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THE EARLIEST FORM OF THE EPICYCLE THEORY

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1. *Introduction*

O. Neugebauer has shown[1] that in Papyrus Michigan 149 an epicycle theory is presented, in which the epicycles of the outer planets Mars, Jupiter and Saturn rotate "in the wrong sense", as Neugebauer puts it. According to Ptolemy, all planets should traverse their epicycles to the left, *i.e.* in the direction of increasing longitudes. In the papyrus, the outer planets are made to rotate to the right. The same theory also underlies the record of Pliny.[2]

According to Neugebauer, such a theory "seems to point to an early state of affairs, perhaps from a time before Apollonius". I fully agree with this view. At the time of Apollonius, theory and observation had been raised to such a high level that a primitive theory of this kind, which is very much at variance with the phenomena, would not be taken into serious consideration.

In what follows, I shall call the theory explained by Pliny and by the papyrus "the primitive epicycle theory". In Section 2 I shall present my arguments in favour of a pre-Apollonius origin of this theory. In Sections 3 and 4 I shall show that the "primitive epicycle theory" is probably due to the Pythagoreans.

In an earlier paper[3] I have defended the hypothesis that Plato knew about epicycle theory and alluded to it in the dialogues *Republic* and *Timaeus*. In the same paper a serious objection was discussed, which may be stated as follows. We learn from Simplicius that Plato asked the astronomers to answer the question: "By what assumptions of uniform, ordered, circular motion can one save the appearances?" The first to answer this question was Eudoxus, who developed his theory of concentric spheres. Now the epicycle theory fits the observations much better than the theory of Eudoxus. So, if Plato had known the theory of epicycles, how could he put his famous question to the astronomers, and how could Eudoxus invent a theory that was worse than an existing theory?

After the publication of Neugebauer's paper, this objection loses its force. The primitive epicycle theory invented by the Pythagoreans was quite good for the Sun, Mercury and Venus, but not for the outer planets, so the search for a better theory was fully justified.

In Section 5 I shall present my interpretation of Plato's text anew. I shall show that this interpretation is confirmed by testimonies of ancient authors, derived from competent sources. At least there were Greek mathematicians who interpreted Plato just as I do.

2. *The Time of Invention of the Primitive Epicycle Model*

According to Ptolemy (*Almagest*, XII, 1) Apollonius proved two theorems, one about eccentres and the other about epicycles. By means of the latter theorem one can determine the stationary points of the apparent motion of any planet as soon as the epicycle radius is known, or conversely, determine the radius as soon as one of the stationary points is known from observation. At the time of

Apollonius (about 200 B.C.) planetary observations were available to which the theory could be applied. In the text of Ptolemy (which seems to derive from a treatise of Apollonius) the proof is given only for the case in which the sense of rotation of the planet on its epicycle is the same as the sense of rotation of the epicycle centre on the deferent, but a similar theorem holds if the two senses are different. Now if one tries to apply the latter model to Mars or Jupiter, the result is in complete disagreement with the observations. Therefore the only reasonable assumption seems to be that Apollonius himself adopted the right model.

Another, independent argument leads to the same conclusion. About 280 B.C., some eighty years before Apollonius, Aristarchus of Samos proposed the hypothesis that the Sun is at rest, while the Earth and the planets rotate about the Sun. By this hypothesis, it is possible to obtain a reasonably good agreement with the observed phenomena. Apollonius, of course, knew about this system. Now if he tried to construct an epicycle system which agreed just as well with experience as the heliocentric system, he was bound to see that this is possible only if both the planets and their epicycle centres are made to rotate to the left, and he could easily construct an epicycle model yielding exactly the same phenomena as the heliocentric model.

Hence we are justified in supposing that Apollonius assumed all planets to rotate to the left, and it is also reasonable to suppose, as Neugebauer did, that the other model, in which the outer planets rotate to the right on their epicycles, was invented before Apollonius, probably even before Aristarchus.

3. *The Pythagoreans as Inventors of Epicycles and Eccentrics*

Proclus states in his astronomical treatise *Hypotyposis:*

> The hypotheses of eccentrics and epicycles commended themselves also, so history tells us, to the famous Pythagoreans as being more simple than all others—for it is necessary in dealing with this question, and Pythagoras himself encouraged his disciples, to try to solve the problem by means of the fewest and most simple hypotheses possible.[4]

In considering this statement, I propose to leave aside the passage about Pythagoras, who "encouraged his disciples to try to solve problems by means of the fewest and most simple hypotheses possible". There was a general tendency among Pythagoreans and Neo-Pythagoreans to ascribe all sorts of inventions in the domain of science to Pythagoras himself, and we do not know to what extent this was justified.

Leaving out the passage about Pythagoras, which may or may not derive from an ancient Pythagorean source, we are left with a quite definite statement about the Pythagoreans, who knew about epicycles and eccentrics and preferred these hypotheses to others because of their simplicity. Concerning his source, Proclus says "as history tells us". It is possible that by "history" he meant the *History of astronomy* of Eudemus; another possibility would be that "history" simply means "tradition". Proclus's usual source of information on the history of the mathematical sciences was Geminus. In his *Commentary on the first book of Euclid's Elements*, Proclus quotes Geminus not less than 20 times. He also quotes Eudemus, but Tannery[5] has shown that these quotations probably come from Geminus. The title of the work of Geminus used by Proclus was "Consideration of the Mathematical Sciences" (τῶν μαθηματῶν θεωρία). This work must have been, according to Tannery, a kind of Encyclopaedia of the Mathematical Sciences. It contained at least six books, for Eutocius quotes the sixth book.[6]

I hope to show on another occasion that the first Proemion of Proclus's *Commentary on Euclid* was also drawn from the work of Geminus. In any case, Proclus was familiar with this work. It is quite possible that he obtained his information on epicycles and eccentric circles from the same source.

As far as the eccentric circles are concerned, the statement of Proclus is confirmed by Simplicius, who writes in his *Commentary to Aristotle's "De caelo"*,[7] speaking of the system of concentric spheres:

> Later astronomers then, rejecting the hypothesis of revolving spheres, mainly because they do not suffice to explain the variations of distance and the irregularity of the movements, dispensed with concentric spheres and assumed eccentrics and epicycles instead—if indeed the hypothesis of eccentric circles was not invented by the Pythagoreans, as some tell us, including Nicomachus and Iamblichus who followed him.

The testimonies of Proclus and Nicomachus are well in accordance with a statement made by Geminus (*ca* 70 B.C.) in his *Astronomical introduction:*

> It is a fundamental assumption in all astronomy that the Sun, the Moon, and the five planets move in circular orbits at uniform speed in a sense contrary to that of the universe. For the Pythagoreans, who were the first to apply themselves to investigations of this kind, assumed the movements of the Sun, the Moon, and the five planets to be circular and uniform. They would not admit, with reference to things divine and eternal, any disorder such as would make them move at one time more swiftly, at one time more slowly, and at another time stand still, as the five planets do at their so-called *stationary points*. For such irregularity of motion would not even be expected of a decent and orderly man in his journeys. With men, of course, the necessities of life are often causes of slowness and swiftness; but with the imperishable stars it is not possible to adduce any cause of swiftness or slowness. Accordingly they posed the problem, how the phenomena could be accounted for by means of circular and uniform movements.[8]

Geminus is a very good source. He explains astronomical theories in a completely clear and competent way. He was familiar with the *History of geometry* of Eudemus, and probably also with the *History of astronomy* of the same author. From his *Isagoge* we know that Geminus was well informed about the astronomical theories of his early predecessors: Euctemon, Callippus and the Chaldeans. What he says about the Chaldeans is in perfect accordance with cuneiform texts.[9]

According to Geminus, the Pythagoreans started with the thesis that the planets are eternal and divine. From this they concluded that their motions must be uniform and circular. Plato, in his *Laws*, argued the other way round. Astronomy, he said, has taught us that the motions of the planets are uniform and circular, and from this he concluded that their motions are governed by a reasonable, divine soul (*Laws* 896E–898C) or possibly by several such souls (899B). Plato could draw this conclusion because by his time the uniform circular motion had been accepted by all competent astronomers. The Pythagoreans could not start with this premise, for in the fifth century the circular motion of the planets was not firmly established. According to Geminus, they took the opposite course: from the assumed divinity of the planets they deduced the postulate of uniform circular motion, and on this postulate they based their astronomy.

This theological deduction of a scientific postulate is not Platonic: it is characteristic of the Pythagoreans. The same tendency to base a science on a theological argument can be observed in Pythagorean harmonics.[10]

According to Geminus, the Pythagoreans knew very well that the apparent motion of the planets is not uniform. They knew about stationary points and retrogradations. Yet they declared that the true motions of the planets must be uniform and circular, because the planets are eternal and divine: "They cannot have a reason for faster or slower motion". For this reason they asked: "How can one explain the phenomena by assuming circular and uniform motions?"

As a workable solution to this problem the eccentric and epicyclic hypotheses presented themselves. Therefore the testimony of Geminus is in accordance with those of Proclus and Simplicius.

In a previous paper[10] I have examined a long fragment drawn by Theon of Smyrna from a book of the Neoplatonist Dercyllides. I have shown that the ultimate source of this fragment is most probably Geminus, and that the astronomical theories explained in it are certainly due to the Pythagoreans. In this fragment an outline of an astronomical theory is given, in which each planet moves on an epicycle between two concentric spheres. It is stated that while the movement of the planets does not appear uniform, it is in truth uniform and circular. Every planet moves freely and without restraint within ordered spheres on a minimum number of circular orbits.

The fragment from Dercyllides confirms the conclusions drawn from the testimonies of Proclus, Simplicius and Geminus. Yet, because these testimonies are rather late, we have to examine their reliability.

4. *How Reliable are Testimonies concerning Pythagorean Science?*

Sir Thomas Heath, after quoting the testimonies of Geminus, Proclus and Simplicius,[11] continues as follows:

> This passage [from Simplicius], it is true, may indicate that it was only eccentric circles and not epicycles also, which the Pythagoreans discovered; but Schiaparelli regards it as conclusive with reference to movable eccentrics. Unfortunately, he has not allowed for the fact that it was the habit of the neo-Pythagoreans to attribute, so far as possible, *every* discovery to the Pythagoreans, and even to Pythagoras himself.

As far as Pythagoras is concerned, I fully agree with Sir Thomas. Many legends about the science of Pythagoras were in circulation. It was, in fact, a habit of the neo-Pythagoreans to attribute all sorts of scientific discoveries to Pythagoras himself.

However, regarding the Pythagoreans, it seems to me that the statement of

Heath is unfounded. In my opinion, the neo-Pythagoreans had no motive and no opportunity to ascribe later discoveries to the Pythagoreans. The scientific doctrines of the ancient Pythagoreans were explained in written texts and were used by later scientists such as Euclid, Geminus and Ptolemy. Eudemus was well-informed about Pythagorean science, and later writers such as Geminus and Proclus used Eudemus as one of their sources. There was just no room for fanciful ascriptions and exaggerations.

To justify this thesis, I shall mention a few facts. For more details I may refer to my earlier papers.[12]

The four sciences. In Plato's *Republic* (510C–536D), Socrates enumerates four sciences existing at his time: Arithmetic, Geometry, Astronomy and Harmonics. In the case of Harmonics and Astronomy, Plato expressly mentions the Pythagoreans, who cultivated these two sciences and regarded them as sisters (530D–531C).

Aristotle reports (*Metaphysics* A5) that the so-called Pythagoreans in Italy, who lived at the time of Leucippus and Democritus and even before, applied themselves to the mathematical sciences (τὰ μαθήματα) and were the first to develop them. We may safely suppose that the Mathemata, of which Aristotle speaks, are just the four sciences mentioned by Plato.

In the first prologue to his commentary on the *Elements* of Euclid,[13] Proclus explains the reasons why the Pythagoreans divided their four sciences into two and two, Arithmetic and Music being concerned with Quantity (ποσόν), Geometry and Astronomy with Magnitude (πηλίκον). This part of the commentary of Proclus is probably due to Geminus, whose name is mentioned right at the beginning of the next section.[14] Geminus was well acquainted with the sciences of the Pythagoreans, as we shall see in the sequel. He probably quoted from a genuine Pythagorean source. In any case, the division of the Mathemata into just four sciences is in full accordance with the testimonies of Plato and Aristotle.[15]

Let us now consider the individual sciences.

(1) *Geometry.* Proclus has preserved three fragments from the *History of geometry* of Eudemus, in which specific inventions are ascribed to the Pythagoreans in two cases and to Archytas in another. In one of these fragments a complete proof of the theorem concerning the sum of the angles of a triangle is reproduced and ascribed to the Pythagoreans.

A scholion to Book 4 of Euclid's *Elements* says that this book is due to the Pythagoreans. In the scholion, as in the two fragments from Eudemus, no author's name is mentioned: the inventions are simply ascribed to "the Pythagoreans". It seems that Eudemus quoted from an existing treatise written by or ascribed to "the Pythagoreans", in which no specific author's name was mentioned.

By an analysis of the first four books of the Elements,[16] Neuenschwander has shown that large parts of these books were drawn from an earlier Pythagorean treatise on elementary geometry. These parts can be recognised by a typical standardised method of exposition and quotation. Euclid must have known this treatise and used it, leaving parts of it unchanged (mainly in Books 2 and 4) and re-writing other parts (notably in Book 1). Probably Eudemus and Euclid both used the same treatise.

A scholion to the thirteenth book of the *Elements* states that the Pythagoreans knew only three regular polyhedra. This shows that even at a much later date detailed information on the geometry of the Pythagoreans was available. The knowledge of all five solids was ascribed to Pythagoras by Proclus and Iamblichus, but never (as far as I know) to "the Pythagoreans".

(2) *Harmonics.* Both Plato and Aristotle were familiar with Pythagorean harmonics. Plato writes: "The numbers they seek are those found in the heard concords" (*Republic* 531C). Aristotle confirms this: "They saw that the properties and ratios of the musical scales are based upon numbers. . ." (*Metaphysics* A5, 986a).

Eudemus gives more detailed information. He states[17] that the Pythagoreans found the ratios of the three symphonic concords quart, quint and octave in the numbers 2, 3 and 4. It follows that a Pythagorean theory of symphonic concords was known to Plato, Aristotle and Eudemus.

A fuller account of such a theory, or rather of two such theories, was given by Ptolemy in his *Harmonics*.[18] Ptolemy ascribes both theories to "the Pythagoreans". I have shown that the first theory, which is based on three axioms, must be more ancient than the second theory, which is based on two axioms only. The second theory is also explained in Euclid's *Sectio canonis;* it is probably due to Archytas.[19] Hence the first theory must be due to Pythagoreans before Archytas.

It follows that Plato, Aristotle, Eudemus, Euclid and Ptolemy were well-informed on the harmonics of the Pythagoreans.

(3) *Arithmetic.* In his *Commentary to Plato's Republic*, Proclus gives an account of the theory of "side-and-diagonal-numbers" due to "the Pythagoreans".[20] The ratios of these numbers are approximations to the ratio of the diagonal of a square to its side. If s_n is the nth side number and d_n the corresponding diagonal-number, the relation

$$d_n^2 = 2s_n^2 \pm 1$$

holds. According to Proclus, the Pythagoreans proved this "from numbers" (δία τῶν αριθμῶν). In this proof they used a theorem which was proved geometrically (γραμμικῶς) by Euclid (*Elements*, II, 10).

Book 7 of Euclid's *Elements* contains a systematic foundation of arithmetic, including the theory of ratios of numbers. I have analysed the logical structure of this book and investigated its connection with the work of Archytas.[21] My conclusion was that this book is due to the Pythagoreans before Archytas.[22]

If this conclusion is admitted, it follows that Euclid and Proclus were familar with the arithmetic of the Pythagoreans.

(4) *Astronomy.* In Section 3, we quoted three testimonies from Proclus, Simplicius, and Geminus, to the effect that the Pythagoreans had a geocentric theory of planetary motion based on the assumption of epicycles and eccentric circles. However, since the reliability of these testimonies has been called in question by Heath and others, we have to look for independent confirmation.

The doxographers' account of the opinions of Alcmaeon includes one important statement, namely Aëtius ii.16.2-3:

> Alcmaeon and the mathematicians hold that the planets have a motion from east to west, in a direction opposite to that of the fixed stars.

On this passage, Heath makes the following comment:

> Incidentally, the assumption of the motion of the fixed stars implies the immobility of the Earth. But this passage is also the first we hear of the important distinction between the diurnal revolution of the fixed stars from east to west and the independent movement of the planets *in the opposite direction*; the Ionians say nothing of it (though perhaps Anaximenes distinguished the planets as having a different movement from that of the fixed stars); Anaxagoras and Democritus did not admit it; the discovery, therefore, belongs to the Pythagorean school. . . .[23]

I fully agree with Heath that the anonymous mathematicians mentioned together with Alcmaeon must have been Pythagoreans. Among the Pythagoreans there were scientists who called themselves "Mathematikoi", and who published their inventions under a collective name, whereas other mathematicians, including Archytas, usually wrote under their own names.

From this testimony we may conclude that the Pythagorean "mathematikoi" had a geocentric theory, in which the planets had a proper motion from west to east, opposite to that of the fixed stars.

Ptolemy informs us (*Almagest*, IX, 1) that "*all first mathematicians*" agreed that the spheres of the planets are inside the sphere of the fixed stars, but outside the sphere of the Moon. They also agreed that the outermost planetary spheres are those of Mars, Jupiter and Saturn. However, "*the earlier mathematicians*" put the spheres of Venus and Mercury inside the sphere of the Sun, whereas "some later ones" put them beyond the sphere of the Sun.

Among the "first mathematicians" we must, of course, include the Pythagoreans. In fact, Eudemus informs us that the Pythagoreans were the first to adopt a definite order of the planets in space. His statement is quoted by Simplicius in his *Commentary* on *De caelo:*

> Anaximander was the first to broach the subject of sizes and distances; this we learn from Eudemus, who however refers to the Pythagoreans the first statement of the order [of the planets] in space.[24]

If we combine the statements of Ptolemy and Eudemus, we may conclude that the Pythagoreans ordered the planetary spheres, starting from the Earth, as follows: first the Moon, next Venus and Mercury (or Mercury and Venus), next the Sun, Mars, *etc.*

Ptolemy's statement that some later authors placed Venus and Mercury beyond the Sun is confirmed by several other sources. We know that Eudoxus, Plato, Callippus, Aristotle, and Eratosthenes adopted this arrangement.[25]

Another testimony is furnished by Nicomachus,[26] who says that the seven planets correspond to the seven chords of a heptachord. His order of the planets, starting from the Earth, is Moon, *Venus*, *Mercury*, *Sun*, *etc.* This order differs from the Platonic order and also from the "Chaldaean" order,[27] which was in general use at the time of Nicomachus. The connection with the heptachord indicates that Nicomachus drew from a Pythagorean tradition. Quite generally, when Nicomachus makes statements without mentioning his source, these statements are mostly drawn from ancient Pythagorean traditions. I hope to show this on another occasion by a detailed analysis of the work of Nicomachus.

If we assume that Nicomachus's order was that of the Pythagoreans, his testimony agrees with that of Eudemus and Ptolemy. According to Eudemus, the Pythagoreans had a quite definite order of the planets; according to Ptolemy they placed Venus and Mercury below the Sun, just as did Nicomachus. Presumably, a Pythagorean treatise on astronomy existed, which was used by Eudemus, Nicomachus, and Ptolemy, and also by Theophrastus, the main source of the doxographers.

I assume the same treatise (or excerpts from it) was also used by Pliny, by the author of the Papyrus Michigan, and by Geminus, Proclus and Simplicius in their statements about epicycles and eccentric circles. It seems to me that there is no reason to doubt the statements of these authors.

5. *Plato and the Epicycle Hypothesis*

In a myth at the end of Plato's *Republic*, the cosmos is represented as a spindle surrounded by eight whorls, which fit together like the boxes children play with. The outer whorl represents the sphere of the fixed stars; therefore we may assume that the other whorls were also supposed to be spheres or segments of spheres.

The astronomical system underlying Plato's myth is essentially the same as that explained in the *Timaeus*. The spherical Earth is at rest in the middle of the cosmos. The sphere of the fixed stars rotates to the right and carries all planets with it in its daily rotation. In addition, every one of the planets (including Sun and Moon) has its own proper motion to the left, *i.e.*, in the direction of the succession of zodiacal signs. The order of the spheres, beginning with the outer one, is (in the *Republic* as well as in the *Timaeus*): Fixed Stars, Saturn, Jupiter, Mars, Mercury, Venus, Sun, Moon.

In the *Republic*, Plato explains at length the order of the breadths of the rims. He says:

> Now the first and outmost whorl had the broadest circular rim, that of the sixth was second, and third was that of the fourth, and fourth was that of the eighth, fifth that of the seventh, sixth that of the fifth, seventh that of the third, eighth that of the second. . . .

This means that the breadth of the sphere of fixed stars is largest of all, the other spheres following in the order:

Venus Mars Moon Sun Mercury Jupiter Saturn.

Now two questions arise:

(1) What does Plato mean by the breadth of the rims, and why does he attach so much importance to them?
(2) How does Plato explain the fact, expressly mentioned in the *Timaeus*, that Venus and Mercury alternately are overtaken by and overtake the Sun?

From Plato's own words (*Timaeus* 38 c–d) it is clear that he has in mind an explanation for this phenomenon. He explains it by means of an ἐναντίαν δύναμιν, a "contrary force" (or impulse), which Venus and Mercury have received. So the question arises:

(3) What does Plato mean by this "contrary force"?

I have elsewhere shown[28] that all three questions can be answered at the same time by assuming that Plato had an epicycle theory in mind. I may add that Heath, in his discussion of the passage *Timaeus* 38 c–d, already considered this possibility.[29] He writes: "The explanation would be quite satisfactory, if Plato could be supposed to have been acquainted with the theory of epicycles."

Heath wrote this in 1913. To-day we are in a better situation, because Neugebauer's investigation has led us to the conclusion that the theory of epicycles was probably known to the Pythagoreans. If we admit this, there is no longer any objection to the assumption that Plato was acquainted with this theory.

On this assumption, we may suppose that according to Plato the planets did not move *on* their spherical surfaces, but in the interspace between two such surfaces. The commentator Dercyllides, who wrote a book *On the spindle and the whorls in Plato's "Republic"*, explains this as follows:

> Every sphere has two surfaces, one concave and one convex, and in the interspace between the two spheres the stars move on epicycles and concentres, and as a consequence of this motion they accidentally describe excentres. As seen from our eye, the motions of the planets are not uniform, but in substance and truth they are uniform.[30]

Dercyllides was no astronomer, and many of his explanations are awkward and unintelligible, but the explanation just quoted is astronomically correct, as I have elsewhere shown.[31] I have also shown that Dercyllides's source was most probably Geminus. Whether or not this is true, in any case Dercyllides must have had a good source: a mathematician who explained the Pythagorean epicycle theory in just this way, and who also interpreted Plato's image of the spindle and the whorls by means of the theory of epicycles.

Even if one leaves aside the testimony of Dercyllides, there are several strong arguments in favour of this interpretation. They may be stated as follows:

A. If Plato's "breadths" are interpreted as widths of interspaces between bounding spheres, we can understand why Plato laid so much stress on these breadths.

B. If this interpretation is correct, the interspaces must be so wide that the epicycles have enough room. I do not know why Plato made the stellar sphere widest of all; perhaps he assumed the fixed stars to have widely different distances to the Earth. I also do not know why he gave the Sun and the Moon more space than was necessary for their epicycles. However, in the case of the five "star-planets" the order of the breadths is exactly the order one would expect from epicycle theory. In fact, if we consider for each of the five the ratio between the radii of the epicycle and the concentre, this ratio is largest for Venus, next comes Mars, next Mercury, next Jupiter and finally Saturn. This is just Plato's order. The probability for such a coincidence to happen by pure chance is 1:120.

C. If the epicycle interpretation is accepted, our question (2) receives a satisfactory answer. In fact, the epicycle hypothesis explains why Venus and Mercury sometimes overtake and sometimes are overtaken by the Sun.

D. The "contrary impulse" too is satisfactorily explained by the epicycle interpretation. In fact, the rotation of the Sun on its epicycle is to the right, whereas Venus and Mercury rotate to the left on their epicycles. It is just as Plato says: the three are "isodromous", which means that their sidereal revolutions are the same, but Venus and Mercury have received an opposite motion impulse on their epicycles as compared with the Sun.

These four arguments are based upon Plato's text alone. Two additional arguments may be drawn from later sources:

E. Theon of Smyrna[32] and Chalcidius[33] both agree that Plato preferred the epicycle hypothesis. Their commentaries probably derive from the same source, namely Adrastus, who wrote a commentary to the *Timaeus*. Chalcidius stresses three times that the Sun rotates on its epicycle in a direction opposite to that of Venus and Mercury. In Chapter 3 Chalcidius explains the epicycle theory for Venus and the Sun by means of a drawing, with which the text is in complete agreement. From this we may safely conclude that text and drawing were copied from a treatise written by a competent astronomer.

F. Proclus writes in his commentary to *Timaeus* 38 D:

> Further we have to investigate by what cause the Sun, Venus and Mercury are isodromous. Some mathematicians say that this is the case because the epicycles of these three stars are connected and their centres lie on a straight line. Now since for a single straight line there can be only one return to the initial position, these three epicycles return to their initial positions simultaneously. Of these three epicycles the outer ones are smaller, whereas the middle one is larger; hence the uniform motions are explained by the same explanation as the non-uniform motions.

In Plato's order of planets, the middle of the three planets in question is Venus, and in fact Venus has the largest epicycle, so the explanation given by these "mathematicians" is astronomically correct and in accordance with Plato's text.

Our sources mention only the epicycles of Venus, Mercury and the Sun in connection with Plato. Regarding the outer planets Plato says (*Timaeus* 38 D):

> As to the rest of the stars, were one to describe in detail the positions in which He set them, and all the reasons therefore, the description, though but subsidiary, would prove a heavier task than the main argument which it subserves. Later on, perhaps, at our leisure these points may receive the attention they merit.

From this text one sees that in Plato's opinion the motion of the outer planets presented a difficult problem. Perhaps he realised the difficulties to which the Pythagorean epicycle model led. This would explain why he asked the astronomers to develop hypotheses of uniform circular motion to "save the appearances" for *all* planets.

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21. B. L. van der Waerden, "Die Arithmetik der Pythagoreer", *Mathematische Annalen*, cxx (1947), 127–53.
22. *Cf.* the article cited in ref. 12.
23. Heath, *Aristarchus of Samos*, 50.
24. Ed. Heiberg, 471.
25. *Cf.* L. Schiaparelli, "Die kozentrischen Sphären des Eudoxos. . .", *Abh. zur Geschichte der Math.*, i (1877), 101–98.
26. Excerpta ex Nicomacho, iii (*Musici scriptores Graeci*, ed. C. Jan, 1895), 271.
27. See B. L. van der Waerden, "Die Aegypter und die Chaldäer", *Sitzungsberichte der Heidelberger Akad.*, 5. Abhandlung (1972).
28. See ref. 3.
29. *Op. cit.* (ref. 23), 257, footnote.
30. Theon Smyrnaeus, *Expositio rerum mathematicarum* . . . (ed. Hiller), 200. *Cf.* the article cited in ref. 12.
31. In the article cited in ref. 10.
32. *Op. cit.* (ref. 30), 188–9.
33. *Commentatio in Timaeum* (ed. Wrobel), chs 81 and 109–11.