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INTERPOLATING MINOR PLANET OR COMET POSITIONS BETWEEN EPHEMERIS DATES

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Ephemerides are normally printed at 0^h Ephemeris Time (ET), that is, for 8 pm Eastern Daylight Time, 7 pm Central Daylight Time, etc., of the preceding day. Ten day intervals are normally used except for rapidly moving objects for which the time interval may be shorter. To predict a position for a time other than 0^h ET on an ephemeris date, the time must first be expressed in days and decimal fractions thereof. With 1440 minutes in a day, every 0.01 day is 14.4 minutes, or for extreme accuracy every 0.00001 day is 0.864 seconds. In the following example days are expressed to hundredths, right ascension to $0^m 1$, declination to $1'$, but the method may be used to any desired level of accuracy.

This writer observed the minor planet 380 Fiducia at 12:05 am CDT Aug. 4, 1975. This converts to 5:05 ET, or 5 hrs 5 min = 305 minutes following the start of Ephemeris Day Aug. 4. This is $305/1440$ of a day; expressed decimally it equals 0.21 days following Aug. 4.00; the time is thus 1975 Aug. 4.21.

A partial ephemeris of 380 Fiducia follows:

1975 Jul 27	21 ^h 44 ^m .6	7.4	-20° 34'	-69
Aug 6	21 37.2	8.4	-21 43	-65
Aug 16	21 28.8	8.2	-22 48	-52
Aug 26	21 20.6		-23 40	

Let α_0 be the right ascension on the ephemeris date preceding observation, α_1 be the right ascension on the following ephemeris date, α_2 be the right ascension on the ephemeris date subsequent to this, etc. This $\alpha_0 = 21^h 44^m .6$, $\alpha_1 = 21^h 37^m .2$, $\alpha_2 = 21^h 28^m .8$, etc., in our example. The first difference $\Delta\alpha_0 = \alpha_1 - \alpha_0 = -7^m .4$. First differences are listed in the Leningrad ephemerides, except that the minus sign is omitted for right ascension because almost all asteroids have retrograde motion near opposition. Other first differences in this example are $\Delta\alpha_1 = \alpha_2 - \alpha_1 = -8^m .4$, $\Delta\alpha_2 = \alpha_3 - \alpha_2 = -8^m .2$.

The use of first differences is the process of linear interpolation. Commonly errors as high as $\pm 0^m .5$, $\pm 5'$ may arise, because first differences replace the actual curved paths by imaginary straight lines. These may be reduced by using second differences, and perhaps even third differences, in second-order and third-order interpolation schemes. The second difference $\Delta_2\alpha_0 = \Delta\alpha_1 - \Delta\alpha_0 = -1^m .0$ in this example; $\Delta_2\alpha_1 = \Delta\alpha_2 - \Delta\alpha_1 = +0^m .2$. The third difference $\Delta_3\alpha_0 = \Delta_2\alpha_1 - \Delta_2\alpha_0 = +1^m .2$. One could continue in this manner if extreme accuracy in ephemerides is available; in most cases the use of second differences is sufficient for accuracy of order $\pm 0^m .1$, $\pm 1'$.

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Let t denote the time between ephemeris date preceding observation and time of observation, in this example 8.21 days. Let T denote the time between successive ephemeris dates, here 10.00 days. Then the formula for third order interpolation in right ascension is:

$$\alpha = \alpha_0 + \frac{t}{T} \Delta\alpha_0 + \frac{1}{2!} \frac{t(t-T)}{T^2} \Delta_2\alpha_0 + \frac{1}{3!} \frac{t(t-T)(t-2T)}{T^3} \Delta_3\alpha_0 + \dots$$

Substituting numbers to illustrate by example gives, for 1975 Aug. 4.21:

$$\begin{aligned} &= 21^h44^m.6 + \frac{8.21}{10}(-7^m.4) + \frac{(8.21)(-1.79)}{2(100)}(-1^m.0) + \frac{(8.21)(-1.79)(-11.79)}{6(1000)}(+1^m.2) + \dots \\ &= 21^h44^m.6 - 6^m.07 + 0^m.07 + 0^m.03 + \dots = 21^h38^m.6, \text{ rounding to the nearest } 0^m.1. \end{aligned}$$

First, second, and third differences in declination are defined in analogous manner. In our example $\delta_0 = -20^\circ34'$; $\delta_1 = -21^\circ43'$; $\delta_2 = -22^\circ48'$; etc.;

$$\Delta\delta_0 = \delta_1 - \delta_0 = -69'; \quad \Delta\delta_1 = \delta_2 - \delta_1 = 65'; \quad \Delta\delta_2 = \delta_3 - \delta_2 = -52';$$

$$\Delta_2\delta_0 = \Delta\delta_1 - \Delta\delta_0 = +4'; \quad \Delta_2\delta_1 = \Delta\delta_2 - \Delta\delta_1 = +13';$$

$$\Delta_3\delta_0 = \Delta_2\delta_1 - \Delta_2\delta_0 = +9'. \quad \text{Then for 1975 Aug. 4.21:}$$

$$\begin{aligned} \delta &= \delta_0 + \frac{t}{T} \Delta\delta_0 + \frac{1}{2!} \frac{t(t-T)}{T^2} \Delta_2\delta_0 + \frac{1}{3!} \frac{t(t-T)(t-2T)}{T^3} \Delta_3\delta_0 + \dots \\ &= -20^\circ34' + \frac{8.21}{10}(-69') + \frac{(8.21)(-1.79)}{2(100)}(+4') + \frac{(8.21)(-1.79)(-11.79)}{6(1000)}(+13') + \dots \\ &= -20^\circ34' - 56'.6 - 0'.3 + 0'.4 + \dots = -21^\circ30', \text{ rounding to the nearest } 1'. \end{aligned}$$

However in this case the position measured with the Vehrenberg Atlas Stellarum grid was $21^h36^m.9$, $-21^\circ37'$. The residual, which is the observed position minus the computed position (O-C), is

$$21^h36^m.9 - 21^h38^m.6, -21^\circ37' - (-21^\circ30') = -1^m.7, -7'.$$

It is this writer's experience that for well over 90% of all the asteroids brighter than magnitude 14 which he has observed with the aid of the Leningrad Ephemerides of Minor Planets and measured off Vehrenberg star atlas grids, the residuals are smaller than $\pm 0^m.4$, $\pm 4'$. Larger residuals occasionally, but not always, appear for orbits published in 1965 or earlier. As these older orbits are being improved, the incidence of appreciable residuals should decrease greatly in the next five to ten years.

Editor's Comment: The Editor wishes to thank Prof. Pilcher for his fine paper, which was written in response to several letters received which asked about interpolation procedures. It is hoped that many readers will be helped by this discussion.