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APPARENT LUMINOSITIES IN A LOCALLY INHOMOGENEOUS UNIVERSE

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ABSTRACT

Apparent luminosities are considered in a locally inhomogeneous universe, with gravitational deflection by individual clumps of matter taken into account. It is shown that as long as the clump radii are sufficiently small, gravitational deflection by the clumps will produce the same average effect as would be produced if the mass were spread out homogeneously. The conventional formulae for luminosity distance as a function of redshift consequently remain valid, despite the presence of any local inhomogeneities of less than galactic dimensions. For clumps of galactic size, the validity of the conventional formulae depends on the selection procedure used and the redshift of the object studied.

Subject headings: cosmology — galaxies: redshifts — gravitation

The observed relation between redshifts and apparent magnitudes continues to play a central role in cosmology. Most often, these measurements have been analyzed under the assumption that the matter of the Universe consists of a homogeneous transparent fluid. For a matter-dominated Friedmann model with zero cosmological constant, this assumption leads to the well-known formula for luminosity distance d_L as a function of redshift z (see, e.g., Weinberg 1972):

$$d_L(z) = H_0^{-1}q_0^{-2}[zq_0 + (q_0 - 1)(-1 + (1 + 2q_0z)^{1/2})]\,, \tag{1}$$

where H_0 and q_0 are the usual Hubble constant and deceleration parameter.

Several authors have argued that this analysis requires modification if the Universe is locally inhomogeneous (Zel'dovich 1964; Bertotti 1965; Dyer and Roeder 1972, 1973, 1974; Roeder 1975). In particular, if all the mass of the Universe is in the form of concentrated clumps, then the lines of sight to distant objects are likely to travel through strictly empty space, and the Ricci tensor term in the optical scalar equations is therefore absent. Dyer and Roeder (1972) find in this case that the expression for luminosity distance becomes

$$d_{L0}(z) = H_0^{-1}(1 + z)^2\int_0^z (1 + z')^{-3}(1 + 2q_0z')^{-1/2}dz'\,. \tag{2}$$

The expansions of equations (1) and (2) in powers of z agree up to order z^2, and only begin to differ in order z^3. However, if q_0 is not too small, the differences between (1) and (2) might be significant even for observed galaxies (Roeder 1975), and become quite important for quasars with $z > 1$.

The purpose of this *Letter* is to point out that in a locally inhomogeneous universe with sufficiently small clump radii, the *average* apparent luminosity (determined by observing objects in various directions) is given by equation (1) rather than by equation (2). The limitation in equation (2) is that (as recognized by Dyer and Roeder) it leaves out the gravitational deflections caused by occasional close encounters with clumps near the line of sight. These gravitational deflections produce a shear which on the average has the same effect in the optical scalar equation as would be produced in a homogeneous universe by the Ricci tensor term in this equation. Thus, as long as the clumps have sufficiently small radii (see below for how small is sufficiently small) the measured redshift-magnitude relation ought to be interpreted in terms of the *conventional* formula, equation (1), rather than the Dyer-Roeder formula, equation (2).

Before giving a general argument for this conclusion, let us see how it works in one interesting special case. We suppose that all the matter of the Universe is concentrated in clumps. For the moment we neglect the size of these clumps; the effects of a finite clump radius are considered below. We will also suppose that $q_0 \ll 1$, and calculate apparent luminosities only to first order in q_0. Under these assumptions, the average number of clumps close enough to the line of sight to produce appreciable deflection effects (such as image splitting) is small, of order q_0 for z of order unity (Press and Gunn 1973). We can therefore consider the deflection produced by just a single clump, and avoid complicated multipath problems. It is in this case, where multiple scattering is unimportant, that the validity of equation (1) is perhaps the most surprising.

Suppose we observe a luminous object with redshift z, and that a small clump of mass m and redshift $z' < z$ lies at a proper distance $l \ll H_0^{-1}$ from the line of sight. There will be two ray paths from the object to us, which pass the clump at proper distances $b_\pm$ given by

$$b_\pm = \tfrac{1}{2}[\pm l + (l^2 + 4f^2)^{1/2}] , \tag{3}$$

where

$$f^2 \equiv 4(1 + z')\lambda(z')[\lambda(z) - \lambda(z')]mG/\lambda(z) . \tag{4}$$

Here $\lambda(z)$ is the affine parameter corresponding to redshift z:

$$\lambda(z) \equiv H_0^{-1}\int_0^z (1 + z')^{-3}(1 + 2q_0 z')^{-1/2}dz' , \tag{5}$$

and likewise for $\lambda(z')$. Press and Gunn (1973) give the intensities of these two rays as

$$I_\pm = \pm I_0[1 - f^4/b_\pm^4]^{-1} , \tag{6}$$

where I_0 is the intensity calculated if gravitational deflection is ignored. Also, the probability that there is a clump in a range $dl\, dz'$ of l and z' is (for $q_0 \ll 1$)

$$dP = \frac{2\pi l dl \rho_0(1 + z')dz'}{mH_0(1 + 2q_0 z')^{1/2}} , \tag{7}$$

where ρ_0 is the present cosmic mass density. If we do not attempt to resolve the split image, then the average observed intensity is increased by the fractional amount

$$\Delta I/I_0 = \int dP(I_+ + I_- - I_0)/I_0 = \frac{2\pi\rho_0}{mH_0}\int_0^z \frac{(1 + z')dz'}{(1 + 2q_0 z')^{1/2}} J[f(z')] , \tag{8}$$

where

$$J(f) \equiv \int_0^\infty l dl\left[\left(\frac{f^4}{b_-^4} - 1\right)^{-1} + \left(1 - \frac{f^4}{b_+^4}\right)^{-1} - 1\right]. \tag{9}$$

It is straightforward to see that $J(f)$ simply equals f^2, so that the clump mass m cancels out of our calculation. The factor $2\pi\rho_0 G/H_0^2$ may be replaced with $3q_0/2$; and since we are taking $q_0 \ll 1$, we can neglect q_0 everywhere else in the z'-integral. We then find, to first order in q_0,

$$\Delta I/I_0 = \frac{2q_0 z^3}{2z + z^2} . \tag{10}$$

Equivalently, the luminosity distance is decreased by the fractional amount

$$\Delta d_L/d_{L0} = \frac{-q_0 z^3}{2z + z^2} . \tag{11}$$

But to first order in q_0, the luminosity distance (2) is

$$d_{L0} = H_0^{-1}[z + \tfrac{1}{2}z^2 - \tfrac{1}{2}q_0 z^2] . \tag{12}$$

Therefore, the luminosity distance, corrected to take account of gravitational deflection, is now

$$d_L = d_{L0} + \Delta d_L = H_0^{-1}[z + \tfrac{1}{2}z^2 - \tfrac{1}{2}q_0(z^2 + z^3)] . \tag{13}$$

This is precisely the same as the conventional result (1) for a homogeneous universe, evaluated to first order in q_0.

We can easily see that the same sort of result will hold for arbitrary q_0, and for models with transparent intergalactic matter as well as clumps. Suppose we observe a number of luminous bodies with Robertson-Walker coordinate r_1 and redshift z, lying in various different directions. Draw a sphere around each object, which passes through our observatory. Even though there may be focusing or defocusing for individual observations, there is nothing special about the location of our telescope, so on the average the fraction of all photons intercepted by our telescope mirror is simply the ratio of the mirror area A to the sphere area $4\pi r_1^2 R^2(t_0)$. The gravitational deflection of light conserves photon energies, so the total power passing through each sphere is the object's luminosity L divided by a frequency redshift factor $(1 + z)$ and an emission-rate redshift factor $(1 + z)$. The average radiant power reaching the telescope mirror is therefore

$$\frac{L}{(1 + z)^2}\frac{A}{4\pi r_1^2 R^2(t_0)} .$$

By definition, this is $LA/4\pi d_L^2$, and solving for d_L gives equation (1).

Of course, if the Universe is filled with large opaque clumps, then when we observe a distant luminous object there *is* something special about our location—we are on a line of sight that happens to miss the clumps. For instance, if we suppose that the clumps are opaque spheres of radius R, then the quantity $J(f)$ in equation (8) becomes

$$J(f) = f^2 - \tfrac{1}{2}R^2 \quad (f > R)$$
$$= f^4/2R^2 \quad (f < R)\,. \tag{14}$$

Clearly we approach the conventional result (1) or the Dyer-Roeder result (2) according to whether R is much less or much greater than a critical value $R_c(z)$, equal to the average value of $f(z')$ for $0 < z' < z$. From equation (4), we see that the critical radius is

$$R_c(z) \approx (mG/H_0)^{1/2} \quad (z \gtrsim 1)$$
$$\approx z(mG/H_0)^{1/2} \quad (z \lesssim 1)\,. \tag{15}$$

If the clumps are dark stars with $m \approx M_\odot$ and $R \approx R_\odot$, then the zero-radius approximation is very good: R is much less than R_c for $z > 10^{-5}$. If the clumps are galaxies themselves, the zero-radius approximation is marginal. In general, R should be taken as the "radius of avoidance," the minimum distance between an intervening galaxy and the line of sight, that would not cause observers to miss a more distant object. As a definite lower bound on R, we might take the typical size of a galaxy nucleus, $R \approx 2$ kpc (Dyer and Roeder 1972). In this case, for a clump mass of $10^{11}\ M_\odot$, we find that $R < R_c$ for $z \gtrsim \frac{1}{3}$, so the Dyer-Roeder result (2) would hold for small redshifts, while the conventional result (1) would apply for large redshifts, say $z > 0.5$. It may be that we should use for R an average projected galaxy radius of order 10 kpc, in which case the Dyer-Roeder result (1) would apply for all z. Also, Dyer and Roeder (1974) suggest that galaxies selected for measurements of redshifts and magnitudes tend to be those for which the line of sight is well away from any intervening galaxy, in which case the radius of avoidance may be even larger than a galactic radius. (In this case, we should really use an angular rather than a linear radius of avoidance, and eq. [14] needs modification.) On the other hand, it is possible that the effective radius of avoidance may be *reduced*, if we tend to select objects that are made unusually bright by the gravitational focusing produced by invisible galaxies lying near the line of sight (e.g., Barnothy 1965). A proper assessment of all these effects would require that we take into account the detailed selection procedures actually followed by observers.

The Dyer-Roeder result (2) may be useful even for the case of small clumps, in setting a lower limit to the apparent luminosity at a given redshift. This lower limit is attained when we observe a distant object which happens to lie along a line of sight near which the number of clumps is anomalously low. In general, there is a statistical spread in luminosity distances, comparable to the difference between equations (1) and (2), caused by differences in the populations of clumps lying near the various lines of sight. (See also Bertotti 1966.) Because of these fluctuations in $d_L(z)$, it is important to note that equation (1) should be used to calculate the mean inverse-square luminosity distance, not the mean luminosity distance itself. It is possible that this spread in luminosity distance at a given redshift accounts for at least part of the observed spread in the apparent luminosities of quasars with similar redshifts.

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