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PARTICLE ACCELERATION BY ASTROPHYSICAL SHOCKS

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ABSTRACT

A new mechanism is proposed for acceleration of a power-law distribution of cosmic rays with approximately the observed slope. High-energy particles in the vicinity of a shock are scattered by Alfvén waves carried by the converging fluid flow leading to a first-order acceleration process in which the escape time is automatically comparable to the acceleration time. Shocks from supernova explosions propagating through the interstellar medium can account for the acceleration of galactic cosmic rays. Similar processes occurring in extragalactic radio sources can lead to efficient in situ acceleration of relativistic electrons.

Subject headings: cosmic rays: general — shock waves

I. INTRODUCTION

Supernovae in our Galaxy produce shocks propagating away from the central explosions which can encompass large volumes before radiative losses begin to drain their energy, $E_{51} \equiv E/10^{51}$ ergs. If the ambient interstellar hydrogen density is $n_0(\mathrm{cm}^{-3})$, then the volume, shock velocity, and remnant age at cooling are $V_{\mathrm{cool}} = 10^{59.17}\ n_0^{-1.23}\ E_{51}^{-0.87} = 10^{63.4}$ cm^3, $v_{\mathrm{cool}} = 10^{2.42}\ n_0^{0.12}\ E_{51}^{0.06} = 120$ km s^{-1}, $t_{\mathrm{cool}} = 10^{4.46}\ n_0^{-0.52}\ E_{51}^{0.22} = 10^{5.9}$ yr for the standard Sedov solution and similar numerically for the case when evaporation from interstellar clouds embedded in the hot gas determine the SNR dynamics (Cox and Smith 1974; McKee and Ostriker 1977; Spitzer 1978). The numerical values used are those appropriate for a model of the interstellar medium (ISM) where much of space is filled with hot, very low-density ($n_0 = 10^{-2.8}$) coronal gas. The ambient interstellar cosmic rays can move only with respect to the background gas at velocities less than the Alfvén velocity (Kulsrud and Pearce 1969), $v_{\mathrm{A}}[=1.3(B/10^{-6})$ gauss $n_0^{-1/2}$ km s^{-1} = 50 km s$^{-1}] < v_{\mathrm{cool}}$. Thus they are effectively frozen into the ISM for $t < t_{\mathrm{cool}}$ and will pass with the gas through the expanding shocks. Since the energy density in cosmic rays, U_{CR}, is $\sim 10^{-12}$ ergs cm^{-3}, the energy in shocked cosmic rays is $V_{\mathrm{cool}} U_{\mathrm{CR}} = 10^{51}$ ergs per supernova. Thus if, on average, the fractional energy increase of a cosmic ray on passing through the strong shock were $\epsilon \sim 10^{-1}$, the total energy input in this form would be $\epsilon\ V_{\mathrm{cool}} U_{\mathrm{CR}} \approx 10^{50}$ ergs SN^{-1}, which is more than sufficient to account for the known energy input requirements. Finding such a mechanism would immediately bypass the several well-known problems associated with point sources like supernovae (which, of course, may still be needed to provide the injection and initial acceleration of cosmic rays).

It has long been recognized that relativistic particles can be accelerated in the vicinity of a strong shock propagating through a nonrelativistic plasma, and several detailed mechanisms have been studied (e.g., Wentzel 1964; Hudson 1965; Jokipii 1966; Fisk 1971; Burn 1976). A new examination of the possibilities is prompted by the realization that the volume shocked by supernovae is so large. In the acceleration scheme discussed here, particles are scattered in pitch angle by wave turbulence on either side of a strong shock so that they can be accelerated by the Fermi (1949) method. We show how this process, which has been independently discovered by Axford, Lear, and Skadron (1977) and Bell (1977), leads naturally to a power-law distribution in momentum with the observed slope. This is especially important because power-law distribution functions of cosmic-ray particles are observed directly at Earth (e.g., Wentzel 1974) or inferred from radio observations in the Galaxy (e.g., Webster 1974), supernova remnants (e.g., Woltjer 1972), and extragalactic radio sources (e.g., De Young 1976). If we define $f(p, x)d^3p d^3x$ to be the number of particles in volume d^3p of momentum space and d^3x of real space, then it has been found that $f \propto p^{-s}$ with $4 \leq s \leq 5$ over many decades of relativistic energy and in quite different physical environments. This suggests that a general mechanism is at work, and acceleration in strong shocks is an attractive possibility in each case.

II. ACCELERATION BY A STRONG SHOCK

Consider a strong shock across which the density increases by a compression ratio r. The angle between the magnetic field and the shock normal ahead of, i.e., $x < 0$ (behind, $x > 0$) the shock is $\theta_-(\theta_+)$, and the fluid velocity in the shock frame is $u_-(u_+)$, so that $r = u_-/u_+ = \tan\theta_+/\tan\theta_-$. We note that, if we can transform to a frame moving with velocity $-u_- \tan\theta_-$ parallel to the shock front, then on both sides $\boldsymbol{E} = 0$ and $\boldsymbol{u} \parallel \boldsymbol{B}$.

Thus, particle acceleration by electric fields will not occur except for the case of shocks nearly perpendicular to the field ($u_- \tan \theta_- > c$) where the field cannot be transformed away. In this special case, the adiabatic invariant $p_\perp^2/B$ will be approximately conserved (Schatzman 1963; Chandrasekhar 1965; see also Chevalier 1977), and there is a net increase $\Delta(p^2)$ in the average squared momentum p^2 even after the density and field return to their preshock values; $\langle\Delta(p^2)/p^2\rangle = (1/3)(1 + 2r)r^{-2/3} - 1$. Returning to the general case, we hypothesize that there exists a source of wave turbulence with wave velocity $w \ll u$ in the vicinity of the shock capable of scattering high-energy particles in pitch angle ϕ at a rate $\langle(\Delta\phi)^2/\Delta t\rangle \equiv \nu$, and tending to make the distribution function isotropic in the frame of the background medium. The acceleration discussed here derives from the entropy generated within the shock front by this diffusive process. The increase in density across the shock requires a convergence in the velocity field and of any hydromagnetic waves being convected with the gas flow. Cosmic rays in the vicinity of the shock will be scattered repeatedly from these converging streams of waves and thereby accelerated by the first-order Fermi process, increasing their kinetic energy T at a rate $dT/dt \sim \nu T(u/v)$, where v is the particle speed. Pitch angle scattering is responsible for diffusion in real space as well as momentum space, and the corresponding coefficient is $D \sim v^2/\nu$; balancing diffusion with convection ahead of the shock establishes a concentration scale length $L \sim D/u_-$; but the thickness within which the first-order process occurs is $\sim(v/\nu)$ and so the net rate at which the shock does work on the high-energy particles of density n is $n(v/\nu)(dT/dt) \sim nTu$ per unit area, independent of ν and w. That is, when a strong shock wave propagates through a medium containing cosmic rays, their energy is roughly doubled, a result implicit in the earlier treatments referred to above.

Before we proceed with the calculation, a word on the four relevant length scales is necessary. We will assume that the shock thickness, δ, the cosmic-ray Larmor radius, r_L, the diffusion length, L, and the postshock fluid scale length, H (e.g., the radius of the SNR), are ordered as follows: $\delta \ll r_L \ll L \ll H$. The first inequality is probably well satisfied because $\delta \sim$ the ion Larmor radius of the thermal medium; violation of the second cannot occur for weak turbulence; and violation of the third, because ν is relatively small for high-energy cosmic rays, can significantly reduce the efficiency of the process.

The equation describing the isotropic part f of a distribution function of cosmic rays coupled by pitch angle scattering to a background medium is

$$\frac{\partial f}{\partial t} + (\boldsymbol{u} \cdot \nabla)f - \nabla \cdot (D_\| \boldsymbol{nn} \cdot \nabla f) = \tfrac{1}{3}[\nabla \cdot \boldsymbol{u}]\left[\frac{\partial f}{\partial \ln p}\right], \tag{1a}$$

or

$$u\frac{\partial f}{\partial x} - \frac{\partial}{\partial x}\left[D_\| \cos^2\theta \frac{\partial f}{\partial x}\right] = \tfrac{1}{3}(u_+ - u_-)\delta(x)\left[\frac{\partial f}{\partial \ln p}\right] \tag{1b}$$

(e.g., Hasselman and Wibberenz 1968; Skilling 1975) for the assumed geometrical conditions. In equation (1a) $\boldsymbol{n}$ is a unit vector in the field direction and $D_\|$, the diffusion coefficient parallel to the field, is given by

$$D_\| = (v^2/4)\int_0^\pi \nu^{-1} \sin^3\phi d\phi .$$

This equation must be solved on either side of the shock and the solutions joined by imposing continuity [by $\delta(x)$] of the distribution function f and the particle energy flux, at a given energy, normal to the shock, $-\boldsymbol{u}\partial f/\partial \ln p^3 - \kappa\nabla f$ [to 0 (u_-/v)], where $\kappa = D_\| \cos^2\theta$.

In a stationary solution, the flux $\boldsymbol{u}f - \kappa\nabla f$ must be constant on either side of the shock; f will approach asymptotic values $f_-(f_+)$ as the distance from the shock $x \to -\infty$ ($x \to +\infty$). As $\kappa > 0$, the solution behind the shock must be $f = f_+$, $x \geq 0$, whereas ahead of the shock

$$f = f_- + (f_+ - f_-)\exp\left[u_-\int_0^x dx'/\kappa_-(x')\right],$$

$x \leq 0$. However, independent of the variation of κ_-, we can relate f_+, f_-, using the junction conditions to obtain $df_+/d\ln p^3 = (f_+ - f_-)u_-/(u_+ - u_-)$, of which the solution is

$$f_+(p) = qp^{-q}\int_0^p f_-(p')p'^{(q-1)}dp' , \tag{2}$$

with $q = 3r/(r-1)$. That is to say, incident cosmic rays of space density n_- and momentum p_0 are Fermi accelerated by the shock to give a power-law distribution $f_+(p) = n_- p_0^{(q-3)}\Theta(p - p_0)/4\pi(q-3)p^{-q}$. If $f_- \propto p^{-s}$ with $s < q$, then $f_+ = q/(q-s)f_- \propto p^{-s}$. However, if $s < q$, lower-energy particles will be accelerated to give $f_+ \propto p^{-q}$, where the coefficient is determined by the number of particles for which Fermi acceleration is more important than thermalization by collisionless processes behind the shock and ionization losses. For particles accelerated in this way, the mean energy gain is $3/(5-2r)$, $r < 2.5$ for nonrelativistic particles, and $3/(4-r)$, $r < 4$ for ultrarelativistic particles. (For shocks that are more compressive than this, the mean energy gain is determined by the maximum momentum for which the isotropization is sufficiently effective.) Examination of time-dependent solutions to equations (1) show that this stationary state will be attained in a characteristic time $\sim\kappa_-/u_-^2$, determined by the scattering rate ahead of the shock.

If the shocked medium eventually expands to the density of the unshocked medium in a time $\tau + H^2/\kappa_+$, then the particles will cool adiabatically so that $f_+(p)$ will be reduced by a factor $r^{-s/3}$, leaving the spectrum unaffected. However, the shock heating will always exceed the adiabatic cooling corresponding to a net gain in entropy of the cosmic rays on crossing the shock. If $\tau \gg H^2/\kappa_+$, the situation is more complicated, but there is still a net gain in energy for particles passed by a shock.

III. ACCELERATION OF COSMIC RAYS

This mechanism may be responsible for the continuous acceleration of interstellar cosmic rays by supernova remnants in a way that avoids the catastrophic adiabatic decompression associated with processes directly related to supernovae (Kulsrud and Zweibel 1975). Cosmic rays streaming with a speed in excess of the Alfvén speed with respect to the background medium will excite Alfvén waves which will scatter them in pitch angle (Kulsrud and Pearce 1969). In this way, the wave turbulence that we have invoked above may be generated self-consistently by the cosmic rays. The condition that the amplitude of the turbulence be adequate to keep the scale length L less than the remnant radius R is simply that the growth time of the waves be small compared with the age of the supernova remnant, R/u^{-}. Taking the standard expression for the growth rate, this condition becomes $n_p \gtrsim c\rho^{1/2}e^{-1}R^{-1}$ (Kulsrud and Pearce 1969; Wentzel 1974) where n_p is the number density of cosmic rays of momentum greater than p, e is the electron charge, and ρ is the density of the background medium. In a low-density interstellar medium, where supernova remnants can expand to $R \gtrsim 100$ pc before becoming both sonic and Alfvénic, self-excited Alfvén waves are adequate to scatter particles of energy of at most 300 GeV. For particles more energetic than this, the turbulence must either exist in the ambient interstellar medium (Lee and Jokipii 1976) or possibly be excited by phase angle anisotropy in the manner suggested by Wentzel (1977). This process certainly fails when the particle Larmor radius exceeds the SNR radius at energies of at least $\sim 10^{18}$ eV and so cannot account for the acceleration of the highest-energy cosmic rays.

For a strong adiabatic shock in an ionized medium, $r = 4$, and so the predicted limiting slope is $s = 4$, about 0.5 less than observed. However, most particles will be accelerated when the shock starts to become Alfvénic. At this stage the efficiency of the process is reduced, as a fraction $\sim w/u_{-}$ of the energy heating the cosmic rays is lost to the waves. The compression will also be reduced, and both effects will tend to steepen the spectrum. Setting $s = 4.5$, corresponding to $r = 3$, we find that the relativistic cosmic-ray energy density is increased by a factor ~ 2 allowing for the adiabatic decompression. At mildly relativistic energies, we see that the spectrum produced at the source should be a simple power law in rigidity (momentum). We do not observe this part of the spectrum directly because of solar modulation, and in any case it will be modified by propagation effects. It is apparent that a more precise predicted spectrum can emerge only from a detailed calculation including these effects. This will be described elsewhere (Blandford, Cassé, and Ostriker, in preparation).

However, we can demonstrate that SNRs are energetically capable of accelerating cosmic rays. If each remnant expands to ~ 100 pc before becoming Alfvénic (or radiative), the energy of the interstellar cosmic rays engulfed by the blast wave is 10^{62} cm^3 $\times$ 10^{-12} ergs cm^{-3} = 10^{50} ergs SN^{-1}. The shock pumping increases this by a factor of ~ 2 (draining 0.1 of the energy of a typical supernova having $E = 10^{51}$ ergs); for a supernova rate of 10^{-68} cm^{-3} yr^{-1} (one per 60 years in the Galaxy), the energy input rate is 10^{-18} ergs cm^{-3} yr^{-1}, which, for a cosmic-ray density of 10^{-12} ergs cm^{-3}, gives an acceleration time of 10^6 years, comparable with the known residence time in the Galaxy. As a steep cosmic-ray spectrum can be flattened to nearly the limiting slope in a single passage of a shock, cosmic-ray ages need not be strongly correlated with their energies. It is interesting that the observed electron-proton ratio (~ 0.03 at 3 GeV) can be explained if we assume that the momentum distribution function slope, $f \propto p^{-4.5}$ for both electrons and protons, extends down to subrelativistic energies within the supernova remnant, and if both particle species build up their total energy densities (dominated by mildly relativistic particles) to approximate equipartition.

IV. RELATIVISTIC ELECTRON ACCELERATION IN EXTRAGALACTIC RADIO SOURCES

Fairly strong arguments (e.g., De Young 1976) can be given for the view that relativistic electrons are being continuously accelerated within the hot spots of strong double radio sources like Cygnus A (Hargrave and Ryle 1974, 1976). In beam models, the necessary power is supplied in the form of a collimated supersonic beam which terminates in a strong shock at the head of the source. Low-energy electrons injected by the beam can be accelerated efficiently by the shock mechanism described above (see Blandford and Rees 1976; Burn 1976) up to an energy fixed by equating the synchrotron cooling time to κ_{-}/u_{-}^{2}. If the scattering ahead of the shock is due to self-excited Alfvén waves, then substituting source values appropriate to Cygnus A ($B_{-} \approx 10^{-4}$ gauss, $u_{-} \sim 30{,}000$ km s^{-1}, and $\rho \sim 10^{-28}$ g cm^{-3}) we find that this maximum energy is approximately 10 GeV. These electrons will radiate at a frequency of at most ~ 100 GHz behind the shock with a spectral index of order unity, roughly what is observed.

In some low-surface-brightness sources (e.g., Coma C, Jaffe 1977; DA 240, Willis, Strom, and Wilson 1974), the inferred relativistic electron transport times across the source are much longer than the estimated synchrotron cooling times, suggesting that these particles are reaccelerated. If a power-law distribution of relativistic electrons is injected by active radio sources into these diffuse sources, then it can be reheated by weak shocks propagating through the source. These shocks, which can form from large-amplitude sound waves, can dissipate a substantial fraction of their energy in this manner. Plausible sources for the shocks are the bow waves associated with the motion of the galaxies in the case of a cluster radio source and the noise emanating from the supersonic beam in the case of a diffuse double source.

Finally, in many compact nonthermal sources associated with quasars and active galactic nuclei, it appears that relativistic electrons must be accelerated in situ, at large distances from the source of the energy.

One method of achieving this, one that has been advocated on other grounds (e.g., Blandford and McKee 1977), involves strong shocks moving at mildly relativistic speeds. Fermi acceleration can also work efficiently in this context.

More detailed application to these three types of source will be described in Blandford (in preparation).

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