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THE ORIGIN OF THE MAGNETIC FIELD AND RELATIVISTIC PARTICLES IN THE CRAB NEBULA

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SUMMARY

The power from the Crab Pulsar NP 0531 probably emerges partly as 30 Hz waves and partly as a relativistic wind containing a toroidal magnetic field. A shock discontinuity is expected where the pressure of the relativistic outflow balances the pressure within the nebula. This shock would occur at a radius R_s which is about 10 per cent of the total nebular radius R_{neb}. The relativistic particles and the magnetic flux would accumulate in the region $R_s < R < R_{neb}$. The magnetic field in this region would build up to equipartition strength, even if this field embodied only a few per cent of the relativistic energy outflow from the pulsar. The 30 Hz waves would be absorbed at $R \simeq R_s$, their energy being transferred to relativistic electrons. The continuum from the bulk of the nebula would then be entirely synchrotron radiation. The relevance of this model to various observed features of the Crab Nebula is briefly explored.

1. INTRODUCTION

The Crab Nebula contains a magnetic field of strength 10^{-3}–10^{-4} G, and a total energy $\sim 10^{49}$ erg in the form of relativistic electrons. These conclusions follow from the assumption that the radio, optical and X-ray continuum is synchrotron-type radiation, and from dynamical considerations which imply that the field cannot be too far away from its equipartition strength. Most of the non-thermal energy emission ($\sim 10^{38}$ erg s^{-1}) is in the ultra-violet and X-ray band, where the relevant synchrotron lifetimes are less than the age of the nebula ($\sim$920 yr), and this implies continuous injection of electrons with Lorentz factors up to at least $\gamma \simeq 10^8$ ($\sim 10^{14}$ eV). It is equally clear that the magnetic field cannot be a relic of the original explosion, because adiabatic losses would have transformed almost all such energy into expansion energy of the nebula. (The detailed arguments leading to these conclusions are deployed in, for example, the proceedings of *IAU Symposium* 44 (Davies & Smith 1971).)

These straightforward considerations have led to a general consensus that the central pulsar NP 0531 is responsible for the present input of fast particles, and that the field and particle energy content of the nebula has been built up or replenished by the pulsar over its lifetime. The striking agreement between the power supply required and the estimated value of $|I\Omega\dot{\Omega}|$ for the pulsar is well known. No fully satisfactory detailed model for the electrodynamic link between pulsar and nebula has yet been given, and we address ourselves to this problem in the present paper.

Our proposal involves, in essence, a modified and extended version of the general scheme first suggested by Piddington (1957) and subsequently discussed by Kardashev (1965), according to which the magnetic field has been ‘ wound up ’ by a central spinning object. It is now clear that this object should be identified with the pulsar NP 0531, and we therefore consider how the 'wound up field' concept can be reconciled with existing ideas on pulsars, and with the view that the pulsar also continuously replenishes the supply of relativistic electrons responsible for the non-thermal optical and X-ray output from the nebula.

We propose that, within a radius $R_s \simeq 3 \times 10^{17}$ cm (~ 10 per cent of the nebular radius), the pulsar environment is unaffected by the surrounding nebula, and that an energy flux $|I\Omega\dot{\Omega}|$ streams outward, partly in the form of a relativistic wind containing a toroidal magnetic field, and partly in the form of electromagnetic waves at frequency Ω. There is a shock discontinuity at R_s (which we can tentatively associated with the wisps) where the wind kinetic energy is randomized. The low frequency waves are absorbed at R_s, and augment the energy of the relativistic electrons being fed into the nebula. The non-oscillatory toroidal magnetic field carried out by the wind is, however, not destroyed: instead, the flux accumulates within the body of the nebula, and the magnetic energy builds up more rapidly than the particle energy content. Outside R_s, the bulk motions are subsonic, and the magnetic stresses and particle pressure gradients come more or less into equilibrium.

2. PULSAR ENCLOSED IN SLOWLY-EXPANDING BOUNDARY

For the purposes of this discussion, we adopt a grossly idealized model, and treat the Crab Nebula as a roughly spherical volume of radius R_{neb}, mainly empty except for relativistic particles and electromagnetic fields, whose (electrically conducting) boundary expands at a uniform speed $\dot{R}_{\text{neb}} \simeq 1500$ km s^{-1}. Hopefully the justification for simplifying things in this extreme fashion will become clear as we go along. We regard this model as applicable at times $\gtrsim 50$ yr after the supernova explosion, and omit any discussion of the early evolution of the Crab Nebula, or of how the filaments might have formed.

As is customary, let us idealize the pulsar as a spinning neutron star with a dipole magnetic field oriented at an angle θ to the rotation axis.

The special case $\theta = 0$—the 'aligned rotator'—has been considered by many authors following the original treatment due to Goldreich & Julian (1969). For a recent discussion see Michel (1974). Although there is still no fully self-consistent solution, it seems likely that a relativistic wind is generated whose density must be *at least* as large as Goldreich and Julian's estimate based on the 'homopolar inductor' mechanism, and that the 'open field lines' (i.e. those that do not close up within the light cylinder (LC) of radius $R_{\text{LC}} = c/\Omega$, where Ω is the angular frequency of the pulsar's rotation) are carried out with the wind to form a field whose strength is $H \simeq H_{\text{LC}}\,(R_{\text{LC}}/R)$. This field is essentially toroidal, and its only reversal occurs in the equatorial plane. The associated currents are radial, and could be supplied by a latitude-dependent charge excess in the wind.

Early discussions of the 'oblique rotator' (Pacini 1968; Ostriker & Gunn 1969) assumed that the rotational energy emerged as electromagnetic waves at frequency Ω. The possibility of a conducting magnetosphere or wind was not discussed, though it was recognized that such waves would rapidly accelerate any individual charged particles exposed to them. For any value of θ, however, Goldreich and Julian's argument still tells us that **E.B.** forces lift a stream of charges off the star, which then escape along open field lines; moreover this argument tells us only the value of the (algebraic) excess of positive over negative charges, and there may be a much higher total particle flux in the wind.

For a general value of θ, we thus expect (Mestel 1971; Goldreich, Pacini & Rees 1971) the energy flowing across a sphere of radius $R > R_{LC}$ surrounding a pulsar to have three ingredients: low frequency electromagnetic waves (carrying an energy flux L_{wave}); a relativistic wind (L_{wind}); and a toroidal magnetic field associated with the wind (L_{mag})*. L_{mag} is defined as the non-time-varying component of the Poynting flux, integrated over a sphere centred on the pulsar. We are unable to make very firm estimates of the relative values of these three contributions to L. One might naively (though without much confidence) guess that $L_{wave}/L \simeq \sin^2\theta$; but we shall not even try to guess L_{wind}, which depends on the particle flux and the Lorentz factor γ_{wind} of the radial outflow.

The expected 'wound up' toroidal magnetic field and the oscillatory magnetic field associated with L_{wave} are of comparable strength unless θ is close to either 0 or $\pi/2$. One might therefore wonder whether the waves could really propagate if the wind density were high enough for the toroidal field to be validly regarded as 'frozen' into it. The wave field reverses its direction every half-wavelength, so the corresponding current density (including the displacement current) is $j_{wave} \simeq H\Omega/c$. The wound-up toroidal field generated by the aligned component of the pulsar's magnetic moment, however, reverses only between the two hemispheres, i.e. on a scale R. The current density at a radius R associated with the wave field is therefore larger by a factor R/R_{LC} than that associated with the wound-up field. For $R \gg R_{LC}$, this means that there exists a wide range of plasma densities between j_{mag}/ec and j_{wave}/ec such that MHD concepts can be applied to the wound-up field, whereas the waves propagate almost as though *in vacuo*. The field reversal in the equatorial plane would not be sharp, so significant field annihilation would not be expected.

If a pulsar were completely isolated, the relativistic wind and low frequency waves would eventually reach arbitrarily large radii. The same cannot, however, hold for a pulsar situated within our idealized nebular cavity, because all the energy must accumulate within the confines of a volume which expands at a rate $\dot{R}_{neb} \ll c$. In this case there will be a characteristic radius R_s at which the ram pressure $L/4\pi c R_s^2$ balances the total magnetic and particle pressure P within the bulk of the nebular volume. (In general, neither the power output from the pulsar nor P will be isotropic, so R_s will depend on orientation.) A very rough estimate of R_s can be made by supposing that the energy reservoir within the nebula accumulated at a steady rate over its lifetime (i.e. we ignore adiabatic and radiation losses on the one hand, and the higher past value of L on the other). This gives

$$\frac{R_s}{R_{neb}} = \left(\frac{\text{light travel time out to } R_s}{\text{age of nebula}}\right)^{1/3} \simeq \left(\frac{\dot{R}_{neb}}{c}\right)^{1/2}. \qquad (1)$$

For the Crab Nebula, (1) yields a ratio ~ 0.07; a more detailed estimate gives a somewhat larger number. This result depends on the assumption that the pulsar causes a relativistic radial outflow, but is insensitive to the precise form this takes.

There must plainly be some kind of shock transition at $R \simeq R_s$. If the energy emerged from the pulsar entirely in the form of an ultra-relativistic wind (i.e. if

* This neglects of course the power emitted by the electromagnetic pulses themselves—a justifiable approximation at least for the case of the Crab.

$L_{\text{wave}} = L_{\text{mag}} = 0$), then the situation would be straightforward. Just outside R_s, the outward velocity would be somewhat less than $c/\sqrt{3}$ (most of the bulk kinetic energy of the wind having been transformed at the shock into relativistic random motions). At $R > R_s$, the motions would be subsonic. For this reason, and also because the sound speed $\sim c/\sqrt{3}$ is $\gg \dot{R}_{\text{neb}}$, the pressure will be more or less uniform throughout most of the volume $R_s < R < R_{\text{neb}}$. Thus the outward velocity of the relativistic gas accumulating in the nebula would decrease smoothly from $\sim c/\sqrt{3}$ at R_s to $\dot{R}_{\text{neb}}$ at R_{neb}, varying roughly as R^{-2}.

3. BUILD-UP OF TOROIDAL FIELD

Consider now the behaviour of a toroidal field attached to the wind. Suppose first that this field is so weak that the magnetic stresses are nowhere significant, but that L_{mag} is a constant (small) fraction of the power output in relativistic particles. The field strength* will undergo a discontinuous increase by a factor $\sim \sqrt{3}$ at R_s (corresponding to the sudden *decrease* by $\sim \sqrt{3}$ in the outward velocity). It will then *increase outwards* in proportion to R, because the usual $1/R$ dependence is outweighed by the amplification resulting from the $\sim R^{-2}$ dependence of the velocity. Along any radius we would then have

$$H(R) \simeq H(R_s)\frac{R}{R_s}\,(R_s < R < R_{\text{neb}}). \qquad (2)$$

The dependence on latitude, at a given R, would be determined by the flux distribution in the wind at $R < R_s$. The behaviour would be modified if the magnetic stresses became comparable with the particle pressure. It is clear from (2) that this is most likely to happen when $R \simeq R_{\text{neb}}$. Relation (2) also ignores the effects of radiative losses by the particles (which would tend to make the flux even more concentrated towards the boundary), as well as diffusion of particles across field lines.

In discussing the magnetic field energy $\mathscr{E}_{\text{mag}}$ in the nebula, it is convenient to think in terms of 'turns' or 'windings' of the magnetic field. The total number of turns is simply

$$\mathscr{N}(t_{\text{now}}) = \int_{t_{\text{min}}}^{t_{\text{now}}} \frac{\Omega}{2\pi}\,dt \qquad (3)$$

and all but a fraction $\sim(\dot{R}_{\text{neb}}/c)^{3/2}$ of these turns will lie outside R_s. The magnetic *flux* stored in the nebula is directly proportional to $\mathscr{N}$. Provided that the dependence of H on R does not change with t—and (2) will continue to hold so long as H remains weak enough for magnetic stresses to be everywhere negligible—then

$$\mathscr{E}_{\text{mag}}(t) \propto (\mathscr{N}(t))^2/R_{\text{neb}}(t). \qquad (4)$$

To progress further we need to know how the pulsar's angular frequency Ω depends on t. Suppose that the electromagnetic torques extract energy at a total rate

$$L(t) = -I\Omega\dot{\Omega}. \qquad (5)$$

For the Crab pulsar, the value of L corresponding to the present rotation rate of 30 Hz ($\Omega \simeq 200\ \text{s}^{-1}$) is probably $\sim 2\times 10^{38}\ \text{erg s}^{-1}$, somewhat more than the

* This refers to the magnetic field measured in a frame sharing the local plasma motion. A stationary observer would of course feel a transverse electric field as well.

nebula's bolometric luminosity. It is normally assumed that

$$L \propto \Omega^{n+1} \tag{6}$$

where the 'braking index' n depends on the nature of the torques acting on the spinning neutron star. The most naive theoretical models predict a law of the form (6) with $n = 3$; analysis by Groth (1974) of precise timing date for NP 0531 suggests, however, that $n \simeq 2{\cdot}5$. In what follows we assume (6) but leave n as a free parameter. We then have

$$L(t) = \frac{L(0)}{\left(1+\dfrac{t}{\tau}\right)^{n+1/n-1}} \tag{7}$$

where τ is the initial slowing-down time scale. If $n = (2{\cdot}5\text{–}3)$, then NP 0531 would initially have been spinning at about twice its present rate, and τ would be ~ 200 yr. An initial rotation rate much faster than this would in fact not be easy to reconcile with the total energy content of the nebula—only $\sim 10^{49}$ erg—nor with its overall dynamics (Trimble & Rees 1970).

$t_{\min}$ (in (3)) should be taken as the earliest time at which we have any confidence in the relevance of our model (perhaps 50 yr). In practice, none of the quantities we wish to calculate are sensitive to the exact value of $t_{\min}$ provided it is $\lesssim \tau$.

We then find that the magnetic energy produced at recent times would dominate that produced when $t \lesssim \tau$ provided that $n > 2$. Similar integrals can in principle be performed for the particle energy content in the nebula. But it is then necessary to allow for radiative losses, and therefore to make specific assumptions about the energy spectrum of the injected particles (see, for example, Pacini & Salvati (1973)). If, however, we assume that the particle energy injected is a constant fraction of L and *neglect* radiative losses, then

$$\mathscr{E}_{\text{part}} \propto \int_{t_{\min}}^{t_{\text{now}}} L \left(\frac{R_{\text{neb}}(t)}{R_{\text{neb}}(t_{\text{now}})}\right) dt. \tag{8}$$

If L is given by (7) and $t \gg \tau$, then the particle energy injected at early times can only make a dominant contribution to the present energy $\mathscr{E}_{\text{part}}$ if $n < 3$.

For $n = 2{\cdot}5$ and $\dot{R}_{\text{neb}} \simeq$ constant, (8) gives $\mathscr{E}_{\text{part}} \propto t^{-1}$, whereas (4) tells us that $\mathscr{E}_{\text{mag}} \propto t^{-1/3}$. There is thus a tendency (at least when $2 < n < 3$) for an initially weak magnetic field to become increasingly important energy-wise relative to the relativistic particle content; and this trend is accentuated if the electrons suffer radiative losses.

(The reason for the differing behaviour of $\mathscr{E}_{\text{mag}}$ and $\mathscr{E}_{\text{part}}$ can perhaps be clarified by considering the simple example of a nebula of fixed size containing a pulsar spinning at a constant rate. In this case $\mathscr{E}_{\text{part}} \propto t$ (and increases even more slowly if radiative losses are important) but the magnetic flux accumulates as the field becomes more and more tightly wound, so that $H \propto t$ and $\mathscr{E}_{\text{mag}} \propto t^2$).

The constant of proportionality in (4) depends on how the flux is distributed in the nebula. (The smallest value of $\mathscr{E}_{\text{mag}}$ when $\mathscr{N}$ and R_{neb} are both given, occurs if $H \propto \varpi^{-1}$, ϖ being the distance from the rotation axis; but when H is weak and the field distribution is such that the particle pressure is almost uniform, we expect it to have the very different dependence (2).) However, (4) obviously admits the possibility that $\dot{\mathscr{E}}_{\text{mag}} > L_{\text{mag}}$. (For the particular case when the magnetic field distribution is given by (2), we have $\dot{\mathscr{E}}_{\mathrm{mag}} \simeq (R_{\mathrm{neb}}/R_{\mathrm{S}})^2 L_{\mathrm{mag}}$.) Provided $\dot{\mathscr{E}}_{\mathrm{mag}}$ remains less than the total pulsar power output L, this is nevertheless a perfectly consistent situation. It implies that the wave and particle pressures have to perform work in compressing the flux already present, and this is of course the reason why $\mathscr{E}_{\mathrm{mag}}$ tends to rise faster than $\mathscr{E}_{\mathrm{part}}$, and why (as we see later) we only need $L_{\mathrm{mag}}/L \simeq 0{\cdot}01$ in order to build the Crab field up to equipartition strength. (The value of R_{S}, and the field distribution at $R > R_{\mathrm{S}}$, would always of course adjust themselves so that $\dot{\mathscr{E}}_{\mathrm{mag}}$ remained below the *total* luminosity L).

4. THE ELECTRON SPECTRUM WITHIN THE CRAB NEBULA

The relativistic electron spectrum in the nebula, inferred from the observed continuum emission (Baldwin 1971) supposing this to be synchrotron radiation in a mean field $\sim 5 \times 10^{-4}$ G, is roughly of the following form:

$$N(\gamma) \propto \gamma^{-3\cdot5} \quad \text{for} \quad \gamma \gtrsim 5 \times 10^6;$$

$$N(\gamma) \propto \gamma^{-2\cdot5} \quad \text{for} \quad 5 \times 10^6 > \gamma > 10^5$$

and

$$N(\gamma) \propto \gamma^{-1\cdot5} \quad \text{for} \quad 10^5 \gtrsim \gamma \gtrsim \gamma_{\mathrm{min}}.$$

The breaks at $\gamma \simeq 5 \times 10^6$ and $\gamma \simeq 10^5$ are inferred from the apparent changes in the slope of the continuum spectrum in the ultra-violet and infra-red bands respectively. The actual frequencies of these spectral features, and hence the corresponding value of γ, are rather ill-determined. The fact that the radio emission extends at least down to ~ 10 MHz implies $\gamma_{\mathrm{min}} \lesssim 100$. A precise calculation should obviously allow for the inhomogeneous distribution of particles and magnetic field within the nebula, but these order-of-magnitude estimates are adequate for our present discussion. The number density of relativistic electrons, in order of magnitude, is

$$N \simeq 10^{-5} \left(\frac{\gamma_{\mathrm{min}}}{100}\right)^{-1/2} \mathrm{cm}^{-3}, \tag{9}$$

and $\langle \gamma^{-1} \rangle$, a quantity we shall need later, is $\sim \frac{1}{3}\gamma_{\mathrm{min}}{}^{-1}$.

Electrons with $\gamma \gtrsim 10^5$ have synchrotron lifetimes $\lesssim 1000$ yr, and the break at this energy is probably attributable to synchrotron losses. (See Pacini & Salvati (1973) for detailed arguments reinforcing this conclusion.) The break at $\gamma \simeq 5 \times 10^6$, however, must be a feature of the injected spectrum. Thus the general features of the radiation from the Crab would be accounted for if electrons were being steadily injected with a differential spectrum

$$N_i(\gamma) \propto \begin{cases} \gamma^{-2\cdot5} & \gamma \gtrsim 5 \times 10^6 \\ \gamma^{-1\cdot5} & 5 \times 10^6 \gtrsim \gamma \gtrsim \gamma_{\mathrm{min}} \end{cases} \tag{10}$$

and losses steepened the particle spectrum in the nebula by one power for $\gamma \gtrsim 10^5$. Note that most of the *injected energy* goes into electrons with $\gamma \simeq 5 \times 10^6$ (although, owing to radiative losses, most of the stored energy—and particle pressure—may be in the form of electrons with $\gamma \simeq 10^5$); whereas most of the injected particles have $\gamma \simeq \gamma_{\mathrm{min}}$. The total injection rate is $\sim 3 \times 10^{40}\ (\gamma_{\mathrm{min}}/100)^{-1/2}$ electrons per second.* Furthermore, if electrons are injected with the spectrum (10), the present

* These general conclusions hold for almost any simple model where the pulsar ' braking index ' is $n < 3$; though the results might be different in more complicated situations where the injected *spectrum* depends on t.

energy content of the nebula $\mathscr{E}_{\text{part}}$ would be only $\sim 50^{-1/2} \simeq \frac{1}{7}$ times that given by (8) owing to the effects of radiative losses.

5. THE FATE OF THE 30 HZ WAVES

What is the fate of 30 Hz electromagnetic waves propagating into a relativistic plasma with the properties inferred from the non-thermal continuum radiation of the Crab Nebula? In the absence of a magnetic field, the refractive index μ for these waves would be

$$(1-10^5 N\langle\gamma^{-1}\rangle)^{1/2}. \tag{11}$$

If $N \simeq 10^{-5}$ cm^{-3} and $\langle\gamma^{-1}\rangle \simeq 10^{-2}$, the waves can propagate: indeed this well-known circumstance was an essential feature of the 'wave field' model (Gunn 1970; Gunn & Ostriker 1971; Rees 1971) in which 30 Hz waves in the Crab Nebula played the role customarily ascribed to a magnetic field, and the non-thermal continuum was 'synchro-Compton' or 'non-linear inverse Compton (NIC)' emission instead of arising from the usual synchrotron process.

If a magnetic field *is* present, then the refractive index is still given roughly by (11) provided that $\nu_L\langle\gamma^{-1}\rangle \lesssim 30$ Hz, $\nu_L \simeq 3\times 10^6 H_{\text{Gauss}}$ being the non-relativistic Larmor frequency. Since this condition would be fulfilled for $H \lesssim 10^{-3}$ G and $\gamma^{-1}{}_{\min} \lesssim 10^{-2}$, one might at first sight conclude that the waves could still propagate.

The presence of a magnetic field introduces, however, the possibility that 30 Hz waves may suffer *synchrotron absorption*, and we can readily show that this process may be an exceedingly efficient one.

The brightness temperature of the Crab at 10 MHz is $\sim 10^9$ K. The electrons responsible for the synchrotron radiation at this frequency have $\gamma \simeq 100$, and effective kinetic temperatures $\simeq 10^{12}$ K. Thus the optical depth of the nebula at 10 MHz is $\tau(10 \text{ MHz}) \simeq 10^{-3}$. Even if there are no lower-energy electrons present (i.e. $\gamma_{\min} \simeq 100$), $\tau(\nu) \propto \nu^{-5/3}$ at frequencies below 10 MHz provided that we do not consider frequencies below the Razin cut-off (see below); and if there are additional electrons with $\gamma < 100$ the dependence on ν is even steeper. This suggests that, if the synchrotron process is the dominant *emission* mechanism in the Crab, the optical depth at 30 Hz is at least 10^6, implying a mean free path $\lesssim 3\times 10^{12}$ cm for these waves.

For this conclusion to be valid, synchrotron emission and absorption must occur at frequencies as low as 30 Hz. Except for electrons with $\gamma \lesssim 100$, 30 Hz exceeds the gyrofrequency $\nu_L\gamma^{-1}$ in a field of 10^{-3} G. 30 Hz is above the tenth harmonic of the gyrofrequency for electrons with $\gamma \gtrsim 10^3$, and these electrons alone would cause the nebula to have an optical depth $\sim 10^4$. For this reason—and also because the electrons span a range of γ, and the magnetic field would not be completely uniform—we are fully justified in assuming that the synchrotron process gives continuous emission and absorption right down to 30 Hz. If the electrons have a range of pitch angles, the efficacy of the absorption would be insensitive to the polarization properties of the 30 Hz waves.

A second requirement is that the so-called Razin cut-off (the frequency below which plasma effects inhibit the synchrotron process) should be *below* 30 Hz. The Razin frequency $\nu_R(\gamma)$ is the value of ν for which

$$(1-\mu(\nu)) \simeq \gamma^{-2}+(\phi(\nu,\gamma))^2 \tag{12}$$

where $\phi(\nu, \gamma)$ is the angular width of the cone into which the radiation emitted at frequency ν by electrons of Lorentz factor γ is beamed.

When $\nu \ll \gamma^2 \nu_L$, $\phi(\nu, \gamma) \simeq \gamma^{-1}(\gamma^2 \nu_L/\nu)^{1/3}$, and the first term on the rhs of (12) can be neglected. The condition that a particle of Lorentz factor γ should still be able to absorb at 30 Hz is then

$$10^5 \, N\langle\gamma^{-1}\rangle < 20\, \gamma^{-2/3} \left(\frac{H}{10^{-3}\,\mathrm{G}}\right)^{2/3}. \qquad (13)^*$$

If $N\langle\gamma^{-1}\rangle$ had its minimum value of $\sim 10^{-7}$ and $H = 10^{-3}$ G this would imply that electrons with $\gamma \lesssim 10^5$ would be able to absorb energy from the 30 Hz waves.

One might worry about the applicability of the foregoing analysis to a case when the 30 Hz waves are so intense that the 'strength parameter' $f = eE_{\mathrm{wave}}/mc\Omega$ exceeds unity. For the parameters appropriate to the Crab, we should have $f \simeq 10$ in the body of the nebula if there were no absorption. When $f > 1$, the motion of an electron with energy γmc^2 is a superposition of periodic oscillations with Lorentz factor $\sim f$, and a 'guiding centre motion' with Lorentz factor $\sim \gamma/f$. If an electron moving in a magnetic field is exposed to 'strong' low frequency waves, its orbit remains basically periodic, with frequency ν_L/γ, provided that $\gamma \gg f$. There are however 'wiggles' of wavelength $\sim 2\pi c/\Omega$ superposed on the overall motion, but one can show that the standard synchrotron theory is still applicable provided that $f < \gamma^{2/5}(\nu_L/\nu)^{1/5}$.

If for some reason synchrotron absorption were *not* effective, there are two other processes which could prevent the 30 Hz waves from penetrating beyond $R \simeq R_s$. (i) It is conceivable that there are actually enough low-γ particle to make μ imaginary (for instance, this would happen if the spectrum $N(\gamma) \propto \gamma^{-1.5}$ extended down to $\gamma_{\min} \simeq 5$†). (ii) Even if this is not so, μ may be sufficiently different from unity, and sufficiently non-uniform, to *scatter* the 30 Hz waves through small angles. This would cause them to lose coherence, and allow the possibility of efficient statistical acceleration (Blandford 1973; Arons *et al.* 1974, in preparation), which could rapidly attenuate the waves, and give individual electrons γ's of up to $\sim 10^5$.

6. A TENTATIVE MODEL FOR THE CRAB NEBULA

We therefore believe that, provided the nebula contains a magnetic field (such as could have been generated in the manner discussed in Section 3), 30 Hz waves from the pulsar will be completely absorbed at $R \simeq R_s$, the associated power L_{wave} being transferred to relativistic electrons. This means that:

(i) the continuum from the nebula must be ordinary synchrotron radiation (rather than synchro-Compton or 'NIC' radiation), and the magnetic field must be comparable with the equipartition value; and

* Note that this expression, in which γ appears explicitly, differs from that normally quoted for the Razin frequency, because we are here concerned with electrons of *given* γ whose synchrotron emission peaks at frequencies $\gg \nu_R$ rather than *at* $\sim \nu_R$. For a fuller discussion of how ν_R depends on γ, see for example McCray (1966).

† In fact, as has been pointed out by Max & Perkins (1971), the propagation requirements for low frequency strong waves are somewhat less stringent if the waves encounter a plasma whose density increases *gradually* rather than suddenly. This correction is only important, however, in situations where the group velocity becomes $\ll c$; but such waves cannot, in our present context, be regarded as propagating at all unless the group velocity is at least greater than $\dot{R}_{\mathrm{neb}}$.

(ii) the power going into relativistic particles is $\sim(L_{\mathrm{wind}}+L_{\mathrm{wave}})$

Can we then estimate the value of L_{mag}/L (presumably a function of the angle θ between the pulsar's rotation and magnetic axes) which this model would entail?

Applying (7) and (8) to the case where $n = 2 \cdot 5$ and $\tau \simeq 200$ yr tells us that $\mathscr{E}_{\mathrm{mag}}/\mathscr{E}_{\mathrm{part}}$ would have been amplified by $5^{2/3} \simeq 3$ over the lifetime of the nebula. A further multiplicative factor of 7 arises from the radiative energy losses associated with an energy spectrum (10). This implies that we require only that L_{mag} should be about 5 per cent of L in order to build up a magnetic energy comparable with the present particle energy content. More detailed considerations which take into account the dependence of field strength on R lead to an even lower estimate of the required L_{mag}. It would be hard to reconcile the observed brightness distribution across the Crab Nebula with a field increasing with R according to (2). Thus the conditions must certainly be such that (2) does not apply right out to R_{neb}. This would merely imply $L_{\mathrm{mag}}/L_{\mathrm{part}} \gtrsim 10^{-2}$ if there were no radiative losses, and an even weaker condition when these losses are allowed for. It seems unlikely, therefore, that L_{mag}/L should be substantially less than $\sim 10^{-2}$. On the other hand, if this ratio were of order unity, the magnetic stresses would, quite early in the nebula's history, have become strong enough to distort it towards a cylindrical configuration.

Note that this kind of field amplification cannot readily give rise to a situation where the magnetic field *exceeds* its equipartition strength. This is because, once the winding-up process gives a field strength approaching equipartition, magnetic stresses tend to contract the field lines (and the associated radial currents) towards the axis. This *reduces* the magnetic energy and *increases* the energy density of particles attached to these field lines. An equilibrium can only be attained when the particle pressure gradient balances the magnetic stresses. After this adjustment, the situation again corresponds to overall equipartition, though neither the particle pressure nor the magnetic field strength would be uniform. This process thus has the attractive feature that it would automatically establish and maintain approximate equipartition.

These (admittedly rough) arguments suggest that L_{mag}/L is probably in the range 0·01–0·1. This indicates that θ is close to $\pi/2$, a result in gratifying accordance with the occurrence of the ' interpulse ' in NP 0531, a fact which independently suggests $\theta \simeq \pi/2$.

In conclusion, we mention how our proposals relate to some specific aspects of the Crab Nebula.

Overall polarization and magnetic field

An excellent review of the polarization data on the Crab Nebula has recently been given by Felten (1974). The fairly uniform linear polarization over the inner parts of the nebula, with the electric vector along the NW–SE (major) axis, is fully consistent with a toroidal field, provided that the pulsar rotation axis points in this same direction and is more or less in the plane of the sky. (The latter circumstance can be inferred independently from the occurrence of the ' interpulse '). The predicted linear polarization is broadly similar to that expected in a ' wave-field ' model. The latter model predicts, however, that the nebula should display several per cent *circular polarization*, at optical as well as radio wavelengths (Rees 1971), especially at points along its major axis. Failure to detect such circular polarization optically even at the 0·1 per cent level (Landstreet & Angel 1971; see also Martin, Illing & Angel 1972) apparently forces us to abandon the idea that the non-thermal continuum is synchro-Compton radiation. Our main aim in the present paper has indeed been to develop a model for the Crab which retains a conventional 'oblique rotator' model for the central pulsar, but accounts for a toroidal field in the bulk of the nebula and explains the absence of 30 Hz waves.

It would be satisfying if one could test whether the field really has a toroidal structure, and whether, as our model predicts, it reverses its direction in the pulsar's equatorial plane (which intersects the plane of the sky in a NE–SW direction). One might think of searching for some systematic differences between Faraday rotation measures in adjoining 'quadrants' of the nebula. Such an effect would, however, only occur if there were sufficient thermal plasma in the amorphous region between filaments; and there seems no reason why this should necessarily be so. A better way of determining the sign of $B_{\parallel}$ in different parts of the nebula might be by detecting the small amount of circular polarization expected on the basis of ordinary synchrotron theory. For an ordered field this is $\sim(\nu_L/\nu)^{1/2}$ (Legg & Westfold 1968), i.e. perhaps $\sim$0·1 per cent at radio frequencies below 1000 MHz —and so may well be detectable by present techniques.

Field structure in outer parts of nebula and around filaments

The polarization near the boundary of the nebula indicates that the field structure there is complex and somewhat irregular. In our model, the importance of magnetic stresses relative to particle pressure increases with R, so we would naturally expect magnetically-driven instabilities, and motions tending to concentrate the flux towards higher latitudes, to manifest themselves in the outer parts of the nebula. Furthermore, the fact that the nebula has an irregular boundary, and the presence of filaments, would inevitably preclude any smooth accumulation of magnetic flux in a toroidal geometry. One feature of the magnetic field structure, noted by Woltjer (1957) and discussed recently by Wilson (1972b), is the tendency for field lines to wrap around the filaments. This finds a natural interpretation in our scheme. The distance of a given filament from the pulsar presumably remains almost a constant fraction of R_{neb} as the nebula expands. But the value of R/R_{neb} associated with a particular 'turn' of the magnetic field *increases*, because each revolution of the pulsar creates extra flux which is transported out by the wind and tends to compress the existing flux towards the boundary of the nebula. If the filaments are good conductors, the field lines will wrap around them, inducing electric currents along the filaments.

Brightness distribution and spectrum of non-thermal continuum

The effective angular extent of the Crab Nebula is about 50 per cent greater at radio than at optical wavelength. There are also variations in the optical spectral index across the nebula, the outer parts being redder than the central regions. Wilson (1972a) interprets these effects fairly satisfactorily by invoking energy-independent diffusion. Electron diffusion almost certainly plays some role in determining the brightness distribution. In our picture, however, the particles would drift outward with the field at a velocity $(R/R_{\mathrm{neb}})^{-2}\,\dot{R}_{\mathrm{neb}}$. Just outside R_s this velocity is a substantial fraction of c. Allowance for this rapid drift would ease the difficulty which Wilson encountered in reconciling the measured size of the X-ray emitting region with his diffusion model. We do not actually need to invoke any diffusion whatsoever for the electrons with $\gamma \lesssim 10^5$ which are responsible for the radio emission. If this diffusion *were* too rapid we would indeed need to modify our ' frozen-in field ' assumption.

In common with other authors, we can offer no convincing explanation for why the injected particles have the spectrum (10). Conceivably, however, the fact that most of the electron energy is in particles near the ' break ' at $\gamma \simeq 5 \times 10^6$, may be connected with the result (13) that the Razin cut-off inhibits absorption of 30 Hz waves by particles once their energy exceeds a value of this general order. (Our ' standard ' parameters yield $\gamma \simeq 10^5$ but are very uncertain.) Also, statistical acceleration may play a role. We would tentatively associate these acceleration processes with the location of the wisps.

The region $R < R_s$ and the wisps

We have argued that L_{wind} and L_{wave} together account for $\gtrsim 90$ per cent of the total pulsar luminosity L, but are unable to assess their relative importance. Nor can we offer any firm estimate for the Lorentz factor γ_{wind} of the outflowing wind at $R < R_{\text{S}}$. Some rough limits may, however, be worth mentioning. The Goldreich–Julian homopolar inductor model allows us to estimate a minimum particle eflux of $\sim 10^{34}\ \text{s}^{-1}$. This corresponds also to the minimum charge density required to carry the currents associated with the magnetic field at $R \simeq R_{\text{LC}}$. Since $L \simeq 10^{38}\ \text{erg s}^{-1}$, the mean particle energy is limited to $\lesssim 10^4$ erg, and the Lorentz factor must be $\lesssim 10^6$ for protons in the wind and $\lesssim 10^9$ for electrons. This is not a very stringent limit. If the electrons ' phase ride ' the wave out to

$$R_{\text{S}} \simeq 10^9\, R_{\text{LC}} \simeq 2 \times 10^8 \left(\frac{2\pi c}{\Omega}\right)$$

their bulk Lorentz factor must be $\gtrsim 10^4$, since otherwise they would accumulate a phase lag exceeding a wavelength before reaching R_{S}. (It is not so clear, however, that the protons need also be in phase with the wave.) If all the relativistic electrons injected with spectrum (10) come from the pulsar, then we require an injection rate of at least $\sim 10^{40}$ electrons s^{-1}. This would imply an *upper limit* of 10^4 for γ_{wind}. It is perhaps suggestive that this is close to the lower limit set by the ' phase riding ' condition.

Any emission from electrons streaming out with the wind would be beamed in the forward direction, and the toroidal field association with the wind would prevent electrons in the body of the nebula (except for any with $\gamma \gtrsim 10^{10}$) from penetrating into the region $R < R_{\text{S}}$. It is conceivable that the pulsar radiation may be generated by electrons in the relativistic wind at $R \gg R_{\text{LC}}$ (Trümper 1970; Michel 1971). There would, however, be no steady isotropic radiation emanating from $R < R_{\text{S}}$. This may be relevant to the apparent ' central hole ' in the nebular brightness distribution noted by Scargle (1969) and others. It also suggests that the region $R < R_{\text{S}}$, though it is pervaded by 30 Hz waves, should emit no synchro-Compton radiation at all. We would not then expect any circular polarization from the nebula, except the small amount $\lesssim (\nu/\nu_{\text{L}})^{-1/2}$ associated with ordinary synchrotron radiation.

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