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Neutrino Rest Mass from Cosmology

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Summary. In standard cosmological models, the overall mass density of the Universe can be calculated from the observed value of the Hubble constant H_0 and the deceleration parameter q_0. Their most recent values suggest a density considerably higher than the estimated density of the known matter in the Universe. The "missing mass" phenomenon is also known in clusters of galaxies. The missing mass may be explained by the relict cosmological neutrinos, produced in the hot era following the Big Bang, if we assume nonvanishing neutrino and neutretto rest masses. The cosmological evolution of the Universe has been calculated in this model. The observed values of H_0, q_0 and t_0 (age of the Universe) agree with the cosmological model, if one chooses an appropriate value for the neutrino mass m. The upper limit on the neutrino and neutretto rest mass obtained in this way is $m = 13.5$ eV. Density fluctuations in the primordial neutrino gas at the temperature $kT = mc^2$ may initiate the formation of clusters of galaxies. The 13.5 eV mass value leads to a separation of clusters at the present time in good agreement with observation. The relict neutrinos with a rest mass could form a halo around clusters of galaxies: this halo would influence the density profile of the cluster in the outer region. Our final conclusion is that a neutrino or neutretto rest mass larger than 15 eV would contradict the astrophysical evidence.

Key words: neutrinos-rest mass — cosmology — clusters of galaxies

Our laboratory information about the neutrino (ν_e) and neutretto (ν_μ) rest masses is rather poor

$$m_{\nu_e} < 60\ \text{eV}, \quad m_{\nu_\mu} < 0.8\ \text{MeV}.$$

A connection between the upper limit on the neutrino charge and the neutrino rest mass has been discussed by Bernstein *et al.* (1963). A straightforward way to learn something more about these elusive particles is offered by cosmological evidence discussed by Zel'dovich (1969). It is hard to escape the conclusion that the temperature of the universe reached the value 10^{12} K some 10^{10} years ago. If this is true, the number of relict neutrinos must be comparable to the number of relict photons, observed in the 2.7 K background radiation. On the other hand, from the upper limit on the deceleration q_0 of extragalactic objects and from the lower limit on the age t_0 of the universe, one can put an upper limit on the overall mass density. Dividing mass density by particle number density one obtains information about the neutrino rest mass m.

When the temperature of the hot universe, following the Big Bang, dropped below 10^{12} K (i.e. $kT < m_\pi c^2$), the unstable hadrons annihilated, the number of the nucleons was negligible, and the leptons were in thermal equilibrium with each other. The muons annihilated soon after and the neutrettos decoupled from the plasma. The main weak neutretto interactions were the following:

$$\nu_\mu + \mu \to \mu + \nu_\mu, \quad \nu_\mu + e^- \to \mu + \nu_e.$$

In this case, the ν_μ decoupling temperature is determined by the disappearance of the muons, so that $T_d = 1.2 \times 10^{11}$ K, as calculated by Chiu (1966) and Graaf (1969, 1971). Another possible coupling between the neutretto and the hot electron plasma is given by electromagnetic scattering

$$e + \nu_\mu \to \nu_\mu + e$$

via the electromagnetic form factor of the neutretto. With a cut-off at 300 GeV, decoupling is expected to have happened at $T_d = 4.1 \times 10^{11}$ K. If the $(e\nu_\mu)$ scattering is due to the neutral Weinberg boson, the decoupling temperature may reduce to $T_d = 3.0 \times 10^{11}$ K. After the ν_μ decoupling, the $e - \gamma - \nu_e$ plasma cools down adiabatically, and the gas loses energy only via the Hubble energy shift. If $E_\nu \gg m_\nu$ the adiabatic cooling and the Hubble shift do not differ significantly, so that the final result will not be sensitive to an accurate value of T_d.

The ν_e decoupling temperature is 0.18×10^{11} K. The disappearance of the positrons and the decoupling of the photons happened at about 5.9×10^9 K.

The occupation number for fermions or bosons is given by the formula

$$n_j(p) = \left[\exp \frac{\sqrt{m_j^2 + p^2}}{kT} \pm 1\right]^{-1}. \tag{1}$$

For the era before decoupling ($T > T_d$), the temperature T may be obtained as a function of the scale factor

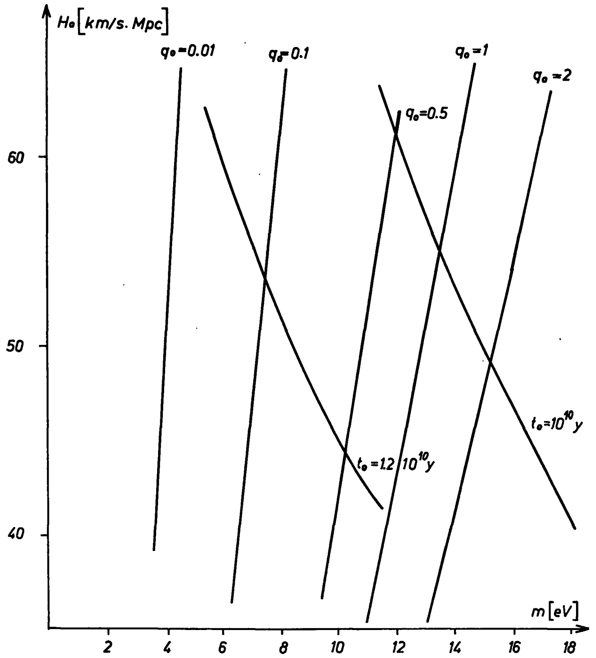

Fig. 1. The dependence of the parameters q_0 and t_0 on H_0 and m

("world radius") R of the homogenous isotropic universe from the adiabatic condition $S(R, T)=\text{const}$. After decoupling ($T<T_d$), one has to write $T=T_d$ in the statistical distribution function of the ν_μ, but the momentum p decreases according to the Hubble law $p\propto\lambda^{-1}\propto R^{-1}$. Thus, one can calculate the pressure P and the mass density ϱ in terms of the scale factor R at any time. This enables us to integrate the Einstein equation for a closed universe:

$$\left(\frac{\dot{R}}{R}\right)^2+\frac{c^2}{R^2}=\frac{8\pi G}{3}\varrho(R) \tag{2}$$

and to obtain the scale factor R, the photon temperature T and the overall mass density ϱ in terms of the time t. The computer integration has been extended from the decoupling temperature $T(0)=T_d$ up to the observed temperature of the present background radiation $T(t_0)=2.7$ K. The corresponding age of the universe t_0, the Hubble constant $H_0=\dot{R}(t_0)/R(t_0)$ and the deceleration value $q_0=-\ddot{R}(t_0)\,R(t_0)/\dot{R}(t_0)^2$ come out as results of computation. If the astronomers supplied us with accurately observed values of these parameters, we would be able to obtain the unknown input values m_{ν_e}, m_{ν_μ} and $R(0)$.

In earlier calculations, Zel'dovich (1969) concluded from the age of moon rocks ($t_0>4.5\times10^9$ years) that $m<200$ eV. From the Hubble time $H_0^{-1}=18.4\times10^9$ years and from a conservative deceleration limit $q_0<2$, the present authors Marx and Szalay (1972, 1973) obtained

$$m_{\nu_\mu}<140\text{ eV}.$$

From an optimistic value $q_0=0.94$, Cowsik and McClelland (1972) concluded that:

$$m_\nu<16\text{ eV}.$$

In the present fluid state of empirical cosmology, it is better to let the reader choose from among the observational parameters. We therefore summarize the results of our calculations in Fig. 1. The values of t_0 and q_0 have been plotted in terms of H_0 and m. This calculation has been performed under the pessimistic assumptions that neutrinos and neutrettos have the same rest mass m and that both the left-handed and right-handed states are filled up.

It is shown by Fig. 1 that the conclusion

$$m<22\text{ eV}$$

is convincing even under moderate use of the empirical values of cosmology given by Sandage (1972)

$$H_0^{-1}=18.4\ \pm2\times10^9\text{ years},$$
$$q_0=\ 0.94\pm0.40.$$

If one accepts that the nuclear age of our galaxy is $(12\pm2)\,10^9$ years, according to Fowler (1973) one can use this as a lower limit to t_0. This gives a more restrictive upper limit

$$m<13.5\text{ eV}. \tag{3}$$

(If one puts $m_{\nu_e}=0$ arbitrarily, our conclusion is evidently $m_{\nu_\mu}<27$ eV. This value is four orders of magnitude smaller than the best available laboratory limit.)

The visible baryonic mass density of the universe is $\varrho_*=3.10^{-31}$ g cm^{-3}, which can be compared with the dynamical mass density $\varrho_d=0.5\times10^{-29}$ g cm^{-3}, suggested by the values of H_0 and q_0 quoted above. The "missing mass" gap $\varrho_d-\varrho_*$ has a tendency to become increasingly smaller with improvement of observational technique, but it still seems to exist. If we were able to state definitely that neutrinos form the main unobserved component of matter, $\varrho_d-\varrho_*=\varrho_\nu$, our mass inequalities would turn into equalities. In this case, the dominating mass in the universe would be a non-relativistic neutrino gas. Evidently the missing mass may be explained also by other phenomena, e.g. by black holes or gravity waves. It is very desirable to learn something more about the rest mass and nature of these "missing" objects.

Let us follow the history of the relict cosmological neutrinos during the evolution of the Universe. Their presence may influence the local gravitational field as discussed by Marx *et al.* (1964, 1967) and Paál (1964). As the expansion of space continues, the wavelength of the neutrinos increases. So long as the kinetic energy is larger than the rest mass, the neutrinos behave like radiation, and fill space homogenously. When the decreasing kinetic energy reaches the rest mass,

$$kT_\nu = mc^2\,, \tag{4}$$

the neutrinos start to behave like a nonrelativistic gas. The homogenous neutrino gas becomes gravitationally unstable. The density fluctuations produce transient concentrations of different size. If a mass concentration is large enough to be stable against thermal motion,

$$G(\varrho_\nu r^3)\, mr^{-1} > \pi k T_\nu$$

it does not resolve, but continues to grow and becomes a protocluster. The minimum radius of a stable concentration is equal to the Jeans length:

$$r_J = \left(\frac{\pi k T_\nu}{G\varrho_\nu m_\nu}\right)^{1/2} = \left(\frac{\pi c^2}{\varrho_\nu G}\right)^{1/2}. \tag{5}$$

The present overall density of the nonrelativistic neutrino gas may be estimated as

$$\varrho_\nu(t_0) = \varrho_d - \varrho_* = 0.5 \times 10^{-29}\ \mathrm{g\,cm^{-3}}\,.$$

At the time given by the Eq. (4) it was

$$\varrho_\nu(t_\nu) = \varrho_\nu(t_0)\, R(t_0)^3 / R(t_\nu)^3\,.$$

By making use of the equation $T=\mathrm{const}$ in Eq. (5) one arrives at

$$r_J = 3.5 \times 10^{23}\ \mathrm{cm} \times \left(\frac{m}{1\ \mathrm{eV}}\right)^{-3/2}$$

at the time of cluster formation as obtained by Szalay (1974). The separation between these clusters can be projected into the present Universe:

$$D(t_0) = \frac{R(t_0)}{R(t_\nu)}\, 2r_J = \frac{T_\gamma(t_\nu)}{2.7}\, 2r_J$$

i.e.

$$D(t_0) = 3 \times 10^{27}\ \mathrm{cm} \times \left(\frac{m}{1\ \mathrm{eV}}\right)^{-1/2}. \tag{6}$$

Abell (1965) estimated the average distance among the 4000 clusters of galaxies, which are nearer than 1200 Mpc, to be $D = 8.10^{26}$ cm. Comparing this number with the theoretical value one gets

$$m < 14\ \mathrm{eV} \tag{7}$$

in surprising agreement with our cosmological estimation.

According to the picture sketched above the first stable concentration was that of the neutrinos. The atomic gas accumulated in the gravitational field of these "neutrino stars". A similar model had been considered by Bludman (1974), where the neutrinos were degenerate. The present equilibrium distribution of the galaxies and of the neutrino halo was shaped by their common gravitational field. The local Newtonian potential ϕ obeys the equation

$$\nabla^2 \phi = 4\pi G[\varrho_G(\phi) + \varrho_\nu(\phi)]\,. \tag{8}$$

The galaxies may be treated as a classical Boltzmann gas, described by the "barometric formula"

$$\varrho_G(\phi) = \varrho_G(\mathrm{at}\, r=\infty) \times \exp[-M\phi/kT_G]\,.$$

M is the average galaxy mass. T_G is the "temperature" for the cluster, to be obtained from the observed velocity distribution of the galaxies:

$$\tfrac{3}{2} kT_G = \tfrac{1}{2} M\langle v^2\rangle = \tfrac{3}{2} M \langle v_{\mathrm{rad}}^2\rangle\,.$$

$\langle v_{\mathrm{rad}}^2\rangle$ is the average spread in the "line of sight" velocities of the galaxies. Consequently,

$$\varrho_G = \varrho_G(\mathrm{at}\, r=\infty) \times \exp(-\phi/\langle v_{\mathrm{rad}}^2\rangle)\,. \tag{9}$$

To discuss the distribution of neutrinos, let us introduce $\alpha = R(t_d)/R(t_0)$, where t_0 indicates the present time and t_d the decoupling time. At t_d, the neutrinos were still extreme relativistic particles ($\varepsilon_\nu \approx pc$), so the neutrino occupation probability at the present time t_0 may be obtained by the substitution $p \to \alpha p$:

$$n_\nu(p) = \left[1 + \exp\frac{\alpha p + m\phi}{kT_d}\right]^{-1}. \tag{10}$$

After decoupling, the form of the occupation probability no longer changes. The neutrino mass density can be written in the following form:

$$\varrho_\nu = g \int_0^\infty \sqrt{m^2 + p^2}\, n(p)\, 4\pi p^2 dp \tag{11}$$

(g is the statistical weight of the neutrinos). In the era of the density fluctuations one has $p^2 \ll m^2$, so the square-root can be neglected. At infinity $\phi \to 0$, and so

$$\varrho_\nu(\phi) = \varrho_\nu(\mathrm{at}\, r=\infty)\, \frac{\int_0^\infty \frac{x^2 dx}{e^{x-y}+1}}{\int_0^\infty \frac{x^2 dx}{e^x+1}} \tag{12}$$

with $y = -m\phi/kT_d$. The neutrino mass density (12) may be computed numerically.

The Coma cluster is a relaxed system of galaxies with spherical symmetry. We may assume that our approximation offers a good description of this cluster. The most recent analysis of the observational material has been performed by Bahcall (1973). Her conclusion was

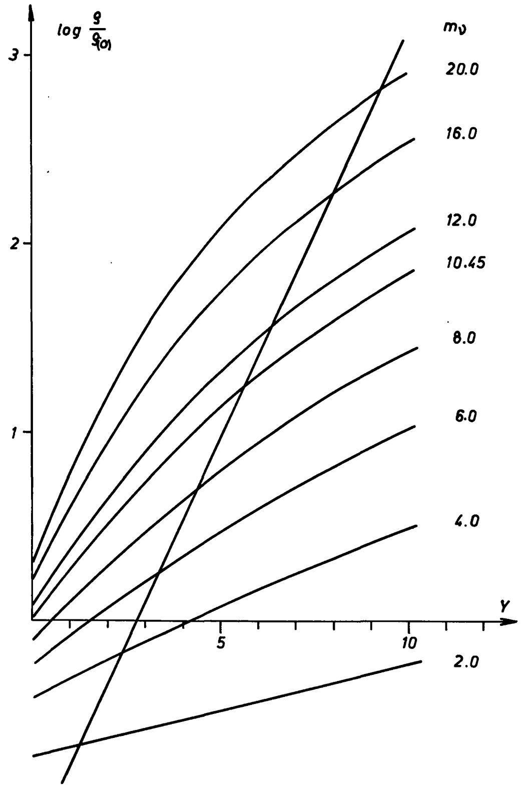

Fig. 2. The dependence of the densities ϱ_G and ϱ_ν on $Y=-\phi/\langle v_{\rm rad}^2\rangle$ for different values of the neutrino mass m

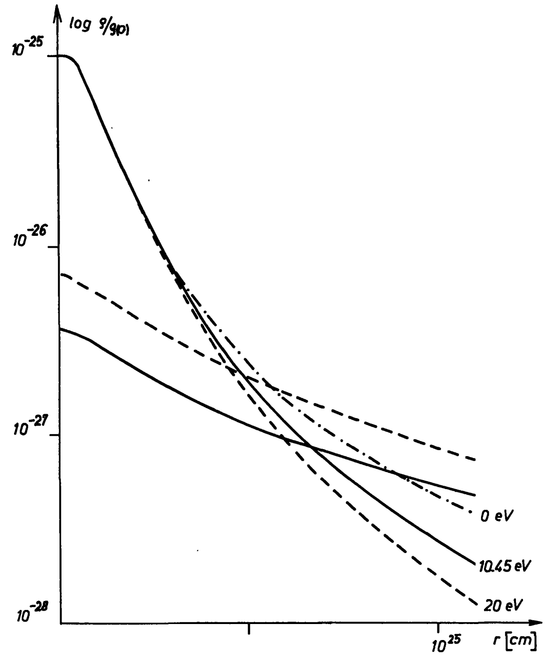

Fig. 3. Self-consistent density profiles of the Coma cluster for the neutrino-mass values 0, 10 eV and 20 eV, respectively

that the density profile of the Coma cluster can be described rather well by an isothermal Emden distribution, after subtracting the contribution of the background. Using the results of this fit, we take the following reasonable values for the parameters:

$$\varrho_G(\text{at}\, r=\infty)=3.10^{-31}\ \text{g cm}^{-3}\,,$$
$$\varrho_\nu(\text{at}\, r=\infty)=5.10^{-30}\ \text{g cm}^{-3}\,,$$
$$\varrho_G(\text{at}\, r=0)=10^{-25}\ \text{g cm}^{-3}\,,$$
$$\langle v_{\rm rad}^2\rangle^{1/2}=1000\ \text{km s}^{-1}\,.$$

Using these data, one can plot $\log[\varrho_G(r)]$ and $\log[\varrho_\nu(r)]$ versus $Y=-\phi/\langle v_{\rm rad}^2\rangle$. According to Eq. (9) the first function will be a straight line. The shape of the second function will depend in a sensitive way on the neutrino mass m, as shown in Fig. 2. In the central region, the mass density will be dominated by galaxies. The profile of the cluster will be influenced by the neutrino halo only in the outer region, where $\phi(r)\simeq 0$.

One can solve the self-consistent Eq. (8) for the case of the Coma cluster numerically. The different density profiles are shown in Fig. 3. For $m=0$, one arrives at the familiar Emden sphere distribution. The "missing mass", needed for the stability of the Coma cluster may be explained by the halo of massive neutrinos. It should be emphasized that no cut-off parameter is needed in this model. A neutrino mass in the region

$$10\ \text{eV}<m<20\ \text{eV} \tag{13}$$

could influence the density distribution in the outer layers $r>5.10^{24}$ cm. Unfortunately, the background galaxies seem to prevent accurate measurements in this region. Accurate information about the outer region or about the total mass of the Coma cluster may in the future offer a new possibility for determining the neutrino mass.

Let us summarize our conclusions in the following way. The most accurate upper limits on neutrino and neutretto rest masses may be collected by observing the gravity of the relict cosmological neutrinos, as a consequence of their enormous and theoretically rather well known number. The neutrino particle density has to be essentially equal to the photon density,

$$n_\nu=\tfrac{7}{4}n_\gamma=\tfrac{7}{4}aT_d^3$$

where $T_d=2\times 10^{10}$ K. The neutrino mass density is given by $\varrho_\nu\approx mn_\nu$. The astrophysical determination of neutrino mass is effective in that mass region where the neutrino mass density is larger than the optical matter density ϱ_*. Thus $mn_\nu>\varrho_*$ gives $m>1$ eV as an interesting region. The present empirical astrophysical evidence supports strongly the conclusion

$$m<15\ \text{eV} \tag{14}$$

for neutrinos and for neutrettos as well. This conclusion has been obtained in different independent ways (age of the Universe, average separation of clusters of galaxies, missing mass in clusters of galaxies). The mass limit (14) is better by several orders of magnitude than that available from laboratory measurements.

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