PULSAR GEOMETRIES. III. THE HOLLOW-CONE MODEL

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ABSTRACT

The relations between observable and geometrical quantities relating to the neutron star (presented in Papers I and II of this series) are extended to include the hollow-cone case, which is attractive on physical grounds. The model by Ruderman and Sutherland, with its specific predictions, has been selected for a detailed comparison of theory and observation, in particular with respect to organized subpulse drifting and objects with double-humped integrated profiles. We found that the hollow-cone model is able to reproduce qualitatively the major trends of observations, but that some crucial aspects of Ruderman and Sutherland's model are at variance with fact and require refinement, especially the predicted frequency dependences. Specific suggestions about an improvement of the physical theory are made.

Subject heading: pulsars

I. INTRODUCTION

In Papers I and II of this series (Oster 1975; Oster and Sieber 1976) the problem of how to extract from observed pulsar characteristics the physical parameters and geometrical conditions of neutron stars, such as the angle between the magnetic and rotational axes, was studied in detail. The restriction placed on these studies was the simplifying assumption that the pulsar radiation is emitted in narrow beams that revolve with a characteristic speed along a great circle perpendicular to the magnetic axis.

On physical grounds, a slightly more complex geometry, namely, that of a hollow cone, is particularly attractive, as has been suggested by several authors, most recently by Ruderman and Sutherland (1975; hereafter RS). Here it is postulated that, near a magnetic pole and the surface of the neutron star, the electric fields cause sparks in which particles are accelerated and, through gamma emission and subsequent electron-positron pair production, form clouds of relativistic particles that move outward along the magnetic field lines. At some distance between the stellar surface and the light cylinder, the maximum frequency of their curvature radiation coincides with the plasma frequency and leads to emission in the radio range, suitably amplified by some plasma mechanism, such as particle bunching. Presumably, at this distance from the star the magnetic fields are not fanning out very much, so that the radiation is concentrated along a cone with a half-angle much less than the 90° chosen in our previous studies.

The question now arises whether it is possible to relate the details of actual observations in the same manner as we did in Papers I and II to the parameters of this type of physical and geometrical model. As before, especially interesting candidates are objects that show systematic subpulse drifts in longitude. The latter are due, according to the RS model, to localized stable sparking areas in the polar cap precessing about the magnetic pole with a well-defined velocity. RS's model is particularly well suited for our purposes, since it makes quantitative predictions on both geometrical aspects (for instance, aperture angle and width of the hollow cone) and physical aspects (period and frequency dependence of parameters).

The purpose of this paper, then, is to present such a detailed comparison and to show that it leads to quite specific suggestions for improving the physical model, as far as both the sparking region and the radiation mechanism are concerned.

We begin in § II by defining the geometrical conditions and relating them to the physical parameters of the neutron star and the relevant quantities extracted from observations. In § III, we compare in detail representative observed data from pulsars that have organized subpulse drifts with the specific predictions of the RS model. A similar comparison is made, in § IV, for pulsars with double-humped integrated profiles. In § V, finally, we translate our results into suggestions for changes and improvements of the physical model.

II. THE GEOMETRY OF THE HOLLOW-CONE RADIATION PATTERN

The basic geometry is one of a neutron star whose rotational and magnetic axes form an angle $\beta$ along a great circle. The observer on Earth then observes the neutron star at a fixed latitude $\mu$, while the radio
emission is supposed to occur in beams of half-width $\Gamma/2$, taken, for simplicity, with a circular cross section; physically this means a probability envelope of more or less random radiation spikes of sometimes very short, that is, $\mu$s, duration (Rickett 1975). The beams' center axes revolve about the magnetic axis along a cone of half-angle $R$ (see Fig. 1); $\beta$, $\mu$, $\Gamma$, and $R$, as well as $G$ and $W$ to be introduced presently, are all defined by great circles on the sphere and measured in degrees.

The detailed geometry on the sphere is sketched in Figure 2a for the "upper case," where the appropriate spherical triangles relate the physical parameters $\beta$, $\mu$, $R$, and $\Gamma$ to the observed values of $W/2$ (half-width of the "window") and $G/2$ (observed half-width of the subpulse in the center of the window). We find that

$$\cos (W/2) = \frac{\cos R - \sin (\mu + \Gamma/2) \cos \beta}{\cos (\mu + \Gamma/2) \sin \beta}$$

and

$$\cos (G/2) = \frac{\cos (\Gamma/2) - \sin \mu \cos (\beta + R)}{\cos \mu \sin (\beta + R)},$$

where $\mu + \Gamma/2$ and $\beta + R$ refer to the alternative geometry of Figure 2b ("lower case").

Similar formulae can be written for an arbitrary position of the beam in the window (defined by a longitude angle $\lambda$, Fig. 3), so that, with a parameter $y$,

$$\cos R = \cos \beta \cos y + \sin \beta \cos \lambda \sin y,$$

we have

$$\cos A = \cos A' = \frac{\cos (\Gamma/2) - \sin \mu \cos y}{\cos \mu \sin y}.$$

$$\cos A = \cos A' = \frac{\cos (\Gamma/2) - \sin \mu \cos y}{\cos \mu \sin y},$$

and
and this in either geometrical setup \( y \geq 90° - \mu \). Note that our previous results (Papers I and II) are recovered from these formulae, if \( R = 90° \), and the geometry around the intersection point is nearly plane, that is, in radians, \( A \ll 1 \). Finally, the central angle \( \phi \) around the magnetic pole is defined through the relation

\[
\cot R \sin \beta = \cos \beta \cos \phi + \sin \phi \cot \lambda .
\]

Equations (1) to (5) contain most of the information we are concerned with in this paper. First, we can now predict observed radiation contours in the sense defined in Paper II: Each subpulse corresponds to a cut (at constant \( \mu \)) through the radiation pattern emerging from the neutron star, separated by one rotation period \( P_1 \). The relative position of the subpulses in drift defines a time scale related to the “phase rotation speed” of Papers I and II that is, in the RS model, an expression of the precession speed of the sparks about the magnetic pole.

The most crucial parameter is the position of the latitude circle \( \mu = \text{const.} \) with respect to the center of the phase circle (magnetic pole). In a “central passage” \( (90° - \mu = \beta \pm R \) for the two alternatives of Fig. 3), the greatest subpulse width will be observed in the center of the window. This is still true in a “below-center passage” \( (90° - \mu < \beta - R \) and \( 90° - \mu > \beta + R \), respectively), but now the maximum observed subpulse width is related to a fraction less than unity of the actual width of the emission pattern. Finally, if we have an “above-center passage” \( (\beta - R < 90° - \mu < \beta + R) \), the observed pattern shows a relative minimum in the center of the window. Included in this case are the truly double-humped patterns predicted for a cut of the latitude circle close to the center of the cone.

As an illustration, in Figure 4 we have plotted, with the aid of equations (3), (4), and (5), the predicted patterns for the geometrical alternative of the upper case and the sample parameters \( \beta = 90° \), \( \mu = 10° \), assuming the window points to be uniformly reached in \( 5 P_1 \) (different precession speeds!), letting \( R = 10° \) (central passage), \( R = 8° \) (below-center passage), \( R = 12° \) (above-center passage), and choosing the three contours \( \Gamma/2 = 5°, 2°.5, \) and \( 1°.25 \) corresponding, respectively, in the real world’s idealized version to intensity contours inside the spherical beam.

The contours of Figure 4 show a wealth of details that may be extremely useful in narrowing the choices of the physical parameters, if highly detailed observational data are available. We point, in particular, to the horizontal slants at the edge of the window, which one is immediately tempted to identify with Backer’s (1970) observation that sometimes subpulses seem to enter and leave the window with infinite speed. It is also noteworthy that a change of just a degree or two in \( R \) transforms a single-humped emission pattern into a double-humped one, or vice versa. In the spark model, this would correspond to a minute change in the spark position with respect to the magnetic axis near the surface. Thus, on the basis of observation, we must postulate a remarkable degree of stability for the sparking areas.

Varying the sample parameters, such as \( \beta \) and \( \mu \), or
Fig. 4.—Predicted radiation pattern for "above-center passage," "central passage," and "below-center passage"; details in text.

$R$, or considering the geometry of the lower rather than the upper case, changes the details, but not the overall features of the predicted radiation patterns. Thus such considerations need not be reproduced here.

For our main purpose of comparing the physical data of the hollow-cone model with the gross structures of observations, we concentrate in the remainder of this paper on one contour representing, say, a certain percentage of intensity above the noise level, its width $G$ at the point of symmetry, and the ratio of this width to the window width $W$. With applications to actual pulsars in mind, we aim, in particular, at $G$-values between $6^\circ$ and $9^\circ$, and ratios $G/W$ between $\frac{1}{2}$ and $1$. Similarly, we restrict our considerations to central passage as far as pulsars with drifting subpulses are concerned.

Sample results of interest are plotted in Figure 5, viz., $G/2$ and $G/W$ as a function of $\beta$, with $\Gamma/2$ ($2^\circ$, $5^\circ$) and $R$ ($15^\circ$, $10^\circ$, $5^\circ$) as parameters, and separately for the geometrical conditions corresponding to upper and lower cases and central passage.

Figure 5 contains the key to the interpretation of observations in terms of the hollow-cone model. Assume that the observations are for a pulsar with an essentially single-humped average emission profile, so that central passage is a good starting approximation, and that we can deduce reasonably stable values of $G/2$ and $G/W$. The physical model will provide us with combinations of $\Gamma/2$ and $R$, whereas the data interpretation is supposed to lead to combinations of $\beta$ and $\mu$.

Obviously, there is the extreme case where $G/2 \approx \Gamma/2$, $\beta \approx 90^\circ$, $\mu \approx 0^\circ$. This solution, in general, will correspond to a $G/W$ ratio that is either too large or too small. Noting the significantly different $\beta$-dependence of $G/2$ and $G/W$, respectively, we expect in most cases a fit with a $G/2 > \Gamma/2$ so that $\beta < 90^\circ$ and $\mu > 0^\circ$. It is conceivable, however, that given $G/2$ in at least one of the geometries, no solution is possible; that is, there is no combination of $\beta$ and $\mu$ for which the prescribed $G/W$ ratio can be reached.

In the same vein, we attempt an interpretation of subpulse drifts. Given a combination of neutron star parameters, that is, $\Gamma$, $R$, $\beta$, and $\mu$, and observational quantities which now, in addition to $G$ and $W$, include the periodicities $P_2$ and $P_3$ referring, as in Paper I, to the point of symmetry, we can express $P_2^\circ$ and $P_3^\circ$ in angular measure on the neutron star; $360^\circ$ equal the rotation period $P_1$. From Figures 1 and 2 and equation (5), it readily follows that

$$\cot R \sin \beta = \cos \beta \cos (\Delta/2) + \sin (\Delta/2) \cot (P_3^\circ/2).$$

(6)

Here, $\Delta$ is the distance between the centers of two neighboring radiation beams, measured in degrees of central angle $\phi$ about the magnetic pole, while the observed $P_3^\circ$ is counted along the latitude circle. For the phase rotation speed $d\phi/dt$ (eq. [4] of Paper I), we obtain

$$d\phi/dt = \Delta/P_3 \approx \frac{\sin (\beta \mp R)}{\sin R} d\lambda/dt.$$ (7)

The approximation holds for $\phi, \lambda \ll 1$. Finally, from equation (7) and the relation $n \Delta = 360^\circ$ follows the number of beams present at any time (assuming some time average over $P_2$ and $P_3$ extracted from observations):

$$n = (360^\circ/P_3)[d\phi/dt]^{-1} = 360^\circ/\Delta,$$ (8)

which in the model by RS equals the number of stable sparking regions present in the polar cap.

We are now ready to compare this specific model with observations.

III. OBSERVATIONAL TEST OF ONE HOLLOW-CONE MODEL: SUBPULSE DRIFT

Of the authors who have proposed hollow-cone models, only Ruderman and Sutherland (1975, RS) have given quantitative predictions for most of the crucial parameters, such as $\Gamma$ and $R$, on the basis of physical deduction. For earlier work, see, for instance, Radhakrishnan and Cooke (1969), Komesaroff (1970), and Sturrock (1971). First, RS derive limits to the angular extent of any radiation that, given the pulsar period $P_3$ in seconds, the observed frequency $\omega_3$ in $10^{15}$ radians per second, emerges from the neutron star (their eqs. [66a] and [66b], where a misprint in the
Fig. 5.—Variation of $G/2$ and $G/W$ as function of $\Gamma/2$, $R$, and $\beta$. Central passage. (a) Upper case: $\beta - R = 90^\circ - \mu$. (b) Lower case: $\beta + R = 90^\circ - \mu$. 
magnetic field dependence of \( \theta_{\text{min}} \) has been corrected:

\[
\theta_{\text{min}} = CP_1^{16/21} B_{12}^{11/7} \rho_6^{-15/7} \omega_{10}^{1/7}
\]

and

\[
\theta_{\text{max}} = CP_1^{-20/42} B_{12}^{17/6} \rho_6^{2/21} \omega_{10}^{-1/3}
\]

Here, \( B_{12} \) is the magnetic field at the neutron star's pole in units of \( 10^{12} \) gauss, \( \rho_6 \) the radius of curvature of the field in the sparking region in units of \( 10^6 \) cm, and \( C = 16^\circ \).

Writing

\[
\theta_{\text{max}} - \theta_{\text{min}} = \Gamma,
\]

we have

\[
R = \theta_{\text{min}} + \Gamma/2.
\]

Since a combination of \( G \) and \( W \), as we have seen, leads to a set of values for \( \Gamma/2 \) and \( R \), while the latter select a \( \theta_{\text{max}} \) and a \( \theta_{\text{min}} \), we end with combinations of \( B_{12} \) and \( \rho_6 \) that are compatible with \( G \) and \( W \).

In practical terms, it is more desirable to start with a \( B_{12} \)-value determined, say, from the relation

\[
B_{12} = \frac{(2\pi^2)}{1} \left[ I c^2 (\dot{P})^2 \right]^{1/2},
\]

where \( I \) is the total moment of inertia of the neutron star and \( r \) the neutron star's radius. Equation (13) is commonly assumed to be at least qualitatively true and implicitly involved in RS's model (see their eq. [40]); however, see below. Taylor and Manchester (1975) have recently derived values of \( B_{12} \) for all pulsars where \( P_1 \) and \( \dot{P} \) are known; their numerical values differ only insignificantly from the ones derived from equation (13) used by RS, because of variances in the assumed \( I \) and \( r \)-values and a slightly different numerical factor. In what follows we have used Taylor and Manchester's \( B_{12} \)-values.

The real problem, in this respect, is the identification of the actual surface field that enters RS's equation (66) (our eqs. [9] and [10]) with the dipole component (13) dominant at great distance from the stellar surface. It is quite conceivable that the two \( B \)-values differ significantly, in view of the fact that RS had to invoke higher multipole fields to bring \( \rho_6 \) down to a value of about 1. We shall come back to these two related points presently.

At this point, \( \Gamma/2 \) and \( R \) are determined for one value of \( \rho_6 \); and, since \( \Gamma/2 \) and \( R \) uniquely determine \( G \) and \( W \) in each of the alternative geometries, we have carried out a computer search for that value of \( \rho_6 \) that leads to the observed values of \( G \) and \( W \). It should be noted that there are, of course, significant error bars around all those quantities—for instance, because of the uncertainty in the observationally deduced values.

We have selected PSRs 0031—07, 0809+74, and 2016+28 from the list of objects with confirmed organized subpulse drifts; they lend themselves readily to this type of comparison. It is then seen that with RS's value of \( C = 16^\circ \), no solution to satisfy the observed \( G \)- and \( W \)-values exists; that is, no combination of \( \rho_6 \), \( \Gamma \), \( R \), \( \beta \), and \( \mu \) reproduces \( G \) and \( W \): for a fixed \( G \), all \( W \)-values are either too large or too small, etc. This happens for both geometries at the same time.

At this point, it is obvious that we have to disregard RS's recommendation for \( C \); in fact, they implicitly suggested that \( \theta_{\text{max}} \) be reduced by a factor of 2 when they argued, in connection with their equation (31), that the sparks, on the average, are at a distance \( r_{\text{sp}}/2 \) from the magnetic pole rather than \( r_{\text{sp}} \) (the horizontal distance of the polar-cap edge) on which their \( C = 16^\circ \) is based. In reality, the numerical factor involved in \( \theta_{\text{max}} \) and \( \theta_{\text{min}} \) is uncertain by something like an order of magnitude, as is revealed by a point-to-point analysis of the string of equations that leads to their numerical result.

Thus, we have varied the value of \( C \) as well and obtained (for the geometry of the upper case, as illustration) the data summarized in Table 1. Obviously all the numbers deduced from the model are somewhat arbitrary under the conditions explained. However, some conclusions appear to us to be independent of all these uncertainties, inaccuracies, and possible observational errors. First, quite reasonable values of \( \beta \), \( \mu \), \( \Gamma \), and \( R \) can reproduce the observations. Second, in practically all cases an oblique rotator (\( \beta < 90^\circ \)) is needed; this, of course, is not a conclusive argument against a uniform value of \( 90^\circ \) for \( \beta \). Third, in order to obtain any consistent solution, the radius of curvature \( \rho_6 \) has to be pushed below RS's standard value of unity. Now, only \( \theta_{\text{min}} \) depends significantly on \( B_{12} \) and \( \rho_6 \) (\( \propto B_{12}^{11/7} \rho_6^{-1/3} \)), whereas

\[
\theta_{\text{max}} \propto B_{12}^{1/7} \rho_6^{2/21}.
\]

Thus, an alternative interpretation of our result is the implication that the effective surface field is in reality stronger than indicated by the dipole value (13). We shall return to these questions in § V.

<table>
<thead>
<tr>
<th>Pulsar</th>
<th>( P_1 )</th>
<th>( B_{12} )</th>
<th>( \omega_{10} )</th>
<th>( G/2 )</th>
<th>( W/2 )</th>
<th>( G/W )</th>
<th>( C )</th>
<th>( \rho_6^{-1} )</th>
<th>( \Gamma/2 )</th>
<th>( R )</th>
<th>( \beta )</th>
<th>( \mu )</th>
</tr>
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<tbody>
<tr>
<td>PSR 2016+28 *</td>
<td>0.558</td>
<td>0.29</td>
<td>0.270</td>
<td>2.0</td>
<td>5.0</td>
<td>0.40</td>
<td>4°</td>
<td>4.0</td>
<td>13°</td>
<td>5.6</td>
<td>46°</td>
<td>49°</td>
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<tr>
<td>PSR 0031—07 †</td>
<td>0.943</td>
<td>0.62</td>
<td>0.091</td>
<td>4.5</td>
<td>15.0</td>
<td>0.30</td>
<td>16°</td>
<td>2.6</td>
<td>40°</td>
<td>27.5</td>
<td>90°</td>
<td>23°</td>
</tr>
<tr>
<td>PSR 0809+74 †</td>
<td>1.292</td>
<td>0.46</td>
<td>0.091</td>
<td>3.5</td>
<td>7.8</td>
<td>0.45</td>
<td>8°</td>
<td>2.1</td>
<td>30°</td>
<td>9.5</td>
<td>69°</td>
<td>31°</td>
</tr>
</tbody>
</table>

* Data from Sieber 1974.
† Data from Huguenin, Taylor, and Troland 1970.
We now turn to a comparison of the predictions based on equations (6) to (8), with observations. Our results are summarized in Table 2. First we note that, except for PSR 0809 + 74, the corresponding \( n \)-values are too high. If the average location of the sparking regions is at about \( r_p \approx h/2 \), where \( h \) is the height of the polar gap, and if the mutual distance of the sparking regions corresponding to \( \Delta \) roughly equals \( h \), then \( n \approx 3 \). The discrepancy between model and observation becomes even more serious when we consider the phase rotation speeds \( d\phi/dt \) derived from observations and their theoretical counterpart, \( \Delta n \), from RS's equation (31). This latter quantity leads to values between 60° and 120° s\(^{-1}\), depending on some minor numerical approximations, or more than an order of magnitude above the observations. Such a factor, however, is hard to digest, since

\[
\Delta v = c\Delta V/\beta \rho, \tag{14}
\]

with \( \Delta v \approx 10^{13} V \), \( B \approx 10^{12} \) gauss, \( \rho \approx 10^4 \) cm. Both discrepancies are obviously related to the model of the sparking process and not to the details of the hollow-cone geometry, and we conclude that the mechanism requires major refining if it is to account for the observed data on the above level of sophistication. The difficulty that we have pointed out in Paper II also remains; the routinely observed variability of \( d\phi/dt \) (note the three systems of PSR 0031—07) is hard to understand on the basis of the simple precession drift of sparking regions, although it does predict a frequency-independent \( P_0 \), a feature which is strongly suggested by observations (Hesse 1974).

Finally, we consider the frequency dependence of \( \theta_{\min} \) and \( \theta_{\max} \). Page (1973) published a strip of simultaneous subpulse observations of PSR 0809 + 74 at 81 and 151 MHz. Deriving \( G/2 \) and \( G/W \)-values is somewhat inaccurate; however, the 151 MHz data and the data of Huguenin, Taylor, and Troland (1970) at 145 MHz that we used in Table 1 are compatible, and so are Page's stated half-power widths and our total widths. In any case, the main point is the obvious decrease of \( G/2 \) and of \( G/W \) with decreasing frequency. This trend was pointed out by Page (1973). Adding our 430 MHz data from Arecibo Observatory\(^2\) to PSR 2016 + 28 data acquired at the Max-Planck-Institut in Bonn at 1720 MHz, we obtain a set of data with a frequency ratio of 4 that is quite well defined and can be analyzed in a uniform way.

In order to conform to the physical model, the previously determined \( \beta \) - and \( \mu \)-values should not be changed; however, we may argue that the "constant" \( C \) of the RS model still in some fashion depends on frequency to accommodate the observations. This procedure leads to the values of \( C, R \), and \( \Gamma/2 \) quoted in Table 3.

The results are in all respects inconsistent. First, we cannot reproduce from equations (1) and (2) the observed values of \( G \) and \( W \). Second, as we have pointed out, small changes in \( R \) at constant \( \mu \) result in the transition from central passage to either above- or below-center passages. This now happens, and for PSR 0809 + 74 the prediction is a disappearance from view which obviously is not observed. PSR 2016 + 28, finally, should not even radiate at 1720 MHz, since we find \( \theta_{\max} < \theta_{\min} \), that is, the cutoff frequency is below 1720 MHz.

At this point we have two alternatives. First, we realize that the main problem is the strong dependence of \( \theta_{\max} \) and \( \theta_{\min} \) on frequency; this was pointed out by Smith (1970), who concluded that all radiation

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\( ^2 \) Arecibo Observatory is operated by Cornell University under contract with the National Science Foundation and with partial support from the Advanced Research Projects Agency.
mechanisms based on curvature (or synchrotron radiation) and plasma effects result in too strong a frequency dependence and thus conflict with fact. Consequently, we can argue that the observed radio emission of pulsars is not described by RS’s model or by similar ones based on curvature radiation, and that we must look for some totally different link between the observed emission and the postulated stream of relativistic electrons and positrons.⁹ The production of these particles in the polar cap, however, may well go on according to RS’s model.

Second, we may insist that, in spite of the failures noted so far, curvature radiation is the link. Then we have to claim that θ_max and θ_min by themselves have no physical meaning; RS argued that the angular radiation pattern might not reach θ_max at all. Then, in a similar ad hoc fashion, we can claim that it may not stretch down to θ_min either, and that it is the angular pattern of the radiation mechanism which is at fault. Conceivably, R, understood as the locus of maximum emission in the cone, may retain an independent reality; we shall investigate this in § IV. In any case, however, even our second alternative requires some major alteration of the basic radiation mechanism.

IV. OBSERVATIONAL TEST OF ONE HOLLOW-CONE MODEL: DOUBLE-HUMPED PROFILES

The second class of pulsars whose characteristics immediately appear to conform to the hollow-cone model is the one with double-humped average profiles. In this case we expect the observer to see a more or less central cut through the radiation pattern, that is, 90° − μ ≈ β.

Since there are no published data to show the subpulse behavior to the degree required by our analysis, we use integrated profiles and assume that some average (Γ) represents θ_max − θ_min, with the separation of peaks ΔS = 2R. Again, at any one given frequency, we can adjust the available parameters (μ, β, C, and ρ₀) to agree with the observations as summarized, for instance, by Sieber, Reinecke, and Wielebinski (1975).

However, as soon as the frequency variation is considered, the procedure leads to vast discrepancies due, as we have seen in the last section, to the strong ω-dependence of the radiation model. Following our previous discussion, we first discount a physical significance of the limits θ_max and θ_min, but retain R = (1/2)(θ_max − θ_min) as the locus of maximum emission without introducing any frequency dependence beyond the one given by RS.

Normalizing at some high frequency (in the several GHz range) R to 1°, we extract from the work of Sieber (1974) and Sieber, Reinecke, and Wielebinski (1975) the data plotted in Figure 6 for the representative PSRs 0525 + 21 [+] and 1133 + 16 [O].

Additional data, some unpublished, were collected by Backer (1975), who specifically compared the behavior of single, double (resolved and unresolved), and multiple pulse shapes. For some reason (his Table 2), he concludes that the width variation with frequency is consistent with the physical hollow-cone models. In reality, almost all his, as well as our, data conform to the scheme below, and the few cases which differ drastically (e.g., PSR 0355 + 54) are obviously influenced by Backer’s definition of width as the total width between the half-power points on leading and trailing edge. For our purposes, this definition is not very useful, especially where the intensity ratio of the two components is comparatively large; see the domains marked I and II in Backer’s Figure 2.

In order to satisfy the observed increase of the separation ΔS in the 100 MHz range, we are forced to choose a cutoff frequency ν ≈ 3.5 GHz where θ_max = θ_min; this leads to the frequency dependence of R depicted in Figure 6 by the solid line. At low frequencies, we have exactly the frequency behavior of the RS model, since here R ∝ ω⁻¹/³; at some intermediate frequency, R turns to the essentially frequency-independent behavior dominant at high frequencies. The cutoff frequency has, of course, in the framework of this ad hoc “model,” no physical meaning, as is obvious from the fact that the two pulsars cited above
and many others are observed above 5 GHz. However, the surprisingly good fit of the $\omega^{-1/3}$ dependence at low frequencies with the frequency-independent behavior at high frequencies is, in our view, a very important indication of the direction in which one may look for refinements of the physical theory.

V. DISCUSSION

In as much detail as is warranted by the available observations, we have compared the geometry of the hollow-cone model and, in particular, the predictions of the physical model by RS, with observational data. We have limited our comparison to two classes of pulsars for which a hollow-cone model would be most obviously suited; whether it is compatible with objects such as PSR 1237 + 25, which show complex structures, is beyond the scope of the present investigation.

In the process, we made the following observations, which in our view clearly indicate the directions in which the physical processes giving rise to hollow-cone emission will have to be worked out in more detail.

1. The observations of organized subpulse drifting in the objects PSRs 0031 - 07, 0809 + 74, and 2016 + 28 are very well represented by a hollow-cone model, provided that the observed variability of the drift rate is somehow accounted for in the physical processes; so far, this has not been possible. We note that a very strong argument here is the predicted shape of the emission contours (see Fig. 5) which, strictly on a geometrical basis, yields an explanation for the otherwise puzzling phase relationships across the window edges reported by Backer (1970). We have not carried this argument further, since Backer's strongest evidence on the basis of a Fourier analysis was obtained for PSR 1919 + 21, which in our 430 MHz data shows typically more than the two submaxima compatible with the simple radiation pattern of the hollow-cone model considered in this paper. Our data on PSR 2016 + 28, on the other hand, do not show the phenomenon. We should recall at this point that the more complex contours investigated in our Paper II can easily mask the flattening of the emission contours at the edge of the window. The simplified description of a singly peaked emission contour clearly does not account for the radiation pattern in the majority of cases, and much work has yet to be done in this area. This is also true for the problem of reconciling observed and theoretical drift rates, and the mutual distance of sparking regions.

2. The observed pulse and window widths in drifting subpulse systems, and the corresponding data in double-humped average profiles, can be easily brought into agreement with a hollow-cone model, resulting in reasonable values for the available parameters. This is also true for RS's model at one given frequency. Here, the parameters in question are the following: (a) the normalizing constant $C$, which we do not think is determined by the various physical processes underlying the model to better than an order of magnitude (this is fairly obvious from RS's paper, where replacing the "approximately equal" sign by the "equal" sign usually involves numerical factors of 2 or 3); (b) the magnetic field strength at the surface of the neutron star, whose derivation from $P_1$ and $\dot{P}_1$ is based on a global energy statement that involves the dipole component dominant at great distance from the star's surface, with the latter conceivably being quite different from the surface field; and (c) the radius of curvature of the field near the pole at the surface, which adds another uncertainty. RS adopted a value ($10^6$ cm) which is, by a factor of 100, smaller than the corresponding value for a dipole field of appropriate strength. The reason is obviously that the larger value would invalidate the physical model. It may or may not be significant that our analysis in § III showed that, for consistency, the radius of curvature has to be pushed below $10^6$ cm, or the strength of the surface field above that of a dipole, or both. RS have argued that there may well be many multipolar components near the surface of the star. However, it is not clear to what extent such components would influence the postulated regularity of the sparking process. For instance, the regularity of subpulse drifting, which, judging from observation, happens in relatively few pulsars, may be the exception, just because in these cases the field structure is less entangled. Nevertheless, one would then want to check for changes of the sparking mechanism as such. It should also be recalled in this respect that the basic model employs an extrapolation of the aligned-rotator case to the oblique-rotator geometry, an extrapolation that may cause a host of as yet unrecognized complications.

3. RS's hollow-cone model, however, fails to represent the frequency dependence of most observed quantities. We have pointed out several times that the reason is the $\omega^{+1/3}$ dependence of $\theta_{\text{max}}$ and $\theta_{\text{min}}$. This in turn is due to the assumption, fundamental to the physical model, that the radio radiation occurs where the local plasma frequency equals the maximum of curvature radiation; the latter now is prescribed by the height variation of the magnetic dipole field. Thus, the discrepancy in frequency variation with observation is fundamental.

If one does not reject the whole idea that the pulsar's radio emission originates near the star, as opposed to its originating near the light cylinder—which would lead to difficulties that in our judgment are even more severe—one is hard pressed to find a plausible substitute for the two major links of the chain, that is, the primary presence of curvature radiation (or synchrotron radiation, for that matter) with its strong frequency dependence, and the importance of the plasma frequency because of the bunching mechanism necessary to explain the observed intensities. Thus, we feel that it would be premature to discard this fundamental portion of the theory on the basis of its failure to lead to a satisfactory representation of the frequency dependence of both cone angle and half-width.

We suspect that the major source of error is in the radiation pattern resulting from the bunching process, possibly because an additional process, such as a maser condition or the like, is in operation and has not yet
been identified. It must be realized that such a process must, in some frequency ranges, be dominant to the extent that it wipes out essentially all the characteristics of the originally proposed curvature-plasma radiation combination. The properties to be postulated which strike us as very important are the frequency behavior of the half-width of the cone and the fact that all observations of double-humped (and equally of more complex) profiles show a sharp outer edge of the angular pattern with a rather filled-in interior, the exact opposite of what is expected from the straight curvature-plasma radiation model. Most significant, however, from the theorist’s point of view, appears to us the result of our Figure 6, that is, the surprisingly uniform data representation by the ad hoc assumption that the half-angle of the cone varies as $\omega^{1/6}$ (as predicted by RS) at low frequencies, turning to an essentially frequency-independent value at high frequencies with no cutoff. One is obviously tempted to speculate at this point about the possibility that two distinct mechanisms operate on radio pulsars, possibly along the lines suggested by Tademaru (1971); however, we feel that our data are not sufficient to justify our making such a rather drastic suggestion.

REFERENCES


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