$, where $f(r)$ is approximately constant for r less than some characteristic radius r_g and declines for $r > r_g$. Then $T \sim 1/r^2$ for $r < r_g$. T may actually increase for $r > r_g$ if $f(r)$ declines faster than $1/r^2$. However, if $r_g > r_{max}$ this region does not exist. This temperature distribution is also convectively unstable for some values of $f(r)$. If $f(r) \sim (1 + r^2/a^2)^{-3/2}$, the resulting $T(r)$ is unstable for $r \lesssim 0.6a$. Stable equilibrium solutions can be constructed in which the gas density distribution is calculated in a self-consistent manner. These solutions have $T = 4GM_c\mu/(ka)$ at $r = 0$ (cf. the polytropic equilibria, e.g., Lea 1975) and $n \sim [y^4 g(y)]^{-1/2}$, where $y \sim r/a$ and

$$g(y) = y^{-3} \ln [y + (1 + y^2)^{1/2}] + (1 + y^2)^{1/2} y^{-2} - \ln \{y^{-1}[1 + (1 + y^2)^{1/2}]\} . \tag{30}$$

Thus $n \to \infty$ as $r \to 0$, which is again a result of the singularity in the cosmic ray density at $r = 0$. Such a density distribution may be compatible with the X-ray emission profile in clusters such as Perseus which have a point source component, but not for clusters like Coma which show no small area enhancements. In general, this profile is too steep to fit the data. However, these solutions do indicate the general property that it is the potential well which determines the gas temperature, and not the heating mechanism.

In discussing the diffusion model however, we must also consider the momentum transferred to the gas by the outward moving electrons. This ensures that the gas is distributed over a region at least as large as the radio source. In some cases in which the total energy supply is adequate, the gas acquires escape velocity and leaves the cluster.

The condition under which this happens is derived in Appendix A. For the Perseus cluster, using the parameters of the inverse Compton model, the wind has a terminal velocity greater than escape velocity for $\gamma_{\min} \ll 110$. In the thermal models, such a wind blows only if $B_{-6}^{(\alpha+1)/2\alpha}\gamma_{\min}$ is much less than 1.

IV. DISCUSSION

We have shown that the Coulomb interactions between nonthermal electrons and thermal plasma in clusters provide sufficient heating to power the X-ray sources in these clusters. However, since the cooling time for the intracluster gas is of the order of the Hubble time, we must also enquire as to whether such heating is necessary.

The recent observation of iron lines in cluster spectra (Mitchell *et al.* 1976; Serlemitsos *et al.* 1977; Mushotzky *et al.* 1978) indicates that the gas is not primordial in origin, but has been shed from the galaxies in the cluster. The gas emerges from the galaxies at a temperature much less than the observed value of $\sim 10^8$ K (see, e.g., Mathews and Baker 1971; Lea and De Young 1976; De Young 1978). This gas will tend to move with the parent galaxy, and will suffer collisions with other such clouds as the galaxies move in the cluster potential (De Young 1978). The gas is heated by these collisions to a temperature

$$\sim \frac{3}{16}\frac{\mu}{k}[2\Delta v + (v_w)]^2, \qquad \text{or} \qquad \frac{1}{4}\left(\frac{\gamma}{\gamma-1}\right)\left[1 + 2\frac{v_w}{\Delta v} + \left(\frac{v_w}{\Delta v}\right)^2\right]$$

of the central temperature of a static equilibrium configuration, where v_w is the velocity of the gas in the galactic wind.[2] For $v_w/\Delta v \sim 0.3$ (cf. De Young 1978), $T \sim 0.4[\gamma/(\gamma-1)]T_c \sim 0.95T_c$ if $\gamma = 5/3$. In the core the gas will therefore be in equilibrium. From De Young's paper, we find that the density in the core rises to $\sim 7 \times 10^{-3}$ cm^{-3} in $\sim$(2–3) times the time scale for star formation (much less than the Hubble time for an elliptical galaxy). The cooling time in the cluster core is then $\sim 4 \times 10^9 T_8^{1/2}$ years. In the absence of heating, the gas then cools and collapses in less than half the Hubble time.

The presence of a steep-spectrum radio source in the cluster is sufficient to prevent this cooling collapse and also ensures that the gas expands to a size $\sim$radio source size, as observed. The density then drops to roughly its observed value of $\sim 2 \times 10^{-3}$ cm^{-3}, and $\tau_{\text{cool}} \sim 10^{10}$ years.

It is possible that such a cooling collapse is responsible for the onset of radio emission (e.g., Silk 1976; Hedrick and Cox 1977). We might then expect that the radio source and gas cloud would expand together. We find from the results in Appendix A that the time to reach a size $\sim$500 kpc is $\sim 10^{10}$ years if τ_H in the final configuration is 10^{10} years.

On the other hand, if we postulate that the X-rays are the result of inverse Compton scattering, then we require that the electron spectrum cut off before $\gamma \sim 100$ in order that high-velocity winds do not result. This corresponds to an inverse Compton photon energy of $\sim$10 eV or a synchroton photon of $\sim$0.3 kHz. The synchroton photons are not accessible to observation. The expected inverse Compton flux at 10 eV is $\sim 10^{-25}$–10^{-26} ergs cm^{-2} s^{-1} Hz^{-1}. Some clusters have been observed in this energy range, although the intent of these experiments was to observe Lα line emission at 10.2 eV (Holberg, Bowyer, and Lampton 1973; Bohlin, Henry, and Swandic 1973). For the Coma cluster, the predicted inverse Compton flux is $\sim 2 \times 10^{-26}$ ergs cm^{-2} s^{-1} Hz^{-1} at 10 eV, while the Holberg *et al.* Lα upper limit gives a continuum upper limit at 10 eV of 8×10^{-20} ergs cm^{-2} s^{-1} Hz^{-1} to within a factor of 2 (Lampton 1977, private communication). The continuum limit from the Bohlin *et al.* experiment is also of this order (Bohlin 1977, private communication), many orders of magnitude above the predicted flux. Further improvement in such measurements will be severely limited by the high background due to galactic and solar system light. Observation of the inverse Compton X-rays in the spectral region corresponding to the observed radio ($\gtrsim$500 keV) remains the best way to decide the question of the contribution of inverse Compton scattering to the observed X-rays.

We stress that regardless of the mechanism for heating the gas, the observed temperature *in a steady state* should be a function only of the cluster parameters, because of the regulation of the temperature by the cluster potential well. If the gas gets too hot, it flows out as a wind. If the gas is too cool, it collapses. Thus we do not expect a tight correlation between the radio and X-ray properties. We do expect some correlation, however. The cooling time in the core $\tau_c \sim a^2(\Delta v)^{-1}$. Thus those clusters which have large velocity dispersions and small core radii have short cooling times. These clusters require additional heating, such as is provided by a steep-spectrum radio source, in order that the gas not collapse to the center. On the other hand, if Δv is small and a is large, the cooling time is long. These clusters do not need a steep-spectrum radio source. In fact, if one exists, the gas may be blown out of the cluster. Put more quantitatively, those sources having

$$\left(\frac{a}{250\text{ kpc}}\right)^2\left(\frac{\Delta v}{1000\text{ km s}^{-1}}\right)^{-1} \lesssim 1.3$$

require a steep-spectrum radio source. The above discussion assumes that all clusters gain gas from their constituent galaxies at the same rate. If this is not the case, then the above number becomes $a_{250}{}^2\Delta v_{1000}{}^{-1} \lesssim 1.3 \times (\Delta M/M) \times 10$, where $\Delta M/M$ is the fraction of the galaxy mass liberated in the early star-formation burst in the

[2] We assume in this section that galaxy velocities are a true indicator of the total cluster gravitational potential.

cluster, and shed into the cluster in the first $\sim 5 \times 10^9$ years of its existence. Further, those clusters having a high Δv must have both a high temperature and a high density, implying a high X-ray luminosity. Thus strong, steep-spectrum radio sources should correlate with high-luminosity, high-temperature X-ray sources. If the cluster does not have a high Δv, then the gas will escape as a wind and X-rays will not be seen, since the gas density in the wind is expected to be low. We are assuming here that gas is produced by the galaxies in a similar fashion in each cluster, so that those clusters which have a wind must have a lower gas density than those clusters which retain all their gas. Conversely, in the absence of a strong radio source (or other heating mechanism) only those clusters having a long cooling time (and therefore low L_x) will be observed as X-ray sources. Bahcall (1977) has recently shown that gas density proportional to galaxy density and $T \propto \Delta v^2$ is compatible with the X-ray data (see also Mushotzky *et al.* 1978). This model thus provides both this correlation and the radio–X-ray correlation simultaneously.

V. CONCLUSIONS

We summarize in Table 2 parameters of inverse Compton models and thermal models such that the observed X-ray and radio properties, and the correlations between them, are reproduced. All the parameters of the inverse Compton model are tightly constrained by the requirements that the inverse Compton flux equal the observed X-ray flux (B), or that the intracluster gas which is needed for the formation of radio tails not be overheated, leading to high-velocity winds (n, $\gamma_{\min}$), or that the thermal X-ray flux be unobservable (n). In addition, these models cannot easily explain the observed 6.7 keV iron line and the observed spectrum can be explained only by invoking ad hoc bends in the electron spectrum. On the other hand, only n is strongly constrained in the thermal models, by the requirement that the thermal X-ray flux equal the observed flux. The regulation of the gas temperature by the cluster potential, coupled with the greater importance of cooling and lower relativistic electron density in these models (compared with the inverse Compton models), allows more freedom in the allowable values of γ. B is constrained only by the requirement that the inverse Compton X-ray flux not be observable in the 2–10 keV band. Inverse Compton X-rays must be present at some level, of course; their contribution to any one individual source remains to be determined.

We therefore conclude that, while inverse Compton models cannot yet be conclusively eliminated, the thermal models discussed here can explain *all* of the observed properties of cluster X-ray sources, including the correlation with radio properties, in a more satisfactory manner.

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TABLE 2

CONSTRAINTS ON MODEL PARAMETERS

Parameter	IC Model	Thermal Model
$\gamma_{\min}$.	$100 \leq \gamma_{\min} \lesssim 500$	$\lesssim 100$
B (gauss).	$(3–10) \times 10^{-8}$	$>(3–10) \times 10^{-8}$
n(cm^{-3}).	$10^{-4} \lesssim n \lesssim 10^{-3}$	$\sim 3 \times 10^{-3}$
Iron line.	No	Yes
Spectrum.	?	Yes

APPENDIX A

CLUSTER WINDS

The steady-state equations for fluid flow in the case considered here are

$$4\pi r^2 \rho u = \dot{M}\,, \qquad \text{conservation of mass}\,, \tag{A1}$$

$$\boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\frac{1}{\rho}\nabla p + \frac{\Gamma}{\rho c}\boldsymbol{r} + \boldsymbol{g}\,, \qquad \text{momentum}\,, \tag{A2}$$

$$\boldsymbol{u}\cdot\nabla\epsilon + \rho\boldsymbol{u}\cdot\nabla\left(\frac{1}{\rho}\right) = (\Gamma - \Lambda)/\rho\,, \qquad \text{energy}\,, \tag{A3}$$

where ϵ is internal energy (see, e.g., Zel'dovich and Raizer 1966). These equations can be combined to yield the integral equation

$$\dot{M}(5c_s^2/2 + \tfrac{1}{2}u^2) = \int [\Gamma(1 + u/c) - \Lambda + \rho g u]dV, \tag{A4}$$

where c_s^2 is the isothermal sound speed and $g = \boldsymbol{g}\cdot\hat{\boldsymbol{r}}$. Now let $\Gamma \gg \Lambda$ and $u \ll c$. Also, noting that $\Gamma \propto n$, we write $\Gamma = \Gamma_1\rho$. Then (A4) becomes

$$5c_s^2/2 + \tfrac{1}{2}u^2 = \int \left[\Gamma_1/u - \frac{3GM_c}{r^2} f_m(r/a)\right] dr,$$

where the mass of the cluster $M(r) = 3M_c f_m(r/a)$ and a is the core radius. Now let $\Gamma_1 = \Gamma_0 f(y)$ and $y = r/a$:

$$(5/2)c_s^2 + \tfrac{1}{2}u^2 = \Gamma_0 \int \frac{a}{u} f(y)dy - \frac{3GM_c}{a} \int f_m(y)y^{-2}dy. \tag{A5}$$

Consider now the case where the wind reaches escape velocity. Then $\exists y$ such that $u^2(y) \gg c_s^2(y)$ and

$$\tfrac{1}{2}u^2 = \Gamma_0 I_{1,r} - \frac{3GM_c}{a} I_{2,r}, \tag{A6}$$

where

$$I_{1,r} = \int_0^{r/a} \frac{a}{u} f(y)dy, \qquad I_{2,r} = \int_0^{r/a} f_m(y)y^{-2}dy.$$

Now let u be $u_0\, w(y)$, where $w(y) \to 1$ as $y \to \infty$, $w(y) \to 0$ as $y \to 0$:

$$\tfrac{1}{2}u_0^2 = \frac{a\Gamma_0}{u_0} \int_0^{\infty} \frac{f(y)}{w} dy - \tfrac{3}{2}v_{\rm ff}^2 I_{2,\infty} \equiv \frac{a\Gamma_0}{u_0} I_{3,\infty} - \tfrac{3}{2}v_{\rm ff}^2 I_{2,\infty} \tag{A7}$$

and we have written $v_{\rm ff}^2 = 2GM_c/a$. This is a cubic equation for u_0. Defining $q = v_{\rm ff}^2 I_{2,\infty}$, $t = a\Gamma_0 I_{3,\infty}$:

$$s_1 = [t + (q^3 + t^2)^{1/2}]^{1/3}, \qquad s_2 = [t - (q^3 + t^2)^{1/2}]^{1/3}. \tag{A8}$$

Then $u_0 = s_1 + s_2$. In the limit

$$(a\Gamma_0 I_{3,\infty})^2 \gg (v_{\rm ff}^2 I_{2,\infty})^3, \tag{A9}$$

$$u_0 = (2t)^{1/3} = (2a\Gamma_0 I_{3,\infty})^{1/3}. \tag{A10}$$

In the opposite limit of $(a\Gamma_0 I_{3,\infty})^2 \ll (v_{\rm ff}^2 I_{2,\infty})^3$ we obtain $u_0 = -q^{1/2} = -v_{\rm ff}(I_{2,\infty})^{1/2}$, or unsupported inward free fall, as required.

Looking at the differential form of (A5), for small y we expect u to be small but u' to be large. We can then approximate the equation by

$$u(\tfrac{1}{2}u^2)' = \Gamma_0 a f(y). \tag{A11}$$

The two forms of $f(y)$ which are of interest are

i)
$$f(y) = 1 \quad (r < r_{\rm max})$$
$$= 0 \quad (r > r_{\rm max}),$$

and

ii)
$$f(y) = \frac{1}{y} \quad (r < r_{\rm max})$$
$$= 0 \quad (r > r_{\rm max}). \tag{A12}$$

This latter assumes the particles diffuse out from a central source, and $r_{\rm max} = r_{\rm max}(E_{\rm min})$. The solution for u in the inner regions takes the form

i)
$$u \sim (3\Gamma_0 r)^{1/3},$$

ii)
$$u \sim (3\Gamma_0 a \ln r/r_{\rm min})^{1/3}, \tag{A13}$$

in the two cases, where the inner cutoff $r_{\min}$ has been introduced to avoid the singularity at $r = 0$ caused by assuming a point source of particles. The quantity $r_{\min}$ is therefore the size of the source. We can use these functional forms to find an estimate of the integral $I_{3,\infty}$:

$$I_{3,\infty} = \left[\frac{3}{2}\left(\frac{r_{\max}}{a}, \ln\frac{r_{\max}}{r_{\min}}\right)\right]$$

in the two cases. Thus the condition for the gravity term to be ignored in computing u_0 is

i) $$\Gamma_0 \gg v_{\mathrm{ff}}{}^3(\tfrac{3}{2}r_{\max}a)^{-1/2} ,$$

ii) $$\Gamma_0 \gg v_{\mathrm{ff}}{}^3[\tfrac{3}{2}r_{\max}{}^2 \ln(r_{\max}/r_{\min})]^{-1/2} . \tag{A14}$$

From Rood *et al.* (1972) we can express v_{ff} as $2^{1/2}\Delta v$, where Δv is the velocity dispersion at the center of the cluster. Then (A14[i]) can be written

$$\Gamma_0 \gg \frac{4}{\sqrt{3}}(\Delta v)^3(r_{\max}a)^{-1/2} . \tag{A15}$$

Now u_0 can be calculated crudely from

$$u_0 \sim (3\Gamma_0 r_{\max})^{1/3} . \tag{A16}$$

As an example, the Perseus cluster inverse Compton model has

$$\Gamma_0 \sim 0.38E_{-3}{}^{-2} , \qquad \Delta v \sim 2.4 \times 10^8 ,$$

so that (A14[i]) becomes

$$E_{-3}{}^{-2} \gg 181$$

and

$$u_0 \sim 4 \times 10^8(E_{-3}{}^{-2}/181)\ \mathrm{cm\ s^{-1}} .$$

Thus we expect an outflowing wind in Perseus with $u > v_{\mathrm{escape}}$ if $\gamma_{\min}$ is much less than about 90. The mass loss rate implied by such a wind is $\gtrsim 10^3\ M_\odot\ \mathrm{yr}^{-1}$, or $\sim 6 \times 10^{-11}\ M_\odot\ \mathrm{yr}^{-1}\ M_\odot{}^{-1}$ in the inner 100 kpc of the cluster. In the diffusion model, with $r_{\max}/r_{\min} = 100$, the limit on $\gamma_{\min}$ is ~ 105.

Consider now the case where $\Gamma \sim \Lambda$ and u is small. The condition that $u \ll c_s$ is a small perturbation to the static equilibrium solution is

$$\int \frac{\Gamma_1}{c}\,dr \ll \int g dr \tag{A17}$$

i) $$\frac{\Gamma_0}{c} \ll \frac{3(\Delta v)^2}{a}\frac{1}{y}\left\{1 - \frac{\ln[y + (1 + y^2)^{1/2}]}{y}\right\} ,$$

ii) $$\frac{\Gamma_0}{c} \ll \frac{3(\Delta v)^2}{\ln r/r_{\min}}\left\{1 - \frac{\ln[y + (1 + y^2)^{1/2}]}{y}\right\} .$$

In case (i) we can write this in terms of τ_8 as

$$\tau_8 \gg 2 \times 10^5 a_{250}(\Delta v_8)^{-2} y\left\{1 - \frac{\ln[y + (1 + y^2)^{1/2}]}{y}\right\}^{-1}\ \mathrm{yr.} \tag{A18}$$

This analysis breaks down for small r, since the time scale for mass loss in the wind, $M/\dot{M}$, becomes very small at small r if u remains small but finite. This is a consequence of our neglect of the mass input from galaxies. Therefore (A18) is a generous limit on τ for the weak wind case.

APPENDIX B

DEPENDENCE ON REDSHIFT

For high-redshift sources our discussion is affected by the redshift of the emitted photons

$$\nu_{\mathrm{observed}} = \nu_{\mathrm{emitted}}(1 + z)^{-1} ,$$

and also by the fact that the energy density in the microwave background increases as $(1 + z)^4$, while the energy of a characteristic photon in this radiation field increases as $(1 + z)$. As a result the following dependences should be noted:

$$\text{eq. (1)}: J_{\nu,0} \propto \nu_0^{-\alpha}(1 + z)^{-\alpha};$$

$$\text{eq. (2)}: K \propto (1 + z)^{\alpha};$$

$$\text{eq. (3)}: k \propto (1 + z)^{\alpha};$$

$$\text{eq. (4)}: j_{\nu} \propto (1 + z)^{3+\alpha};$$

$$\text{eq. (5)}: F_{\text{ic}} \propto (1 + z)^{3+\alpha};$$

$$\text{eq. (6)}: \nu_{s,\text{observed}} = (1 + z)^{-1}10^{18}BE^2;$$

$$\text{eq. (7)}: \nu_{\text{ic,emitted}} \propto (1 + z); \quad \nu_{\text{ic,observed}} \propto (1 + z)^0;$$

$$\text{eqs. (11), (12)}: L_x \propto (1 + z)^{\alpha} \quad \text{and} \quad E_{\min} \propto (1 + z)^{1/2};$$

$$\text{eq. (13)}: \Gamma \propto (1 + z)^{\alpha};$$

$$\text{eq. (14)}: \tau_H \propto (1 + z)^{-\alpha};$$

$$\text{eq. (15)}: T_{\text{eq}} \propto (1 + z)^{2\alpha};$$

$$\text{eq. (18)}: \tau_{\text{ic}} \propto (1 + z)^{-4};$$

$$\text{eq. (29)}: T \propto (1 + z)^{2\alpha}.$$

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