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Massive Compact Objects in Elliptical Galaxy and Their Dynamical Relation to the Halo Formation

Takao SAITO

Department of Physics, Nagoya University, Chikusa-ku, Nagoya 464

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Abstract

A large number of massive compact objects are assumed to be distributed throughout an elliptical galaxy at its formation. The strong scattering of field stars is considered to be a result of their binary encounter with these objects. We evaluated the contribution of the scattered stars to the formation of the halo around the e liptical galaxy. In the elliptical galaxy M 87, about one tenth of the observed halo mass is explained by the present model.

Key words: Black hole; Elliptical galaxy; Galactic halo.

1. *Introduction*

According to recent observations and theories of active phenomena of the galactic nucleus and accompanying radio sources, some people have come to believe that the activity is due to the interaction between gas and a huge black hole at the center (see Lynden-Bell 1969). More recently this model was extended to a dynamical state of the galactic nucleus in which three or more massive objects (black holes) interact strongly, and this massive multiple system becomes unstable to split up and ejects massive compact objects (Saslaw et al. 1974). They claimed that this model could naturally explain the double nature of many radio sources, their association with the plane of a central elliptical galaxy, and the presence of very compact radio components. It was also noted that a rapidly revolving massive compact binary at the nucleus could strongly scatter a large number of ordinary stars. Thus there seems to be the possibility that this spray of stars forms a halo around the giant elliptical galaxies, such as those detected by the deep sky survey by de Vaucouleurs (1969), Arp and Bertola (1971), Welch and Sastry (1971), and Kormendy and Bahcall (1974). Arp and O'Connell (1975) suggested that a giant bright spot in CG 1116+51 and CG 1124+54 is due to a massive compact object ejected from its nucleus by the gravitational slingshot of Saslaw et al. (1974).

In the present paper we extend the model of Saslaw et al. (1974) to include a large number of massive compact objects (it will be hereafter referred to a group of MCOs) at the formation of a giant elliptical galaxy. The encounter between MCOs and ordinary field stars and the resultant scattering of stars are discussed. The halo formation by the scattered stars is examined and compared with observations. We also follow the dynamical evolution of this group of MCOs in a giant elliptical galaxy.

2. *Massive Compact Objects in an Elliptical Galaxy*

2.1. *The Number of Stars Scattered by a Massive Compact Object*

When an ordinary star of mass m_s, and velocity $\boldsymbol{v}_s$ encounters an MCO of mass m_o, and velocity $\boldsymbol{v}_o$ at an impact parameter D, the star gains the energy (Chandrasekhar 1960, p. 54)

$$\Delta E = -2m_s v_o V \cos(\phi-\psi)\cos\psi\cos i\ , \tag{2.1}$$

with

$$\cos\psi = \frac{1}{\left(1+\dfrac{D^2V^4}{G^2m_o^{\,2}}\right)^{1/2}}\ , \tag{2.2}$$

where G is the gravitational constant, V the relative velocity, $\pi-2\psi$ the deflection of the relative velocity in the orbital plane, and the angles ϕ and i are defined in figure 1. Since the inequality $m_s \ll m_o$ holds, the velocity $v_o=|\boldsymbol{v}_o|$ is equal to that of the center of gravity. We introduce a frame moving with the MCO, in which we take the polar coordinates whose z-axis is directed parallel to $-\boldsymbol{v}_o$. The geometry in figure 1 gives

$$\left.\begin{aligned} \cos i &= (\sin^2\theta\cos^2\Theta+\cos^2\theta)^{1/2}\ ,\\ \cos\phi &= -\cos\theta\,(\sin^2\theta\cos^2\Theta+\cos^2\theta)^{-1/2}\ . \end{aligned}\right\} \tag{2.3}$$

Substituting these relations into equation (2.1), we have

$$\Delta E = 2m_s v_o V\left[\cos\theta\left(1+\frac{D^2V^4}{G^2m_o^{\,2}}\right)^{-1} - \sin\theta\cos\Theta\frac{DV^2}{Gm_o}\left(1+\frac{D^2V^4}{G^2m_o^{\,2}}\right)^{-1}\right]. \tag{2.4}$$

The necessary energy for a field star to leave the central region of the galaxy and form a halo is given from the velocity distribution of the halo. The extension of the halo is so large that the velocity for the field star to contribute to it is nearly the same as the escape velocity from the galaxy, $v_\infty=2v_s$, where v_s is the root-mean-square velocity of the member stars. A reliable lower limit of

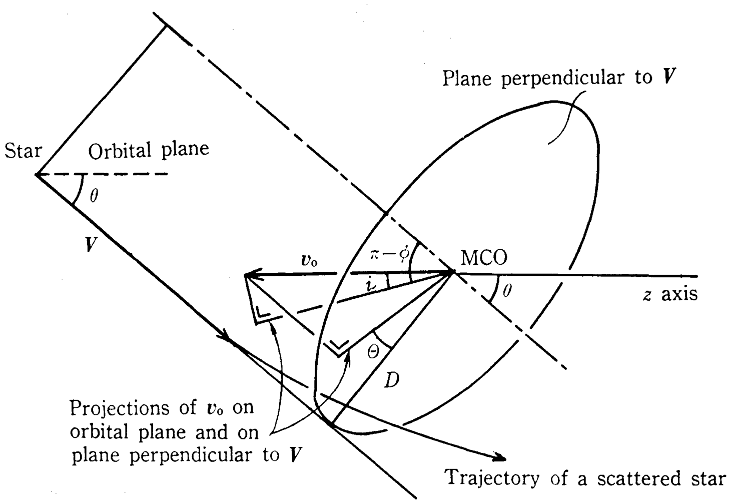

Fig. 1. A frame moving with the MCO and the trajectory of the field star.

the velocity for the halo formation may exist between v_s and v_∞, but in this paper, we take v_∞ as the lower limit for simplicity. Therefore, the field star that forms the halo must gain the energy, $\Delta E \gtrsim (3/2) m_s v_s^2$, at the encounter with an MCO. Statistically a significant fraction of the field stars encounter with the MCO at lower incident angles. Therefore, we consider the following situation for the order estimation that all the field stars run along one direction, $\theta=0$, with the root-mean-square velocity, v_s. [In this situation, the impact parameter D has a maximum value for the fixed ΔE in equation (2.4).] Then we obtain the impact parameter for the strong scattering:

$$D^2 = \frac{G^2 m_o^2}{V^4}\left(\frac{4}{3}\frac{v_o V}{v_s^2}-1\right), \tag{2.5}$$

and the number of stars scattered to form the halo per unit time, and per one MCO:

$$\begin{aligned} n\left(\Delta E \gtrsim \frac{3}{2} m_s v_s^2\right) &\simeq \pi D^2 n_s V \\ &= \pi G^2 m_o^2 \frac{n_s}{V^3}\left(\frac{4}{3}\frac{v_o V}{v_s^2}-1\right). \end{aligned} \tag{2.6}$$

The total number of stars scattered to infinity during a time-interval T is

$$N \approx n N_o T, \tag{2.7}$$

where N_o is the total number of MCOs in the elliptical galaxy, and T is the contraction time of a group of MCOs, which will be evaluated in the next subsection.

2.2. *Contraction of the Group of Massive Compact Objects*

(a) We consider a dynamical state in which a number of MCOs (a group of MCOs) are initially distributed throughout the giant elliptical galaxy and are in an equilibrium state satisfying the virial theorem. Then the group of MCOs starts to contract toward the center through the dynamical friction with field stars. In a state where the gravitational interactions among MCOs are negligible (we call this state a), we can estimate the contraction time scale of the MCO group by following a typical motion of a member. To do so, we follow the stellar-dynamical method used for the spiral motion of globular clusters in M 31 (Tremaine et al. 1975). When an MCO falls gradually to the galactic center on a spiral orbit, the quasi-circular velocity v_c is

$$v_c^2 = \frac{GM(r)}{r}, \tag{2.8}$$

where $M(r)$ is the mass of the galaxy inside r. Since the density distribution of the elliptical galaxy is closely approximated by an isothermal gas sphere with a central density ρ_0 and a structural length α, the mass $M(r)$ in equation (2.8) may be represented as $M(r) = 8\pi\rho_0\alpha^2 r$. Thus the quasi-circular velocity in equation (2.8) is constant with respect to r, $v_c = \sqrt{2}\,\sigma_s$, where σ_s is the line-of-sight velocity dispersion of field stars:

$$\sigma_s^2 = \frac{1}{3} v_s^2 = \frac{1}{2 j_s^2} = 4\pi G \rho_0 \alpha^2. \tag{2.9}$$

Let $\varepsilon_o(r)$ be the energy per unit mass on a quasi-circular orbit of radius r, then we have

$$\frac{d\varepsilon_o}{dr}=\frac{dU(r)}{dr}=\frac{2\sigma_s^2}{r}\,, \tag{2.10}$$

where $U(r)$ is the gravitational potential. Here the energy dissipation is, of course, due to the dynamical friction on an MCO (Chandrasekhar 1943),

$$\frac{d\varepsilon_o}{dt}=v_o\langle\Delta v_{o\parallel}\rangle=-2\sqrt{2}\,G\ln\Lambda g(j_s v_o)\frac{m_o\sigma_s}{r^2}\,, \tag{2.11}$$

where we use the diffusion coefficient,

$$\langle\Delta v_{o\parallel}\rangle=\frac{dv_{o\parallel}}{dt}=-4\pi G^2 m_o\rho_s\ln\Lambda\frac{\varphi(j_s v_o)-j_s v_o\varphi'(j_s v_o)}{v_o^2}\,,$$

with Λ, the mean radius of the galaxy divided by the impact parameter of the nearest approach 1 pc ($\sim$ the average distance between the field stars), and $g(j_s v_o)$ is defined as

$$g(j_s v_o)\equiv\frac{\varphi(j_s v_o)-j_s v_o\varphi'(j_s v_o)}{2(j_s v_o)^2}$$

with the error function φ (Spitzer 1962). Finally from equations (2.10) and (2.11), we obtain the change of the radius of the quasi-circular orbit of an MCO,

$$\frac{dr}{dt}=-\sqrt{2}\,\ln\Lambda g(j_s v_o)\frac{Gm_o}{\sigma_s r}\,. \tag{2.12}$$

Integrating from the initial radius to the final radius gives the time-interval T, mentioned in the preceding subsection. The total mass of stars M scattered during this time-interval T is, from equations (2.6) and (2.7) with $V=v_s+v_o$ and $v_s^2=3\sigma_s^2$,

$$M\approx 0.04\pi G^2 m_o^2(m_s n_s)\frac{1}{\sigma_s^3}N_o T\,, \tag{2.13}$$

where we have used as v_o the circular velocity of MCO, $v_c=\sqrt{2}\,\sigma_s$, and we have assumed that the mean spatial density of field stars remains unchanged during the contraction.

(b) As the contraction of the MCO group proceeds, its spatial density becomes comparable to or more than that of the galaxy. Then the dynamical state of the MCO group comes to be governed by the gravitational interactions among the members of this group, and consequently the following relations hold:

$$v_o^2=\frac{GM_o}{2R} \tag{2.14}$$

and

$$E_o=-\frac{GM_o^2}{4R}\,, \tag{2.15}$$

where $M_o=m_o N_o$, and R and E_o are the typical radius and the total binding energy of the cluster respectively.

Let us follow the behavior of the MCO group interacting with field stars. From equation (2.15), we have

$$\frac{dE_o}{dR}=\frac{GM_o^2}{4R^2}\,. \tag{2.16}$$

On the other hand, the dynamical friction on an MCO is represented in the same way as in the state a, $m_o v_o \langle \Delta v_{o\parallel} \rangle$, and then the dissipation of the total binding energy of the MCO group is obtained from equation (2.11):

$$\frac{dE_o}{dt}=M_o v_o \langle \Delta v_{o\parallel} \rangle = -4\pi G^2 m_o \rho_s \ln \Lambda(2j_s^2) g(j_s v_o) M_o v_o\,, \tag{2.17}$$

or combining equations (2.14), (2.16), and (2.17),

$$\frac{dR}{dt}=-48\sqrt{2}\,\pi G^{1/2}\rho_s \ln \Lambda g(j_s v_o)\left(\frac{m_o^2}{M_o^3}\right)^{1/2} R^{5/2}\,, \tag{2.18}$$

where we have assumed that the density of the field stars ρ_s remains unchanged during the contraction of the MCO group and the velocity dispersion of the field stars is the same as that of MCOs, namely, $j_s^2=1/2\sigma_s^2=3/2v_o^2$ or $(j_s v_o)^2=3/2$. The total mass of stars scattered to infinity by the MCO group is, from equations (2.6) and (2.7) with $v_s=v_o$ and $V=2v_o$,

$$M \approx 0.2\pi G^2 m_o^2 (m_s n_s)\frac{1}{v_o^3} N_o T\,, \tag{2.19}$$

where T is evaluated by integrating equation (2.18) from the initial radius to the final radius of our interest.

However, the present state b, is sooner or later replaced by the next dynamical state, at which the evaporation of MCOs becomes dominant to enhance the contraction of the group. Therefore the final radius of the present state b, is approximately determined by equating $|dR/dt|$ in equation (2.18) to $|dR/dt|$ due to the evaporation of MCOs, for which the following arguments are available (Chandrasekhar 1960, p. 207). The evaporation of the MCOs is

$$\frac{dN_o}{dt}=-\frac{0.0074}{T_R}N_o\,, \tag{2.20}$$

with the relaxation time T_R defined by

$$T_R=8.3\times 10^5\left(\frac{N_o R^3}{m_o}\right)^{1/2}(\log_{10} N_o-0.3)^{-1}\ \ (\mathrm{yr})\,, \tag{2.21}$$

where m_o and R are expressed in units of solar mass and parsec, respectively. From equations (2.17), (2.20), (2.21), and the fact that the escaping MCO carries the zero energy to infinity, we obtain

$$\frac{dR}{dt}=-\,2.9\times 10^{12}\left(\frac{m_o}{N_o}\right)^{1/2}(\log_{10} N_o-0.3)R^{-1/2}\ \ (\mathrm{cm\,s^{-1}})\,. \tag{2.22}$$

Now, equating equation (2.18) to equation (2.22), we can determine the final radius of the present state b.

(c) The life-time of the evaporation-dominant stage can be obtained by the usual method for the evolution of the globular cluster (see Spitzer and Saslaw 1966),

$$N_o(t)=N_{o*}\left(1-\frac{0.04}{T_{R*}}t\right)^{2/7}, \tag{2.23}$$

where the quantities with superscript * refer to the initial values of the group, at which the evaporation becomes most effective for the contraction of the MCO group. Thus the life-time is

$$T_E=T_{R*}/0.04\,. \tag{2.24}$$

3. *Application to the Giant Elliptical Galaxy M 87*

An extended halo of M 87 is observed by deep sky survey to reach as far as 0.3 Mpc from the center of the galaxy. The density has been determined as 10^{-28} g cm^{-3} (Arp and Bertola 1971) and the halo mass is about $10^{11}M_\odot$. Since the qnantitative investigation of the galactic halo has been made only for M 87, we apply our result to this galaxy. For our numerical calculations, we adopt the data in table 1 from Einasto (1974), and Brandt and Roosen (1965), where Einasto (1974) classified the central part of the galaxy into three parts, namely bulge, core, and nucleus. The mass of field stars is assumed to be identical with $1M_\odot$.

Table 1. Parameters of M 87 (NGC 4486).

Population	Total mass ($10^9M_\odot$)	Radius (kpc)	Density (g cm^{-3})
Nucleus	15	0.03	2.0×10^{-18}
Core	1090	1.4	1.6×10^{-21}
Bulge	1510	10	6.5×10^{-24}
Halo	226	76	5.7×10^{-27}
Line-of-sight dispersion: $\sigma_S=550$ km s^{-1}			

We consider the following six cases that the initial total masses of a group of MCOs are $10^{11}M_\odot$ and $10^{12}M_\odot$, for which the individual masses of the members are $10^6M_\odot$, $10^8M_\odot$, and $10^{10}M_\odot$. We take the radius of the bulge as the initial mean radius of the MCO group.

We obtained the radii R_1 and R_2 at which the spatial mass density of the MCOs $[M_o/(4\pi R_1^3/3)]$ is equal to that of the field stars in M 87 and the evaporation of the MCOs becomes a dominant mechanism of the contraction by equating equation (2.18) to equation (2.22); here we applied formally equation (2.22) to the cases that the total number of members is small. This application would make the value of R_2 uncertain but the contraction time does not depend significantly on R_2 (table 2). We computed the time of contraction to these typical radii with equations (2.12) and (2.18), and the total mass of the stars scattered to infinity during these time-intervals with equations (2.13) and (2.19) (table 3). The relaxation time of the MCO group at R_2, and the life-time of this evaporation-dominant stage are given in table 4.

Table 2. Typical radius of the MCO group.

Case		R_{Bulge}*	R_{Core}*	R_1	R_2
M_o ($M_\odot$)	m_o ($M_\odot$)	(kpc)	(kpc)	(kpc)	(kpc)
10^{11}	10^6	10	1.4	0.98	0.25
	10^8	10	1.4	0.98	0.21
	10^{10}	10	1.4	0.98	0.14
10^{12}	10^6	10	1.4	2.1	0.58
	10^8	10	1.4	2.1	0.50
	10^{10}	10	1.4	2.1	0.38

* Mean radii of the bulge R_{Bulge} and the core R_{Core} are given from Einasto (1974) in table 1.

Table 3. Contraction time and total mass of scattered stars.

Case		Stage			
		$R_{\mathrm{Bulge}} \to R_{\mathrm{Core}}$*		$R_{\mathrm{Core}} \to R_2$*	
M_o ($M_\odot$)	m ($M_\odot$)	T (yr)	M ($M_\odot$)	T (yr)	M ($M_\odot$)
10^{11}	10^6	$>10^{10}$	$<10^8$	$>10^{10}$	$<10^9$
	10^8	$>10^{10}$	$<10^8$	4×10^8	2×10^9
	10^{10}	2×10^8	1×10^9	6×10^6	6×10^9
10^{12}	10^6	$>10^{10}$	$<10^9$	$>10^{10}$	8×10^9
	10^8	$>10^{10}$	$<10^9$	9×10^9	1×10^{11}
	10^{10}	2×10^8	4×10^9	2×10^7	2×10^{10}

* The radius R_1 is defined in the text such that the spatial density of the MCO group is equal to that of the field stars at this radius. It is of course dependent on M_o. For $M_o=10^{11}$ or $10^{12} M_\odot$, R_1 is between R_{Core} and R_2 or between R_{Bulge} and R_{Core}, respectively.

Table 4. Life-time of evaporation-dominant stage.

Case		R_2	T_R*	T_E
M_o ($M_\odot$)	m_o ($M_\odot$)	(kpc)	(yr)	(yr)
10^{11}	10^6	0.25	2×10^8	6×10^9
	18^8	0.21	3×10^6	7×10^7
	10^{10}	0.14	6×10^4	2×10^6
10^{12}	10^6	0.58	2×10^9	5×10^{10}
	10^8	0.50	3×10^7	6×10^8
	10^{10}	0.38	4×10^5	9×10^6

4. *Discussion and Conclusion*

The initial total mass of a group of MCOs could be less than $10^{12}M_\odot$, in view of the observed mass of the central part of M 87. As to the upper limit of mass of the members, we can take $10^{10}M_\odot$, because a few MCOs left behind at the center from the evaporation cannot be more massive than the observed total mass of the nucleus of M 87.

As shown in table 3, the most efficient scattering of the field stars is realized for $M_o=10^{12}M_\odot$ and $m_o=10^{10}M_\odot$ (therefore the initial number of the MCOs is 100). Therefore the mass of the stars contributing to the halo formation is about one tenth of the halo mass claimed to be observed.

The author does not consider that this mass discrepancy is so large as to invalidate our dynamical process of the MCOs in the giant elliptical galaxy. Since the observational accuracy of the halo mass is still poor and the result may vary within one order of magnitude, our mechanism of the halo formation is promising.

The scattering of stars at the evaporation-dominant stage is not so efficient as compared with the preceding one, because the life-time of the system at this stage is short and the scattering cross-section is much reduced due to the increase in the velocity dispersion. As Saslaw et al. (1974) supposed, if a rapidly revolving binary of MCOs at the galactic center works to form the halo, it is expected from the above discussion that each mass of a binary must be $>10^{11}M_\odot$, and also it must lose its potential energy effectively by scattering the field stars during, say, 10^{10} yr.

We have assumed the existence of a number of MCOs at the galaxy formation. However, we have not enough evidence to support it. This problem should be solved by means of investigations such as by Arp and O'Connell (1975), and Saslaw et al. (1974).

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