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THE DISTRIBUTION OF MASS IN THE PLANETARY SYSTEM AND SOLAR NEBULA

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Abstract. A model 'solar nebula' is constructed by adding the solar complement of light elements to each planet, using recent models of planetary compositions. Uncertainties in this approach are estimated. The computed surface density varies approximately as $r^{-3/2}$. Mercury, Mars and the asteroid belt are anomalously low in mass, but processes exist which would preferentially remove matter from these regions. Planetary masses and compositions are generally consistent with a monotonic density distribution in the primordial solar nebula.

1. Introduction

Most theories of cosmogony assume that the planetary system formed from a nebula of solar composition, but assumptions as to its mass and structure vary widely. A lower limit on the mass is set by augmenting the planets with H and He to restore a solar composition. One may also try to 'reconstruct' the solar nebula by spreading the augmented planetary masses through zones surrounding their orbits. Such an *ad hoc* nebula can resemble the original only to the extent that the formation of the planets was the reverse of the spreading process. Alternatively, some of its features may provide information about planetary formation, rather than the nebula. Kuiper (1956) and Kusaka *et al.* (1970) modeled the solar nebula in this fashion, on the assumption that the reconstructions represented the original nebula. Similar calculations for various purposes were performed by Edgeworth (1949), Safronov (1967), Alfvén and Arrhenius (1970), and Lecar and Franklin (1973). The results of these efforts differ significantly. While the calculations are simple, the assumptions used are, unfortunately, rarely described in detail. Also, our knowledge of solar elemental abundances and planetary compositions has improved significantly within the last few years. Another calculation of this sort, with assumptions clearly stated, appears justified. I shall try to estimate the range of uncertainties involved, and the types of information which may be derived.

2. Assumptions

Equivalent solar-composition masses of the terrestrial planets are computed from their iron contents. By use of Cameron's 1973 abundances, the mass fraction of Fe in solar matter is 1.2×10^{-3}. Nominal Fe contents by mass are 0.62 for Mercury (Siegfried and Solomon, 1974), 0.35 and 0.38 for Venus and Earth (Reynolds and Summers, 1969),

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and 0.30 for Mars (Johnston *et al.*, 1974). The range of values for the models of each planet suggests that their Fe contents are uncertain by about ±10% of the total amounts. For the asteroid belt, I assume a mass of 5×10^{-4} earth masses, with Fe mass fraction of 0.25, consistent with a mainly carbonaceous chondritic composition (McCord and Chapman, 1975). The collisional evolution model of Chapman and Davis (1975) suggests an initial population of asteroids some 300 times larger than at present; this is taken as a rough upper limit.

Models of Jupiter and Saturn used are by Podolak and Cameron (1974), and Zharkov *et al.* (1975). These models assume that the planets consist of H and He in solar ratio, with cores of rocky and icy matter. The computed compositions are rather uncertain, depending on the assumed H/He ratio, equations of state, and temperature boundary conditions. Both planets appear to be enriched in heavy elements over solar composition. The enrichment factors lie between 2 and 40 for Jupiter, and 10 and 60 for Saturn. Note that the overlap of these ranges is due to the variety of models considered. For any consistent set of assumptions, Saturn contains a larger proportion of heavy elements than Jupiter.

Uranus and Neptune models are by Podolak and Cameron (1974), Makalkin (1973), and Reynolds and Summers (1973, unpublished). Using Podolak and Cameron's value of 0.003 43 for the mass fraction of metal plus silicates in solar composition, the core masses of these planets correspond to about 700–2000 earth masses of solar material. The adopted ranges for the giant planets cover all the models given, without any choice of 'best' models.

The choice of zones is essentially arbitrary. I assume that the nebula was continuous, and that neighboring zones touched without overlap. Obviously, the actual situation was more complex. During accretion of the terrestrial planets, their perturbations could cause some exchange of matter among their zones (Wetherill, 1975; Hartmann, 1976). While mixing may have been significant in the final stages of accretion, it was not sufficient to erase the differences in planetary bulk compositions. Wetherill's results also show that this process would result in a net addition of matter to Mercury and Mars; their small masses place a limit on the amount of possible mixing. In the absence of an accepted, detailed model for planetary formation, I take as zone boundaries the arithmetic means of adjacent orbits (use of geometric means would change the computed areas by only a few percent). The zones of Mars and Jupiter extend to the boundaries of the main asteroid belt. The zones of Mercury and Neptune are assumed to extend equal distances inward and outward from their orbits.

3. Results and Discussion

The results are summarized in Table I. The computed mass, actually the lower bound for the mass of the original nebula, is 0.01 to 0.07 solar masses. The total angular momentum is 3×10^{51} to 2×10^{52} g cm^2 s^{-1}, compared with the present value, 3×10^{50} g cm^2 s^{-1}. Most of the uncertainty in these quantities comes from the wide

TABLE I

Planetary zones: masses and surface densities

	Mass ($M_\oplus$)	Fe mass fraction	Solar comp. mass ($M_\oplus$)	Zone limits (AU)	Surface density (g cm^{-2})
Mercury	0.053	0.62	27	0.22	880
Venus	0.815	0.35	235	0.56	4750
Earth	1	0.38	320	0.86	3200
Mars	0.107	0.30	27	1.26	95
Asteroids					
present	0.0005	0.25	0.1	2.0	0.13
original	0.15?		30		40
				3.3	
Jupiter	318	–	600–12 000	7.4	120–2400
Saturn	95	–	1000–6000	14.4	55–330
Uranus	14.6	–	700–2000	24.7	15–40
Neptune	17.2	–	800–2000	35.5	10–25

range of models for the giant planets. Surface densities, σ, rather than volume densities, are used to avoid additional assumptions about the vertical structure of the nebula. Figure 1 displays these results graphically. The vertical 'error bars' reflect only the estimated uncertainties in the planetary compositions. The horizontal bars mark the mid-range of log σ in each zone, and show the zone widths. Note that a change in the adopted solar Fe abundance would not change the relative values in the terrestrial planet zones. The values of σ for the giant planets, relative to each other and to the terrestrial zones, depend somewhat on the adopted solar abundances of ice- and rock-forming elements. However, their uncertainties are much smaller than those associated with the compositions of the giant planets.

The trend from Venus to Neptune is approximately $\sigma \propto r^{-3/2}$. This profile is similar to model nebulae by Cameron and Pine (1973), but at much lower absolute values. The zones of Mercury, Mars, and the asteroids appear strongly depleted in mass. The fit of Venus and Earth with the trend set by the outer planets suggests their common origin from a nebula with an initially monotonic variation of σ. Such a scenario requires preferential removal of matter from the anomalous zones. However, plausible mechanisms for such removal do exist, and are described below.

The zones of Mars and the asteroids comprise a single density minimum. Interpolation between Earth and Jupiter suggests that several Earth masses of solid matter should have been present in that region originally. Safronov (1969) suggested that planetesimals from Jupiter's zone would be perturbed by that planet into eccentric orbits. They would pass through the Mars/asteroids region, colliding with and disrupting the planetesimals which had formed there. Weidenschilling (1975) showed that such a bombardment would be most intense in that region. Ip (1977) has shown that

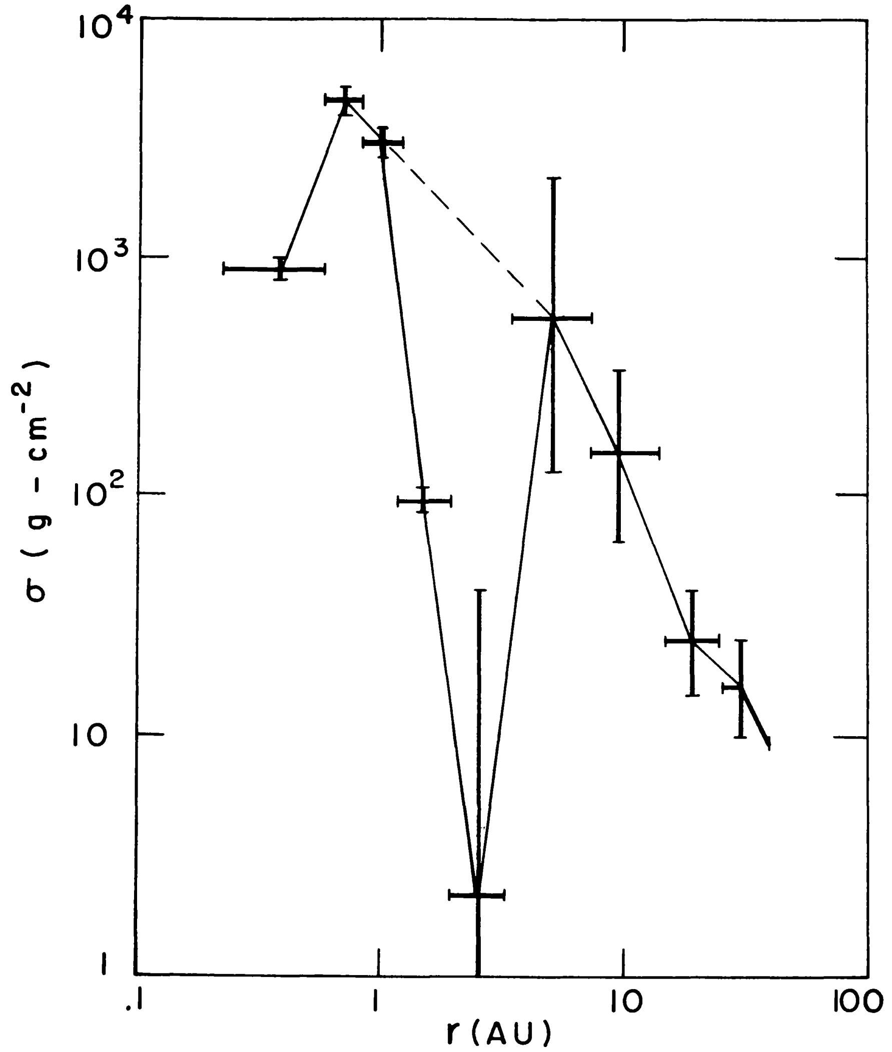

Fig. 1. Surface densities, σ, obtained by restoring the planets to solar composition and spreading the resulting masses through contiguous zones surrounding their orbits. The meaning of the 'error bars' is discussed in the text.

most planetesimals originating as far out as Uranus would eventually become Jupiter-crossing. Much of this bombarding material would be of icy composition. If a significant fraction of the total condensible matter remained after formation of the giant planets, the bombarding mass could exceed the original mass of planetesimals in the Mars/asteroids region. This 'bombardment hypothesis' can also account, at least qualitatively, for the large velocity dispersion in the asteroid belt.

The low surface density in Mercury's zone cannot be due simply to an overestimate of the zone size. Using just the area between the planet's perihelion and aphelion distances would only raise σ by a factor of two. Since σ is computed from the planetary iron content, it appears that Fe is depleted in Mercury's zone. Its high mass fraction is due to a still greater depletion of silicates. A naive extrapolation in Figure 1 would suggest that Mercury's mass should be similar to that of Venus or Earth. Some or all of this mass anomaly may reflect an actual drop in density at the inner edge of the solar nebula. The planetary composition could then be due to incomplete condensation in this high-temperature region (Lewis, 1972). However, the apparent Fe depletion suggests physical fractionation. Any process which preferentially removed silicates would not require unreasonably high efficiency, since much Fe was apparently removed as well. In principle, Poynting-Robertson drag can accomplish this, since particles of lower density have shorter lifetimes for orbital decay; it would also be most effective nearest the sun. However, this mechanism places severe constraints on particles sizes and time scale. With the most favorable assumptions the silicates must remain in the form of fine dust for millions of years without being incorporated into larger bodies (Herczeg, 1969).

Another fractionation mechanism, effective for large bodies on much shorter time scales, is aerodynamic drag (Whipple, 1972; Weidenschilling, 1977a). In a centrally condensed solar nebula, the pressure gradient in the gas tends to support it against the central gravity. Therefore, the gas rotates at slightly less than the Kepler velocity. Solid bodies are not supported by the pressure gradient; they move with respect to the gas, under the influence of gravitational and drag forces. In general, they spiral inward. This effect is greatest in the innermost part of the nebula, and is significant even in the 'minimum mass' nebula discussed here (Weidenschilling, 1977a). Density fractionation may occur in either sense, depending on particle size. For bodies larger than about a meter, those of lower density are eliminated more rapidly. In Mercury's zone, the time scale in years for effective fractionation is on the order of the particle size in centimeters (Weidenschilling, 1977b).

A 'reconstruction' of this type can yield no information about absolute efficiencies in planetary formation; only relative depletions of mass can be identified in Figure 1. Any loss of heavy elements is undetectable if it varied smoothly with heliocentric distance, so we cannot rule out a more massive nebula or a somewhat different mass distribution. However, a process that could remove from the solar system a mass of condensed matter greater than that in the planetary system has not been demonstrated quantitatively. Perturbations by the giant planets could have ejected cometary bodies from the solar system. The ejected bodies carry off excess angular momentum (Safronov, 1969). Conservation of angular momentum requires the orbital radii of the planets to decrease. If Neptune had formed at a distance of 40 AU in agreement with Bode's 'Law', it could have reached its present orbit by ejecting about five earth masses from its zone (Safronov, 1967). The value of σ for the larger mass and zone is shown by Neptune's bent crossbar in Figure 1. The change is roughly parallel to the general trend, and would be undetectable. However, Ip (1977) has shown that most objects in the outer solar system would be ejected by an encounter with either Jupiter or Neptune, regardless of their original orbits. The orbits of Saturn and Uranus would not be changed significantly. If Jupiter's orbit had ever been larger than about 6 AU, Saturn's orbit would have been unstable (Delibaltas, 1976). This condition limits the mass ejected by Jupiter to a few tens of earth masses. Assuming the ejected matter to include ices, the corresponding amount of solar-composition gas is a few percent of the solar mass.

4. Conclusions

Reconstruction of a minimal-mass solar nebula, based on current knowledge, suggests that the surface density of the original nebula was roughly proportional to $r^{-3/2}$. By this model, the regions of Mercury, Mars and the asteroids are anomalously low in mass. The Mars and asteroids anomalies form a single feature, and require a common explanation. Plausible mechanisms exist for selective removal of solid matter from these regions; therefore, an initially monotonic density distribution cannot be excluded. The minimum mass of the nebula lies between 0.01 and 0.1 solar masses. Most of the uncertainty is associated with the compositions of the giant planets and the possible ejection of comets from the outer solar system. Theories which involve a more massive nebula or a radically different mass distribution require additional assumptions to account for the planetary system as it exists today.

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