RELIABILITY OF THE METHOD OF CONSTANT INTENSITY CUTS FOR RECONSTRUCTING THE AVERAGE DEVELOPMENT OF VERTICAL SHOWERS

T. K. Gaisser
Bartol Research Foundation of The Franklin Institute
University of Delaware
Newark, Delaware 19711

A. M. Hillas
Department of Physics, University of Leeds
Leeds, U.K.

Abstract. We present the results of some simple model calculations that investigate the validity of the intensity-cut method of reconstructing the shower development curve. We find that $N_{cut} \sim N_{rms} > \overline{N}$.

1. Introduction.

The longitudinal development of extensive air showers is undoubtedly an essential feature that reflects both the composition of the primary cosmic rays and the gross features of particle interactions for $E \geqslant 10^{15}$ eV. In the past, however, this crucial property of showers has only been determined indirectly by taking cuts of constant intensity in size spectra for showers of different zenith angle bins. The question thus arises, how well does the development curve obtained in this way (call it N_{cut}) represent the true, average longitudinal development ($\overline{N}$) of showers of fixed primary energy? Dedenko has argued that the method of constant intensity cuts seriously misrepresents the true average development of showers. He calculates [Dedenko, 1975] that the depth of maximum of N_{cut} is 100 gm/cm^2 (or more) higher in the atmosphere than the maximum of $\overline{N}$. This discrepancy is comparable to that between $\overline{N}$ calculated with scaling and proton primaries [Fishbane et al., 1974] and N_{cut} obtained in the Chacaltaya experiment [LaPointe, et al., 1968]. If this discrepancy is largely due simply to a difference between N_{cut} and $\overline{N}$, then arguments against scaling and proton primaries based on the Chacaltaya data would have to be revised.

We have therefore investigated the relation between N_{cut} and $\overline{N}$ from several points of view in an effort to clarify both the meaning of N_{cut} and the circumstances under which such a large discrepancy as found by Dedenko may exist.

2. Analytic Estimate of N_{cut}.

We first carry out a calculation of N_{cut} along the lines of Dedenko (1975) but using a simple analytic parametrization of the size of a shower initiated by a proton of energy E at depth t=0 (t measured in units of λ=70 gm/cm^2):

$$S_1(E,t) = S_o \frac{E}{\varepsilon} \exp[t_{max}] (t/t_{max})^{t_{max}} e^{-t} , \quad (1)$$

with $t_{max} = .51 \ln(E/\varepsilon) - 1$, $S_o = 0.045$, and $\varepsilon = 0.074$ GeV.

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The first step is to construct the probability, $\psi(>N|E,t_o,\theta)$ that a shower of energy E and angle θ will produce more than N particles at slant depth $t_o=y_o \sec\theta$, where y_o is the vertical depth of observation. In the simple model defined by Eq. (1)

$$\psi(>N|E,t_o) = H(E-\frac{N}{S_o}\varepsilon)\int_0^{t_o}\frac{dt}{\lambda_F}e^{-t/\lambda_F}H(t-t_1)H(t_2-t) \quad (2)$$

(Where H(x) is a step function)

In Eq. (2), $\frac{N}{S_o}\varepsilon$ is the minimum energy required to produce a shower with N particles at its maximum of development. To give a shower with size >N at t_o the cascade must be initiated at slant depth t between t_1 and t_2 ($0 \leq t_1 \leq t_2$) where t_1 and t_2 are roots of

$$N = S_o\frac{E}{\varepsilon}\exp[t_{max}]\left(\frac{t_o-t}{t_{max}}\right)^{t_{max}}e^{-(t_o-t)} .$$

In Eq. (2) $\lambda_F \geq \lambda$ to allow for the fact that there are other sources of fluctuation than point of first interaction.

One next calculates the size spectra at various slant depths and takes constant intensity cuts to construct $N_{cut}(E,t)$. We have used an integral primary energy spectrum

$$I(>E) = I_o(E/E_o)^{-\gamma} \quad (3)$$

We have taken $I_o=10^{-9}$ m^{-2} sec^{-1} sr^{-1}, $E_o=5\times10^7$ GeV and $\gamma=2$, and we have compared N_{cut} for $I(>N) = 10^{-9}$ m^{-2} sec^{-1} sr^{-1} with $\overline{N}$ for $\lambda_F/\lambda=1$, $\lambda_F/\lambda=1.43$ and $\lambda_F/\lambda=2.14$. To compute $\overline{N}$ we have taken E to be the solution of Eq. (3) with $I(>E) = I(>N)$. For $I(>N) = 10^{-9}$ m^{-2} sec^{-1} sr^{-1} this gives $E = 5\times10^7$ GeV. The result for $\lambda_F/\lambda = 1.43$ is shown in Fig. 1.

Our result differs from that of Dedenko in three significant ways: 1) We find a depth of maximum about 20 gm/cm^2 higher in the atmosphere for N_{cut} than for $\overline{N}$ (rather than 100 gm/cm^2 higher). 2) We find N_{cut} $(t_{max}) > \overline{N}$ (t_{max}), whereas Dedenko has N_{cut} $(t_{max}) = \overline{N}$ (t_{max}). 3) Our N_{cut} is broader than $\overline{N}$ but the two curves do not cross, as Dedenko finds. To explore the source of these differences we look at the general relation between N_{cut} and $\overline{N}$. We also must consider the effect of the fact that the simple model discussed so far probably underestimates fluctuations [Watson, 1976].

3. General relation between N_{cut} and $\overline{N}$. It can be shown that N_{cut} $(t,E) \sim N_{r.m.s.}$ (t,E) (rather than $\overline{N}$ (t,E)) under rather general conditions, including that of a mixed composition.

First, if the shower development curve has a shape which fluctuates from one shower to another, but has no dependence on energy, one can establish this result accurately. Thus let E_c be the "calorimetric energy", which is deposited in ionization (somewhat less than E, because of neutrinos, etc., and probably not exactly proportional to E). Then we take $N(t,E)=n(t).E_c$ in a particular shower, and assume that the probability distribution $p(n,t)dn$ is independent of E_c. Let the integral flux of primary particles be expressed as in Eq. (3) with $E \to E_c$. Then the integral flux of showers of size $\geq N$ at depth t is

$$I(t,N) = \int_0^\infty p(n,t) \cdot I_o \cdot (N/nE_o)^{-\gamma} dn = I_o \cdot (N/E_o)^{-\gamma} \int_0^\infty n^\gamma p(n,t) dn$$

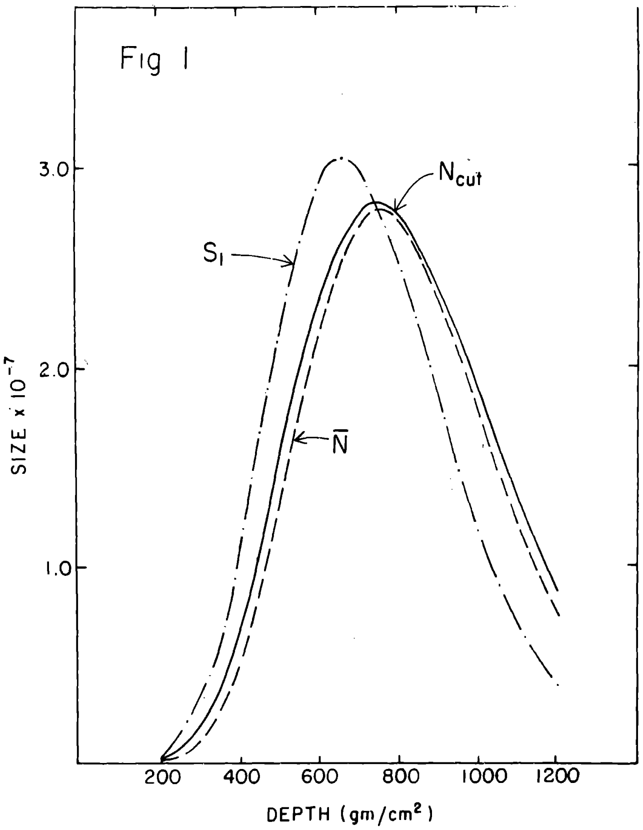
$$= I_o \cdot (N/E_o)^{-\gamma} \langle n(t)^\gamma \rangle .$$

Hence, the shower size N_{cut} for which $I(t,N) = I_o$ is given by

$$N_{cut} = E_o \langle n(t)^\gamma \rangle^{1/\gamma} = \langle N(t,E_o)^\gamma \rangle^{1/\gamma} = N_{\gamma-av}(t,E_o).$$

$$= N(t,E_o)_{r.m.s.} \text{ if } \gamma \doteq 2.$$

Thus $N_{cut}(t,E)$ should always be larger than $\bar{N}(t,E)$ if there are any fluctuations, as the example in Section 2 illustrates. The actual variance in N at depth t could, if known, be used to correct the value N_{cut} ($=N_{rms}$ if $\gamma = 2$) to obtain $\bar{N}$. Alternatively, one can compare the experimental values $N_{cut}(t,E)$ with theoretical curves for $N_{rms}(t,E)$, rather than with $\bar{N}(t,E)$. Also, if one has estimates for the difference between N_{rms} and N_{av} one can find the ratio of areas under these two curves, and hence find the error in the calorimetric energy which is deduced for these showers, from this area.



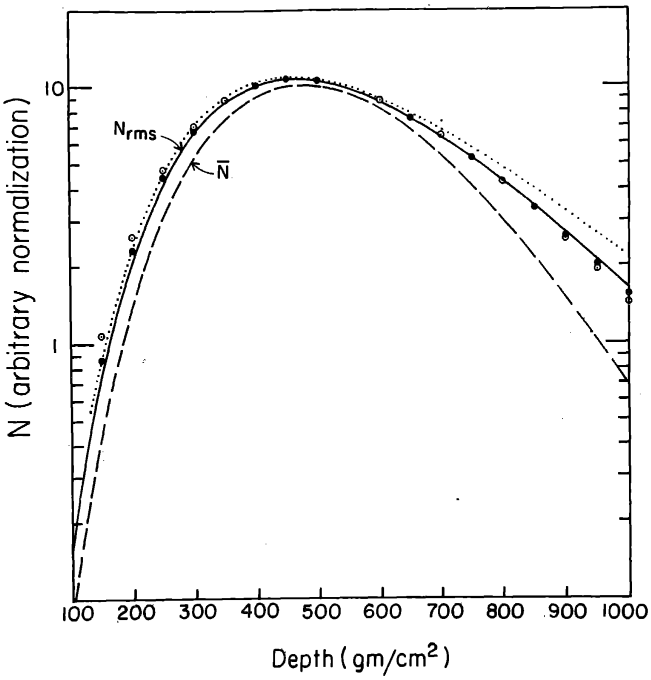


Fig. 2

The general nature, but not the correct magnitude, of the differences between $\bar{N}(t)$, $N_{cut}(t)$ and $N_{rms}(t)$ can be illustrated by taking a rather extreme model of shower development which gives unrealistically large fluctuations (in contrast to the model of Section 2, which gave rather low fluctuations). At the same time we investigate the effects of a shape that changes with E. For this illustration, each shower will be taken to have a shape corresponding to an electromagnetic cascade generated by π° mesons of energy E_{π° injected at a depth x which fluctuates with the distribution exp $(-x/L)dx/L$ for $x<L$, but $(x/L)\exp(-x/L)dx/L$ for $x \geq L$. The latter part roughly corresponds to the fact that in reality many cascades are generated, but that the mean point of injection of energy is at the second interaction of the primary particle, whilst the first part allows for extra fluctuation at high altitudes as though all the energy can go into the products of the first collision ($L=80$ g/cm^2).

The worst assumption, for the case of energy-independent composition, is to assume a scaling-type model, in which the effective π° energy E_{π° rises proportionately to E. The case $E_{\pi^\circ} = (E/E_0) \times 3.10^{13}$ eV has been taken to produce the results plotted in Figure 2 (for $\gamma = 2$) as the points Θ, which show the calculated values of N_{cut} for the intensity I_0 (which corresponds to the primary energy E_0). The results do not differ much from N_{rms}. If instead one takes the slower variation of shower shape given by $E_{\pi^\circ} = (E/E_0)^{0.5} \times 3.10^{13}$ eV, which is rather like that obtained when particle multiplicities rise as $E^{1/4}$ (noting that E_{π° is not just the energy of pions from the first generation), one gets the values of N_{cut} shown by the solid points, very close to N_{rms}. So N_{rms} is still a good match to N_{cut}, even when the shower shape is varying systematically with E. If $\gamma = 2.5$ rather than 2, the curve $N_{\gamma-av}$ is of course rather further displaced from N_{av}, as shown for this extreme shower model by the dotted line in Figure 2.

4. Discussion. The calculations of Section 3 confirm the results of Section 2 and show in general that $N_{cut} \sim N_{rms}$ if $\gamma \sim 2$. We have also confirmed the latter result for a mixed composition of Fe and p. Even in the model with rather extreme fluctuations, the depth of maximum was shifted up typically only by 20-40 gm/cm^2 and the area under the curve increased by ∿18-25% for N_{cut} as compared to $\bar{N}$. We have been able to think of two possible sources of the difference between our result and that of Dedenko: 1) in averaging over the zenith angle bin 0-30° Dedenko has plotted the resulting N_{cut} at $t_{vertical}$ rather than at $\langle t\rangle_{\theta=0-30°}$. We have found that this can lead to a shift upward of the depth of max. of N_{cut} by ∿30 gm/cm^2. 2) Dedenko has $N_{cut}(t_{max}) = \bar{N}(t_{max})$. He may have defined $\bar{E}$ (the energy used to compute $\bar{N}$) to achieve this equality. But this would require $\bar{E} > E_{cut}$ and hence cause the depth of max for $\bar{N}$ to be shifted down in the atmosphere (by ∿10 gm/cm^2) relative to $N_{cut}(E_{cut})$. We can thus account for about 40 gm/cm^2 of the discrepancy between our result and Dedenko's.

5. Conclusion. The main point of this work is to present a different way of viewing $N_{cut}(t,I_0)$, which can be carried over into

situations where one has many other complicating factors. We are measuring $N_{rms}(t,E_o)$ for the whole group of primaries of energy E_o, and it could be more satisfactory to compare with theoretical N_{rms} curves. However, when $\overline{N}(t,E_o)$ really is the quantity of interest, as when estimating the area under the curve for calorimetric purposes, one may either use experimental (scanty) data on the variance in N(t,E) to correct N_{rms} to $\overline{N}$, or one may apply a more realistic model to estimate these differences. Even with the crude model of Section 3, the area without such correction exceeds the "correct" area by typically 18-25%: in reality the error should be much less. Thus it seems unlikely that previous conclusions based on a comparison between N_{cut} and $\overline{N}$ will be qualitatively changed, although the difference is large enough to be significant, and it should be taken into account in future calculations.

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