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THE DISTRIBUTION OF PERIODS OF THE MAGNETIC A-TYPE STARS

SIDNEY C. WOLFF
Institute for Astronomy, University of Hawaii

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ABSTRACT

Four-color (*uvby*) photoelectric observations are used to obtain periods for several Ap stars with $v \sin i < 10 \text{ km s}^{-1}$. The derived periods are 69.0 days for HD 5797, 10.61 days for HD 22374, 69 days for HR 3724, and 5.07 days for HD 115708. The magnetic field strengths measured from Zeeman spectrograms of 33 Lib vary with a period of 23.26 days. Photoelectric and Zeeman spectroscopic observations yield a period of 525 days for HD 2453. Evidence is given that suggests that HR 4816 may vary on a time scale of 10 years.

The present observations, combined with results published previously, make it possible to derive the distribution of periods for the cool (non-Si) Ap-type stars. It is argued that the data support the hypothesis that the period of variation is to be identified in all cases with the period of rotation, and that the existence of Ap stars with extremely long periods is due to the loss of angular momentum through some form of magnetic braking.

Subject headings: magnetic stars — peculiar A stars — rotation, stellar

I. INTRODUCTION

The oblique dipole rotator model has provided a satisfactory explanation for many of the observed properties of Ap stars. Among its more notable successes are its ability to account for the period–line width relation, the crossover effect, and the phase relationship between spectrum and radial-velocity variations. One recent significant test has been the successful prediction of the order of magnitude of the surface field in β Crb, given the observed longitudinal field and the derived angles between the line of sight and the rotation axis and between the rotation and magnetic axes (Preston 1971).

The discovery that HR 465 is periodic with a period of 22–24 years (Preston and Wolff 1970) is of great significance for the oblique dipole rotator model. The variable properties of HR 465 are quite similar to those observed in Ap stars with shorter periods. The Eu and Cr lines vary in antiphase, as is typical of many Eu-Cr stars; the Eu lines are at maximum strength when the star is reddest, as is true also for such stars as 73 Dra (Preston 1967; Stępień 1968) and HD 221568 (Osawa 1967); the extrema of the spectrum, magnetic, and photometric variations coincide in phase, as is the general rule. The magnetic field, which varies from about +1000 to −1000 gauss, is not atypical in strength, and the spectral peculiarities, while remarkable, occur to almost the same degree in some stars of shorter period (e.g., Jones *et al.* 1974). Because of these similarities, it seems likely that the same basic mechanism that is at work in the shorter period Ap stars must cause the variations of HR 465 as well, and that we must either accept that the rotation period of HR 465 is about 23 years, or else we must abandon the oblique rotator model for all Ap stars.

The extremely long period of HR 465 is not unique, since Babcock (unpublished) had previously found a period of 6.7 years for HD 187474. In an effort to encourage the search for additional Ap stars with very long periods, Preston (1970*b*) published a list of all the Ap stars (excluding Hg-Mn stars) known to him to have $v \sin i < 10 \text{ km s}^{-1}$; on the oblique dipole rotator hypothesis, only stars with such sharp lines can have periods greater than about 16 days. Wolff and Morrison (1973) have already published photoelectric observations for about half the stars in Preston's list. Observations have now been obtained for all but one of the remaining sharp-line stars for which no previous determinations of period were available. The new observations, which are described in the present paper, make it possible to derive the period-frequency distribution for the Ap stars.

II. OBSERVATIONS

All of the photometric observations, which are on the four-color (*uvby*) system, were obtained with a 24-inch (61 cm) telescope on Mauna Kea, and were made and reduced with techniques that have been described elsewhere (Wolff and Morrison 1973). Two comparison stars were used for each variable, and their mean colors are given in Table 1. The standard deviation of a single differential magnitude, i.e., of a measurement of the brightness of the variable star relative to the comparison stars, ranges from about ±0.004 mag for the brighter stars to about ±0.007 mag for the faintest ones. The standard deviations of the transformed colors, as estimated from a comparison of the measurements in Table 1 with those of other observers, are about ±0.02 mag in V, ±0.01 in $b - y$, and ±0.015 in m_1 and c_1. The technique described by Lafler and Kinman (1965) was used to derive periods from the photometric data.

TABLE 1

Photometric Data for Comparison Stars

Name	V	$b - y$	m_1	c_1
HR 44.........	6.218	−0.012	+0.166	+1.018
HR 71.........	5.878	−0.014	+0.146	+1.079
HD 5380.......	7.632	+0.033	+0.090	+0.774
HD 5813.......	7.110	+0.094	+0.190	+1.097
HR 1086.......	5.960	+0.073	+0.158	+1.129
HR 1137.......	6.089	+0.000	+0.186	+1.003
HR 3702.......	6.614	+0.044	+0.178	+1.120
HR 3744.......	6.507	+0.031	+0.146	+1.107
HR 5057.......	5.736	+0.037	+0.186	+1.074
HR 114520.....	6.819	+0.267	+0.169	+0.578

For three of the stars, HD 2453, HD 22374, and HD 137949, a limited number of Zeeman spectrograms (dispersion 6.7 Å mm^{-1}) have been obtained with the coudé spectrograph of the 2.2 m telescope on Mauna Kea. Since there are no plans to continue the spectroscopic observations of these stars, the available data are presented here. About 30–50 lines on each spectrogram were measured in the usual way (Wolff and Bonsack 1972) in order to derive magnetic field strengths and radial velocities. Typical internal standard deviations for the magnetic field strengths are ±75 gauss, and for the radial velocities, ±0.3 km s^{-1}.

III. DESCRIPTION OF INDIVIDUAL STARS

a) *HD 2453*

Photometry of HD 2453 obtained in 1970 and 1971 (Wolff and Morrison 1973) suggested that this star is probably variable on a time scale of about two years. In an effort to establish the period of HD 2453, additional photometric observations were made in 1972; measurements were also made of several Zeeman spectrograms obtained at Lick Observatory by George Preston and at Mauna Kea.

The magnitudes and colors of HD 2453 were derived from differential photometry relative to two comparison stars, for which the colors derived from the 1971 observations were adopted. The results for the three observing seasons are given in Table 2. The values given are means of all the observations in each year. The only clearly significant variation is in c_1, and an examination of the individual c_1 indices (see Table 3) indicates that the most likely period for HD 2453 is about 1.5 years.

The magnetic field data, which are given in Table 4, confirm the photometric period. The amplitude of the magnetic variation is slightly larger than 700 gauss, with no reversal in sign, and all of the observations, both from Lick and Mauna Kea, can be represented by the elements

$$\mathrm{HJD\ (magnetic\ minimum)} = 2{,}442{,}288 + 525E\,. \quad (1)$$

In Figure 1 are plotted the observed magnetic field intensities and the c_1 indices according to these elements. (Since the c_1 observations in 1971 and 1972 were made with a better data acquisition system [Wolff and Morrison 1973] than was available in 1970, the later observations exhibit less scatter and are more accurate.) There may be a slight phase shift between magnetic and photometric extrema, a result that is also found in HD 188041 (Babcock 1954; Jones and Wolff 1973), but additional observations are required to establish the details of the variations. Nevertheless, the fact that both sets of data, photometric and spectroscopic, can be represented by the same period suggests that 525 days is indeed approximately the correct period for HD 2453. Visual inspection of the spectrograms indicates that there are no conspicuous spectrum variations.

TABLE 2

Results for HD 2453 from Three Observing Seasons

Year	V	$b - y$	m_1	c_1
1970...........	6.877	+0.015	+0.252	+0.924
1971...........	6.867	+0.020	+0.273	+0.878
1972...........	6.870	+0.016	+0.252	+0.906

TABLE 3

c_1 Data for HD 2453

HJD 2,440,000+	Phase	c_1
784...........	0.14	+0.919
788...........	0.14	+0.916
790...........	0.15	+0.929
821...........	0.21	+0.938
822...........	0.21	+0.935
833...........	0.23	+0.922
860...........	0.28	+0.915
902...........	0.36	+0.928
1170...........	0.87	+0.868
1171...........	0.87	+0.874
1184...........	0.90	+0.864
1186...........	0.90	+0.874
1204...........	0.94	+0.876
1206...........	0.94	+0.872
1234...........	0.99	+0.892
1246...........	0.02	+0.882
1528...........	0.55	+0.922
1530...........	0.56	+0.919
1591...........	0.67	+0.895
1594...........	0.68	+0.898
1612...........	0.71	+0.890

b) *HD 5797*

Photometric observations of HD 5797 were obtained during two observing seasons, and are listed in Table 5. The photometry indicates that the period is between 40 and 70 days, with the best representation of the data being given by the elements

$$\mathrm{HJD}\ (u_{\mathrm{max}}) = 2{,}441{,}206 + 69\overset{\mathrm{d}}{.}0E\,. \quad (2)$$

Periods of 45.5, 57, and 67.5 days, although not as satisfactory as 69 days, cannot be eliminated on the basis of these data.

TABLE 4

Magnetic Field Data for HD 2453

Plate Number	HJD 2,430,000+	Phase	H_e (gauss)	V(km s^{-1})
ECZ 5094....	9333.98	0.37	− 420	−19.7
5116....	9339.00	0.38	− 280	−18.0
5143....	9342.91	0.39	− 440	−20.2
5156....	9343.98	0.39	− 250	−18.1
5213....	9371.77	0.45	− 360	−21.5
5269....	9402.77	0.50	− 530	−20.0
5372....	9453.61	0.60	− 610	−20.5
5383....	9454.58	0.60	− 520	−18.6
5409....	9456.77	0.61	− 610	−18.5
5414....	9459.58	0.61	− 750	−20.9
5462....	9491.61	0.67	− 740	−21.3
5838....	9720.96	0.11	− 760	−20.4
5851....	9721.97	0.11	− 650	−20.7
KE 577....	11536.10	0.57	− 700	−21.8
592....	11564.00	0.62	− 560	−20.9
1035....	11844.08	0.14	− 520	−20.2
1048....	11845.08	0.16	− 630	−19.9
1131....	11935.96	0.33	− 530	−19.8
1183....	11947.97	0.35	− 480	−19.5
1542....	12224.09	0.88	− 860	−18.1
1660....	12288.00	0.00	−1030	−17.8

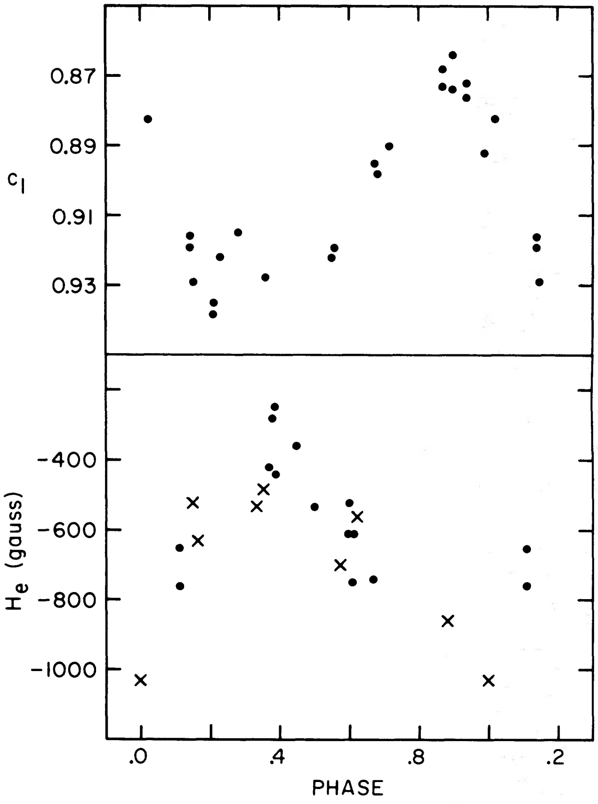

Fig. 1.—Variations in c_1 and magnetic field for HD 2453 plotted according to the ephemeris HJD (magnetic minimum) = 2,442,288 + 525E. In the lower panel, filled circles and crosses represent observations made at Lick and Mauna Kea, respectively.

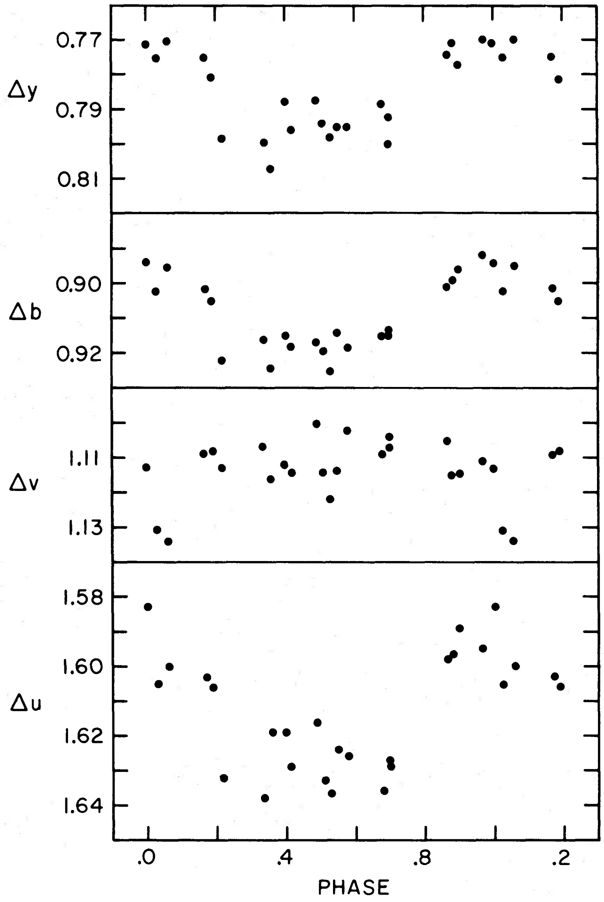

Fig. 2.—Photometric observations of HD 5797 plotted according to the ephemeris HJD ($u_{\rm max}$) = 2,441,206 + 69$^{\rm d}$0E. Magnitude differences are in the sense HD 5797 − HD 5380.

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TABLE 5

PHOTOMETRIC DATA FOR HD 5797

HJD 2,440,000+	PHASE	HD 5797 − HD 5380			
		Δy	Δb	Δv	Δu
1171.07......	0.49	+0.787	+0.917	+1.110	+1.616
1172.06......	0.51	+0.794	+0.919	+1.115	+1.633
1185.09......	0.70	+0.800	+0.913	+1.107	+1.629
1204.06......	0.97	+0.770	+0.892	+1.111	+1.595
1206.08......	0.00	+0.771	+0.894	+1.113	+1.583
1208.46......	0.04	+0.775	+0.902	+1.131	+1.605
1210.03......	0.06	+0.770	+0.895	+1.134	+1.600
1233.90......	0.40	+0.788	+0.915	+1.112	+1.619
1234.94......	0.42	+0.796	+0.918	+1.114	+1.629
1242.93......	0.54	+0.798	+0.925	+1.122	+1.637
1243.94......	0.55	+0.795	+0.914	+1.114	+1.624
1530.11......	0.70	+0.792	+0.915	+1.104	+1.627
1563.05......	0.17	+0.775	+0.901	+1.109	+1.603
1563.98......	0.19	+0.781	+0.905	+1.108	+1.606
1591.04......	0.58	+0.795	+0.918	+1.102	+1.626
1610.99......	0.87	+0.774	+0.901	+1.105	+1.598
1611.96......	0.88	+0.771	+0.899	+1.115	+1.597
1612.96......	0.90	+0.777	+0.896	+1.115	+1.589
1643.84......	0.35	+0.800	+0.916	+1.107	+1.638
1644.81......	0.36	+0.807	+0.924	+1.116	+1.619
1874.12......	0.68	+0.788	+0.915	+1.109	+1.636
1911.04......	0.22	+0.798	+0.922	+1.113	+1.632

The light curves for HD 5797 are shown in Figure 2. The *y*, *b*, and *u* curves are in phase, with the largest amplitude in *u*. The *v* curve may be in antiphase to the other three curves, but the scatter in the data is large enough to prevent a definite conclusion. The unusually large scatter is due to the relative faintness ($V = 8.4$) of HD 5797.

c) HD 22374

From *uvby* photometry obtained in 1970, Wolff and Morrison (1971) suggested that the period of HD 22374 (=9 Tau) was 10.6 days. This period was subsequently confirmed by Winzer (1974), who derived the elements:

$$\mathrm{HJD}\ (U_{\max}) = 2{,}441{,}252.12 + 10\overset{\mathrm{d}}{.}61E\,. \quad (3)$$

Table 6 lists *uvby* observations of HD 22374, and the resulting light curves are shown in Figure 3. These light curves agree well in shape and amplitude with those that Winzer obtained from *UBV* photometry, except that he found a secondary minimum in the *V* curve only, near phase 0.8. The amplitude of the variability is quite small (~0.02 mag) at all wavelengths observed, and as a consequence the period must be considered as tentative, despite the generally good agreement of the two sets of observations.

Zeeman spectrograms of HD 22374 were obtained at phases 0.86, 0.94, 0.04, and 0.63. There is no evidence of spectroscopic or magnetic variations, and the average field strength, which is +230 gauss, and the average radial velocity, which is -1.0 km s^{-1}, agree well with the values of +140 gauss and -0.2 km

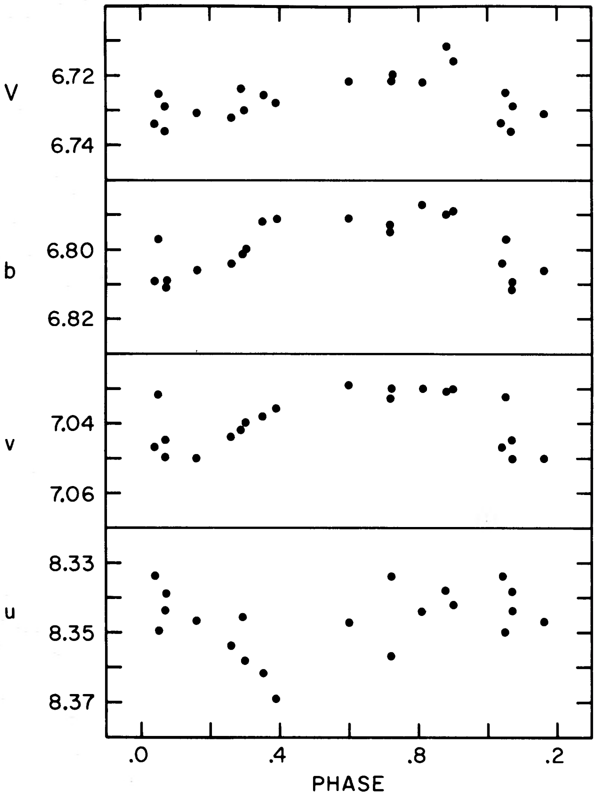

FIG. 3.—Photometric observations of HD 22374 plotted according to the ephemeris HJD ($U_{\max}$) = 2,441,252.12 + $10\overset{\mathrm{d}}{.}61$E.

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TABLE 6

PHOTOMETRIC DATA FOR HD 22374

HJD 2,440,000+	PHASE	HD 22374 − HR 1086			
		V	$b-y$	m_1	c_1
839.03......	0.07	6.736	+0.074	+0.162	+1.062
845.99......	0.72	6.721	+0.071	+0.166	+1.067
851.98......	0.29	6.724	+0.077	+0.164	+1.063
860.00......	0.04	6.734	+0.075	+0.163	+1.049
865.96......	0.60	6.722	+0.069	+0.170	+1.078
899.00......	0.72	6.720	+0.073	+0.166	+1.085
899.97......	0.81	6.722	+0.065	+0.178	+1.071
900.93......	0.90	6.716	+0.073	+0.168	+1.071
953.76......	0.88	6.712	+0.078	+0.163	+1.066
955.79......	0.07	6.729	+0.081	+0.158	+1.047
956.78......	0.16	6.731	+0.075	+0.170	+1.051
1223.03......	0.26	6.732	+0.071	+0.170	+1.069
1224.00......	0.35	6.726	+0.065	+0.180	+1.077
1234.09......	0.30	6.730	+0.069	+0.171	+1.077
1235.09......	0.39	6.728	+0.063	+0.180	+1.089
1242.04......	0.05	6.725	+0.071	+0.162	+1.081

s^{-1} derived by Babcock (1958) from a single spectrogram.

d) *HR 3724*

The photometric observations (Table 7) of HR 3724, which span the interval 1973 January–May, exhibit two pronounced minima in v. Superposition of these minima leads to the elements

$$\text{HJD}\ (v_{\min}) = 2{,}441{,}783 + 69\text{E}\,. \qquad (4)$$

The observations are equally compatible with a period of half this value; since no observations were made in the time interval HJD 2,441,733–2,441,766, it is impossible to determine whether the correct period is 69 or 34.5 days.

The observed light curves (see Fig. 4), which exhibit a large amplitude in v, smaller but in-phase variation in u, and little variation in b and y, are similar to those observed for HD 188041 (Jones and Wolff 1973), HD 71866 (Wolff and Wolff 1971), and HR 5153 (Wolff and Morrison 1975). The first two of these stars are pronounced rare-earth variables, while HR 5153 exhibits marked variations in Cr line strengths. On the basis of the photometry of HR 3724, one would expect that it, too, should be a conspicuous spectrum variable with either rare-earth maximum or Cr minimum coinciding in phase with v minimum.

e) *HR 4816*

Eight observations of HR 4816 (HD 110066) were obtained during the spring of 1973 (two observations

TABLE 7

PHOTOMETRIC DATA FOR HR 3724

HJD 2,440,000+	PHASE	HR 3724 − HR 3702			
		Δy	Δb	Δv	Δu
1691.09......	0.67	−0.111	−0.061	+0.077	−0.097
1694.04......	0.71	−0.112	−0.062	+0.072	−0.102
1695.09......	0.73	−0.110	−0.062	+0.072	−0.099
1697.09......	0.75	−0.111	−0.063	+0.068	−0.098
1698.08......	0.77	−0.108	−0.060	+0.075	−0.091
1706.04......	0.88	−0.106	−0.054	+0.093	−0.095
1711.08......	0.96	−0.111	−0.048	+0.115	−0.092
1730.89......	0.24	−0.106	−0.063	+0.070	−0.101
1732.00......	0.26	−0.113	−0.058	+0.073	−0.098
1732.90......	0.27	−0.112	−0.061	+0.074	−0.099
1766.81......	0.77	−0.107	−0.061	+0.074	−0.098
1767.81......	0.78	−0.111	−0.059	+0.073	−0.106
1773.87......	0.87	−0.107	−0.056	+0.092	−0.092
1774.82......	0.88	−0.107	−0.054	+0.099	−0.099
1782.80......	0.00	−0.109	−0.043	+0.127	−0.083
1784.81......	0.03	−0.109	−0.050	+0.117	−0.093
1786.81......	0.06	−0.107	−0.052	+0.107	−0.090
1789.81......	0.10	−0.106	−0.055	+0.091	−0.096
1793.81......	0.16	−0.106	−0.058	+0.075	−0.101
1796.82......	0.20	−0.115	−0.058	+0.075	−0.101
1798.82......	0.23	−0.107	−0.056	+0.073	−0.099

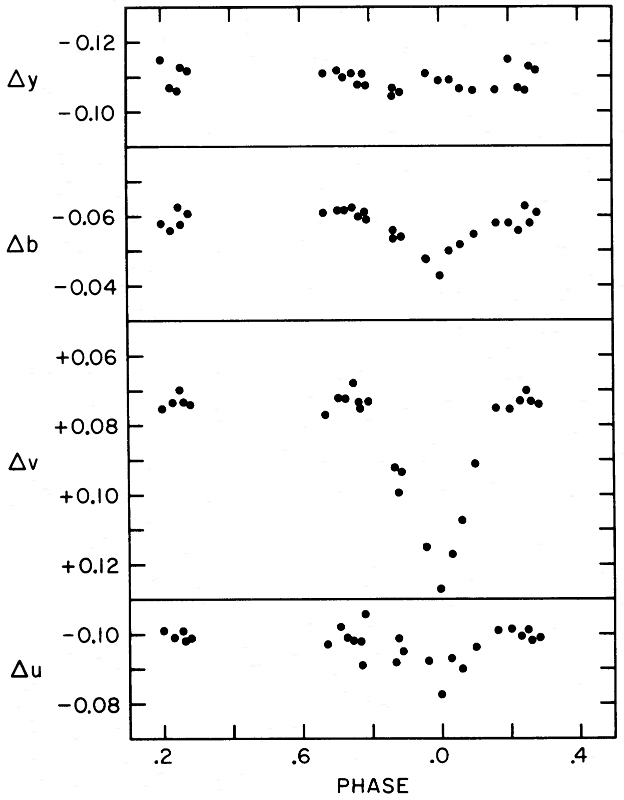

FIG. 4.—Photometric observations of HR 3724 plotted according to the ephemeris HJD ($v_{\min}$) = 2,441,783 + 69E. Magnitude differences are in the sense HR 3724 − HR 3702.

in February, one each in March and April, four in May), and there is no evidence of variability as large as 0.01 mag at any of the four wavelengths observed. Contemporaneous photometric measurements by Winzer (1974) and spectroscopic observations by Bonsack (1974) also indicate that HR 4816 does not exhibit variability on a time scale of several months.

An intercomparison of photometry obtained by various observers during the past 15 years suggests, however, that HR 4816 may be variable on a much longer time scale. Table 8 lists the published Strömgren indices, together with some additional measurements of V. The range of observed values of V exceeds 0.2 mag, and is unlikely to be due to observational error alone. The observations, which are plotted as a function of time in Figure 5, suggest that the maximum possible period for HR 4816 is about 20 years. If the apparent increases in brightness during the early 1960s and 1970s are real, then the most likely period is about 10 years. A program of regular monitoring of this star should be undertaken in order to establish whether it is, in fact, a variable with a period of several years.

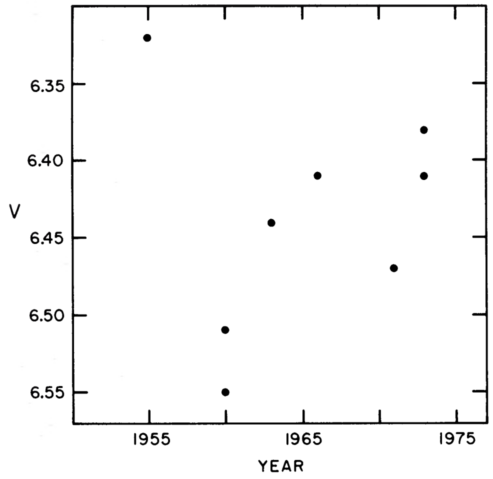

FIG. 5.—V magnitudes of HR 4816 plotted as a function of time.

f) *HD 115708*

The photometric observations of HD 115708, which were made on 23 nights in 1973, are listed in Table 9. Because the star is fairly faint ($V = 7.8$), the standard deviation of a single measurement ($\sim \pm 0.007$ mag) is about one-fourth the rather small amplitudes observed in v and u, where the variability is most conspicuous. The data are best represented by the elements

$$\mathrm{HJD}\,(v_{\min}) = 2{,}441{,}731.0 + 5.0E\,, \qquad (5)$$

and the resulting light curves are shown in Figure 6. A number of other periods less than 5 days, including 1.2422 and 2.536 days, cannot be ruled out on the basis of these observations, but periods longer than 10 days are definitely eliminated. The low $v \sin i$ of HD

TABLE 8

PHOTOMETRIC DATA FOR HR 4816

Date	V	$(b-y)$	m_1	c_1	Source
1955.....	6.32	...	...	...	Osawa (1959)
1960.....	6.55	...	...	...	Slettebak *et al.* (1961)
~1960.....	6.51	−0.002	+0.276	+0.862	Perry (1969)
1963.....	6.44	+0.004	+0.256	+0.898	Cameron (1966)
~1966.....	6.41	+0.014	+0.236	+0.904	Philip (1968)
1971.....	6.47	+0.027	+0.209	+0.907	Warren (1973)
1973.....	6.38	+0.019	+0.236	+0.891	Present paper
1973.....	6.41	...	...	...	Winzer (1974)

TABLE 9

Photometric Data for HD 115708

HJD 2,440,000+	Phase	HR 115708 − HD 114520			
		Δy	Δb	Δv	Δu
1691.15......	0.14	+0.945	+0.840	+0.755	+0.846
1694.13......	0.73	+0.957	+0.849	+0.750	+0.846
1697.14......	0.32	+0.960	+0.850	+0.754	+0.846
1698.14......	0.52	+0.961	+0.853	+0.756	+0.848
1706.10......	0.09	+0.960	+0.850	+0.754	+0.846
1711.08......	0.07	+0.949	+0.846	+0.769	+0.859
1731.06......	0.01	+0.956	+0.842	+0.778	+0.853
1733.06......	0.41	+0.961	+0.850	+0.761	+0.842
1767.90......	0.28	+0.950	+0.838	+0.754	+0.842
1771.04......	0.90	+0.949	+0.844	+0.764	+0.849
1775.00......	0.68	+0.952	+0.836	+0.745	+0.840
1782.85......	0.23	+0.951	+0.834	+0.750	+0.844
1784.94......	0.64	+0.960	+0.845	+0.750	+0.839
1786.89......	0.02	+0.948	+0.848	+0.775	+0.862
1789.87......	0.61	+0.958	+0.845	+0.746	+0.842
1793.95......	0.42	+0.956	+0.849	+0.763	+0.847
1796.88......	0.99	+0.942	+0.848	+0.772	+0.863
1798.85......	0.38	+0.959	+0.842	+0.750	+0.844
1825.83......	0.70	+0.954	+0.839	+0.743	+0.835
1826.83......	0.90	+0.949	+0.844	+0.772	+0.856
1827.82......	0.10	+0.941	+0.837	+0.765	+0.851
1828.82......	0.29	+0.947	+0.837	+0.750	+0.833
1832.83......	0.08	+0.945	+0.840	+0.767	+0.852

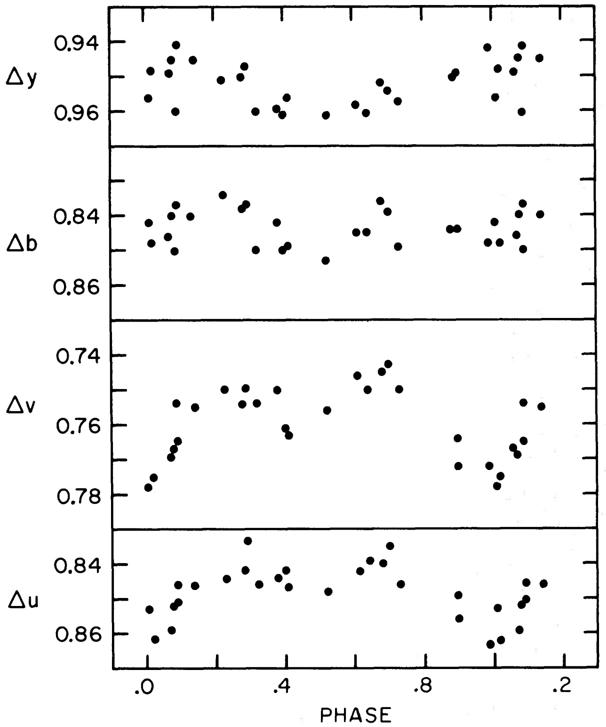

Fig. 6.—Photometric observations of HD 115708 plotted according to the ephemeris HJD ($v_{\min}$) = 2,441,731.0 + 5.07E. Magnitude differences are in the sense HD 115708 − HD 114520.

115708 is evidently primarily an aspect effect, and not the consequence of unusually slow rotation.

g) *HD 137949*

Photoelectric observations of HD 137949 (=33 Lib) were obtained on eight nights in 1973 April and May, and the amplitude of any variability does not exceed 0.01 mag. The mean colors agree well with those derived by Cameron (1966) from observations made in 1963 (see Table 10).

Nine Zeeman spectrograms of 33 Lib were measured for magnetic field strength and radial velocity, and the results are given in Table 11. From an analysis of five Zeeman spectrograms of 33 Lib, van den Heuvel (1971) suggested that the period of this star is 18.4 days. The present data suggest a slightly longer time scale for the variability, and the best representation of all the data is given by the elements

$$\text{HJD (magnetic maximum)} = 2{,}441{,}468 + 23.26E\,. \tag{6}$$

The data are plotted in Figure 7. The changes in the

TABLE 10

Photometric Data for 33 Librae

Year	V	$b - y$	m_1	c_1
1963............	6.71	+0.209	+0.302	+0.558
1973............	6.69	+0.202	+0.288	+0.571

1975ApJ...202..127W

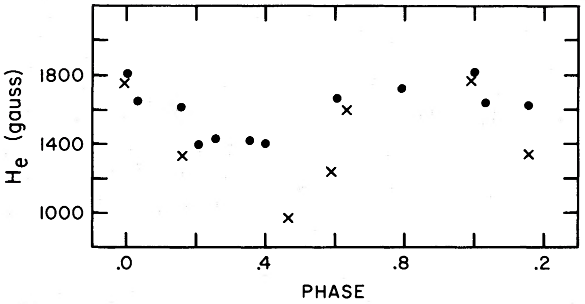

FIG. 7.—Magnetic field strengths of 33 Lib plotted according to the ephemeris HJD (magnetic maximum) = 2,441,468 + 23.26E. Crosses and filled circles represent observations made at Lick and Mauna Kea, respectively.

magnetic field are large enough that variation on a time scale of 15–25 days is fairly well established, but additional confirmation of the exact value of the period is desirable.

IV. DISCUSSION

Table 12 lists the stars that Preston (1970*b*) reported to have $v \sin i < 10$ km s^{-1}, together with the best available values of their periods. The star HD 89069 could not be observed at Mauna Kea because it was too far north. Four of the 24 stars which have been observed exhibit no detectable variability, and there are two possible explanations for this fact. Either the periods greatly exceed the time interval spanned by the observations, or else the amplitudes are so small as to be undetectable, even though the periods may be fairly short. The second possibility is entirely plausible. Winzer (1974) has recently compiled a list of all the Ap stars in the northern hemisphere that are bright enough to be listed in the Bright Star Catalog, and has obtained *UBV* observations for all the ones with previously undetermined periods. If all Si stars are excluded from Winzer's list, since none of the stars in Preston's list of objects with $v \sin i < 10$ km s^{-1} is a Si star, then 9/48 or 19 percent of Winzer's stars are nonvariable, a number comparable to the number of nonvariables in Table 12.

It should be explicitly noted that the Si Ap stars are neglected throughout the present discussion. Only a very few Si stars have $v \sin i < 10$ km s^{-1}, and in these few cases the sharp lines may be primarily due to an aspect effect rather than to intrinsically slow rotation. No Si star is known to have a period greater than 20 days. The Hg-Mn stars are also omitted. Although many have extremely sharp lines, none is known to be variable.

Half of the 20 periods listed in Table 12 are firmly established. Of the remainder, only three, those for HD 18078, HD 110066, and γ Equ, may be substantially in error, since none of these has been accurately observed during a full cycle. Although the listed periods of the remaining seven stars may be revised after additional observations are made, the time scale of the variations seems to be established within a factor of 2, and so these data are accurate enough to be used in deriving the frequency distribution of the periods of Ap stars.

Unfortunately, the period-frequency distribution cannot be obtained directly, since Preston has not published the details of his survey, including the size of his original sample. However, if Preston's sample is unbiased with respect to period, as it should be, since his sample included most of the known Ap stars brighter than $V = 9.0$, then the data in Table 12 give the correct frequency distribution for periods greater than 16 days. Winzer's list of all the bright Ap stars in the northern hemisphere can be used to derive the ratio of the number of non-Si stars with $P < 16$ days to the number of stars with $P > 16$ days. Winzer's list

TABLE 11

MAGNETIC FIELD AND RADIAL VELOCITY DATA FOR 33 LIBRAE

Plate Number	HJD 2,440,000+	Phase	H_e (gauss)	Velocity (km s^{-1})
KE 499.......	1458.85	0.61	+1680	−27.8
520.......	1467.91	0.00	+1810	−28.7
526.......	1468.81	0.03	+1650	−28.8
920.......	1801.91	0.36	+1420	−29.0
933.......	1803.03	0.40	+1400	−29.4
959.......	1811.92	0.79	+1720	−28.8
984.......	1822.96	0.26	+1430	−28.8
1031.......	1843.92	0.16	+1610	−29.3
1041.......	1844.86	0.20	+1400	−29.2

TABLE 12

PERIODS OF Ap STARS WITH $V \sin i < 10$ km s^{-1}

HD	Name	Period (days)	Source
2453........	...	525	1
5797........	...	69.0	1
8441........	...	69.5	12
9996........	HR 465	∼8000	9
12288........	...	34.9	12
18078........	...	3000?	12
22374........	9 Tau	10.61	10
24712........	HR 1217	12.448	7
81009........	HR 3724	69	1
89069........	...	Not observed	
110066........	HR 4816	3000?	1
115708........	...	5.07	1
126515........	...	130	6
137909........	β CrB	18.487	8
137949........	33 Lib	23.26	1
176232........	10 Aql	Not variable	12
187474........	HR 7552	2500	2
188041........	HR 7575	224.5	11
191742........	...	Not variable	12
192678........	...	Not variable	12
196502........	73 Dra	20.2754	5
201601........	γ Equ	26000:	3
204411........	HR 8216	Not variable	12
216533........	...	17.20	12
221568........	...	160	4

REFERENCES.—[1] Present paper; [2] Babcock (unpublished); [3] Bonsack & Pilachowski 1974; [4] Osawa 1967; [5] Preston 1967; [6] Preston 1970*a*; [7] Preston 1972; [8] Preston & Sturch 1967; [9] Preston & Wolff 1970; [10] Winzer 1974; [11] Wolff 1969; [12] Wolff & Morrison 1973.

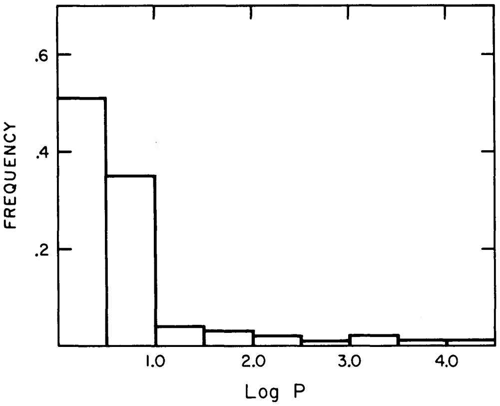

FIG. 8.—Distribution of periods for non-Si Ap stars

includes 55 stars, of which eight have $V \sin i < 10$ km s^{-1}; of these eight, two stars (10 Aql and HR 8216) exhibit no variations, and their periods are unknown, so an upper limit on the percentage of stars with $P > 16$ days is 14 percent. The ratio of the number of stars with $P < 3$ days to the number with $3 < P < 10$ days has been derived from the stars with known periods in Winzer's list. Since there is a tendency for short-period stars to have smaller amplitudes (Wolff 1975), it is more difficult to determine periods for these objects, and consequently the number of stars with $P < 3$ days may be underestimated.

The resulting period-frequency distribution is shown in Figure 8. More than half the Ap stars have periods less than 3 days, and the number of Ap stars decreases rapidly with increasing period. The decline is fairly smooth, with no discontinuity between $P < 30$ days and $P > 30$ days. Rather, Ap stars are found at all values of period from days to several years. Because of the continuous nature of the period-frequency distribution, and because the variable properties of the long-period Ap stars are in every way similar to those of Ap stars with shorter periods, the most reasonable hypothesis is that the same process is responsible for the variations of all the Ap stars. The evidence in favor of the oblique rotator model is very persuasive for the stars with $P < 30$ days, and arguments of similarity and continuity suggest that for all Ap stars the period of variation should be identified with the period of rotation.

If the periods are in fact rotation periods, then the number of slowly rotating, long-period Ap stars greatly exceeds that predicted for a normal distribution of rotational velocities (Preston 1970*c*). A Maxwellian distribution gives a satisfactory representation for normal stars on the upper main sequence (e.g., Deutsch 1967). If we assume that the distribution of rotational velocities for the Ap stars is also Maxwellian, and adopt $\langle (v \sin i)^2 \rangle^{1/2} = 30$ km s^{-1}, then 0.1 percent of the Ap stars would be expected to have $P > 160$ days ($v \sin i < 1$ km s^{-1}). The observed number is 6 percent, a discrepancy of a factor of 60. The observed distribution of periods therefore suggests that some kind of powerful deceleration mechanism has operated to slow the rotational velocities of the Ap stars.

In the following paper (Wolff 1975), arguments are offered to support the hypothesis that magnetic braking takes place throughout the main-sequence lifetime of Ap stars. If this is true, then it is possible to estimate the time scale for rotational deceleration from the data in Table 12. The $(b - y)$ colors of these stars, which are not strongly affected by blanketing (Wolff 1967), can be used to determine the temperature. The bluest star known to have a long period is HD 126515, with $(b - y) = -0.03$ (Cameron 1966), corresponding to $T_e = 12{,}000$ K (Osmer and Peterson 1974), and to a mass of about 3 $M_\odot$, if this star has a normal main-sequence mass. This last assumption is compatible with the only direct evidence on the mass of an Ap star (Abt *et al.* 1968) and with spectroscopic determinations of atmospheric parameters (e.g. Wolff 1967; Jugaku and Sargent 1968). From a study of Ap stars with reliable periods, Preston (1970*c*) has estimated that the upper limit on the radii of Ap stars is almost twice the radius on the zero-age main sequence, and the time required for a 3 $M_\odot$ star to reach this radius is about 2.3×10^8 years (Iben 1965). Evolution during this time proceeds with little change in color, and the increase in radius is approximately linear with time. Since no bluer (i.e., younger) magnetic stars are known to have very long periods, 2.3×10^8 years is an approximate lower limit on the time required for magnetic braking to reduce rotational velocities to values less than 10 km s^{-1}. On the average the stars in Table 12 are cooler and presumably less massive than HD 126515, so that typical braking times are probably larger than this value.

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SIDNEY C. WOLFF: Institute for Astronomy, 2680 Woodlawn Drive, Honolulu, HI 96822