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Tidal Friction in Close Binary Stars

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Summary. We examine various physical mechanisms which may produce tidal friction in close binary stars. We find that the most efficient in stars with convective envelopes is turbulent viscosity retarding the equilibrium tide, and in stars with radiative envelopes the action of radiative damping on the dynamical tide. Theoretical predictions based on these dissipative processes are in good agreement with the rotational velocities and orbital eccentricities observed in close binaries. The results are applied to the X-ray binaries Her X-1 and Cen X-3.

Key words: stellar structure — close binaries — X-ray binaries

1. Introduction

A well established property of close binary stars is that they rotate in general more slowly than single stars of the same type. In fact, the observations reveal that in close pairs the rotation tends to be synchronized with the orbital motion, and that the two stars need not be in contact to achieve this. To explain it, one is naturally tempted to invoke tidal friction, a mechanism which is so effective in the Earth-Moon system. Starting with Jeans (1929), many authors have addressed the problems posed by stellar tides, and the recent discovery of the X-ray binaries has provided a new impetus for such studies.

The main difficulty has been to identify the physical processes which are actually responsible for the tidal torque. A first step was to recognize the prominent role of turbulent friction in stars possessing an extended outer convection zone (Zahn, 1966b). But the tidal evolution of binaries with radiative envelopes was not understood until recently, when it was shown that radiative damping could produce the required torque by retarding the dynamical tide (Zahn, 1975b).

The purpose of this article is to analyze the theoretical results which are presently available, and to compare them with the observations. We first establish in §2 the gravitational potential of a tidally distorted star in terms of its structural parameters. Next, we reformulate in §3 the secular equations which govern the exchanges of energy and angular momentum in a binary system. Then we examine in §4 the various dissipation mechanisms that can retard the equilibrium tide. In §5 we describe some properties of the dynamical tide experienced by a star with a radiative envelope. In §6 the observations of rotational velocities and orbital eccentricities in close binary stars are discussed in terms of the previously outlined theoretical predictions. Finally, §7 applies these results to the X-ray binaries Her X-1 and Cen X-3.

2. The Perturbing Potential

If one wishes to analyze the dynamical effects caused by the tides in a binary star, it is necessary first to determine the outer gravity field created by the two components. Beside the familiar graviational pull between mass centers, each star experiences an additional force arising from the non-spherical part of the mass distribution within the tidally distorted companion. It is the work done by this extra force, as the stars revolve around each other, that is responsible for the changes in energy and angular momentum of the orbital motion.

Let us concentrate on one of the stars in a binary system, and refer to it from now on as the primary. Its tidal distortion is produced by the gravity field of the other star (the secondary). This perturbing potential can be conveniently expanded in terms of spherical functions, as described for instance by Kopal (1959). In the general case where the orbit is not circular, it is necessary to decompose further each spherical harmonic in Fourier series of the mean anomaly. This is illustrated here only for the dominant term of the tidal potential, which involves the spherical harmonic of order $n=2$ (Zahn, 1966a):

$$U=\frac{GM_2}{a}\left(\frac{r}{a}\right)^2\cdot\Big[-\frac{1}{2}P_2(\cos\theta)\cdot\left(1+\frac{3}{2}e^2+3e\cos\omega t+\frac{9}{2}e^2\cos 2\omega t\right)+\frac{1}{4}P_2^2(\cos\theta)\left(\left(1-\frac{5}{2}e^2\right)\cos(2\omega t-2\phi)-\frac{e}{2}\cos(\omega t-2\phi)+\frac{7}{2}e\cos(3\omega t-2\phi)+\frac{17}{2}e^2\cos(4\omega t-2\phi)\right)+O(e^3)\Big]. \quad (2.1)$$

In this expression M_2 is the mass of the secondary star, ω the mean orbital motion, and a and e are the semi-major axis and the eccentricity of the orbit. The spherical coordinates r, θ and ϕ are measured in a reference frame whose origin is the center of the primary and whose polar axis is perpendicular to the orbital plane; the mean anomaly ωt and the azimutal angle ϕ are counted from the direction of the periastron.

Let us for simplicity assume that the star rotates uniformly with an angular velocity Ω and that its spin axis is perpendicular to the orbital plane; in a similar reference frame, but which is now corotating with the star, the Fourier components of the external gravitational field take the general form

$$U_u^{lm}(r/R)^n P_n^m(\cos\theta)\exp i(\sigma_{lm}t-m\phi), \quad (2.2)$$

where R is the radius of the primary star, and $\sigma_{lm}=l\omega-m\Omega$ is the apparent frequency of that tidal component. The amplitude coefficients U_n^{lm} are readily determined; from expression (2.1) we see for instance that the largest of them is

$$U_2^{22}=\frac{1}{4}\frac{GM_2}{R}\left(\frac{R}{a}\right)^3\left(1-\frac{5}{2}e^2+O(e^4)\right). \quad (2.3)$$

The main reason for breaking the external gravity field into such Fourier components is that one can then readily study the oscillations of a star forced by a potential which varies sinusoidally in time. If it is further assumed that the amplitudes of the oscillations remain small enough to justify a linear treatment of the problem, the total response of the star is just the sum of its responses to each individual Fourier component. In such a linear approach, the tidal problem is then reduced to determining the outer gravitational potential created by the star when it is submitted to a single component of the perturbing field.

The spatial dependence of that outer potential is imposed by the condition that it be harmonic and vanish at infinity; it must therefore be of the form

$$\Phi_n^{lm}(r/R)^{-n-1}P_n^m(\cos\theta)\exp i(\sigma_{lm}t-m\phi). \quad (2.4)$$

The problem is then to evaluate, as functions of the tidal frequencies σ_{lm}, the amplitude coefficients Φ_n^{lm} which are in general complex quantities.

The most complete solution available applies to stars with early-type main sequence structure, possessing convective cores and radiative envelopes. It is further assumed that the effects of the Coriolis force on the tidal oscillation are negligible; the validity of this assumption however remains to be justified. Under these conditions, an asymptotic treatment can be used in the limit of small tidal frequencies (Zahn, 1975b), to yield

$$\Phi_n^{lm}/U_n^{lm}=2k_n\exp i\alpha(s_{lm})+E_n s_{lm}^{8/3}(q_n(s_{lm})+ip_n(s_{lm})). \quad (2.5)$$

This expression is valid for $s_{lm}\ll 1$, with s_{lm} being the dimensionsless form of the tidal frequency σ_{lm} defined as

$$s_{lm}=\sigma_{lm}(R^3/GM)^{1/2}=(l\omega-m\Omega)(R^3/GM)^{1/2}. \quad (2.6)$$

Here M (without subscript) designates the mass of the primary.

The first term in the expression (2.5) for Φ_n^{lm}/U_n^{lm} represents the contribution of the equilibrium tide. This tide can be described by simply assuming that the star is always in hydrostatic equilibrium; k_n is the familiar apsidal motion constant (see Kopal, 1959). The angle α determines the phase shift of the equilibrium tide that is produced by the dissipative processes which operate in the star; we will see in §4 how this angle can be estimated.

The second term in expression (2.5) results from the dynamical tide and will be analyzed in some detail in §5. It suffices to mention here that E_n is the analog of the apsidal motion constant, that the function $p_n(s)$ is always positive and that it tends to unity for small tidal frequencies s.

It is well known that the real part of Φ_n^{lm}/U_n^{lm} is responsible for the advance of the apsidal line, a property for which the centrifugal distortion is also partly responsible. Here, however, we shall be dealing with the exchanges of energy and angular momentum due to the tides, and these are exclusively caused by the imaginary part of the outer potential. For the present purpose, it is thus not necessary to describe the deformation of the star due to the rotation and to the stationary part ($l=m=0$) of the tidal potential; in other words, the star is assumed spherically symmetrical in zero order.

In the next section, we shall derive the differential equations that describe the secular changes of the orbital parameters. Before doing so, it is convenient to rewrite the imaginary part of expression (2.5) as

$$\mathrm{Im}(\Phi_n^{lm}/U_n^{lm})=\varepsilon_n^{lm}, \quad (2.7)$$

with the understanding that ε_n^{lm} contains the contributions of both the equilibrium and the dynamical tides:

$$\varepsilon_n^{lm}=2k_n\sin\alpha(s_{lm})+E_n s_{lm}^{8/3}p_n(s_{lm}). \quad (2.8)$$

This simplified notation will be used in the equations that govern the rotational and orbital evolution of binary system, which will now be presented.

3. The Secular Equations

The next step in determining the secular effects of the tides is to evaluate the perturbing force exerted by the primary star upon the secondary. To do so, we take the gradient of the outer potential created by the primary at the center of the secondary, whose polar coordinates are $(r, O, v-\Omega t)$ in the reference frame of the primary. Replacing the true anomaly v by its expansion in terms of the mean anomaly ωt:

$$v=\omega t+2e\sin\omega t+\tfrac{5}{4}e^2\sin 2\omega t+O(e^3)\,, \qquad (3.1)$$

we can likewise express the perturbing acceleration experienced by the secondary as a Fourier series in ωt. In this series, we retain here only the terms that are capable of producing a net work over a complete orbital revolution; these are the terms $\boldsymbol{R}$, odd in ωt, for the radial component of the acceleration, and the even terms, $\boldsymbol{S}$, for its tangential component. We thereby obtain

$$\boldsymbol{R}=-\frac{9}{4}\frac{GM_2}{R^2}\left(\frac{R}{a}\right)^7\left(\frac{a}{r}\right)^4$$

$$\cdot\left[e\sin\omega t\left(\varepsilon_2^{10}+\frac{1}{4}\varepsilon_2^{12}-4\varepsilon_2^{22}+\frac{7}{2}\varepsilon_2^{32}\right)+O(e^2)\right] \qquad (3.2)$$

and

$$\boldsymbol{S}=-\frac{3}{2}\frac{GM_2}{R^2}\left(\frac{R}{a}\right)^7\left(\frac{a}{r}\right)^4$$

$$\cdot\left[\varepsilon_2^{22}+e\cos\omega t\left(-\frac{1}{2}\varepsilon_2^{12}+\frac{7}{2}\varepsilon_2^{32}\right)\right.$$

$$+e^2\left(\varepsilon_2^{12}-\frac{13}{2}\varepsilon_2^{22}+7\varepsilon_2^{32}\right)$$

$$\left.+e^2\cos 2\omega t\left(-\varepsilon_2^{12}+4\varepsilon_2^{22}-7\varepsilon_2^{32}+\frac{17}{2}\varepsilon_2^{42}\right)+O(e^3)\right]. \qquad (3.3)$$

To simplify the presentation, we limit ourselves to the spherical harmonic of order $n=2$ of the perturbing acceleration, and keep only quadratic terms in the eccentricity e. The first of these truncations is well justified in most situations, since the apsidal motion constants k_n decrease rapidly with order n (Kopal, 1959), and even more so the coefficients E_n (Zahn, 1975b). It is necessary to include terms of higher order in e in the expansions when the orbit under consideration has an appreciable eccentricity (say $e>0.3$), but this presents no essential difficulty.

The perturbing acceleration modifies the orbit in a manner which is described by the classical variational equations for the semi-major axis a and the eccentricity e (see for instance Sterne 1960):

$$\frac{da}{dt}=\frac{M+M_2}{M}\frac{2(1-e^2)^{1/2}}{\omega}\left[\frac{\boldsymbol{R}^2 e\sin v}{(1-e^2)}+\frac{a}{r}\boldsymbol{S}\right]. \qquad (3.4)$$

$$\frac{de}{dt}=\frac{M+M_2}{M}\frac{(1-e^2)^{1/2}}{\omega a}\left[\boldsymbol{R}\sin v+\frac{\boldsymbol{S}}{e}\left((1-e^2)\frac{a^2}{r^2}-1\right)\right]. \qquad (3.5)$$

If we now substitute the components $\boldsymbol{R}$ and $\boldsymbol{S}$ of the perturbing acceleration in these variational equations, and take the time averages over one orbital period, we obtain the following expressions for the secular variation of a and e

$$\frac{da}{dt}=-\frac{3}{\omega}\frac{M+M_2}{M}\frac{GM_2}{R^2}\left(\frac{R}{a}\right)^7$$

$$\cdot\left[\varepsilon_2^{22}+e^2\left(\frac{3}{4}\varepsilon_2^{10}+\frac{1}{8}\varepsilon_2^{12}-5\varepsilon_2^{22}+\frac{147}{8}\varepsilon_2^{32}\right)+O(e^4)\right], \qquad (3.6)$$

$$\frac{de}{dt}=-\frac{3}{4}\frac{e}{\omega}\frac{M+M_2}{M}\frac{GM_2}{R^3}\left(\frac{R}{a}\right)^8$$

$$\cdot\left[\frac{3}{2}\varepsilon_2^{10}-\frac{1}{4}\varepsilon_2^{12}-\varepsilon_2^{22}+\frac{49}{4}\varepsilon_2^{32}+O(e^2)\right]. \qquad (3.7)$$

In order to simplify the presentation as much as possible, these equations contain only the contribution of the primary star, but they can be readily extended to include the tidal friction of the secondary.

Similarly, by taking the time average of the torque $-M_2 r\boldsymbol{S}$ applied to the primary star, we find that its rotational velocity Ω varies with time according to

$$\frac{d}{dt}(I\Omega)=\frac{3}{2}\frac{GM_2^2}{R}\left(\frac{R}{a}\right)^6$$

$$\cdot\left[\varepsilon_2^{22}+e^2\left(\tfrac{1}{4}\varepsilon_2^{12}-5\varepsilon_2^{22}+\tfrac{49}{4}\varepsilon_2^{32}\right)+O(e^4)\right], \qquad (3.8)$$

I being the momentum of inertia of the primary about its axis of rotation. One readily verifies that the total angular momentum of the system is conserved since

$$\frac{d}{dt}\left(I\Omega+\frac{MM_2}{M+M_2}(1-e^2)^{1/2}[G(M+M_2)]^{1/2}a^{1/2}\right)=0\,, \qquad (3.9)$$

where again only the contribution of the primary has been included.

The coupled differential Equations (3.6)–(3.8), together with an equation similar to (3.8) for the rotational velocity of the secondary, may be integrated numerically to follow the dynamical evolution of a close binary, once the coefficients ε_n^{lm} are known. We now proceed to evaluate those tidal coefficients; we will examine first, in §4, the dissipative mechanisms acting upon the equilibrium tide and thereafter, in §5, those operating on the dynamical tide.

4. The Equilibrium Tide

When all forms of dissipation are neglected, the description of the equilibrium tide becomes simply that of the hydrostatic equilibrium achieved by a star when it is submitted to a harmonic gravity potential of the form defined in Equation (2.2). For a thorough account on this classical problem, we refer the reader to Kopal's (1959) treatise. It will suffice here to recall that the tidal coefficient ε_n^{lm} reduces then to twice the apsidal motion constant k_n, and that its value is determined by the structure of the star.

In a star that is not corotating with the perturbing potential, the effect of dissipation is to render this tidal coefficient complex, as in

$$\varepsilon_n^{lm} = 2k_n \exp i\alpha(\sigma_{lm}) . \tag{4.1}$$

The value of the phase angle α depends on the dissipation mechanisms that are at work, and these will now be discussed.

a) Viscous Dissipation

Darwin (1879) was the first to establish the value of this angle in the case of a viscous body. In the limit of small viscosities, he found it to be proportional to the tidal frequency σ_{lm}, as in

$$\alpha = t_F^{-1}(R^3/GM)\sigma_{lm} . \tag{4.2}$$

We introduce here a friction time t_F to measure the efficiency of viscous dissipation (the smaller this time t_F, the more efficient the dissipation). In general, t_F depends on the order of the tidal potential, but we will omit the subscript n for simplicity.

Using this parameter t_F and replacing the tidal frequency by its value $l\omega - m\Omega$, we can reformulate the differential equations that govern the variation of the semi-major axis a (3.6), the eccentricity e (3.7) and the rotational velocity Ω (3.8) as

$$\frac{1}{a}\frac{da}{dt} = -12\frac{k_2}{t_F}q(1+q)\left(\frac{R}{a}\right)^8 \cdot \left[(1-\Omega/\omega) + e^2\left(23 - \frac{27}{2}\Omega/\omega\right) + O(e^4)\right], \tag{4.3}$$

$$\frac{1}{e}\frac{de}{dt} = -\frac{3}{2}\frac{k_2}{t_F}q(1+q)\left(\frac{R}{a}\right)^8 \left[\left(27 - \frac{33}{2}\Omega/\omega\right) + O(e^2)\right], \tag{4.4}$$

$$\frac{d}{dt}(I\Omega) = 6\frac{k_2}{t_F}q^2 MR^2\left(\frac{R}{a}\right)^6 \cdot [(\omega-\Omega) + e^2(26\omega - 15\Omega) + O(e^4)] \tag{4.5}$$

(q is the mass ratio $q = M_2/M$). From these equations, we can deduce the two time scales which characterize the tidal evolution of a binary system. These are:

i) the *synchronization time* t_{sync} defined as

$$\frac{1}{t_{\text{sync}}} = -\frac{1}{(\Omega-\omega)}\frac{d\Omega}{dt} = 6\frac{k_2}{t_F}q^2\frac{MR^2}{I}\left(\frac{R}{a}\right)^6, \tag{4.6}$$

where the corrections due to a finite eccentricity has been neglected[1];

ii) the *circularization time* t_{circ} given by

$$\frac{1}{t_{\text{circ}}} = -\frac{1}{e}\frac{de}{dt} = \frac{63}{4}\frac{k_2}{t_F}q(1+q)\left(\frac{R}{a}\right)^8, \tag{4.7}$$

assuming that corotation has already been achieved ($\Omega = \omega$).

When the phase angle α is proportional to the tidal frequency, as in expression (4.2), with no further modification of the amplitude of the outer potential, this has been sometimes identified as the weak-friction case. One easily verifies that the phase-shift of each Fourier component of the potential can then be interpreted in terms of a time delay $(R^3/GM)/t_F$, which does not depend on l or m. Making use of this property, Alexander (1973) recently treated the tidal problem in a very elegant way; dispensing with the usual Fourier expansions, he derived secular equations similar to (4.3) and (4.4), but that are valid for any value of the eccentricity e. (It is unfortunate that his method cannot be used when the phase angle α ceases to be proportional to the tidal frequency, as will be the case with the other dissipation mechanisms.) One readily verifies that Alexander's equations are identical to ours at the lowest order in e. The same conclusions may therefore be drawn from both. For instance, a circular orbit is stable (i.e. $de/dt < 0$) only if the ratio of the rotational to the orbital velocities satisfies the inequality $\Omega/\omega < 18/11$, a result first established by Darwin (1879). Also, the semi-major axis of an eccentric orbit will be stationary if

$$\Omega/\omega = 1 + \tfrac{19}{2}e^2 + O(e^4) . \tag{4.8}$$

We now turn to the estimation of the viscous friction time t_F. According to Darwin this time is given by

$$t_F = t_V = R^2/\nu \tag{4.9}$$

in a star of typical kinematical viscosity ν, if one neglects numerical factors of order unity. This is a time of the order of 10^{12} or 10^{13} years; viscous dissipation can therefore be completely neglected when describing the equilibrium tide.

b) Radiative Dissipation

Radiative dissipation looks at first sight a more promising mechanism, since one may then expect the friction time to be comparable to the Kelvin-Helmholtz time

$$t_{\text{KH}} = RL/GM^2 , \tag{4.10}$$

[1] There appears to be an error in Kopal's (1972) result concerning this synchronization time. His prediction of a $(a/R)^3$ dependence for t_{sync} can be traced to a wrong upper limit applied to his integral (6.71) representing the viscous torque

L being the luminosity of the star. However, a more detailed analysis reveals that the analog of t_F is larger by a factor of 10, at least (Zahn, 1965, 1966c; Dziembowski, 1967). Introducing this value of t_F in expressions (4.6) and (4.7) for the characteristic time scales, we see that radiative damping, when acting upon the equilibrium tide, is ineffectual during the nuclear life span of the system. (We will see in §5 however that the same radiative dissipation is much more powerful when operating on the dynamical tide.)

c) Turbulent Dissipation in Stars Possessing a Convective Envelope

A much more effective mechanism is that of turbulent friction occurring in the convective regions of a star. Unfortunately, no satisfactory description for ordinary stellar convection is even available, let alone for the coupling between convection and large scale oscillations. If one wishes nonetheless to estimate the interaction between the tides and the convective motions, one has to resort to some approximate phenomenological procedure. For instance, one may use the artifact of eddy-viscosity to evaluate the tidal torque exerted on the convective regions. If the convective region occupies a substantial fraction of the star, and if convection transports most of the energy flux, one finds that the friction time is given by (Zahn, 1966b)

$$t_F \sim t_{EV} = (MR^2/L)^{1/3} . \quad (4.11)$$

This time scale turns out to be typically of the order of one year, leading us to the important conclusion that turbulent convection is by far the most powerful agent retarding the equilibrium tide. Using this friction time, the characteristic times for synchronization and circularization introduced in Equations (4.6) and (4.7) can be expressed as

$$t_{\mathrm{sync}} = \frac{1}{6q^2 k_2} \left(\frac{MR^2}{L} \right)^{1/3} \frac{I}{MR^2} \left(\frac{a}{R} \right)^6 \quad (4.12)$$

and

$$t_{\mathrm{circ}} = \frac{1}{63q(1+q)k_2} \left(\frac{MR^2}{L} \right)^{1/3} \left(\frac{a}{R} \right)^8 . \quad (4.13)$$

Highly conjectural as they may appear, such estimates based on the eddy-viscosity nevertheless seem to represent adequately the tidal time scales, but only for those stars possessing a thick convective envelope, such as the Sun. For brevity, we shall designate such stars from now on as the CE stars (for convective envelope); in §6 we shall see how our theoretical predictions compare with the observations of binary stars of that kind.

d) Turbulent Dissipation in Stars Possessing a Convective Core

The crudeness of the eddy-viscosity approach should perhaps deter one from trying to refine further the expression (4.11) estimating the friction time t_F. However, when dealing with stars in which convection plays a lesser role, one is obliged to proceed with a somewhat more detailed analysis. This is the case with stars possessing a convective core and a radiative envelope, such as early-type main sequence stars; we shall call them for brevity the CC stars (for convective core). It was found (Zahn, 1966b) that the friction time is then considerably increased, since it scales roughly as $(R/R_C)^7$, R_C being the radius of the convective core. Another loss of efficiency arises from the fact that the convective motions are then so slow that they cover during one tidal period a distance which is often much shorter than the mixing length. It is this actual distance that must then be taken as the mean-free path of the turbulent elements, and hence the eddy-viscosity coefficient will be accordingly reduced. The outcome is that for those CC stars the friction time may be approximated by

$$t_F \sim t_{EV} = (MR^2/L)^{1/3} (R/R_C)^7 , \quad (4.14)$$

and that the phase angle α is then given by

$$\alpha = t_{EV}^{-1} (R^3/GM) \quad \sigma_{lm} \eta(\sigma_{lm}) . \quad (4.15)$$

The efficiency factor η in this expression (4.15) is a function of the tidal frequency; in terms of the dimensionless frequency s, it is defined by

$$\eta = 1 \qquad \text{for} \quad s < s_{EV} , \quad (4.16a)$$

and

$$\eta = 2(s_{EV}/s) - (s_{EV}/s)^2 \quad \text{for} \quad s > s_{EV} . \quad (4.16b)$$

The cut-off frequency s_{EV} corresponds essentially to the time needed by a turbulent element to travel across the convective core. In Table 1 we present numerical values of t_{EV} and s_{EV} for main-sequence stars of various masses.

In those CC stars, the phase angle α thus ceases to be proportional to the tidal frequency s as soon as $s > s_{EV}$. The weak friction approximation then no longer holds and one cannot use the simple expressions (4.6) and (4.7) to predict the evolution of the binary system.

Table 1. Parameters for the equilibrium tide. The eddy-viscous friction time t_{EV} and the cut-off frequency s_{EV} (in dimensionless units) serve to evaluate the time scales which characterize the equilibrium tide damped through turbulent friction in stars possessing a convective core [cf. Eqs. (4.14) and (4.16)]. These two parameters are given here for zero age main sequence stars of various masses

Mass ($M_\odot$)	t_{EV} (years)	s_{EV}
1.6	$8.14\ 10^5$	$4.70\ 10^{-3}$
2	$1.22\ 10^5$	$3.80\ 10^{-3}$
3	$3.07\ 10^4$	$4.64\ 10^{-3}$
5	$8.00\ 10^3$	$6.24\ 10^{-3}$
7	$3.25\ 10^3$	$7.03\ 10^{-3}$
10	$1.27\ 10^3$	$7.90\ 10^{-3}$
15	$4.41\ 10^2$	$7.53\ 10^{-3}$

Instead, one has to go back to the original Equations (3.7) and (3.8) and substitute there the tidal coefficients by

$$\varepsilon_n^{lm} = 2k_2 t_{EV}^{-1} (R^3/GM)^{1/2} s_{lm} \eta(s_{lm}) . \tag{4.17}$$

In the asymptotic regime of $s_{22} \gg s_{EV}$, the rotational velocity then varies as

$$-\frac{1}{\omega}\frac{d\Omega}{dt} = \frac{1}{t_{\text{sync}}} = 3\frac{k_2}{t_{EV}} s_{EV} q^2 (1+q)^{1/2} \frac{MR^2}{I} \left(\frac{R}{a}\right)^{9/2} , \tag{4.18}$$

where we have neglected the correction in e^2. Likewise, the eccentricity of the orbit follows the law

$$-\frac{1}{e}\frac{de}{dt} = \frac{1}{t_{\text{circ}}} = \frac{81}{4}\frac{k_2}{t_{EV}} s_{EV} q(1+q)^{1/2} \left(\frac{R}{a}\right)^{13/2} , \tag{4.19}$$

again with the assumption of corotation ($\Omega = \omega$).

These characteristic times turn out to be very large. The synchronization time of a $10 M_\odot$ main-sequence star, for instance, would exceed the nuclear life-time as soon as the fractional separation of the system becomes larger than $a/R = 2.85$; this semi-major axis corresponds to the very short period of 0.93 day, assuming a mass ratio q of unity. Such estimates disagree with the observations, which instead suggest that corotation can be achieved at separations which are typically two or three times larger than those inferred from expression (4.18).

e) Turbulent Dissipation Generated by the Tide Itself

Another dissipation mechanism has been recently proposed by Horedt (1975), Press et al. (1975) and Lecar et al. (1976). They notice that the tidal flow itself could generate turbulence, since the Reynolds number which characterizes it is very high (10^{12} or more), and that this turbulence could well retard the tide, much like the convective turbulence discussed above.

However, the shear induced by the tides oscillates periodically in time around a zero mean value; it is therefore extremely unlikely that it becomes unstable, as was pointed out by Seguin (1976), since the growth rate of small (linear) perturbations is much smaller than typical tidal frequencies. Moreover, the density stratification exerts a strong stabilizing influence on the horizontal shear, even if one takes radiative damping into account [as outlined, for instance, in Zahn (1975a)].

The observations seem also to rule out this instability. It is well known that close binary components evolve towards the giants branch, like single stars of the same type, and thus that they too build up a helium-rich core. This would not be possible with tidally induced turbulence, which would maintain a star in a homogeneous state.

Let us summarize the results of this section. The most efficient form of dissipation which may operate on the equilibrium tide is clearly turbulent viscosity in the convective regions of a star. This mechanism provides a satisfactory interpretation of the behavior of stars possessing a convective envelope, as we will see in §6. But when convection is confined in the core of a star, as is the case in stars with radiative envelopes, this process is not powerful enough to explain the observed synchronization of close binaries of that type. In the next section we propose another mechanism which we believe to be responsible for this.

5. The Dynamical Tide with Radiative Damping

Due to the elastic properties of the stellar material, a star can experience a variety of oscillations. Those oscillations that are self-excited through some instability mechanism have been extensively studied in connection with variable stars. Yet the oscillations that are driven by an outer perturbing force, such as arise from the tides, have received much less attention.

Cowling (1941) was the first to describe those forced oscillations in a binary component, which include both the dynamical and the equilibrium part of the tide. But his study was restricted to the non-dissipative case, and hence it cannot be used to evaluate the exchanges of energy and momentum. The effects of viscous dissipation, be it of molecular or turbulent origin, have been analyzed by Kopal (1968a–d); his results have however limited application since they are derived for a stellar model that is everywhere strongly superadiabatic. It is well known that in such a model all the gravity modes are unstable; they will therefore overwhelm the oscillation that is forced by the tidal potential, and it makes little sense to calculate the tidal lag under those conditions. More realistic models are required to estimate the effects of viscosity on the dynamical tide, but it is unlikely that these effects are as important as those experienced by the equilibrium tide, which have been considered in the preceding section.

We have examined recently the effects of radiative damping on the dynamical tide (Zahn, 1975b); our results apply to stars possessing a convective core and a radiative envelope (the CC stars). The physical process may be sketched in the following way. The tidal potential generates a variety of gravity waves in the star under consideration; if dissipation were negligible, these waves would be totally reflected by the surface and they would result in standing waves, locked in phase with the tidal potential. But in the outer region of the star, the radiative cooling time is comparable with the tidal period and thus the waves undergo some amount of damping. Only a fraction of the incident wave is reflected by the surface, phase-shifted, and a net flux of mechanical energy is therefore transported from the adiabatic interior to the dissipative region near the surface. This serves to transfer angular momentum from the rotation of the star to the orbital motion.

The properties of the dynamical tide acting on those CC stars, when radiative damping is taken in account, are presented in the original paper. They may be summarized as follows:

i) The bulge raised by the dynamical tide is much smaller than that produced by the equilibrium tide, but unlike the latter, it can take any orientation with respect to the companion star, depending on the tidal frequency.

ii) This tidal bulge does not coincide in general with an equipotential surface; the phase lag of the outer potential created by the dynamical tide also depends on the tidal frequency, and is given by

$$\phi_n^{lm} = \tan^{-1}(\gamma_n \tan(\pi/3 - \psi_n^{lm})) + \tan^{-1}(\gamma_n \tan\psi_n^{lm}) . \quad (5.1)$$

Here ψ_n^{lm} is a linear function of the tidal period $2\pi/\sigma_{lm}$ which takes the values $\pi/2$, $3\pi/2$, $5\pi/2$,... at the eigenperiods of the free modes of oscillation. The damping factor γ_n increases monotonically with the tidal period, from zero (vanishing period) to unity (very large periods). Notice that in this latter case of $\gamma_n=1$ the phase angle ϕ_n^{lm} ceases to be a function of ψ_n^{lm} and that it then takes its asymptotic value $\pi/3$.

The function p_n introduced in Equation (2.5) can also be expressed in terms of those two arguments ψ_n^{lm} and γ_n as

$$p_n(s_{lm}) = \gamma_n/(\cos^2\psi_n^{lm} + \gamma_n^2 \sin^2\psi_n^{lm}) . \quad (5.2)$$

For most applications, such as time integrations of the secular Equations (3.6)–(3.8), the function p_n may be replaced by its sliding mean defined as

$$\overline{p_n}(s) = \int_{\psi-\pi/2}^{\psi+\pi/2} p_n(s) d\psi/\pi = 1 . \quad (5.3)$$

This comes about because all functions $f(s)$ involved in such integrations will vary very slowly with ψ, as also does γ_n, so that integrals of the type

$$\int p_n(s) f(s) dt , \quad (5.4a)$$

over large enough intervals ($\Delta\psi \gg 1$) can be approximated by

$$\int \overline{p_n}(s) f(s) dt = \int f(s) dt . \quad (5.4b)$$

Remarkably enough, this sliding mean $\overline{p_n}$ does not depend on the tidal frequency s, nor on the details of radiative damping represented by the damping factor γ_n.

Replacing p_n by its constant value 1, we find that the contribution of the dynamical tide to the imaginary part of Φ_n^{lm}/U_n^{lm}, in Equation (2.5), may be approximated by

$$\varepsilon_n^{lm} = E_n s_{lm}^{8/3} . \quad (5.5)$$

The tidal coefficients E_n are very sensitive to the structure of the star, particularly to the size of the convective core: to first approximation, they scale as $(R_C/R)^{2n+4}$.

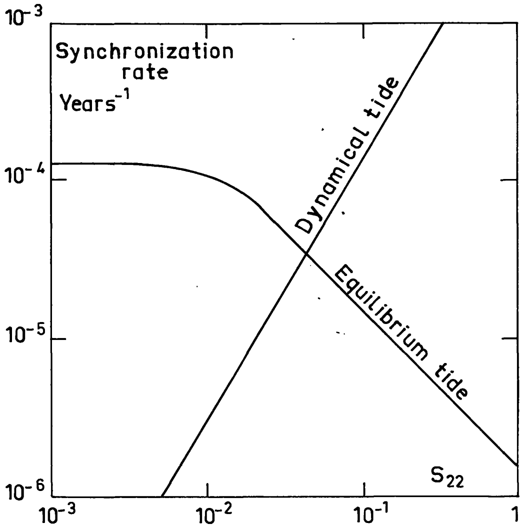

Fig. 1. The intrinsic synchronization rate versus the tidal frequency $s_{22}=2(\Omega-\omega)(R^3/GM)^{1/2}$ for a $5M_\odot$ star. This intrinsic rate must be multiplied by $q^2(R/a)^6$ to yield the actual synchronization rate $(\Omega-\omega)^{-1} d\Omega/dt$ [see Eq. (5.6)]. The contributions of the equilibrium tide (damped by turbulent friction in the convective core) and of the dynamical tide (damped by radiative dissipation) are shown separately

In a fully radiative star, the E_n would vanish altogether, and one would have to carry the asymptotic treatment to a higher order in s. As for the apsidal constants k_n, these coefficients must be determined through numerical integration, using stellar structure models; this is explained in the original paper (Zahn, 1975b), where numerical values of the E_n for zero age main sequence stars are also provided.

With the approximate expression (5.5) for the coefficients ε_2^{lm}, we can now evaluate the time-scales characterizing the dynamical tide. For instance, the rotational velocity of a close binary component varies as

$$-\frac{1}{\Omega-\omega}\frac{d\Omega}{dt} = \frac{1}{t_{\rm rot}} = 3\left(\frac{GM}{R^3}\right)^{1/2} \frac{MR^2}{I} E_2 \left[q^2\left(\frac{R}{a}\right)^6\right] s_{22}^{5/3} \quad (5.6)$$

again neglecting here the correction called for when the orbit is eccentric. The synchronization rate $t_{\rm rot}^{-1}$ is a function of both the strength of the perturbing potential, as measured by the quantity in brackets $[q^2(R/a)^6]$, and of the tidal frequency $s_{22}=2(\Omega-\omega)(R^3/GM)^{1/2}$. In Figure 1, we have singled out the latter dependence by displaying the "intrinsic" rate $t_{\rm rot}^{-1}[q^2(R/a)^6]^{-1}$ versus the dimensionless frequency s_{22}; the model chosen is that of a $5M_\odot$ main sequence star. For comparison, we also show that same intrinsic synchronization rate for the equilibrium tide, with this tide being damped by turbulent viscosity in the convective core, as explained in §4d. Notice that it is the dynamical tide which is the more efficient of the two, except for very small tidal frequencies; this property holds for all stars with radiative envelopes.

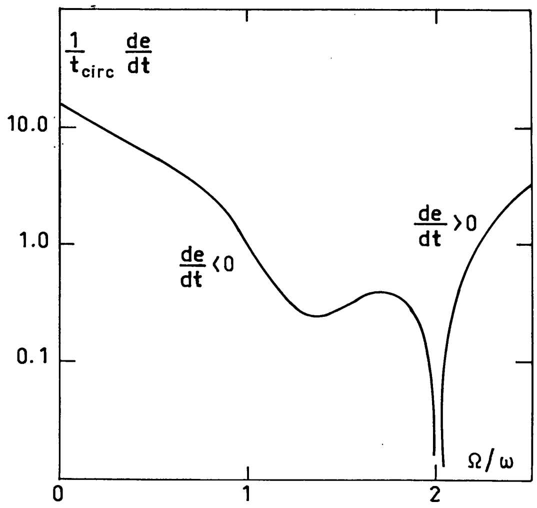

Fig. 2. Variation of the circularization rate characterizing the dynamical tide, as a function of Ω/ω, the ratio between rotational and orbital velocities. The semi-major axis is fixed [see Eq. (5.10)]

If we neglect entirely the contribution of the equilibrium tide, it is more meaningful to introduce a new synchronization time t_{sync} defined as

$$1/t_{sync} = 5\, 2^{5/3} (GM/R^3)^{1/2} (MR^2/I) q^2 (1+q)^{5/6} E_2 (R/a)^{17/2}. \tag{5.7}$$

In most binary systems, the momentum of inertia I of the stars is much smaller than the orbital momentum $[q/(1+q)]Ma^2$, so that the variations of the orbital velocity ω can be neglected compared to those of the rotational velocity Ω. Under such conditions, the relative departure from synchronism $(\Omega-\omega)/\omega$ varies in time according to

$$\frac{d}{dt} |(\Omega-\omega)/\omega|^{-5/3} = 1/t_{sync}, \tag{5.8}$$

if one further neglects the possible variations of the momentum of inertia.

We shall characterize the eccentricity variations, as before with the CE stars, by the circularization time $t_{circ} = e(de/dt)^{-1}$. In most circumstances this time is larger than the synchronization time t_{sync}, so that the star approaches synchronism before the eccentricity has been appreciably modified. We can thus evaluate the time t_{circ} for $\Omega = \omega$, which results in

$$1/t_{circ} = 21/2 (GM/R^3)^{1/2} q(1+q)^{11/6} E_2 (R/a)^{21/2}. \tag{5.9}$$

But we must keep in mind that the circularization rate varies with the relative departure from synchronism. According to (3.7) and (5.5) we have

$$\frac{1}{e}\frac{de}{dt} = \frac{1}{t_{circ}} \frac{1}{14} \left[\frac{3}{2} + \frac{1}{4}(1+\zeta)^{8/3} + \zeta^{8/3} + \frac{49}{4}(1-\zeta)^{8/3} \right] \tag{5.10}$$

with $\zeta = 2(\Omega-\omega)/\omega$. The variation of this function with Ω/ω is displayed in Figure 2. Notice that for $\Omega/\omega > 2.007$ the eccentricity would increase with time, if the dynamical tide were operating alone; one may compare this value with that found by Darwin for the equilibrium tide with viscous damping ($\Omega/\omega = 18/11 = 1.636$).

Due to the high exponents of (R/a) in expressions (5.7) and (5.9), the time scales for the synchronization and circularization of CC binary stars are generally orders of magnitude longer than those characterizing the equilibrium tide in the CE stars. We may therefore conclude that the strongest tidal effects are to be found in binary systems that contain stars possessing a convective envelope. We will see in the next section that this is borne out by the observations.

6. Comparison with the Observations

In this section, we shall compare the theoretical predictions of §§4 and 5 with the observational results that are available. This comparison can be made only in a broad statistical sense, for we lack information about the initial rotation speeds and often also about the ages of the systems. Nevertheless, the observations bring forth separations or periods that divide the close binaries in two somewhat overlapping groups; those which have experienced synchronization, and those which have not. The same occurs also for circularization. These limiting separations and periods are the parameters which can be conveniently confronted with the theoretical predictions.

We will restrict here our attention to those stars which have not envolved too far from the main sequence. One reason for doing this is that the stars generally loose very little mass prior to the end of the hydrogen-burning phase, and thus exchanges of energy and momentum in a binary system are then due solely to tidal friction.

Even more important is the fact that on the main sequence the rotation of the stars is likely not to deviate much from solid body rotation, and this has been implicitly assumed in the preceding sections. The tidal torque per unit mass varies with depth and latitude in a star, and therefore it tends to induce differential rotation. However, as long as a star remains homogeneous, it is liable to various instabilities which tend to restore uniform rotation. Among these, the most efficient appear to be the shear instabilities which are only partly hindered by the stabilizing density stratification (Zahn, 1975a). This permits us to treat a homogeneous star as a solid rotator, at least to a first approximation, thereby avoiding having to deal with the intricacies of differential rotation. However such a simple treatment is not possible once the star has left the main sequence, since it then develops a helium-rich core whose rotation is likely to decouple from that of the envelope.

When examining the tidal evolution of close binaries, one has to distinguish between the CE stars which possess a convective envelope and the CC stars which do not. We have seen that the major contribution to the tidal torque has a different physical origin in the two classes of stars: in the CE stars, the dissipation is caused by turbulent friction in the convective envelope, whereas in the CC stars, it is due to radiative damping in the non-adiabatic layers located near the surface. As we have already noticed, these two physical processes have different efficiencies, and one should expect CE stars in binary systems to display more signs of tidal friction than CC stars.

The observations fully confirm this dissimilar behavior. Take the detached systems listed in the Kopal and Shapley (1956) catalogue of eclipsing binaries: with only one exception to be discussed later, all systems that contain a CE star have circular orbits, and the systems which have eccentric orbits all involve two CC stars. This property is so striking that one is tempted to use such eccentricity determinations to locate the boundary between the CE and CC stars. An upper limit for the mass of the CE stars seems to be $1.6M_\odot$ (Zahn, 1966c), and this agrees well with stellar structure calculations: according to Baker and Temesvary (1966), main sequence stars with masses smaller than about $1.5M_\odot$ have extensive outer convection zones (assuming a standard chemical composition and a ratio of 2 between mixing length and pressure scale height).

We shall now proceed with a more quantitative comparison between the observations and our theoretical predictions; we shall first consider systems with CE stars, and thereafter with CC stars.

a) Stars with Convective Envelopes

We have given in §4 the tidal time-scales that apply to CE stars. Let us emphasize again that, due to the uncertainties of the eddy-viscosity treatment, the numerical coefficients in expressions (4.12) and (4.13) must not be taken literally. Suitable approximations for these formulae, probably well within the error margin, are therefore

$$t_{\text{sync}} \sim q^{-2}(a/R)^6 \sim 10^4((1+q)/2q)^2 P^4 \quad \text{years} \tag{6.1}$$

for the synchronization time, and

$$t_{\text{circ}} \sim (q(1+q)/2)^{-1}(a/R)^8 \sim 10^6 q^{-1}((1+q)/2)^{5/3} P^{16/3} \quad \text{years} \tag{6.2}$$

for the circularization time, the orbital period P being expressed in days. According to those estimates, a binary component of the age of the Sun should be in nearly synchronous rotation if the fractional separation (a/R) is less than about 40. Likewise, its orbit should be circular by this time if the separation is less than 15.

This limiting separation for synchronization seems to be confirmed by the observations. One of Levato's (1975) conclusions is that binary stars around the spectral type F5, which possess a convective envelope, are found in synchronism in systems which have a period of up to 17 days. This period corresponds to a separation of $(a/R)=35$ in this mass range (assuming equal components, i.e. $q=1$), which is in good agreement with the predicted separation of 40 mentioned previously. Further substantiation that the limiting period for synchronization should be around 17 days is provided by the system αCrB: with its period of 17.36 days, it contains a CE star (of spectral type G6) which is not yet synchronized since Kron and Gordon (1953) find it to be in rapid rotation. It is this eclipsing binary which does not obey the rule mentioned earlier that systems with CE stars have circular orbits: αCrB has an eccentricity of 0.37. But due to its extremely long period (for an eclipsing binary), the characteristic time is of the order of 10^{12} years, according to our estimate (6.2). (Furthermore, the eccentricity probably increases at this very slow rate, since the ratio of rotational to orbital velocities Ω/ω very likely exceeds the critical value 18/11 found by Darwin.)

b) Stars with Convective Cores and Radiative Envelopes

With the CC stars, the discussion is somewhat more delicate, and this for two reasons. First, the rotation of the surface layers of such stars is less strongly coupled to that of the interior than in the CE stars which have a turbulent envelope; the rotation observed at the surface of the CC stars could well be a poor indicator of the total angular momentum $I\Omega$ stored in those stars. But let us suppose again that these stars do not deviate much from uniform rotation; we will check later if this simple assumption is not contradicted by the observations.

Second, the tidal coefficient E_2, which plays such a crucial role in the expression of the tidal torque, is much more sensitive to the structure of the CC stars than the apsidal constant is to that of the CE stars. As the star evolves off the main sequence, its convective core shrinks and the coefficient E_2 decreases considerably. To illustrate this, we have evaluated E_2 for a $15M_\odot$ star, using an evolutionary sequence calculated by Stothers and Chin (1975) (which is based on the Ledoux criterion for convection). We find that in these models the coefficient E_2 drops from $1.46\,10^{-6}$ to only $4.31\,10^{-8}$ at half life, and further to $6.53\,10^{-11}$ at the end of the main sequence phase. This decrease of E_2 is only partly compensated by the parallel increase in radius: the synchronization time t_{sync} in expression (5.7) varies as $(I/MR^2)/E_2R^7$, and this function is multiplied by a factor of 6 at half life and by a factor of 50 at the end of the main sequence stage. For the circularization time t_{circ}, which scales as $(E_2R^9)^{-1}$, these factors are respectively 4.5 and 20.

We may thus conclude that most of the tidal braking actually occurs during a rather short period at the beginning of the main sequence phase. To illustrate this further, we have performed a few time integrations

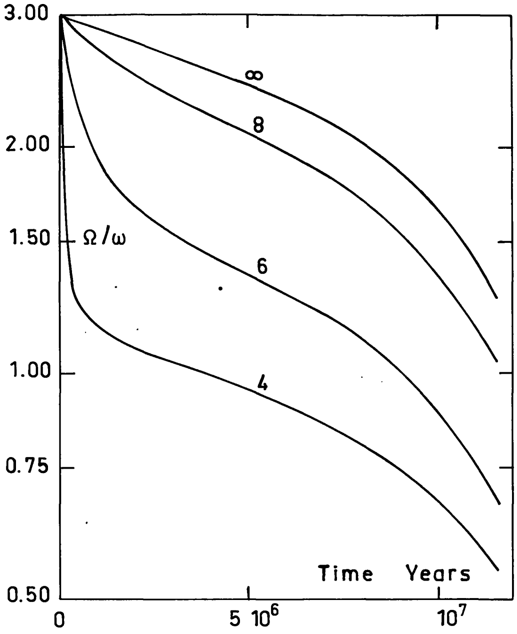

Fig. 3. Synchronization of a $15M_\odot$ star. The variation with time of the ratio Ω/ω is shown for three values of the initial separation, $(a/R)=4$, 6 and 8. The mass ratio is assumed to be unity and the orbital eccentricity zero. The curve labelled ∞ displays the decrease of Ω/ω in the absence of tidal braking; this results simply from the conservation of angular momentum in the expanding star. With our definition (see caption of Table 2), this star has a synchronization time $t_{\text{sync}}=6.54$ days

Table 2. Limiting separations and periods for the dynamical tide. This table gives the parameters which describe the dynamical tide damped by radiative dissipation in stars possessing a convective core and a radiative envelope. The limiting separations $(a/R)_{\text{sync}}$ and $(a/R)_{\text{circ}}$ are the fractional separations at which respectively the synchronization time t_{sync} and the circularization time t_{circ} are equal to 1/4 of the main sequence life span; P_{sync} and P_{circ} are the corresponding periods. These parameters have been calculated for a mass ratio of unity. For a different mass ratio q, the correction factor to be applied to $(a/R)_{\text{sync}}$ is $q^{4/17}[(1+q)/2]^{5/51}$; to $(a/R)_{\text{circ}}$, it is $(q/2)^{2/21}[(1+q)/2]^{11/63}$, taking then only the contribution of one star into account. There are two entries for $15M_\odot$, corresponding respectively to the model of Aizenman (see Zahn, 1975b) and to that of Stothers and Chin (1975)

Mass ($M_\odot$)	Synchronization		Circularization	
	$(a/R)_{\text{sync}}$	P_{circ} (days)	$(a/R)_{\text{circ}}$	P_{circ} (days)
1.6	6.11	1.21	4.44	0.75
2	7.05	1.59	4.99	0.95
33	6.81	1.92	4.85	1.10
5	6.52	2.19	4.68	1.33
7	6.72	2.69	4.80	1.62
10	6.67	3.30	4.77	2.00
15 A	7.04	3.98	4.99	2.38
15 SC	6.54	3.62	4.69	2.20

of Equations (3.6) and (3.8), using the same models for this $15M_\odot$ star; the variation of the ratio Ω/ω between the rotational and orbital velocities is shown in Figure 3. The small contribution of the equilibrium tide is included in the calculations; we have assumed that the eccentricity is zero and the mass ratio unity. The initial conditions are $\Omega/\omega=3$ and $(a/R)=4$, 6 and 8. For reference we have added the variation of Ω/ω in the absence of tidal braking; the corresponding curve is labelled $(a/R)=\infty$. These integrations confirm that the tidal torque operates only at the beginning of the main sequence life; thereafter the rotational velocity of the star continues to decrease, but this is just a consequence of the conservation of angular momentum as the star expands.

We are therefore led to define the limiting separations for synchronization and circularization as the initial separations $(a/R)_{\text{sync}}$ and $(a/R)_{\text{circ}}$ for which the corresponding characteristic times are equal, say, to one quarter of the main sequence life span. Numerical values of these limiting separations, together with the corresponding periods, are given in Table 2 for the same stars. These values are calculated for a mass ratio of unity; the corrections to be applied for other values of q are given in the caption of the table.

Let us emphasize here that the limiting periods P_{sync} and P_{circ} are much more model-dependent than the limiting separations from which they are derived, since for a given (a/R) they vary as $R^{3/2}$. There are some indications that the radii of our theoretical models (computed by Aizenman, see Zahn, 1975b) are somewhat smaller than the observed ones. For instance, the radius of our $5M_\odot$ model is $R=2.35R_\odot$, whereas the observations give about $3R_\odot$ (Popper, 1974); the limiting period for synchronization P_{sync} should thus lie around 3.15 days, instead of 2.19 quoted in Table 2. It is therefore safer, when comparing observations and theory, to use the fractional separation (a/R) whenever this parameter is directly measurable, as it is in eclipsing binaries.

The observations of rotational velocities in close binaries have been presented and discussed in several articles during the past decade (Olson, 1968; Plavec, 1970; Nariai, 1971; Stothers, 1973; Levato, 1975): one finds that the stars are synchronized up to a separation (a/R) of 7 or 8. This limiting separation seems quite independent of the mass of the star; however, it should be mentioned that the observational material available is very scarce for masses greater than $10M_\odot$. Levato quotes somewhat higher limiting periods than the others, but his criterion for synchronization, namely that $\Omega/\omega<2$, is less severe than those adopted elsewhere.

The observational values for the limiting separation are in satisfactory agreement with the theoretical values of Table 2, which range from 6.52 to 7.05 (with the exception of the model of lowest mass, $1.6M_\odot$). However, if further observations confirm this average limiting separation of about 7.5, one would have to conclude that the tidal coefficients E_2 exceed by a factor 2 or 3 those computed from current models. This in turn could lead to a revision of these models: for instance, an increase of 15% in the mass of their convective core could make up for the observed difference. It is tempting to see in this further evidence for substantial overshooting from convective cores (cf. Maeder, 1976), even though it is certainly too early to jump to this conclusion.

The reasonable agreement between the observations and our predictions is an indirect confirmation of the assumption made for the rotation: we may conclude that the interior rotation of the CC stars is fairly uniform and thus that the mechanisms which redistribute the angular momentum in those stars operate on a time scale which is short compared with the synchronization time.

However a few stars present a considerable excess of rotational velocity. Two well known examples are AR Cas and U Cep, in which the primary rotates respectively with four and five times the speed required for synchronism (Olson, 1968); the separation in both binaries lies around $(a/R)=5$ (Koch et al., 1970), which is well below our theoretical limit of about 7. In most cases, the surface layers are probably accelerated by matter coming from the companion star. The secondary of U Cep, for instance, seems to fill its Roche lobe and transfer of mass has been observed in late 1974 (Batten et al., 1975; Plavec and Polidan, 1975; Olson, 1976). The case of AR Cas is more puzzling, since there is no indication for mass exchange; the only explanation we can offer is that the system be very young.

The observed orbital eccentricities provide another verification for the tidal theory in those CC stars. Koch (1976) has thoroughly analyzed the data contained in Batten's Sixth Catalogue and in the two lists of the Toulouse Observatory. He finds that in the spectral range B7–F0 (2 to $6M_\odot$) the systems with a period shorter than 1.5 day all have circular orbits. The situation is less clear at higher masses: some very close pairs have such a large eccentricity that they cannot avoid encounter at periastron. It remains to be verified however whether what one measures there is a genuine eccentricity: by shifting the brightness distribution at the surface of the stars, the dynamical tide modifies the light- and velocity curves in the same way as would an orbital eccentricity (Zahn, 1975b).

Let us come back to the limiting period of 1.5 day revealed by Koch's investigation. This is higher than our theoretical periods of Table 2 which, for the same mass range, lie between 0.95 and 1.33 day. Part of the discrepancy is certainly due to the underestimated radii of our models: if one takes $3R_\odot$ as the radius of the $5M_\odot$ star, instead of $2.35R_\odot$ as in our model, one increases its limiting period from 1.33 to 1.90 day. It would thus be desirable to calibrate the observed eccentricities versus the separation (a/R) of the most massive component: we have already mentioned that this parameter is much less model-sensitive than the period. Also, the theory predicts that the limiting separation is almost independent of the mass of the star, whereas the period increases with mass as the inverse square-root of the mean density.

7. Application to X-ray Binaries

The current interpretation of the X-ray binaries is that the X-ray source be the neutron star remnant of a supernova explosion. If this is true, one has to explain why these binaries now have circular orbits, since the sudden mass loss would have produced a sizeable eccentricity.

It is natural again to invoke tidal braking as the circularization mechanism, and this has been done by several authors (cf. Chevalier, 1975; Wheeler et al., 1975; Lecar et al., 1976). Let us verify whether the theoretical predictions presented here confirm such a hypothesis.

This would be the case if the fractional separation (a/R) characterizing the primary component of a particular X-ray binary is less than the limiting separation given by expression (6.2) for the CE stars or in Table 2 for the CC stars. However, the separation which must be used for this comparison is the initial one, achieved just after the explosion; that which is presently observed has been generally inflated by the expansion of the star accompanying its evolution, and we know that the tidal torque is efficient only during the first part of the main sequence phase.

It must also be stressed that the circularization time has been evaluated under the assumption of corotation, which is not necessarily fulfilled: the mass ratio of X-ray binaries can be so small as to render the circularization time shorter than the synchronization time. Thus numerical time integrations may be required to solve some dubious cases.

But we shall be content here with simple comparisons between actual and limiting separations. We choose as examples the binaries Her X-1 and Cen X-3, which have also been considered by Lecar et al. (1976).

a) Her X-1

Following these authors, we take for that binary the parameters:

$$M=2.0M_\odot\,,\quad M_2=1.0M_\odot\,,\quad a=8.64R_\odot\,.$$

We may assume that the primary started with an initial radius of $1.5R_\odot$; the initial separation thus was $(a/R)\sim 5.75$, if we further neglect a probable increase of the semi-major axis. This is substantially larger than the limiting separation of 4.12, which is the value of Table 2 corrected by the factor $(q/2)^{2/21}[(1+q)/2]^{11/63}$ to allow for the mass ratio differing from unity. We thus conclude that the dynamical tide was not efficient enough to circularize the orbit.

One must therefore rely upon the equilibrium tide, which can operate here on the subphotospheric convection zone. It is difficult to assess how efficient the tidal braking will be in this rather shallow convection zone, and one can hardly do better than Lecar et al. (1976) who estimate that the circularization time is of the order of 10^7 years. It should also be mentioned that the energy flux must vary considerably on the surface of this highly distorted star (it fills the Roche lobe!), and that the thickness of the convection zone, and thus the efficiency of tidal braking, may be underestimated when taking a mean effective temperature.

b) CenX-3

We assume that this binary is characterized by

$$M=17M_\odot, \quad q=1/20, \quad a=17R_\odot,$$

and that the initial radius was $R=7.5R_\odot$. Neglecting again the variation of a, we get for the initial separation $(a/R)\sim 2.25$, which is appreciably less than the limiting separation $(a/R)_{\text{circ}}=3.15$ drawn from Table 2, after correction for the small mass ratio. We may therefore conclude that the dynamical tide was indeed capable of circularizing the orbit.

It has been conjectured that this system should be unstable due to the very small mass ratio, and that the compact star should be spiraling into the primary (Chevalier, 1975; Wheeler et al., 1975). To decide if this is true, one needs to know the rotational velocity of the primary; only if Ω is smaller than ω, the orbital velocity, does the semi-major axis decrease secularly. This result has been established by Counselman (1973) for circular orbits; it also holds for moderate eccentricities, according to Equation (3.6). In the case of Cen X-3, the rotational velocity is not known. If the primary was corotating before the explosion, then $\Omega>\omega$ immediately thereafter, but this inequality may well be reversed by now due to the star's expansion.

8. Conclusion

It appears that the two principal mechanisms which are at the origin of the tidal torque in close binaries have now been identified. These are the turbulent friction acting on the equilibrium tide in stars possessing a convective envelope, and the radiative damping retarding the dynamical tide in stars with radiative envelopes. The agreement between the observations and the theoretical predictions based on these physical processes is quite satisfactory; it can possibly be improved by further refinements of the theory.

Among such refinements, a more realistic treatment of the coupling between convective motions and tidal oscillations should be given the highest priority. It would be desirable also to verify whether the effects of rotation, which have been neglected in our description of the dynamical tide, are negligible or not. We hope that future work will bring these problems to a successful conclusion.

The dynamical tide provides us with yet another mean to probe the interior of stars with radiative envelopes. The tidal coefficients E_2 are much more sensitive than the apsidal constants to the structure of those stars, and we anticipate that they too will eventually be used to check the accuracy of theoretical models.

Acknowledgements. I wish to express my thanks to R. H. Koch and R. Stothers, with whom I had many stimulating discussions, and to D. O. Gough and J. Toomre, who kindly read and improved the manuscript.

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