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SURFACE BRIGHTNESS AND EVOLUTION OF GALAXIES

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ABSTRACT

It is well known that before the redshift-magnitude diagram of galaxies could be used for determination of the cosmological parameters one must know the evolution of the galaxies. We propose use of apparent surface brightness—which depends only on the redshift and is independent of the cosmological model and the inhomogeneities in the universe—for observational determination of the evolution of galaxies. The needed observations are isophotal angular diameters and apparent magnitudes within this or any other reasonable angular diameter. The application of the results for determination of q_0 is discussed briefly.

Subject headings: galaxies: redshifts — galaxies: evolution

I. INTRODUCTION

A good fraction of the efforts of observational cosmologists during the past three decades has been devoted to the determination of the value of the deceleration parameter q_0. The primary method for this has been the redshift-magnitude relation of the first-ranked cluster galaxies (Humason, Mayall, and Sandage 1956; Sandage 1972*a* and references therein; Gunn and Oke 1975).

However, in an evolving (non–steady state) universe the observations of discrete sources contain information on both the evolution of the universe (i.e., cosmological parameters such as q_0) and the evolution of sources (such as luminosity evolution of first-rank cluster galaxies or evolution of luminosity function of galaxies in clusters). Consequently, without knowledge of the evolution of sources, determination of the cosmological parameters through the redshift-magnitude test is impossible. Assuming no evolution, the redshift-magnitude data of the brightest galaxies in clusters give $q_0 \sim 1$ (Sandage 1972*a*) or $q_0 \sim 0$ (Gunn and Oke 1975); but as stressed by Gunn and Oke, these results are "meaningless without good evolutionary corrections." Theoretical studies by Tinsley (1972) indicate that evolutionary corrections are not negligible and when included can reduce the value of q_0 by 1 (cf. however, Ostriker and Tremaine 1975). There is, however, no direct observational evidence on the luminosity evolution of these galaxies. In this *Letter* we propose use of variation of the apparent surface brightness of galaxies (which is independent of cosmological parameters) with redshift as a test for evolution of actual surface brightness (and hence luminosity) of galaxies. In § II we review the aperture correction which is closely related to the surface brightness. The surface brightness test is discussed in § III, and its application for determination of q_0 in § IV.

II. THE APERTURE CORRECTION

Galaxies are not point sources. For a fixed aperture, different portions of galaxies are observed at different redshifts. Thus, the observed magnitudes must be corrected for this effect before they can be used in the redshift-magnitude test. The procedure for this correction has been to correct the magnitudes so that they refer to a standard physical diameter. However, to apply this correction, one needs to know the cosmological model—the very thing the test is supposed to determine.

Two different methods have been suggested to overcome this difficulty. Sandage (1972*b*) used an iterative procedure whereby aperture correction is applied assuming a cosmological model (or a value of q_0). Then a new value of q_0 is determined using the standard redshift-magnitude relation. The procedure is repeated until it converges. Gunn and Oke (1975) apply the aperture correction assuming a fiducial cosmological model and use a modified redshift-magnitude relation for determination of q_0. In both cases, in addition to the assumption about the cosmological model, one requires a knowledge of the variation of surface brightness $B(r)$ with r, the projected radial distance from the center of the galaxy. We show here that it is not necessary to make any *a priori* assumption about the cosmological model, an assumption which could cause some confusion.

The observed flux density $f_\nu(\theta, z)$ at frequency ν and within a circular aperture of radius θ is given by[1]

$$f_\nu(\theta, z) = \frac{L_\nu(r_\theta, z)k(z)g(\Omega, z)}{4\pi(1+z)^2\mathfrak{L}^2(q_0, \Lambda, z)}, \quad (1)$$

where $g(\Omega, z)$ takes account of galactic and intergalactic (if any) absorption in the direction Ω. For spherically symmetric galaxies,

$$L_\nu(r, z) = \int_0^r 2\pi r' B_\nu(r', z)dr' \quad (2)$$

is the luminosity at ν and redshift z within the projected radius r, where $B_\nu(r, z)$ describes the variation of

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[1] The function $\mathfrak{L}$, called the "proper motion distance" by Weinberg (1972), depends on the redshift, on the Hubble constant, and on the cosmological parameters such as q_0 and the cosmological constant Λ. Since the form of $\mathfrak{L}$ does not enter our discussion, we refer the reader to Weinberg (1972, p. 485) for $\Lambda = 0$ models and to Petrosian and Salpeter (1968) for $\Lambda \neq 0$ models.

surface brightness with ν, r, and z. The projected radial distance corresponding to angular distance θ is given by

$$r_\theta = \theta \mathfrak{R}(q_0, \Lambda, z)/(1 + z) \,. \tag{3}$$

The k-correction term $k(z)$ is defined such that $K(z) = -2.5 \log k(z)$ is the usual K-term applied to magnitudes: $k(z)/(1 + z) = B_{\nu(1+z)}/B_\nu$.

The above equations make a complete set which in principle could be fitted to observations of $f_\nu(\theta)$, θ, and z of galaxies to determine the value of q_0 if $B_\nu(r, z)$ is known. It is not necessary to assume any cosmological model for aperture correction. For example, for a power-law surface brightness, $L_\nu(r) \propto r^\alpha$, the luminosity $L_\nu(r_\theta, z)$ in equation (1) is replaced by $L_\nu(r_0, z)[\theta\mathfrak{R}/r_0(1 + z)]^\alpha$ (cf. eq. [3]) so that the flux density is expressed in terms of the luminosity within a standard radius r_0 and in terms of observables θ and z. Similar relations are obtained for different brightness distributions.

III. SURFACE BRIGHTNESS AND EVOLUTION

In this section we derive relation between the surface brightness and cosmological parameters of galaxies. It is well known (e.g., Hubble and Tolman 1935) that the apparent surface brightness b_ν of a source depends only on the absolute surface brightness B_ν and redshift of the source independent of the cause of this redshift (cosmological, Doppler, or gravitational) and independent of curvature of spacetime (i.e., cosmological parameters) or degree of inhomogeneity of matter,

$$b_\nu(r) = B_{\nu(1+z)}(r)/(1 + z)^3 \,. \tag{4}$$

Thus from measurement of the apparent surface brightness at different redshifts one can obtain the evolution of the surface brightness and, with some assumptions, the evolution of the total luminosity.

Hubble and Tolman (1935) proposed measurement of the central surface brightness $b(0)$ and its variation with redshift as a test of expansion of the universe, since in the nonexpanding, tired-light cosmological models the factor $(1 + z)^3$ is absent from the above equation (cf. Sandage 1974 for a more recent discussion). Recently Gudehus (1975) has discussed the possibility of measurement of $b(0)$ for determination of evolution. There are, however, some difficulties with measurement of $b(0)$. The essential difficulty is that one must infer the value of $b(0)$ from actual measurements which always refer to average surface brightness. Such extrapolations give rise to uncertainties. For example, for the Hubble (1930) surface brightness distribution, $B(r) = B(0)\ (1 + r/r_H)^{-2}$, the difference between the average central 1″ surface brightness at $z = 0$ and $z = 0.3$ would be 2–3 magnitudes depending on the value of r_H. Furthermore, since r_H is much less than the "size" of the galaxy, the evaluation of the angular radius a (cf. Hubble and Tolman 1935; Gudehus 1975) corresponding to r_H, which is needed for inference of the value of $b(0)$, would be difficult at high redshifts.

Because of these difficulties, we propose a measurement of average surface brightness for test of evolution. The simplest measurement would be measurement of average isophotal surface brightness.

Let θ_s be the isophotal angular radius, defined as the radius where the apparent surface brightness drops to a prescribed limiting value, $b_{\lim,\nu}$. It then follows from equations (1)–(3) that the isophotal metric radial distance $r_s = \theta_s\mathfrak{R}/(1 + z)$ from the center of a galaxy at redshift z is obtained from (cf. also Weinberg 1972, p. 424)

$$B_\nu(r_s, z) = \frac{4\pi b_{\lim,\nu}(1 + z)^4}{k(z)g(\Omega, z)} \,, \tag{5}$$

and that the average apparent isophotal surface brightness is given by

$$b_\nu(\theta_s, z) \equiv \frac{f_\nu(\theta_s, z)}{\pi\theta_s^2} = \eta_\nu(r_s, z) b_{\lim,\nu} \,, \tag{6}$$

where $\eta_\nu(r, z)$ is the ratio of the average surface brightness up to a radius r to the surface brightness at r:

$$\eta_\nu(r, z) = 2\int_0^r B_\nu(r', z) r' dr'/r^2 B(r, z) = \frac{2d \log r}{d \log L_\nu(r, z)} \,. \tag{7}$$

As is evident, the average apparent surface brightness is independent of cosmological parameters; only the structure and redshift of the galaxy enter these equations. Consequently, comparison of this equation with observations could in principle reveal variation with redshift of the intrinsic surface brightness distribution.

We outline the procedure here. On Figure 1 we plot

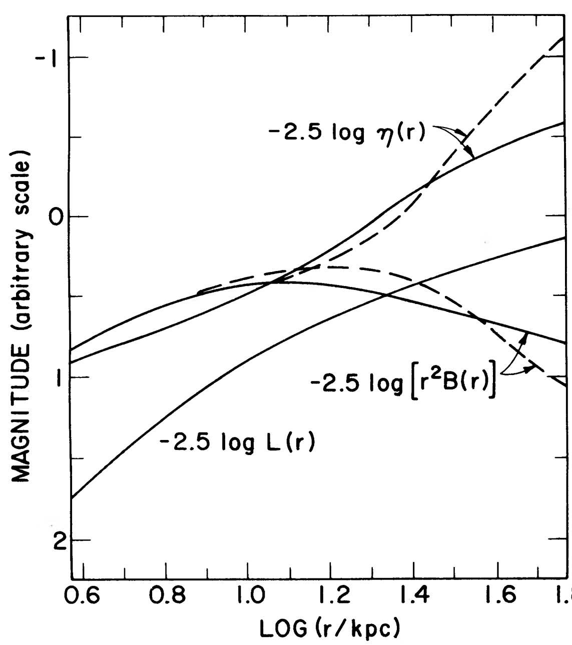

FIG. 1.—Variation with projected radius of surface brightness $B(r)$, the luminosity $L(r)$ inside radius r, and $\eta(r)$. *Solid lines*, from Sandage's 1972*b* composite curve of growth. *Dashed lines*, from Gudehus's analytic approximation. Note the deviations from Hubble's law $B(r) \propto r^{-2}$. The variation of $\eta(r)$ indicates deviations from a power-law surface brightness. $H_0 = 50$ km s^{-1} Mpc^{-1}.

variation of $L(r)$, $B(r)$, and $\eta(r)$ at the visual band, where for illustration $L(r)$ is taken from Sandage's (1972*b*) composite diagram[2] (Fig. 6 and Table 3) and $B(r)$ and $\eta(r)$ are calculated according to equations (2) and (7). For comparison we also show $B(r)$ and $\eta(r)$ for the analytic surface brightness distribution given by Gudehus (1975), which is a modified version of the Hubble (1930) law. Similar curves are obtained for the empirical $B(r)$ curve given by Oemler (1973). Here we have assumed $H_0 = 50$ km s^{-1} Mpc^{-1}.

The first step in this procedure is evaluation of r_s from equation (5). For this we need to know the galactic and intergalactic absorption and the *k*-correction. The galactic absorption is independent of redshift and produces no difficulty. However, as pointed out by Gudehus (1975), redshift-dependent intergalactic absorption could cause confusion. All evidence is against existence of intergalactic absorption which, even if present, could in principle be accounted for through detailed spectroscopic studies. We therefore set $g(\Omega, z) = 1$ and use the *k*-correction given by Schild and Oke (1971). For the purpose of illustration we assume a limiting surface brightness $b_{\rm lim}$ such that in the limit $z \to 0$, $r_s \to r_0 = 30$ kpc. For the Hubble (1930) law, $B(r) = B_0(1 + r/r_{\rm H})^{-2}$, we set $r_0/r_{\rm H} \approx 10$.

[2] Note that this diagram is obtained assuming $q_0 = 1$. The effects due to the difference between the actual cosmological model and the $q_0 = 1$ model will be smaller than the scatter in the data.

Now assuming no evolution, $B(r, z) = B(r)$ from equation (5), and $B(r)$ curves given on Figure 1, we derive the variation of the isophotal radius r_s with redshift and the expected value of $\eta_\nu(r_s, z)$ (cf. Fig. 2, *dashed lines*). If the assumed surface brightness and the assumption of no evolution are correct, then the ratio of measured average isophotal surface brightness $b_\nu(\theta_s, z)$ to $\eta_\nu(r_s, z)$ should be independent of redshift. Any systematic variation of this quantity with redshift would imply evolution of the surface brightness, once the selection effects are properly accounted for.

In order to determine the sensitivity of the test to evolution, we have also calculated r_s and the corresponding $\eta(r_s, z)$ for $B(r, z) = B(r)(1 + z)^\beta$, $\beta = -1$, 1, 2, and 3, the results of which are shown by solid lines on Figure 2 (a value of $\beta \approx 1$ is what is expected in Tinsley's 1972 theoretical models, and $\beta = 3$ is the expected value for the so-called tired-light model). As is evident from equations (5)–(7), evolutionary effects manifest themselves through equation (5). As a result, the final value of η is not directly proportional to the evolutionary factor $(1 + z)^\beta$. Inspection of Figure 2 shows that $\eta_\beta/\eta_0 \propto (1 + z)^{\beta/n}$, where $n \approx 2$ to 4. The sensitivity of the test will be different for different values of r_0 and for different $B(r)$ curves.

It should be noted that this is a consistency test. Comparison of observation with the expected η for an assumed evolution shows whether or not the assumption

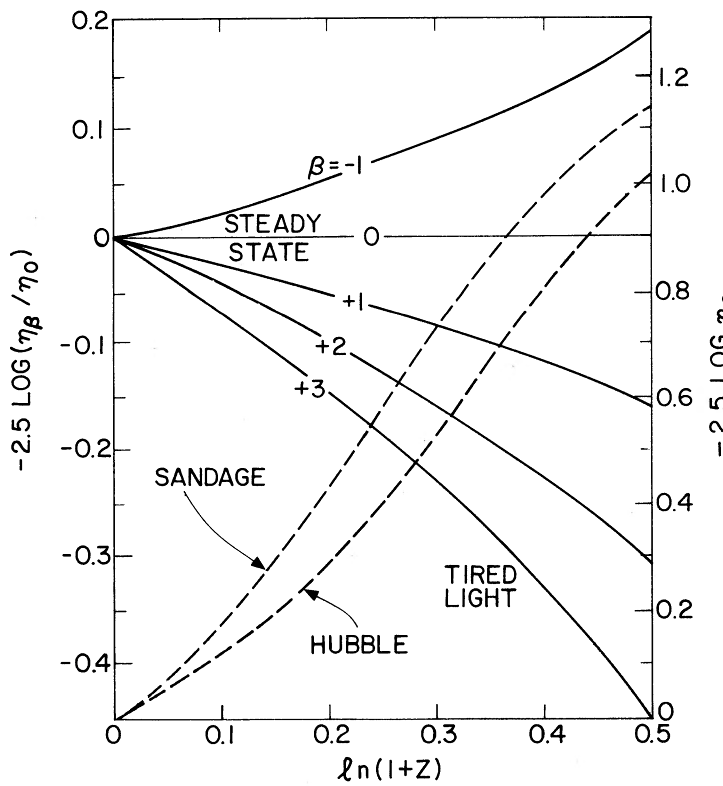

FIG. 2.—Variation of the parameter η with redshift, assuming the *k*-correction given by Schild and Oke (1971) and neglecting the absorption term $g(\Omega, z)$. The two dashed lines (*right-hand scale*) give variation of η for the Sandage composite curve of Fig. 1, with $b_{\rm lim}$ chosen so that as $z \to 0$, $r_s \to r_0 = 30$ kpc, and for the Hubble law with $r_0/r_{\rm H} \approx 10$. The solid curve gives the ratio of η with evolutionary law $(1 + z)^\beta$ to η in absence of evolution for the Hubble law. The results for the Sandage data are similar.

is consistent with data. However, absence of systematic deviation of observed values of η from the expected ones would not necessarily imply the correctness of the assumed evolution. For example, absence of systematic difference between the observed isophotal surface brightness and the expected η assuming no evolution could come about if two opposing evolutionary effects fortuitously cancel each other. It should also be noted that for a power-law dependence of luminosity on r, $L(r, z) \propto r^{\alpha}$, the parameter η is equal to $2/\alpha$ and therefore is independent of r_s and any evolution. Gudehus (1975) has suggested that since for $r/r_{\rm H} \gg 1$ the Hubble law brightness distribution can be approximated by a power law, the isophotal surface brightness test cannot be used for evolutionary studies; instead, he suggests indirect inference of the central brightness B_0 for test of evolution. The lower dashed line on Figure 2 clearly indicates the feasibility of the test suggested here. Furthermore, in the case of a power-law luminosity not only the test proposed here but all tests based on surface brightness would fail to determine evolution. For example, for a pure power law the parameter a in Gudehus's equation (3) becomes indeterminate.

Sandage (1972*b*) has estimated isophotal diameters $2\theta_s$ for a sample of first-rank elliptical galaxies in clusters. Unfortunately, the magnitudes within the isophotal diameters are not available for carrying out the test suggested here. However, it is not essential to have isophotal magnitudes. If the magnitudes within another reasonable angular diameter 2θ are known, then one can define a pseudo–surface brightness $b_\nu(\theta, z) = f_\nu(\theta, z)/\pi\theta_s^2$ which with the help of equations (1), (3), and (5) can be written as

$$b_\nu(\theta, z) = \frac{\eta_\nu(r_s, z) b_{\lim,\nu} L_\nu(r_s\theta/\theta_s, z)}{L_\nu(r_s, z)}\,. \quad (8)$$

This quantity is again independent of the cosmological model, since r_s depends only on z and the assumed surface brightness distribution and its evolution. Depending on the value of θ/θ_s, this quantity may be more or less sensitive to evolution. For $\theta/\theta_s < 1$, $b_\nu(\theta, z)$ varies more rapidly with redshift and consequently is more sensitive to evolution than the isophotal surface brightness $b_\nu(\theta_s, z)$ in equation (5). The opposite is true for $\theta/\theta_s > 1$. For the sample of galaxies with known θ_s, the magnitudes are known within angular diameters larger than $2\theta_s$ so that, as pointed out by Dr. Tinsley (private communication), the sensitivity of the test to evolution is reduced considerably, and no conclusion can be reached from these data.

IV. DETERMINATION OF q_0

The proposed test, when carried out, not only would provide information about evolution of galaxies but, more important, would provide a more reliable method for determination of q_0. Once the above test is carried out and the isophotal radius $r_s(z)$, $\eta(r_s, z)$ and the evolutionary law $E(z) = B_\nu(r, z)/B_\nu(r, 0)$ which are consistent with observation are determined, then application of this evolutionary correction to equation (1) with $\theta = \theta_s$ could be used for determination of q_0 or other cosmological parameters. Alternatively, one could use equation (3) with $\theta = \theta_s$ and r_θ equal to the $r_s(z)$. These two methods are identical.

The evolutionary law $E(z)$, and the k-correction and absorption terms $k(z)$ and $g(\Omega, z)$, do not enter equation (3) directly. Consequently, in principle, evaluation of these terms is not necessary for determination of q_0. For example, as is evident from equation (5), one need not define isophotal diameters $2\theta_s$ at the same limiting value $b_{\lim,\nu}$ for all galaxies. By choosing $b_{\lim,\nu}(z)$ and $\theta_s(z)$ so that $b_\nu(\theta_s, z)/b_{\lim,\nu}(z)$ is the same for all galaxies, one ensures (according to eq. [6]) constancy of $\eta(r_s)$ and r_s, so that equation (3) becomes

$$\theta_s(z) \propto (1+z)/\mathfrak{L}(q_0, \Lambda, H_0) \quad (9)$$

and could be used for determination of q_0 without evaluation of $k(z)$, $g(\Omega, z)$, or the evolutionary law $f(z)$ individually or collectively. In fact, for this test one does not have to limit the observation to first-rank cluster members. However, one is limited to galaxies which satisfy the basic assumptions of the test, which is independence of the shape of the surface brightness distribution from redshift (i.e., no size variation; e.g., $r_{\rm H}$ = const. in case of the Hubble law) and frequency (i.e., no color differentials). In practice, however, for ascertaining the compliance of sources with these assumptions one must observe the surface brightness distribution, to as much detail as possible, at few frequencies.

V. SUMMARY AND CONCLUSIONS

We have proposed use of variation of apparent average surface brightness with redshift for determination of evolution of galaxies. In § II we have clarified the procedure for correcting magnitudes of galaxies for aperture effect and have shown that no *a priori* assumptions about the deceleration parameter are necessary. In § III we have discussed the variation of surface brightness with redshift and its independence from the cosmological model. We have shown that measurements of isophotal diameters and magnitudes within these diameters are sufficient for determination of the variation of absolute average surface brightness with redshift.

This is the simplest procedure for determination of evolution of surface brightness. However, as shown, measurements of isophotal diameters and magnitudes within a diameter smaller than the isophotal would be sufficient and would increase the sensitivity of the test. In fact, instead of measuring the isophotal diameters, one could use measured angular diameters to different limiting surface brightness for different galaxies as long as the limiting surface brightnesses are known. It is shown that by following such a procedure for galaxies with the same surface brightness distribution one can directly determine q_0 without evaluation of the redshift-dependent terms such as evolution, k-correction, or galactic and intergalactic absorption.

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