Mon. Not. R. astr. Soc. (1975) **170**, 333–342.

A DETERMINATION OF THE RATE OF CHANGE OF *G*

Thomas C. Van Flandern

(Received 1974 August 29)

SUMMARY

A new analysis of lunar occultation observations from 1955 to 1974 utilizing Atomic Time gives a value for the empirical part of the secular acceleration of the Moon's mean longitude of $(-65'' \pm 18'')/\text{cy}^2$. This differs from other determinations which utilized the Ephemeris Time scale, based on the apparent annual motion of the Sun about the Earth. These latter, which give $(-38'' \pm 4'')/\text{cy}^2$, measure only the tidal component of the Moon's longitude acceleration. The remaining acceleration $(-27'' \pm 18'')/\text{cy}^2$, has as its most probable cause a decrease in the Universal Gravitational Constant at the rate of $\dot{G}/G = (-8 \pm 5) \times 10^{-11}$/yr. There is a large body of supporting evidence for a rate of about this size, including the disappearance of the artificial 'non-tidal' component in the deceleration of the Earth's rotation, geophysical evidence for an observed expansion of the Earth's radius, and generally good agreement with recent determination of the Hubble constant, which is related to $\dot{G}/G$. The observed rate is also consistent with the Dirac and the Hoyle–Narlikar cosmological theories, and to a lesser degree, with the Brans–Dicke theory. Other possible interpretations of the observed excess (negative) lunar acceleration are also discussed; but only those with cosmological significance seem at all plausible.

INTRODUCTION

No variation in a fundamental constant of physics has yet been demonstrated. However, the expansion of the Universe has suggested to many theoreticians that the Universal Gravitational Constant, *G*, may be decreasing, either as a cause or an effect of the expansion. Indeed, at least three currently plausible cosmological theories, Brans–Dicke (1961), Hoyle–Narlikar (1972), and Dirac (1973), demand that *G* be decreasing. In the strictest classical form of the widely-accepted general relativity theory, however, *G* must remain constant. This paper deals with observational evidence that *G* is, in fact, decreasing at a rate of about one part in 10^{10} per year, and summarizes some implications of this result, along with other possible interpretations of the observational data.

PRINCIPLE BEHIND DETERMINATION

An immediate consequence of a decreasing gravitational constant is a slow, adiabatic expansion of all orbits in the solar system about their primaries. Vinti (1974) has shown that, for two-body motion with *G* varying inversely with time and averaging over a complete revolution,

$$\frac{\dot{G}}{G} = -\frac{\dot{a}}{a} = \frac{1}{2}\frac{\dot{n}}{n} = -\frac{1}{2}\frac{\dot{P}}{P}, \tag{1}$$

where a dot denotes the time derivative, a = semi-major axis, n = mean motion, P = period of revolution. Hoyle (1972) has shown how such a change in the mean motion of the Moon about the Earth can be distinguished from similar changes due to other causes, such as tidal friction. Since equation (1) holds for the Earth's orbit around the Sun, as well as for the Moon's orbit around the Earth, it follows that an astronomical time scale derived from the observed motion of the Sun about the Earth, such as Ephemeris Time, will suffer a slowing at the same proportional rate, $\dot{G}/G$. Hence, when we measure the acceleration of the Moon's mean longitude utilizing Atomic Time (presumed uniform), we obtain the total acceleration from all causes. But when we measure the same acceleration utilizing Ephemeris Time, any contribution due to $\dot{G}$ would be excluded, since it would be absorbed into the time scale. The difference between the two acceleration values would then indicate the amount due exclusively to a decreasing gravitational constant.

OBSERVATIONAL DATA UTILIZING EPHEMERIS TIME

There have been five determinations of the acceleration of the Moon's mean longitude utilizing Ephemeris Time. The first is due to Spencer–Jones (1939), who analysed observations of the Sun, Moon, Mercury, and Venus over the last three centuries. His results were converted to their Ephemeris Time equivalent by Clemence (1948), and the mean error was estimated by Morrison (1972a), obtaining a negative lunar acceleration of $(-22'' \pm 7'')/\mathrm{cy}^2$ (cy = century), over and above the dynamical acceleration induced by the slowly decreasing eccentricity of the Earth's orbit around the Sun. The second determination is due to Murray (1957), who analysed ancient observations and obtained $(-42'' \pm 6'')/\mathrm{cy}^2$. The third determination is Newton's (1969), who analysed ancient observations at two different epochs, primarily of solar eclipses, to obtain independent estimates of $(-41''\cdot 6 \pm 4''\cdot 3)/\mathrm{cy}^2$ and $(-42''\cdot 3 \pm 6''\cdot 1)/\mathrm{cy}^2$. The fourth determination was made by Oesterwinter & Cohen (1972), who analysed meridian circle observations of the Sun, Moon and planets since 1913 to obtain $(-38'' \pm 8'')/\mathrm{cy}^2$.* The fifth determination is the recent result of Muller & Stephenson (1974), who utilized more extensive ancient eclipse data than Newton or Murray, and used an improved method of analysis. Their result is $(-37''\cdot 5 \pm 5''\cdot 0)/\mathrm{cy}^2$.

Of these five determinations, the most suspect is the Spencer–Jones value, which, as has been pointed out by Morrison (1972b), is influenced greatly by seventeenth century observations of transits of Mercury across the Sun. Omitting this first determination because of the high probability of systematic error, and assuming that the Muller & Stephenson result obsoletes the earlier results from ancient observations, the average of these last two determinations gives

$$\dot{n}_{\mathrm{M}}^{\mathrm{ET}} = (-38'' \pm 4'')/\mathrm{cy}^2 \tag{2}$$

as the acceleration of the Moon's mean longitude, utilizing Ephemeris Time. All of the determinations, except that of Spencer–Jones, lie within their own mean errors of this adopted value. As will become evident later, adopting any value closer to the Spencer–Jones result would make the derived rate of decrease of G larger.

* The statement in the Oesterwinter & Cohen paper that the Atomic Time scale was extrapolated backwards to 1913 is incorrect in the context of the present discussion—if G changes, then the time scale which was extrapolated backwards was an Ephemeris Time scale, adjusted in epoch and rate to fit the Atomic Time scale from 1955 to 1969. Fig. 8 of their paper should be re-labelled, for present purposes, as '*New* Ephemeris Time minus *old* Ephemeris Time, . . .'.

OBSERVATIONAL DATA UTILIZING ATOMIC TIME

The analysis of lunar occultation data utilizing Atomic Time to determine the Moon's mean longitude acceleration has been discussed earlier by Van Flandern (1970), obtaining $(-52'' \pm 16'')/\text{cy}^2$, and by Morrison (1973), who obtained $(-42'' \pm 6'')/\text{cy}^2$. The results of a new analysis, presented here, differ from both earlier results primarily because of the use of a numerically-integrated lunar ephemeris (see Appendix), instead of the Brown–Eckert analytical theory; and because of the inclusion of new observations.

The occultation observations in the new analysis cover the period 1955–1974, during which Atomic Time was available. To guard against an obvious source of systematic error, a solution was performed only for photoelectric timings (1786 observations), because insufficient information was available to us to remove completely the effects of the reaction delay of the observer for visual timings before 1967. Also, the effect of the non-uniform time distribution of the observations was minimized by a simple weighting scheme, such that the total weight of all observations within each year was kept nearly the same. The complete solution contained 21 unknowns, including selected lunar and solar elements, lunar size and shape parameters, and star reference system corrections, for which correlations were kept to a minimum (Van Flandern 1971). The total acceleration not accounted for by ordinary gravitational theory is determined to be $(-65'' \pm 10'')/\text{cy}^2$. The formal mean error is given. This result was subjected to significance tests and vulnerability to various known sources of systematic error. Some of these are discussed in the Appendix. As a result of such tests, a confidence level more realistic than the formal mean error was determined, giving

$$\dot{n}_M{}^{AT} = (-65'' \pm 18'')/\text{cy}^2 \tag{3}$$

for the acceleration utilizing Atomic Time.

THE RATE OF CHANGE OF *G*

Differencing the lunar acceleration utilizing Atomic Time, equation (3), and the acceleration utilizing Ephemeris Time, equation (2), we obtain the acceleration presumed due to a changing gravitational constant, that is $(-27'' \pm 19'')/\text{cy}^2$. Taking this result, call it $\dot{n}_M{}^G$, together with the lunar mean motion, $n_M = 17''\cdot33 \times 10^8/\text{cy}$, we obtain

$$\frac{1}{2}\frac{\dot{n}_M{}^G}{n_M} = (-0\cdot8 \pm 0\cdot5) \times 10^{-8}/\text{cy} = (-8 \pm 5) \times 10^{-11}/\text{yr}. \tag{4}$$

Finally, by equation (1), we have the result that $\dot{G}/G = \frac{1}{2}(\dot{n}_M{}^G/n_M)$. In what follows, we will use the symbol A for $\frac{1}{2}(\dot{n}_M{}^G/n_M)$, the observed numerical rate given by equation (4).

EXAMINATION OF ALTERNATIVE HYPOTHESES

In the next section, we will discuss evidence that a secular decrease in G is a reasonable hypothesis, and is in fact consistent with other observational data. In this section, we will consider the alternative possibilities for explaining the observed excess (negative) lunar acceleration when Atomic Time is utilized.

Examining the variation of Kepler's third law,

$$2\frac{\dot{n}}{n}+3\frac{\dot{a}}{a}=\frac{\dot{G}}{G}+\frac{\dot{M}}{M}, \tag{5}$$

where M is the mass of the two-body system being considered, we see that there are several possible interpretations of an observed non-zero value for $\dot{n}/n$. The interpretation preferred here with $\dot{G}/G \neq 0$ and $\dot{M}/M = 0$ is given by equation (1), and will subsequently be referred to as 'Hypothesis G'. However, there are other interpretations possible, including some for which $\dot{G}/G$ remains zero.

Let us first consider the case for which $\dot{G}/G = 0$ and $\dot{M}/M = A$. If all masses decrease at the same rate A, this situation is not meaningfully different from decreasing G. Indeed, all celestial mechanics experiments measure only the product GM, and cannot separate the two. If only the mass of the Earth (or Earth plus Moon) were decreasing at the rate A, then the acceleration given by equations (2) and (3) should be equal, which is clearly not the case. The most interesting possibility for mass variation is that only the mass of the Sun decreases at the rate A. Let us call this 'Hypothesis M (mass)'. In this case, the orbits of all the major planets expand, and their periods increase, as before; and therefore, Ephemeris Time will decelerate with respect to Atomic Time. However, the orbit of the Moon about the Earth would not expand, since the Earth's mass would not be changing significantly. Under this assumption, all of the acceleration of the Moon's longitude in equation (3) must be ascribed to tidal friction, or some such mechanism; and the acceleration in equation (2) is smaller because of the slowing of the Ephemeris Time scale. This hypothesis has the attractive feature that it would explain the discordance of the Spencer–Jones value for the lunar acceleration to the extent that it is actually dependent upon 17th century transits of Mercury, as discussed by Morrison (1973). To test Hypothesis M, consider the possible ways in which the Sun's mass could be decreasing. From $E = mc^2$ and the solar constant, we can compute that the rate of conversion of matter to energy in the Sun causes a mass loss of about 7×10^{-14} solar masses per year. (A solar mass is 2×10^{33} g.) Assuming that the solar wind is isotropic, the mass transported away from the Sun by the wind is at most about 3×10^{-13} solar masses per year. Using recent *Skylab* results that solar flares which remove 10^{-17} solar masses from the Sun per event occur with a frequency of one per 100 hr (Newkirk 1974, private communication), the loss rate might be 10^{-15} solar masses per year. Even the largest solar prominences on record are unlikely to remove more than about 10^{-16} solar masses. Therefore, the requirement of a mass loss rate of $A \approx 10^{-10}$/yr for the Sun would seemingly rule out Hypothesis M.

A second possible interpretation, in which $\dot{G}/G = 0$ and $\dot{M}/M = 0$, is $\dot{a}/a = -\frac{4}{3}A$. Call this 'Hypothesis S (space)'. This implies a uniform expansion of all space, including solid bodies, at the rate of $-\frac{2}{3}A$. For orbiting bodies only, the expansion rate for the semi-major axis is twice the radial expansion rate because of the increase in angular momentum. If we change the units of $-\frac{2}{3}A$ for comparison with the Hubble constant, we get 52 km^{-1} s^{-1} Mpc^{-1}. (The observed Hubble constant is apparently between 40 and 100 km^{-1} s^{-1} Mpc^{-1}.) In support of this hypothesis, Wesson (1973) has summarized observational data which suggest that the Earth has been expanding at about the Hubble rate throughout its existence. The hypothesis is made more attractive by the prediction that the Earth would originally have had only half its present radius, and that all of the present continents would then have formed a covering over the entire Earth. Sea-floor spreading is readily understandable with such an expanding model for the Earth. To test Hypothesis S, we need a direct measure of the rate of expansion of some orbit in the solar system. The laser ranging project for the Moon will probably make such a measurement before radar ranging to the planets does. But at the present time, we cannot observationally distinguish between Hypotheses G and S. The principal reasons for preferring Hypothesis G are the lack of a comprehensive cosmological theory incorporating Hypothesis S, and the continuous increase in angular momentum of all orbiting and rotating bodies which it implies.

A third possibility is that the length of the Atomic second is changing relative to uniform time. Call this ' Hypothesis T (time) '. In essence, we are postulating a secular increase in the number of oscillations of the caesium atoms per Ephemeris second (here, of constant length). This would seem to require a variation in some fundamental constant of physics other than G. Moreover, it is probably immaterial whether we consider the Atomic second as constant and other physical constants as varying, or vice versa.

Dicke (private communication) has pointed out that variable tidal friction is another possible explanation—' Hypothesis F '. This is quite true. However, variation by a factor of 50 per cent within a few decades is not likely. This can be stated with even greater confidence in the light of the new Muller & Stephenson results, which show that the tidal deceleration has remained essentially constant for over 3000 yr. Any extremely short term variations should average out over the Moon's nodal period of 18·6 yr.

There are an unlimited number of possible ways in which Kepler's law can be violated, due to the action of small forces which have not been considered in the equations of motion. In this category we might consider solar radiation pressure, solar wind, magnetic forces, meteoritic impacts, passing comets, the tidal action of the Sun on the Earth, and undiscovered masses in the solar system. In each of the cases just mentioned, estimates of the reasonable upper limit for the size of these effects are negligible. None the less, the main reason for preferring Hypothesis G to the ' unmodelled force ' hypothesis is the supporting evidence which follows.

DISCUSSION OF SUPPORTING EVIDENCE

Hoyle (1972) has reviewed much of the evidence in support of Hypothesis G, that the gravitational constant is decreasing at about a part in 10^{10} per year. Perhaps the most persuasive implication is that of conservation of angular momentum in the Earth–Moon system. If we take Muller & Stephenson's (1974) results, for example, then $10^9(\dot{\omega}/\omega)$ for the Earth's rotation is about (-29 ± 3)/cy, when measured utilizing Ephemeris Time. (Newton's (1973) result was -25 ± 3.) This breaks down into a frictional part of (-44 ± 5)/cy from the tidal action of the Sun and Moon, and a non-frictional part of $(+15 \pm 6)$/cy of unknown origin. (The frictional part comes from equation (2), by applying Newton's relation

$$10^9(\dot{\omega}/\omega) = 1{\cdot}165\, \dot{n}_M{}^{ET},$$

as required to conserve angular momentum.) When Atomic Time is utilized instead, observed values of $\dot{\omega}/\omega$ utilizing Ephemeris Time must be corrected by $2A$, where A is given by equation (4). (The slowing of the Earth's rotation due to the expansion which accompanies a decreasing G is a much smaller effect.) Hence, Muller & Stephenson's results for $10^9(\dot{\omega}/\omega)$ would change from (-29 ± 3)/cy to (-45 ± 10)/cy, in excellent agreement with the frictional part of (-44 ± 5)/cy. No unexplained ' non-frictional ' change in the Earth's rotation remains.

Other arguments cited by Hoyle in support of Hypothesis G include the thermal history of the Earth, which would have been much hotter in its early history than at present (consistent with the evolutionary trend of maximum survival temperatures for the most primitive life forms on Earth); the calculation that the initial helium content of the Sun would have to be about 15 per cent rather than about 25 per cent to reach its present-day luminosity (consistent with solar wind measures that the outer solar layers, which are believed to have remained essentially unchanged over the lifetime of the solar system, contain 15 per cent helium); and the prediction of a blue, starlike class of quasi-stellar galaxies (such as recently found by Sandage) due to an enhancement of the rate of stellar evolution as a function of mass.

Wesson (1973) cites further geophysical evidence, including an illuminating discussion of continental drift and sea-floor spreading. If G is decreasing, the Earth must expand somewhat due to the decreased weight of the surface layers, but at perhaps only 10–20 per cent of the rate in Hypothesis S. Wesson points out that the expanding globe model makes it easier to understand how a mid-oceanic ridge system can almost surround a continent like Antarctica.

Morganstern (1972) has examined a pressure-filled curved-space cosmological situation, and finds that positively-curved spaces with pressure predict a $\dot{G}/G$ rate in agreement with equation (4).

Newton (1968) also came to the conclusion that the Earth's polar moment of inertia is increasing and that G is decreasing at rates quite consistent with equation (4), and with the rate of upward transport of material at the mid-ocean ridges.

It should also be mentioned that if Dicke (1966) had used the modern data for $\dot{n}_M^{ET}$, instead of the Spencer–Jones value, in his study of the Earth's rotational history, his estimated $\dot{G}/G$ values would have been in good agreement with the present result, instead of the smaller value suggested by the Scalar-tensor theory (Brans & Dicke 1961).

Cosmological theories consistent with Hypothesis G are those of Hoyle–Narlikar (1972), Dirac (1973), and Brans-Dicke (1961). These all have the important features that they are nearly consistent with general relativity in so far as the solar light bending and Mercury perihelion advance tests are concerned; and they are characterized by a properly integrated form of Mach's principle. In the Hoyle–Narlikar theory, equation (4) gives twice the Hubble expansion rate. Therefore, the Hubble constant implied by equation (4) would be (39 ± 24) km^{-1} s^{-1} Mpc^{-1}. For the other two cosmologies, the Hubble constant would equal the solar system expansion rate of (78 ± 49) km^{-1} s^{-1} Mpc^{-1}, according to equation (1). These are in reasonable agreement with some of the most recent determinations of that parameter. Sandage (1972), for example, has obtained (55 ± 7) km^{-1} s^{-1} Mpc^{-1}. The Hubble age of the Universe, from equation (4), would be $(12^{+21}_{-4}) \times 10^9$ yr.

FUTURE OBSERVATIONAL DATA

In the solar system, many of the observational consequences of a decreasing G increase with time squared. In addition, correlations between accelerations and other parameters decrease rapidly with time. In the case of lunar occultation data, the time base begins in 1955, when Atomic Time came into use. About two more years of data will result in halving the mean error of the present determination.

Independent determinations are now nearly possible from planetary radar ranging results. A determination by Shapiro *et al.* (1971) set an upper limit of $\dot{G}/G \leqq 4 \times 10^{-10}$/yr. A more recent estimate (Shapiro 1974) from Mercury ranging had decreased this estimate to $(+4 \pm 8) \times 10^{-11}$/yr, but with the separation of the acceleration from the topography of Mercury not yet complete. The lunar laser ranging program is also very close to obtaining a lunar acceleration measure, and should have preliminary results by the end of 1974. Moreover, some laboratory experiments are now contemplated for the next few years which can reach better than 1 part in 10^{11}/yr.

On the theoretical front, the consequences of G varying must be reviewed carefully in the fields of geophysics, astrophysics and cosmology for possible contradiction with observational data, as in the discussion of the Hoyle–Narlikar theory by Barnothy & Tinsley (1973). Since the possibility of G decreasing has now become of significant probability, it will be important to make the effort to incorporate varying G into stellar evolution calculations, for example, as was done by Pochoda & Schwarzschild (1964). If the G Hypothesis is contradicted, then alternate hypotheses, especially Hypothesis S, should be tested.

CONCLUSION

If Atomic Time is a uniform time scale, then the observed discordance between the acceleration of the Moon's mean longitude utilizing Ephemeris Time with the same acceleration utilizing Atomic Time necessarily implies a deceleration in the Ephemeris Time scale; or equivalently, there must be an unmodelled acceleration of the Sun's apparent annual motion about the Earth. If Ephemeris Time, T_E, and Atomic Time, T_A, are adjusted so as to be equal in epoch and rate at some moment T_0 (with time, T, in centuries), then

$$T_E - T_A = A(T - T_0)^2, \tag{6}$$

where A is given by equation (4). If we wish to express the difference $(T_E - T_A) \equiv \Delta T_A$ in seconds, we would have

$$\Delta T_A = -25^s{\cdot}25(T - T_0)^2. \tag{7}$$

The corresponding acceleration in the Sun's mean longitude utilizing Atomic Time is given by

$$\dot{n}_S{}^{AT} = -2''{\cdot}07/\text{cy}^2, \tag{8}$$

$\dot{n}_S{}^{ET}$ being zero by definition. The evidence presented here suggests that the most probable cause of the anomalous accelerations is a secular decrease in the Universal Gravitational Constant, G, at the rate of $(-8 \pm 5) \times 10^{-11}$/yr. This interpretation of the cause is not compelling; but it seems to be strongly indicated, since the anomalous accelerations almost certainly have a cosmological cause, and since there is other supporting evidence.

ACKNOWLEDGMENTS

The author is indebted to Dr John Vinti, whose presentation before the Division on Dynamical Astronomy of the American Astronomical Society first directed the author's attention to the possibility of explaining the anomalous accelerations by a decrease in the gravitational constant; to Professor Sir Fred Hoyle and Mr Leslie Morrison, whose papers showed how gravitational acceleration could be distinguished from a tidal acceleration; to Drs J. D. Mulholland, C. Oesterwinter, C. Cohen and J. Williams for making the numerical integrations used in this analysis available to the author; to Drs Evans and Dunham of the University of Texas at Austin, for supplying their latest photoelectric occultation timings in advance of publication; and to the many other individuals whose comments have materially influenced the contents of this paper.

US Naval Observatory, Washington DC 20390

Received in original form 1974 *May* 20

REFERENCES

Barnothy, J. M. & Tinsley, B. M., 1973. A critique of Hoyle and Narlikar's new cosmology, *Astrophys. J.*, **182**, 343.
Bender, P. L. *et al.*, 1973. The lunar laser ranging experiment, *Science*, **182**, 229.
Brans, C. & Dicke, R. H., 1961. Mach's principle and a relativistic theory of gravitation, *Phys. Rev.*, **124**, 925.
Clemence, G. M., 1948. On the system of astronomical constants, *Astr. J.*, **53**, 169.
Dicke, R. H., 1966. *The secular acceleration of the Earth's rotation and cosmology in the Earth–Moon system*, eds, B. G. Marsden & A. G. W. Cameron, Plenum Press.
Dirac, P. A. M., 1973. Long range forces and broken symmetries, *Proc. R. Soc. Lond. A*, **333**, 403.
Garthwaite, K., Holdridge, D. B. & Mulholland, J. D., 1970. A preliminary special perturbation theory for the lunar motion, *Astr. J.*, **75**, 1133.
Hoyle, F., 1972. The Presidential Address, The history of the Earth, *Q. Jl R. astr. Soc.*, **13**, 328.
Hoyle, F. & Narlikar, J. V., 1972. Cosmological models in a conformally invariant gravitational theory—II. A new model, *Mon. Not. R. astr. Soc.*, **155**, 323.
Morganstern, R. E., 1972. Curved space cosmological bounds on the time variation of G, *Nature*, **237**, 70.
Morrison, L., 1972a. The secular accelerations of the Moon's orbital motion and the Earth's rotation, *The Moon*, **5**, 253.
Morrison, L., 1972b. The role of occultations in the improvement of the lunar ephemeris, Urey and Runcorn (eds), *The Moon*, 395.
Morrison, L., 1973. Rotation of the Earth from AD 1663–1972 and the constancy of G, *Nature*, **241**, 519.
Muller, P. M. & Stephenson, F. R., 1974. The accelerations of the Earth and Moon from early astronomical observations, in *Growth rhythms and history of the Earth's rotation*, eds G. D. Rosenberg and S. K. Runcorn, John Wiley & Sons, London
Murray, C. A., 1957. The secular acceleration of the Moon, and the lunar tidal couple, *Mon. Not. R. astr. Soc.*, **117**, 478.
Newton, R. R., 1968. Experimental evidence for a secular decrease in the gravitational constant G, *J. geophys. Res.*, **73**, 3765.
Newton, R. R., 1969. Secular accelerations of the Earth and Moon, *Science*, **166**, 825.
Newton, R. R., 1973. The historical acceleration of the Earth, *Geophys. Surv.*, **1**, 123.
Oesterwinter, C. & Cohen, C. J., 1972. New orbital elements for Moon and planets, *Celest. Mech.*, **5**, 317.
Pochoda, P. & Schwarzschild, M., 1964. Variation of the gravitational constant and the evolution of the Sun, *Astrophys. J.*, **139**, 587.
Sandage, A. R., 1972. Distance to the galaxies: The Hubble constant, the Friedmann time, and the edge of the world, *Q. Jl R. astr. Soc.*, **13**, 282.
Shapiro, I. I., Smith, M. B., Ash, M. B., Ingalls, R. P. & Pettengill, G. H., 1971. Gravitational constant: experimental bound on its time variation, *Phys. Rev. Lett.*, **26**, 273.

Shapiro, I. I., 1974. Paper delivered to 5th Cambridge Conference on Relativity, June 10.
Spencer-Jones, H., 1939. The rotation of the Earth, and the secular accelerations of the Sun, Moon and planets, *Mon. Not. R. astr. Soc.*, **99**, 541.
Van Flandern, T. C., 1970. The secular acceleration of the Moon, *Astr. J.*, **75**, 657.
Van Flandern, T. C., 1971. Lunar ephemeris and astrometric corrections from occultations, *Astr. J.*, **76**, 81.
Vinti, J. P., 1974. Classical solution of the two-body problem if the gravitational constant diminishes inversely with the age of the Universe, *Mon. Not. R. astr. Soc.*, **169**, 417.
Wesson, P., 1973. The implications for geophysics of modern cosmologies in which G is variable, *Q. Jl R. astr. Soc.*, **14**, 9.

APPENDIX

This analysis utilizes a numerically-integrated lunar ephemeris in the comparison with occultation observations. The integrated ephemeris originally adopted was that of Garthwaite, Holdridge & Mulholland (1970). This will be referred to subsequently as LE16. The result for $\dot{G}/G$ originally obtained, using LE16, was $(-12 \pm 3) \times 10^{-11}$/yr. It was pointed out by a number of people that the validity of LE16 was of crucial importance to the principal result of this paper. Furthermore, there was evidence in the literature (see, e.g. Oesterwinter & Cohen 1972; Bender *et al.* 1973) that there were discrepancies between LE16 and the Oesterwinter & Cohen lunar integration, which we will call NWL. Therefore, a comparison of the two integrations was performed.

The differences NWL–LE16 were formed during the 23-yr period (1950–73) common to the two integrations. The effects of completely different starting conditions or elements for both the Earth and the Moon had to be removed, along with the effects of differing adopted masses and other constants, utilizing analytical partial derivatives. The remaining signal was still highly systematic. A power spectrum analysis was then performed to identify the precise periodicities, which in turn led to an understanding of their causes.

It was found that LE16 had used a force function for the perturbations on the Moon due to the oblateness of the Earth which omitted the effects of the finite mass of the Moon. This had been discovered earlier by Jim Williams at the Jet Propulsion Laboratory in connection with shorter numerical integrations used for the laser ranging project. Moreover, it was found that the NWL integration had omitted the effect of the precession of the Earth's figure, when calculating the same perturbations. After correction for these two effects, the remaining differences between the two integrations were all numerically less than $0''\cdot02$.

Since a problem had arisen with each of the integrations, a third comparison integration was considered to be of significant value. The only integration resulting from the laser ranging project which had been extended over a sufficiently long time period was LE25, which was kindly supplied by Jim Williams. The differences LE25–LE16 were formed over the interval 1950–75, and analysed as before. The results differed from the other two integrations in the crucial acceleration term. Consultations with Jim Williams again led to the cause—in this case, a libration theory truncated too soon in LE25, so that the mean orientation of the lunar bulge toward the Earth was not exactly in the mean Earth direction. This problem was not yet of any significant importance to the laser ranging project. After applying this correction, the three integrations mutually agreed to within $0''\cdot02$.

The three force function problems, discovered in this analysis (one in each integration) resulted in discordances which looked like $\sin \Omega$, $\cos \Omega$, and T^2, respectively, where Ω is the mean longitude of the Moon's ascending node on the ecliptic (period 18·6 yr). Because of their long period, both Ω terms are correlated with the T^2 term, which measures the acceleration. Hence, all three problems would have influenced the determination of $\dot{G}/G$.

Although the full effect of each of these known problems has been removed, and the three integrations now agree with each other, it is clear in hindsight that the original mean error estimate ($\pm 3 \times 10^{-11}$/yr) did not adequately allow for such possible sources of systematic error. A mean error of ($\pm 5 \times 10^{-11}$/yr) would have covered each of the errors in $\dot{G}/G$ resulting from force function problems. Moreover, systematic errors in star positions also have the potential to introduce $\sin \Omega$ terms, because the Moon occults differing declination zones of stars with an Ω period. Therefore, based upon the above experience, and upon detailed studies of possible systematic error sources in the observations, the author believes that ($\pm 5 \times 10^{-11}$/yr) is a more realistic mean error than the formal solution mean error.