

Energy spectrum of particles accelerated in the neighborhood of a line of zero magnetic field

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A study is made of the acceleration of charged particles in a hyperbolic magnetic field and uniform electric field, these fields arising from the decay of a neutral current sheet. The acceleration process consists of direct acceleration by the electric field in the nonadiabatic region near the zero line and betatron acceleration in the drift region. The characteristic energy of the accelerated particles is found in the nonrelativistic and ultrarelativistic limits. At high energies, the spectra are exponential.

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1. In the theory of the origin of cosmic rays, an important place is occupied by the question of the acceleration mechanisms: How do the particles acquire the observed very high energies?¹ In refs. 2-4, the acceleration of charged particles is attributed to the collisionless dissipation of magnetic field in neutral current sheets. The formation of current sheets is considered in refs. 2-5. They arise in a strong magnetic field in the neighborhood of points where the magnetic field is zero but the electric field is not. In ref. 5 the plasma flow near a current sheet was calculated numerically; in ref. 4, this problem was solved analytically. A very important consequence flows from these two studies: During the development of the neutral sheet the plasma density near the sheet decreases and may fall to very low values. Under these conditions, the neutral current sheet may be a plasma sheet in vacuum confined by a magnetic field that in its turn is generated by the current flowing in the sheet.

Such a sheet is unstable. Of greatest interest for the problem of acceleration of cosmic rays is the tearing-mode instability.⁶⁻⁹ As a result of its development, the current sheet can decay, the magnetic field is effectively annihilated, and an electric field appears with acceleration of charged particles.

Strong electric fields $E \sim (v/c)H$ (v is the decay velocity of the sheet) can appear when the particles at the "tear" cannot be treated as a plasma. We have in mind the property of a plasma of screening external electric and magnetic fields of frequency lower than the plasma frequency.^{2,10} This case can be realized if the external fields are sufficiently strong. The condition of quasi-stationarity of the magnetic fields is then not satisfied, in contrast to the usual situation in a cosmic plasma, for which it is satisfied with a large margin.

In ref. 11, a study was made of a simple model of decay of a neutral current sheet. It was assumed that the current sheet breaks up into two halves, which move in accordance with a given law. It was assumed that the magnitudes of the currents as well as the velocities of their motion remain constant. The electric and magnetic fields were determined from the solution of the initial-value problem for the Maxwell equations. In a certain sense the opposite case was considered in ref. 12, in which it was also assumed that the sheet decays in accordance with a definite law. The field outside the sheet was

determined from the condition of its being electrostatic. This means that it must satisfy the Laplace equation. In contrast to ref. 11, complete screening of the field on the surface of the neutral sheet was assumed. In both limiting cases, similar configurations of the electric and magnetic fields in the "tear" are obtained. There arises a hyperbolic zero point of the magnetic field of singular type^{11,12}:

$$E = Ee_z; \quad H = h(-ye_z - xe_y). \quad (1)$$

The electric field strength is $E \sim (v/c)H_0$. Here, v is the decay velocity, equal to $\dot{a}(t)$, where $a(t)$ is the half-width of the "tear" (see Fig. 1); H_0 is the magnetic field outside the sheet at the initial time; $h \sim H_0/a(t)$ is the magnetic field gradient. Below, for the sake of simplicity, we assume, as in ref. 11, that $v = \dot{a}(t)$ is constant.

Below, we shall consider the acceleration of charged particles in such fields. The motion of individual particles in the field (1) with constant E and h was considered in ref. 13. In ref. 14, on the basis of a numerical solution of the equations of motion, the energy spectrum of the particles was found, and it was shown that it has the exponential form $n(\mathcal{E}) \sim \exp(-\mathcal{E}/\mathcal{E}_0)$, where $\mathcal{E}$ is the particle energy. For the characteristic energy $\mathcal{E}_0$, a numerical value was given. The question of its dependence on the parameters characterizing the acceleration region — the electric and magnetic fields, their gradients, etc. — was not solved.

2. Below, we attempt to obtain analytic expressions for the energy spectrum of cosmic rays accelerated in fields described by the expressions (1). We ignore the interaction of the particles with one another and the surrounding plasma. In other words, the spectrum is determined in the approximation of individual particles. We are interested in energies appreciably higher than the thermal energy of the accelerated plasma. We shall therefore assume that the initial velocities are low (the particles are at rest).

We shall describe the plasma by means of the distribution function $f(\mathbf{r}, \mathbf{p}, t)$. Suppose that at $t = 0$ it is $f_0(\mathbf{r}_0, \mathbf{p}_0, 0)$. The distribution function for all times can be found if we know the solution of its equation of motion. Suppose we know the coordinates and momenta as functions of the time:

$$\mathbf{r} = \mathbf{r}(t, \mathbf{r}_0, \mathbf{p}_0), \quad (2)$$

$$\mathbf{p} = \mathbf{p}(t, \mathbf{r}_0, \mathbf{p}_0). \quad (3)$$

We can then find the dependences $\mathbf{r}_0(t, \mathbf{r}, \mathbf{p})$ and $\mathbf{p}_0(t, \mathbf{r}, \mathbf{p})$. In this case, the distribution function is

$$f(\mathbf{r}, \mathbf{p}, t) = f_0(0, \mathbf{r}_0(t, \mathbf{r}, \mathbf{p}), \mathbf{p}_0(t, \mathbf{r}, \mathbf{p})) \left| \frac{D(\mathbf{r}_0, \mathbf{p}_0)}{D(\mathbf{r}, \mathbf{p})} \right|. \quad (4)$$

On the right-hand side of (4), the initial function f_0 , which depends on t , $\mathbf{r}$, and $\mathbf{p}$ through $\mathbf{p}_0$ and $\mathbf{r}_0$, is multiplied by the Jacobian of the transformation from the initial coordinates and momenta to the final ones, and this is equal to unity if the particles do not lose energy.

3. Let us consider the electric and magnetic fields described by the expressions (1). In the general case, the gradients of the magnetic field ($\mathbf{h} \sim H_0/v\tau$) depend on the time. In these fields, there are two distinguished regions. Immediately next to the zero line ($(x^2 + y^2)^{1/2} = \rho < R$) on Fig. 1 [see Eq. (10)] the electric field is strong, and the magnetic field can be regarded as a perturbation. Here, the charged particles are directly accelerated by the electric field. If the acceleration time of one particle in this region is less than the characteristic time of variation of the field, we can assume that the electric field strengths and the gradient of the magnetic field in Eqs. (11) depend on the time as on a parameter. We shall denote the time in this case by τ . One can show the condition of slow variation of the field is equivalent to the inequality

$$\tau > \omega_H^{-1}. \quad (5)$$

Here, $\omega_H = eH_0/me$ is the Larmor frequency in the field outside the sheet at the initial time H_0 . One can also show that during the initial period of the decay of the sheet $\tau < \omega_H^{-1}$ the acceleration is ineffective.

The other characteristic region in the field (1) is the region of drift motion. Here the magnetic moment $\mu = 3ep^2/2mH$ is conserved. In the quadrants II and IV (Fig. 1) the particles move in the direction of stronger magnetic fields, so that, the adiabatic invariant being conserved, the transverse momentum increases as

$$p_{\perp} = p_{\perp 0} \left(\frac{H(\tau)}{H(\tau=0)} \right)^{1/2} = p_{\perp 0} \left(\frac{\rho(\tau)}{\rho(\tau=0)} \right)^{1/2}. \quad (6)$$

Here, $H(\tau)$ is the magnetic field as a function of the time in the region in which the given particle is found; $\rho(\tau)$ is the distance traversed by the particle from the zero line in

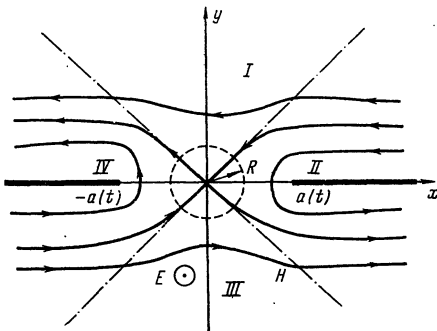


Fig. 1. Form of the electric and magnetic fields in the "tear" of the current sheet.

the time τ . The acceleration in the drift region depends on the initial energy.

Thus, this process of acceleration can be represented as follows. The plasma flows from quadrants I and III of Fig. 1 into quadrants II and IV. Only the particles that pass through the region $\rho < R$ are strongly excited. Here, they are accelerated to certain initial energies and the spectrum in the high-energy region is formed. Further acceleration takes place in the drift region. For particles whose paths do not intersect the region $\rho < R$, the acceleration in the drift region is ineffective since it depends on the initial energy (6).

4. As we have already said above, we consider energies appreciably exceeding the thermal. We shall therefore assume that at the initial time $\mathbf{p}(t=0) = 0$ (the plasma is at rest). In addition, we consider the approximation of independent particles. The equations of motion are

$$\dot{p}_x = \frac{eh}{c} \dot{x}, \quad (7)$$

$$\dot{p}_y = -\frac{eh}{c} \dot{y}, \quad (8)$$

$$p_z = eEt - \frac{eh}{2c} [(x^2 - x_0^2) - (y^2 - y_0^2)]. \quad (9)$$

The dot denotes differentiation with respect to the time. The subscript 0 is appended to the initial values. Equation (9) follows from conservation of the generalized momentum, the problem being homogeneous with respect to $\mathbf{e}_z$. The velocities and the momenta are related in the usual manner: $\dot{\mathbf{v}} = c\mathbf{p}(m^2c^2 + p^2)^{-1/2}$.

We consider separately the nonrelativistic and the relativistic approximations.

5. In the nonrelativistic limit, the characteristic parameters of the problem are¹³

$$R = \left(\frac{mc^2 E}{eh^2} \right)^{1/2}, \quad T = \left(\frac{m^2 c}{Ee^2 h} \right)^{1/2}. \quad (10)$$

In our case, R is the radius of the region of initial acceleration and T is the characteristic time during which a particle remains in this region. Equations (8)-(10) in the nonrelativistic limit $p \ll mc$ have the form¹³

$$\ddot{x} = ix \frac{eh}{mc}, \quad (11)$$

$$\ddot{y} = -y \frac{eh}{mc}, \quad (12)$$

$$\dot{z} = \frac{eEt}{m} - \frac{eh}{2mc} [(x^2 - x_0^2) - (y^2 - y_0^2)]. \quad (13)$$

The region $\rho < R$ is the region in which the particles are not magnetized. In it, the solution of the system (11)-(13) for zero initial velocities [$\mathbf{p}(t=0) = 0$] can be written as^{13,15}

$$x(t) = x_0 \frac{\Gamma(2/3)}{3^{1/2}} \left(\frac{t}{T} \right)^{1/2} I_{-1/2} \left(\frac{2}{3} \left(\frac{t}{T} \right)^{3/2} \right), \quad (14)$$

$$y(t) = y_0 \frac{\Gamma(2/3)}{3^{1/2}} \left(\frac{t}{T} \right)^{1/2} J_{-1/2} \left(\frac{2}{3} \left(\frac{t}{T} \right)^{3/2} \right), \quad (15)$$

$$z(t) = \frac{R}{2} \left(\frac{t}{T} \right)^2. \quad (16)$$

Here, $\Gamma(x)$ is the gamma function and $I_\nu^{(x)}$ and $J_\nu^{(x)}$ are the modified and the ordinary gamma function of order¹⁶ ν . These solutions hold as long as the second term on the right-hand side of Eq. (13) does not exceed the first.

Only particles from quadrants I and III in Fig. 1 can reach the region $\rho < R$. In it, they are accelerated along the axis e_z . After a time t' they leave this region. Since the acceleration is basically along e_z , the energy of the particle when it leaves the region $\rho < R$ is approximately

$$\mathcal{E} \simeq \frac{e^2 E^2}{2m} t'^2. \quad (17)$$

The time t' can be determined from the condition that at this instant the particle is at the boundary of the region

$$x^2(t') + y^2(t') = R^2 \quad (18)$$

and from the expressions (14) and (15). The high-energy part of the spectrum, which is what we want, corresponds to the limit of large t' . As $t \rightarrow \infty$, $x(t)$ and $y(t)$ behave as follows¹⁶:

$$x(t) \approx x_0 \frac{3^{1/2} e^{-\pi/12} \Gamma\left(\frac{2}{3}\right)}{\pi^{1/2}} \left(\frac{t}{T}\right)^{-1/2} \exp\left\{\frac{2}{3} \left(\frac{t}{T}\right)^{3/2}\right\}, \quad (19)$$

$$y(t) \approx y_0 \frac{3^{1/2} \Gamma\left(\frac{2}{3}\right)}{\pi^{1/2}} \left(\frac{t}{T}\right)^{-1/2} \cos\left(\frac{2}{3} \left(\frac{t}{T}\right)^{3/2} - \frac{\pi}{12}\right). \quad (20)$$

The time t' is

$$t' \approx T \left(1.5 \ln 1.36 \frac{R}{x_0} \right)^{2/3}. \quad (21)$$

The particle energy is

$$\mathcal{E} \approx \left(\frac{3}{2^{1/2}} \ln 1.36 \frac{R}{x_0} \right)^{4/3} \frac{e^2 E^2 T^2}{m}. \quad (22)$$

It can be seen that the particles whose coordinates on the boundary of the region $\rho < R$ are near the plane $x_0 = 0$ acquire the greatest energy.

We introduce the function $n(\mathcal{E})$, which is equal to the total number of particles accelerated in the region $\rho < R$ to the energy $\mathcal{E}$ in unit time divided by unit energy interval and unit length of the current sheet.

It can be found from the expressions (4), (14), (15), and (17) for the distribution function. However, in the approximation under consideration, this problem can be solved more easily. Consider particles that enter the region $\rho < R$. Small initial values x_0 ($x_0 \ll R$) correspond to the maximal energies. For them one can assume that the particles are distributed uniformly with respect to x_0 for $y_0 \approx R$, i.e., the particle flux at the boundary, $\mathcal{J}_0$, depends solely on the time and not on the coordinates as well. Then $n(\mathcal{E})$ can be written as

$$n(\mathcal{E}) d\mathcal{E} dz = \mathcal{J}_0 \left| \frac{dx_0}{d\mathcal{E}} \right| d\mathcal{E} dz = \frac{\pi^{1/2} e^{\pi/12}}{2 \cdot 3^{1/2} \Gamma(2/3)} \frac{\mathcal{J}_0 R}{\mathcal{E}_0} \left(\frac{\mathcal{E}_0}{\mathcal{E}} \right)^{1/4} \exp\left\{-\frac{2^{1/4}}{3} \left(\frac{\mathcal{E}}{\mathcal{E}_0} \right)^{3/4}\right\} d\mathcal{E} dz. \quad (23)$$

The values of $\mathcal{J}_0$ and $\mathcal{E}_0$ depend on the time as a parameter. The characteristic energy in the spectrum $\mathcal{E}_0$ is determined by the variables which characterize the "tear" of the sheet:

$$\mathcal{E}_0 = k \left(\frac{e^2 E^4 m c^2}{2 \hbar^2} \right)^{1/4} = k \left(\frac{e^2 H_0^2 m v^6 \tau^2}{2 c^2} \right)^{1/4}. \quad (24)$$

With the course of time, $\mathcal{E}_0$ increases as $\tau^{2/3}$. Therefore, with the course of time the acceleration proceeds to ever higher energies. In this model, the particles with highest energy are formed in the final stage of decay of the neutral current sheet. The dimensionless numerical factor k in (24) arises because the relation (17) between the energy $\mathcal{E}$ and the time t' is approximate. In addition, the condition (18) that $\rho(t') = R$ has a formal meaning. These equations, which determine the energy $\mathcal{E}$ and the time t' , can pretend only to a qualitative explanation of the acceleration process. The exact numerical value of k in (24) and also the pre-exponential factor in the particle spectrum (23) can be found by a more rigorous analytic solution of the problem or numerical calculations. Bearing in mind these comments, we can rewrite the expression (23) as

$$n(\mathcal{E}) d\mathcal{E} dz = A \frac{\mathcal{J}_0 c}{e H_0 v} \exp\left\{-\frac{2^{1/4}}{3} \left(\frac{\mathcal{E}}{\mathcal{E}_0} \right)^{3/4}\right\} d\mathcal{E} dz, \quad (25)$$

where A is a numerical factor of order unity.

6. Let us now consider the ultrarelativistic case.

The characteristic temporal and spatial scales in this problem are

$$T = \frac{E}{ch}, \quad R = \frac{E}{h}. \quad (26)$$

The equations of motion in this limit can be conveniently represented in the form

$$\ddot{x} + \frac{\dot{\mathcal{E}}}{\mathcal{E}} \dot{x} = -\frac{ch\dot{\mathcal{E}}}{E\mathcal{E}} x, \quad (27)$$

$$\ddot{y} + \frac{\dot{\mathcal{E}}}{\mathcal{E}} \dot{y} = -\frac{ch\dot{\mathcal{E}}}{E\mathcal{E}} y. \quad (28)$$

The equation for the z component of the momentum can be conveniently rewritten in the form

$$\frac{\mathcal{E}\dot{z}}{ec^2 E} = eEt - \frac{eh}{2c} [(x^2 - x_0^2) - (y^2 - y_0^2)]. \quad (29)$$

We now consider the motion of particles within the region $\rho < R$. Here, the second term on the right-hand side of Eq. (29) is smaller than the first. The motion along the axis e_z occurs with constant velocity of the order of that of the velocity of light. The particle energy increases with the time in accordance with the law

$$\mathcal{E} = (\mathcal{E}^2(t=0) + e^2 E^2 c^2 t^2)^{1/2}. \quad (30)$$

In the asymptotic region $t \rightarrow \infty$, $\mathcal{E} = eEt$. A solution of Eqs. (27) and (28) in this limit is given by

$$x(t) = x_0 I_0 \left(2 \left(\frac{\hbar c t}{E} \right)^{1/2} \right), \quad (31)$$

$$y(t) = y_0 J_0 \left(2 \left(\frac{\hbar c t}{E} \right)^{1/2} \right). \quad (32)$$

Here, $J_0(x)$ and $I_0(x)$ are the ordinary and the modified Bessel function of zeroth order.¹⁶ As $t \rightarrow \infty$, $x(t)$ increases exponentially and $y(t)$ oscillates with the time.

We now find the energy spectrum of ultrarelativistic cosmic rays at the boundary of the region $\rho \approx R$. The particle coordinates depend on the time in accordance with (31) and (32). In the same way as we obtained Eqs. (23)–(25), we find

$$n(\mathcal{E}) d\mathcal{E} dz = \frac{D \mathcal{F}_0 R}{\mathcal{E}_0} \exp \left\{ - \left(\frac{\mathcal{E}}{\mathcal{E}_0} \right)^{1/2} \right\} d\mathcal{E} dz. \quad (33)$$

Here, D is a pre-exponential factor, whose value we do not particularize. The characteristic energy in the particle spectrum $\mathcal{E}_0$ in the ultrarelativistic limit is

$$\mathcal{E}_0 = q \frac{2eE^2}{\hbar} = q \frac{2eH_0 v^2 \tau}{c^2}, \quad (34)$$

where q , a numerical factor, is analogous to k in (24). As in the nonrelativistic limit, the characteristic energy in the cosmic-ray spectrum increases with the time. The maximal energies are reached during the last stage of decay of the sheet.

7. We now calculate the particle flux on the boundary of the region $\rho = R$ as a function of the time. We assume that far from the "tear" in the current sheet the magnetic field and the plasma density n_0 are constant. If $\rho \ll v\tau$, the influence of the nonstationarity of the magnetic field on the motion of the particles can be ignored. For this case, the plasma density was found as a function of the coordinates and the time in the field (1) in refs. 4 and 8. Using these results, we find that

$$\mathcal{F}_0(\tau) = n_0 v \left(\frac{v\tau}{R(\tau)} \right)^{1-\sqrt{2}}, \quad (35)$$

where R is determined by the expression (10) if $\mathcal{E}, \mathcal{E}_0 \ll mc^2$, and by (26) if $\mathcal{E}, \mathcal{E}_0 \gg mc^2$.

Substituting (35) into (25) and (33), we can obtain the cosmic-ray spectrum as a function of the time on the boundary of the region $\rho \approx R$.

In addition, the energy spectrum is further changed by the motion of the particles in the drift region.

8. We now consider the acceleration of the particles in the drift region. This corresponds to values $\rho > R$. The general nature of the motion of the particles in this region was discussed in ref. 13. If the particles move in quadrants II and IV in Fig. 1, there is an additional acceleration in accordance with Eq. (6). The initial energy

is equal to the energy of the particle at the boundary $\rho \approx R$ of the region. The initial value $\rho(\tau=0) = R$ is the condition that for $\tau=0$ the particle is on the boundary of the drift region. At long times, the particle is at the point $\rho \approx v\tau$, where v is the decay velocity of the current sheet. The motion in the drift region in the field (1) occurs in such a way that in the limit $\tau \rightarrow \infty$ the charge particle "catches up" with the current sheet. As follows from Eqs. (6), the energy of the particle increases. This means that the spectrum is "stretched" along the energy axis by the factor $(\omega_H \tau)^{1/3}$ in the nonrelativistic limit and by $(c/v)^{1/2}$ in the ultrarelativistic case.

9. We now write down the final expressions for the time-integrated energy spectrum of the cosmic rays. If $\mathcal{E}, \mathcal{E}_0 \ll mc^2$ then

$$N(\mathcal{E}) = \frac{B c \tau n_0}{e H_0 (\omega_H \tau)^{1/3}} \exp \left\{ -1.1 \left(\frac{\mathcal{E}}{\mathcal{E}_0} \right)^{1/4} \frac{1}{(\omega_H \tau)^{1/4}} \right\}. \quad (36)$$

If $\mathcal{E}, \mathcal{E}_0 \gg mc^2$ then

$$N(\mathcal{E}) = \frac{F n_0 c \tau}{e H_0} \left(\frac{v}{c} \right)^{\frac{2\sqrt{2}-3}{2}} \exp \left\{ - \left(\frac{v}{c} \right)^{1/4} \left(\frac{\mathcal{E}}{\mathcal{E}_0} \right)^{1/4} \right\}. \quad (37)$$

Here, $N(\mathcal{E}) = \int n(\mathcal{E}, \tau) d\tau$ is the time-integrated cosmic-

ray spectrum per unit length of the current sheet; B and F are numerical factors of order unity. For the expressions for $\mathcal{E}_0$ in Eqs. (36) and (37) see (24) and (34), respectively. The characteristic energies in the spectra (36)–(37) of the accelerated particles are as follows: for $\mathcal{E}, \mathcal{E}_0$

$$\tilde{\mathcal{E}}_0 = k \left(\frac{m^2 v^2 e H_0 \tau}{2c} \right)^{1/4} \approx k \frac{e H_0 v^2 \tau}{c}, \quad (38)$$

and for $\mathcal{E}, \tilde{\mathcal{E}}_0$

$$\tilde{\mathcal{E}}_0 = 2q \left(\frac{v}{c} \right)^{1/4} e H_0 v \tau. \quad (39)$$

10. Let us compare our results with numerical solutions of the equations of motion of the charged particles. In ref. 13, a computer was used to solve Eqs. (11)–(13). Figure 2 shows the logarithm of the particle density as a function of the energy plotted on the basis of a numerical calculation. It confirms the exponential spectrum of the accelerated particles (see also ref. 14). From the slope of $\ln N(\mathcal{E})$ as a function of the energy $\mathcal{E}$ the value of k in (24) and (38) can be determined. The numerical value, to within the accuracy of the numerical calculation, is $k = 1$.

Let us make some estimates. Let us consider the parameters typical of polar flares. We take magnetic field strength $H_0 = 300$ G, "tear" region 10^8 cm across, and decay velocity of the order of the Alfvén velocity in the corona: $v_A \approx 3 \cdot 10^8$ cm/sec. The relativistic approximation holds for both the protons and the electrons. Their maximal energies are approximately 9 BeV. The nonrelativistic approximation holds for protons for weaker flares. Energy is accumulated over a distance of about

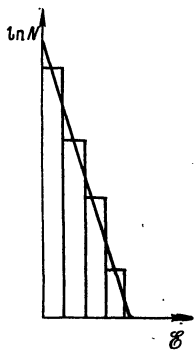


Fig. 2. Logarithm of the particle density as a function of the particle energy on the basis of the numerical calculations of ref. 13.

10^7 cm. The particles traverse about a tenth of this distance in the region $\rho < R$.

This maximal cosmic-ray energy, 9 BeV, is characteristic for intermediate flare parameters. If the maximal allowed magnetic field strengths, size of the acceleration region, etc., are taken, one can obtain energies up to a few hundred BeV. This is in qualitative agreement with the conclusions of ref. 17.

We now estimate the number of accelerated particles and their total energy in this range of energies. We assume that in the neighborhood of the current sheet the plasma density at the initial time is $n_0 = 3 \cdot 10^5$. From the condition of validity of the vacuum approximation in the acceleration region and the expression (35) we can obtain the bound $n < H_0(c/v)^{1/2}/4\pi ea$. The total number of particles accelerated to energies $\varepsilon > \varepsilon_0$, is $N_{\text{tot}} = H_0 a L / 4\pi e$. Here, L is the length of the current sheet ($\sim 2 \cdot 10^8$ cm). In our case, $N_{\text{tot}} = 10^{27}$. The ratio of the energy of the cosmic rays, $W_{\text{cr}} (\varepsilon > \varepsilon_0) \approx N_{\text{tot}} \cdot \varepsilon_0$, to the energy of the flare, $W_{\text{fl}} = H_0^2 a^2 L^2 / 8\pi$, is of order $W_{\text{cr}}/W_{\text{fl}} \approx (v/c)^{3/2} \sim 10^{-3}$.

In the general case, additional restrictions on the acceleration mechanism (energies of the particles, their number, spectrum, etc.) can appear when one takes into account the possibility of plasma instabilities developing in the acceleration region. As this is outside our present scope, we restrict ourselves to some comments. In the region $\rho < R$, in which the particles are not magnetized, the electrons and ions are accelerated by the electric field, and there is a directed motion of the electrons relative to the ions. Here, there may be a Buneman instability. However, in the situation under consideration in the relativistic approximation and under conditions of applicability of the vacuum approximation it is hard for it (and the other two-stream instabilities) to develop. This is because the characteristic length R_{rel} in our case is shorter than the Debye radius, which is the characteristic length over which the instability develops.

Let us now consider the energies for pulsars. The magnetic field is $H_0 = 10^{12}$ G, the velocity v is of the order of the rotation velocity at the pulsar radius $r \approx 10^8$ cm, i.e., $v \approx 3 \cdot 10^8$ cm. Then for the maximal energies we obtain the estimate $\varepsilon_0 \approx 2 \cdot 10^{17}$ eV and if $v \approx c$ then $\varepsilon_0 = 3 \cdot 10^{20}$ eV (ref. 18).

11. In conclusion, let us summarize the main features of the acceleration processes. Cosmic rays are accele-

rated in nonstationary and inhomogeneous electromagnetic fields. These fields can arise when a neutral current sheet in a plasma decays. A line of zero magnetic field is then formed with electric field along this line. The plasma surrounding the current sheet flows into the neighborhood of the zero line. The acceleration process can be split into two stages. Initially, the cosmic rays are accelerated to certain initial energies near the zero line, where the particles are not magnetized. Further acceleration takes place in the drift region. Here, the energy of the cosmic rays is increased in proportion to their initial energy. The resulting spectra are exponential.

These dependences refer to the most energetic section of the spectrum. It is obvious that the exponential dependence cannot be continued into the region of lower energies. It is known, for example, that in this region solar cosmic rays have a power spectrum with exponent $\gamma \approx 3-5$.

This dependence can be obtained most probably if allowance is made for diffusion in the momentum space resulting from scattering by turbulent pulsations.¹⁹ The power spectrum in the less energetic, $\varepsilon < \varepsilon_0$, region is formed during the initial stages of decay of the neutral sheet, when the vacuum approximation with the fields (1) is not yet satisfied. During this stage, various instabilities leading to turbulence of the plasma of the current sheet can develop. For this case, the value $\gamma \approx 3-5$ was obtained for the power spectrum in ref. 20.

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¹V. L. Ginzburg and S. I. Syrovatskii, *Origin of Cosmic Rays*, Pergamon, Oxford (1965).

²S. I. Syrovatskii, *Astron. Zh.*, **43**, 340 (1966) [*Sov. Astron.*, **10**, 270 (1966)].

³S. I. Syrovatskii, In: *Solar Flares and Space Research*, Svestka (editor), North-Holland, Amsterdam (1969), p. 346.

⁴S. I. Syrovatskii, *Tr. Fiz. Inst. Akad. Nauk SSSR*, **74**, 3 (1974).

⁵B. V. Somov and S. I. Syrovatskii, *Tr. Fiz. Inst. Akad. Nauk SSSR*, **74**, 14 (1974).

⁶H. P. Furth, I. Killen, and M. N. Rosenbluth, *Phys. Fluids*, **6**, 459 (1963).

⁷G. Laval, R. Pellat, and M. Vuillemin, *Proc. of Conf. on Plasma Phys. and Contr. Nucl. Fus.*, **II**, Vienna (1965), p. 259.

⁸S. I. Syrovatskii, *Izv. Akad. Nauk SSSR, Ser. Fiz.*, **39**, 359 (1975).

⁹V. K. Neil, *Phys. Fluids*, **5**, 14 (1962).

¹⁰S. I. Syrovatskii, *Izv. Akad. Nauk SSSR, Ser. Fiz.*, **31**, 1303 (1967).

¹¹S. V. Bulanov and S. I. Syrovatskii, *Tr. Fiz. Inst. Akad. Nauk SSSR*, **74**, 88 (1971).

¹²B. V. Somov and S. I. Syrovatskii, *Izv. Akad. Nauk SSSR, Ser. Fiz.*, **39**, 375 (1975).

¹³S. V. Bulanov and S. I. Syrovatskii, *Tr. Fiz. Inst. Akad. Nauk SSSR*, **90**, (1975).

¹⁴M. Friedman, *Phys. Rev.*, **182**, 1408 (1969).

¹⁵T. W. Speiser, *J. Geophys. Res.*, **70**, 4219-4226 (1965).

¹⁶E. Jahnke, F. Ende, and F. Lötch, *Tables of Higher Functions*, McGraw-Hill, New York (1960).

¹⁷E. V. Kolomeets, *Proc. Intern. Seminar on Solar Cosmic Rays and Their Penetration into the Earth's Magnetosphere* [in Russian], Leningrad (1973), p. 205.

¹⁸S. I. Syrovatskii, *Proc. 11th Intern. Conf. on Cosmic Rays*, Budapest (1969), 06-2; *Acta Physica*, **29**, Suppl. 1, 17-21 (1970).

¹⁹D. Smith, *Inv. Rev. to appl. in Proceed. of IAU Symposium*, No. 57 (1973).

²⁰S. I. Syrovatskii, *Zh. Eksp. Teor. Fiz.*, **40**, 1788 (1961) [*Sov. Phys.-JETP*, **13**, 1257 (1961)].

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