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RADIOMETRIC DIAMETERS AND ALBEDOS OF 40 ASTEROIDS

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ABSTRACT

New radiometric observations at wavelengths of 10 and 20 μ are presented for 34 asteroids. When combined with similar data for nine asteroids published by Cruikshank and Morrison, these observations yield a sample of 38 mainbelt asteroids and two Mars-crossing asteroids for which diameters and geometric albedos can be determined. Within this sample, the five largest asteroids are 1 Ceres, 2 Pallas, 4 Vesta, 10 Hygiea, and 511 Davida. The highest albedo (0.24) is that of 4 Vesta; the lowest (0.03) is found for 19 Fortuna, 324 Bamberga, and 747 Winchester. No significant correlation exists between diameter and albedo or between either of these and the orbital elements. When the albedo and diameter are plotted against $(B - V)$ color, however, clear groupings into three classes are evident. The most numerous class is apparently of silicate mineralogy ($0.8 < B - V < 0.9$; $0.08 < p_V < 0.18$; $D < 250$ km); the second class, which includes the largest asteroids, is apparently carbonaceous ($B - V < 0.74$; $p_V < 0.11$); and the third class contains but a single known member, 4 Vesta. The three asteroids with measured masses have mean densities of 2.1–3.1 g cm^{-3}.

Subject headings: asteroids — infrared — meteorites, meteors

I. INTRODUCTION

Although many thousands of minor planets are known, only a few dozen have been the subject of physical investigations, and until very recently there was little evidence to suggest that the asteroids do not constitute a mineralogically homogeneous group. One of the impediments to detailed studies has been the small angular sizes of these objects, which place even the largest at the resolution limit of the best telescopes. Only during the past three or four years has the application of two new techniques of determining the reflectivities of the asteroid surfaces—from polarimetric and from radiometric observations—begun to reveal information on their sizes and albedos (cf. Morrison 1973*a*; Chapman and Morrison 1974). In this paper, the results of a two-year program of infrared radiometry of 40 asteroids are presented.

The radiometric technique of determining albedos of small objects that are in equilibrium with the insolation was first applied to minor planets by Allen (1970, 1971) and Matson (1971, 1972). The method is based upon the complementarity of photometric and radiometric observations, which measure the reflected and the absorbed components of the insolation, respectively. For dark objects in particular, broadband observations in the visible and in the infrared near the wavelength of peak thermal emission yield directly the *bolometric geometric albedo* (called p in this paper), and the solutions are relatively insensitive to the choice of such parameters of the models as the phase integral (q) and the infrared emissivity of the surface. For a detailed description of this technique and of its sources of uncertainty, the reader is referred to Morrison (1973*b*) and Jones and Morrison (1974).

The albedos and sizes of 11 minor planets based on radiometric observations have been published: four by Allen (1971), nine by Cruikshank and Morrison (1973), and three by Matson (1972), who also presents fragmentary data on 23 additional asteroids. Two important conclusions can be drawn from these published results. (1) The diameters of the first four asteroids, as measured with a filar micrometer by Barnard in 1894 and 1895 and long accepted as basically reliable, are too small. (2) There is a wide range in the albedos of asteroids. These conclusions are further supported by contemporaneous polarimetric determinations of asteroid sizes and albedos by Veverka (1971*a*, *b*, 1973), Zellner (1973), and Bowell and Zellner (1974). The discrediting of the Barnard diameters, in particular, has had a salutary effect on many physical studies of minor planets. However, there is clearly a need for determinations of albedo and size of a much larger sample of asteroids. Since one of the virtues of both the radiometric and polarimetric methods is the fact that the accuracy of the determinations is independent of the size of the asteroid (as long as it is bright enough to be measured photometrically or radiometrically), it is particularly appropriate that these techniques be applied to a broad sample of objects. In this paper, I discuss a homogeneous set of radiometric observations of 40 asteroids, for 29 of which there exists no previously published radiometry. Of these, 38 are in the main asteroid belt and constitute a statistically significant sample of the largest 100 or so minor planets.

II. OBSERVATIONS

The infrared radiometric observations were made with the 2.2-m telescope of Mauna Kea Observatory between 1973 February and 1974 January. The infrared photometer is that described previously by Morrison and Cruikshank (1973) and Morrison and Simon (1973). Most objects were observed in two broad bands, the first from 8 to 14 μ and the second from 17 to 28 μ. All observations were calibrated with respect to a set of bright, nonvariable standard stars.

TABLE 1

MAGNITUDES OF STANDARD STARS

Star	m_{10}	m_{20}
α Boo.......	−3.2	−3.3
α Her.......	−4.0	−4.3
β Peg.......	−2.5	−2.7
α Tau.......	−3.1	−3.2
α Ori.......	−5.2	−5.7

The magnitudes adopted for these standards are listed in table 1; the flux density for zero magnitude is 1.2×10^{-16} W cm^{-2} μ^{-1} at 10 μ and 7.3×10^{-18} at 20 μ. These flux and magnitude scales are consistent with those generally in use in infrared photometry (Becklin *et al.* 1973), and are identical to those used in the calibration of the radiometric technique by Morrison (1973*b*) and Jones and Morrison (1974).

Table 2 is a log of the asteroid observations. Given are the object, month of observation, phase angle (α), distance from the Sun in a.u. (R), number of observations at each wavelength (N_{10} and N_{20}), and monochromatic magnitudes at $\Delta = 1$ a.u. in the two bands with their estimated standard errors. The broad-band observations have been converted to monochromatic magnitudes as described by Morrison (1973*b*). They are also corrected to unit distance from Earth; however, as noted by Morrison (1973*b*), it is not possible to scale them to arbitrary distance from the Sun as is usually done for visible photometry, and I will analyze these infrared observations for the value of R at which they were obtained.

Observations of 34 asteroids are given in table 2. However, since the observations of nine asteroids (of which three are in common with the list of 34) published by Cruikshank and Morrison (1973) were made with the same equipment and are on the same photometric system, we will include them with the new data in the following analysis. Thus a homogeneous set of observations of 40 asteroids can be discussed.

A comment on possible selection effects in this sample of asteroids is in order. Except for the two Mars-crossing objects, 433 Eros and 887 Alinda, they

TABLE 2

LOG OF OBSERVATIONS

Name	Date	α	R	N_{10}	N_{20}	m_{10}	m_{20}
4 Vesta..........	01/74	−25	1.90	4	4	−3.15 ± 0.10	−5.90 ± 0.10
5 Astraea.........	09/73	−17	2.65	5	4	−0.40 ± 0.15	−2.45 ± 0.10
7 Iris............	01/74	+12	2.03	5	5	−1.60 ± 0.10	−4.07 ± 0.10
8 Flora..........	04/73	+16	2.53	3	3	−0.30 ± 0.15	−3.20 ± 0.15
9 Metis..........	04/73	+11	2.49	9	9	−1.08 ± 0.05	−3.85 ± 0.05
10 Hygiea.........	06/73	−16	3.00	4	4	−1.40 ± 0.15	−4.35 ± 0.10
	09/73	+12	3.06	2	1	−2.19 ± 0.15	−5.40 ± 0.15
11 Parthenope.....	04/73	−18	2.40	3	3	−0.25 ± 0.10	−3.12 ± 0.10
	06/73	+13	2.33	1	1	−0.80 ± 0.20	−3.30 ± 0.20
14 Irene...........	09/73	− 6	3.01	6	5	+0.37 ± 0.05	−2.80 ± 0.10
15 Eunomia.......	01/74	−13	2.75	1	...	−1.20 ± 0.20	...
16 Psyche.........	04/73	− 7	3.28	2	4	−0.62 ± 0.15	−3.72 ± 0.05
	06/73	+15	3.23	2	2	−0.75 ± 0.10	−3.81 ± 0.10
17 Thetis..........	06/73	− 4	2.14	3	5	+0.15 ± 0.15	−2.46 ± 0.10
18 Melpomene.....	04/73	− 6	2.80	2	2	−0.15 ± 0.15	−3.13 ± 0.10
19 Fortuna........	03/73	−13	2.83	2	1	−1.47 ± 0.20	−3.92 ± 0.15
	04/73	+ 1	2.83	4	4	−0.85 ± 0.05	−3.93 ± 0.05
21 Lutetia.........	06/73	+ 8	1.15	4	5	−0.20 ± 0.10	−2.65 ± 0.05
22 Kalliope........	02/73	−13	2.90	11	7	−0.05 ± 0.15	−3.30 ± 0.10
23 Thalia..........	04/73	+ 9	2.31	5	5	+0.10 ± 0.05	−2.70 ± 0.05
27 Euterpe........	02/73	+ 4	1.02	9	8	−0.40 ± 0.05	−3.05 ± 0.05
29 Amphitrite......	04/73	−18	2.74	3	3	−0.30 ± 0.15	−3.40 ± 0.10
	06/73	+ 9	2.73	1	1	−0.71 ± 0.15	−3.69 ± 0.15
30 Urania.........	04/73	+ 7	2.65	2	2	+1.02 ± 0.10	−1.92 ± 0.10
37 Fides...........	02/73	...	2.59	12	7	+0.80 ± 0.05	−2.10 ± 0.05
40 Harmonia......	09/73	+12	2.17	3	3	−0.20 ± 0.10	−2.80 ± 0.15
43 Ariadne........	01/74	+ 3	2.56	5	...	+0.85 ± 0.20	...
63 Ausonia........	06/73	+12	2.10	5	4	−0.20 ± 0.05	−2.55 ± 0.20
89 Julia...........	01/74	+ 4	2.70	4	3	−0.40 ± 0.20	−3.20 ± 0.10
140 Siwa...........	09/73	+10	2.19	4	5	0.00 ± 0.05	−2.54 ± 0.05
192 Nausikaa.......	06/73	+17	2.73	...	6	...	−1.80 ± 0.10
230 Athamantis.....	04/73	− 6	2.52	2	2	−0.06 ± 0.10	−2.91 ± 0.10
	06/73	+21	2.50	2	2	+0.40 ± 0.25	−2.50 ± 0.20
324 Bamberga......	04/73	−19	2.79	1	1	−1.10 ± 0.20	−3.90 ± 0.20
471 Papagena.......	04/73	+14	3.25	5	5	+0.85 ± 0.20	−2.50 ± 0.15
511 Davida.........	09/73	+13	2.92	5	5	−1.25 ± 0.15	−4.15 ± 0.15
532 Herculina.......	03/73	+12	2.29	4	2	−1.92 ± 0.10	−4.00 ± 0.10
	04/73	+21	2.28	5	5	−0.95 ± 0.05	−3.80 ± 0.05
554 Peraga.........	09/83	−12	2.11	4	5	−0.15 ± 0.10	−2.45 ± 0.10
747 Winchester......	09/73	+11	2.42	3	3	−1.25 ± 0.10	−3.80 ± 0.10
887 Alinda.........	01/74	+ 7	1.14	6	2	+4.80 ± 0.10	+2.85 ± 0.20

were selected on the basis only of their visible brightness. During 1973 I attempted to observe every asteroid with mean opposition magnitude less than 11.5 that came to opposition after February and was north of declination $-40°$. Of these 36 objects, 33 were either successfully observed in this program or had been observed previously by Cruikshank and Morrison (1973); only 354 Eleonora (due to a faulty ephemeris), 39 Laetitia, and 129 Antigone were missed. An additional five asteroids were selected from among those with *1973* opposition magnitudes of less than 11.5. No objects other than the three named above were searched for but not located. I therefore argue that the only bias in this sample is toward photometrically bright objects; there is no significant selection, for instance, of low-albedo objects that would be more easily detected radiometrically.

III. RESULTS

In order to derive albedos and diameters from the radiometric observations, we must know the photometric brightness of the asteroid at the time of the observations, and we must have a model to describe the relationship between the absorbed solar radiation at the asteroid surface and the infrared brightness as seen from Earth. The absolute visual magnitudes $V(1, 0)$ are taken from a list prepared by Zellner, Gehrels, and Gradie (1974); most of these are in turn derived from photometry published by Gehrels (1970) and Taylor (1971). In order to relate the visual magnitudes to the bolometric magnitudes, I have used a summary of asteroid spectrophotometry between 0.3 and 1.1 μ prepared by C. R. Chapman, drawn largely from observations published by Chapman, McCord, and Johnson (1973) and McCord and Chapman (1974). No allowance has been made for the influence of rotation or observing aspect on these published opposition magnitudes.

The model for the thermal behavior of the asteroids is that described by Jones and Morrison (1974). The asteroid is assumed to be spherical, nonrotating, and viewed at zero phase angle. The observed radiometric brightness temperature will in general then be a function of the distance of the asteroid from the Sun (R), the bolometric Bond albedo (A), the departure of the thermal limb darkening from a $\cos^{1/4}\theta$ law, and the emissivity. Jones and Morrison assume that the flux absorbed at the surface will scale as $(1 - A)R^{-2}$, and they characterize the other two dependences of the emitted thermal radiation by a single variable, the brightness temperature at disk center at $R = 1$, $\Delta = 1$. This parameter is called T_0. For a smooth sphere with unit emissivity, $T_0 = 395°$ K. However, Jones and Morrison treat T_0 as a free parameter to be varied in order to obtain correct albedos and diameters of the Galilean satellites from radiometric observations, and they conclude that T_0 is $400° (+10°, -5°)$ K. Accordingly, I adopt $T_0 = 400°$ K for the analysis presented in this paper. However, it should be noted that the occultation diameter of Iapetus recently obtained by Elliot, Veverka, and Goguen (private communication) is about 10 percent smaller than the value obtained for this satellite with the present model, suggesting that the results given here for asteroids may be systematically in error by about this amount.

The main observational parameter from which the geometric albedo is to be derived is the difference between the visual and radiometric magnitudes. The data in table 2 determine two such differences, $V - m_{10}$ and $V - m_{20}$, for most of these asteroids. Both the 10- and 20-μ magnitudes measure the radiation near the peak of the Planck curve for the asteroids, and both should yield albedos and diameters of comparable precision. This redundancy can be used to examine possible departures from the assumptions of the thermal models.

Before discussing the thermal model needed to derive albedos and diameters from combined visual and radiometric magnitudes, let us consider what can be learned from the 10- and 20-μ magnitudes alone. The difference $(m_{10} - m_{20})$ should be a function of the temperature of the object (and hence of both its Bond albedo and its distance from the Sun) and of any differences between the broad-band thermal emissivities at 10 and 20 μ. In figure 1 are plotted the observed radiometric magnitude differences as a function of the solar distance R. The minimum standard error in this color index is about 0.1 mag, and for most of these objects the uncertainty is between 0.15 and 0.20 mag. To within this rather low level of precision, the asteroid observations define a smooth, monotonic curve in figure 1. One can therefore immediately conclude that there are no large variations (say of 20% or more) in the ratio of the 10- and 20-μ emissivities of this sample of objects.

If we assume that the emissivities at 10 and 20 μ are equal, we can interpret the $(m_{10} - m_{20})$ index in terms of the color temperature of the asteroid. The solid curve in figure 1 is the calculated value of this index for a sphere with a black surface. If the object has a nonzero Bond albedo, it will be to the left of this theoretical curve. The scale at the lower right of the figure relates the horizontal displacement of the observed point from the curve to the bolometric Bond albedo of the asteroid. Most of the observed points do fall to the left of the curve, typically by amounts that correspond to albedos of the order of 0.1. At the upper right of figure 1 I have plotted the Galilean satellite data from Morrison and Cruikshank (1974). These objects lie significantly to the left of the curve, as is to be expected from their relatively high albedos. It is clear from figure 1 that none of the asteroids has a color temperature that suggests a Bond albedo as high as that of Europa, Io, or Ganymede.

Since the displacement of each point in figure 1 from the theoretical curve is proportional to the Bond albedo A, while either radiometric magnitude in combination with the visual magnitude can yield the geometric albedo p, it would be possible in principle to determine both albedos and hence to derive the phase integral q of an asteroid. To do so, of course, requires that the emissivity be known. It is obvious that this method of deriving q will be most accurate

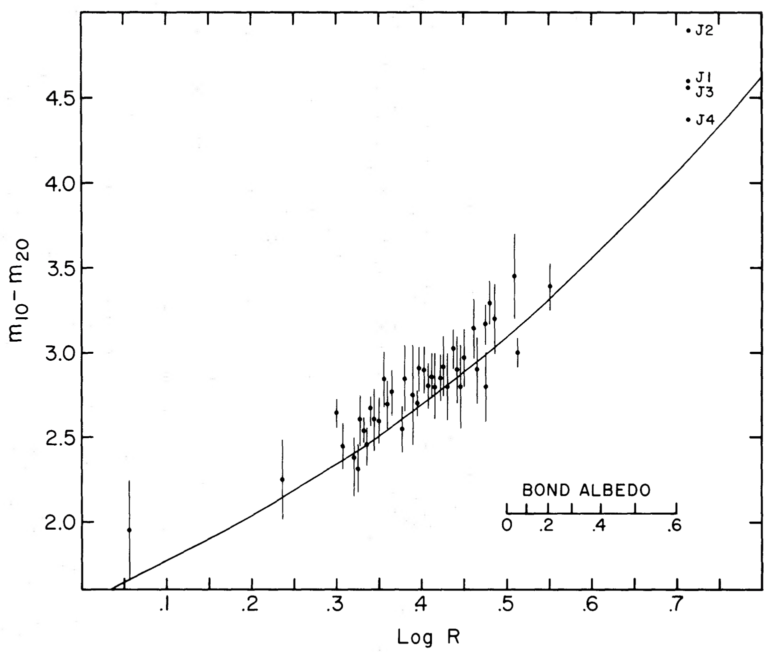

FIG. 1.—Color of the asteroids and the Galilean satellites from broad-band 10- and 20-μ radiometry. The solid line represents the $(m_{10} - m_{20})$ index calculated for a black sphere as a function of distance from the Sun. Objects that are not black but that have unit emissivities at 10 and 20 μ will lie to the left of the curve. The horizontal displacement from the curve is proportional to the log of the Bond albedo, as illustrated by the scale in the lower right part of the figure. The asteroids, unlike the Galilean satellites, all appear on the basis of this figure to have small Bond albedos.

for large values of A, whereas the asteroids are all dark objects. Indeed, the error bars plotted in figure 1 indicate that none of these asteroids has a color temperature significantly different from that of a blackbody, and therefore that none has a Bond albedo significantly different from zero. Nevertheless, a formal comparison of these Bond albedos of the 20 best-observed asteroids determined from figure 1 with the geometric albedos computed below yields a mean phase integral of $q = 0.8 \pm 0.4$. This value is consistent with what we would expect for these objects, but is of insufficient precision to be of much consequence.

The diameters and geometric albedos calculated from the models and the observations in table 2 are given in table 3. Tabulated are the absolute visual magnitude $V(1, 0)$, the ratio of bolometric to visual albedo p/p_V, the solar distance R, the two magnitude differences $(V - m_{10})$ and $(V - m_{20})$, the corresponding derived diameters, the mean of these two diameters, the visual geometric albedo p_V, and the logarithms of the diameter and of p_V. In these calculations it was assumed that the phase integral $q = 0.6$, but the results are not sensitive to departures from this value. Note that this method is not capable of determining the Bond albedos of these asteroids. The radiometric magnitudes in table 3 have been corrected to zero phase on the assumption that the infrared phase coefficient is $\sim$0.008 mag per degree (cf. Matson 1972). However, further observations of a few asteroids obtained over a wide range in α are needed to establish more accurately the dependence of m_{10} and m_{20} on α.

Jones and Morrison (1974) and Morrison (1973*b*) conclude that the radiometric method has an inherent uncertainty of about 10 percent in the diameter and 20 percent in the albedo as a result of uncertainties in the definitions of the photometric systems and of the basic parameters of the models. The internal uncertainties in most of the observations yield errors smaller than this; therefore, in most cases we are limited more by the inherent problems of the technique than by observational uncertainties. However, it is possible that the errors in diameter for some of these minor planets may exceed 10 percent. A few of the objects were observed on only a single night, and a few of the asteroids in table 2 either have poorly determined V-magnitudes or else are known to vary substantially as they rotate. Among the objects with high-amplitude light curves are 15 Eunomia, 17 Thetis, 43 Ariadne, and 433 Eros (Taylor 1971; Johnson and Matson 1973). Since I did not measure the V-magnitudes of these asteroids at the same time as the 10- and 20-μ magnitudes, my results could be substantially in error. However, I expect that the great majority of the listed diameters have relative values correct within 10 percent. If the asteroids have thermal emission properties similar to those of the Galilean satellites, which served as calibrators, the absolute values should also be accurate to 10 percent; however, if these darker objects have emissivities more strongly peaked in the forward direction (as does the Moon), the diameters

TABLE 3

SUMMARY OF RESULTS

Name	$V(1,0)$	p/p_V	R	$V - m_{10}$	$V - m_{20}$	D_{10}	D_{20}	D	p_V	$\log D$	$-\log p_V$
1 Ceres*	3.42	0.9	2.56	9.95	12.76	1016.	1067.	1041.	0.07	3.01	1.16
2 Pallas*	4.53	1.0	2.40	9.90	12.43	579.	560.	569.	0.08	2.75	1.08
3 Juno*	5.64	1.0	3.05	8.66	11.96	239.	265.	252.	0.15	2.40	0.81:
4 Vesta*	3.55	1.1	2.57	8.38	11.24	522.	530.	526.	0.24	2.72	0.62
4 Vesta	3.55	1.1	2.32	8.60	11.37	539.	566.	553.	0.22	2.74	0.66
5 Astraea	7.17	1.1	2.65	9.03	11.88	127.	129.	128.	0.14	2.10	0.84
6 Hebe*	5.87	1.1	2.15	9.07	11.61	213.	217.	215.	0.17	2.33	0.77
7 Iris	6.03	1.1	2.03	9.21	11.66	205.	209.	207.	0.16	2.31	0.80
8 Flora	6.67	1.1	2.53	9.08	11.98	159.	171.	165.	0.14	2.21	0.86:
9 Metis	6.45	1.1	2.49	9.58	12.28	213.	215.	214.	0.10	2.33	1.00
10 Hygiea	5.87	0.9	3.06	10.49	13.69	465.	514.	489.	0.03	2.68	1.48:
10 Hygiea	5.87	0.9	3.00	9.95	12.75	361.	337.	349.	0.06	2.54	1.19:
11 Parthenope	6.98	1.0	2.40	9.23	12.08	141.	155.	148.	0.13	2.17	0.89
14 Irene	6.60	1.1	3.01	8.62	11.79	151.	159.	155.	0.17	2.19	0.78
15 Eunomia*	5.48	1.0	2.19	9.12	11.79	260.	278.	269.	0.15	2.43	0.81
15 Eunomia	5.48	1.0	2.75	8.97	...	274.	...	274.	0.15	2.43	0.82:
16 Psyche	6.19	1.0	3.26	9.50	12.50	273.	259.	266.	0.08	2.42	1.08
17 Thetis	7.86	1.1	2.14	9.36	11.97	95.	101.	98.	0.13	1.99	0.88
18 Melpomene	6.97	1.1	2.80	9.35	12.33	163.	171.	167.	0.10	2.22	0.99
19 Fortuna	7.63	1.0	2.83	10.78	13.81	224.	243.	234.	0.03	2.36	1.54
21 Lutetia	8.00	0.9	2.14	9.85	12.30	108.	109.	109.	0.09	2.03	1.03
22 Kalliope	6.76	0.9	2.90	9.17	12.32	168.	185.	176.	0.11	2.24	0.95:
23 Thalia	7.47	1.1	2.31	9.27	11.98	114.	120.	117.	0.13	2.06	0.88
27 Euterpe	7.66	1.1	2.00	9.56	12.21	110.	125.	118.	0.11	2.07	0.96:
29 Amphitrite	6.42	1.1	2.74	9.20	12.10	195.	200.	198.	0.12	2.29	0.91:
30 Urania	7.90	1.1	2.65	9.01	11.93	90.	94.	92.	0.14	1.96	0.84
37 Fides	7.60	1.1	2.59	8.91	11.71	98.	99.	98.	0.16	1.99	0.78
40 Harmonia	7.63	1.1	2.17	9.61	12.21	118.	124.	121.	0.10	2.08	0.97
43 Ariadne	8.22	1.1	2.56	9.41	...	89.	...	89.	0.11	1.95	0.95:
51 Nemausa*	7.86	1.0	2.52	10.17	13.08	144.	160.	152.	0.05	2.18	1.26:
63 Ausonia	7.25	1.1	2.10	9.08	11.46	112.	108.	110.	0.18	2.04	0.74
89 Julia	7.00	1.1	2.70	9.55	12.35	171.	171.	171.	0.09	2.23	1.02
140 Siwa	(8.65)	1.0	2.19	10.31	12.89	99.	105.	102.	0.06	2.00	1.23:
192 Nausikaa	7.53	1.1	2.73	...	11.61	...	97.	97.	0.18	1.98	0.74:
230 Athamantis	7.77	1.1	2.50	9.70	12.45	123.	126.	125.	0.09	2.09	1.06
324 Bamberga	7.41	1.0	2.79	10.78	13.58	246.	243.	244.	0.03	2.38	1.49:
433 Eros*	11.52	1.1	1.72	10.24	12.49	23.	24.	24.	0.07	1.38	1.14:
471 Papagena	(6.95)	1.0	3.25	8.60	12.05	133.	150.	142.	0.14	2.15	0.84:
511 Davida*	6.43	1.0	3.56	10.13	13.52	342.	363.	353.	0.04	2.54	1.42:
511 Davida	6.43	1.0	2.92	10.05	12.95	287.	286.	287.	0.06	2.45	1.24:
532 Herculina	7.14	1.1	2.28	9.92	12.77	172.	198.	185.	0.07	2.26	1.15:
554 Peraga	8.74	1.0	2.11	10.56	12.86	104.	100.	102.	0.05	2.01	1.27
747 Winchester	(8.05)	1.0	2.42	11.21	13.76	205.	201.	203.	0.03	2.30	1.59:
887 Alinda	14.50	1.0	1.14	9.98	11.93	5.	5.	5.	0.10	0.72	1.01

* Observations taken from Cruikshank and Morrison 1973.

given here may be systematically large by of the order of 10 percent. In table 3, a colon follows the final entry for those objects that are subject to the greatest uncertainty, as judged by a difference of more than 10 percent in the diameters derived at the two wavelengths or by unusually large uncertainties in either the radiometric or the visual magnitudes used in the computations.

IV. DISCUSSION

a) *Comparison with Other Observations*

Ten of the asteroids studied in this paper were also observed radiometrically by Matson (1972), and 14 are on the list of objects with derived polarimetric diameters and albedos published by Zellner (1973). Matson (1972) does not list diameters or albedos for any of his asteroids except 4 Vesta, 7 Iris, and 324 Bamberga, but diameters derived from his radiometric observations at 11.6 μ can be extracted from figure 33 of his thesis. His model used to derive these radii is a "rough, nonrotating" one similar to that by Jones and Morrison (1974) used as the basis for calculations in this paper. The polarimetric diameters given by Zellner (1973) are based on preliminary results from his major program of asteroid observations (cf. Zellner *et al.* 1974). The data are interpreted according to the calibration of the asteroid "slope-albedo law" given by Bowell and Zellner (1974). Figure 2 compares the diameters from these sources with those given in table 3.

The agreement of the data in figure 2 is worse than might have been hoped. My diameters are generally in good accord with Matson's, as would be expected in view of the similar techniques used. The two objects for which we differ substantially—7 Iris and 15 Eunomia—presumably represent poor-quality observations. As illustrated in the upper part of figure 2,

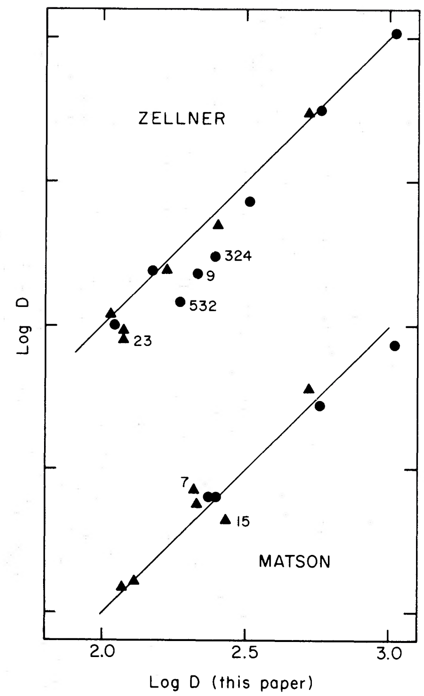

FIG. 2.—Comparison of the radiometric diameters presented in this paper with diameters derived by Zellner (1973) from polarimetric measurements and those derived by Matson (1972) from radiometry at 11.6 μ. The diagonal lines represent perfect agreement among the results. All objects for which my diameters differ from those measured by Zellner or Matson by greater than 0.10 dex are labeled. The circles in both plots are high-albedo asteroids (p_V > 0.10), and the triangles are dark asteroids (p_V < 0.10).

however, there are a number of cases where the polarimetric diameters are substantially smaller than the radiometric. Among the cases of greatest discrepancy are the very dark objects, such as 324 Bamberga, for which I obtain an albedo of $p_V = 0.03$ (in excellent agreement with Matson's value), whereas Zellner derives $p_V = 0.06$. The differences between the radiometric results and the polarimetric will be discussed in more detail in a forthcoming publication (Chapman, Morrison, and Zellner 1974). Since, however, the radiometric method should be most accurate for low-albedo objects whereas the polarimetric slope-albedo law is not well calibrated for samples with albedos less than 0.05, I will assume for purposes of the present discussion that there are no important systematic errors in the radiometric diameters in table 3.

b) *Distribution of Diameters and Albedos*

The selection of main-belt asteroids for study only on the basis of their opposition magnitudes discriminates in favor of large objects, high albedos, and small semimajor axes. Keeping these selection effects in mind, we can now examine the distribution of diameter and albedo within this 40-asteroid sample.

The number of asteroids as a function of log D is shown in figure 3. The two Mars-crossing objects in the sample are not included. Since the observations include about 70 percent of the asteroids with mean opposition magnitude of 11.5 or smaller, this list should be very nearly complete down to log $D \simeq 2.35$. Thus the five largest asteroids—1 Ceres, 2 Pallas, 4 Vesta, 10 Hygiea, and 511 Davida—shown in figure 3 are probably (barring observational errors) the largest objects in the main asteroid belt, and further investigation is unlikely to reveal more than one other asteroid in this size range (D > 300 km). On the other hand, for diameters of less than 200 km this list is clearly incomplete, and the cutoff at D = 100 km shown in the figure is entirely a selection effect.

The number of asteroids as a function of log p_V is illustrated in figure 4. Selection effects are less important here than in the previous figure, although they will reduce the number of objects with low albedos. It should also be remembered that the albedos plotted are (with the exception of 433 Eros and 887 Alinda) those of the larger asteroids. Most of the asteroids observed have albedos between 0.09 and 0.18, i.e., values similar to the range of geometric albedos on the Moon. A significant number of dark objects is also apparent, however, with albedos as low as 0.03. Presumably at least 25 percent of the asteroids in a sample that did not discriminate against low albedo

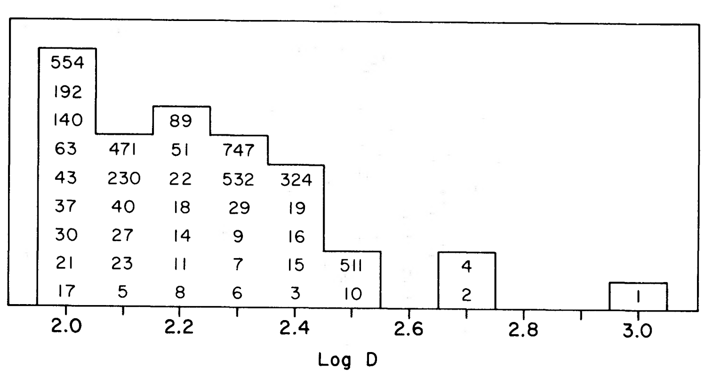

FIG. 3—Distribution of diameters derived for the 38 main-belt asteroids in this study. The cutoff at $D \simeq$ 100 km is an effect of observational selection. Typical uncertainties in the diameters are about one-half the width of a column in the histogram.

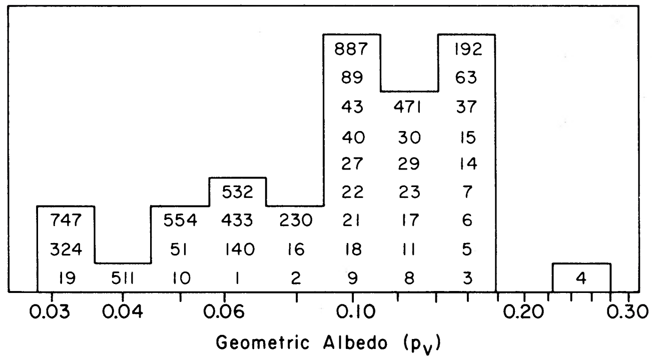

FIG. 4.—Distribution of geometric visual albedos derived for the 40 asteroids in this study. Selection effects reduce the number of dark objects observed. Typical uncertainties in the albedos are about the width of one column in the histogram.

would be found to have $p_V \leq 0.05$. Such low reflectivities suggest carbonaceous surfaces for these objects (cf. Chapman and Salisbury 1973; Johnson and Fanale 1973).

c) Correlations among Diameter, Albedo, Color, and Orbital Elements

It is of interest to see whether the parameters derived in this study—diameter and geometric albedo—are correlated with each other, with the $(B - V)$ color index, or with the orbital elements a (the semimajor axis), e (the eccentricity), or $\sin i$ (the inclination). Table 4 lists the linear correlation coefficients among these parameters calculated for the asteroids in this sample. The logarithm of the diameter and of the albedo were used in this computation rather than the values themselves. The two smallest objects—433 Eros and 887 Alinda—and the largest—1 Ceres—were omitted from calculations involving the diameter, and the three objects without measured colors were omitted from calculations involving $B - V$. No rigorous analysis of the significance of the correlations has been attempted, but I note that in a random sample of 40 objects from an uncorrelated parent population, a correlation coefficient of 0.3 or greater occurs with a probability of only 0.05. Several of the coefficients in table 4 are greater than 0.3; these cases will be discussed below.

It is clear from the coefficient in table 4 that there exists no significant correlation between asteroid albedo and size. This point is important, since large-scale statistical studies of the distribution of size and mass in the asteroid belt have been and will continue to be based on magnitudes only, not on measured sizes.

TABLE 4

CORRELATION COEFFICIENTS

Parameter	$\log p_V$	$B - V$	a	e	$\sin i$
$\log D$........	−0.18	−0.61	0.44	0.08	0.52
$\log p_V$........	...	0.49	−0.27	−0.16	−0.15
$B - V$........	...	...	−0.54	0.13	−0.30

If there existed a significant dependence of albedo on size, such studies would need to take it into account.

The possible correlations of D and p_V with a indicated in table 4 are probably not real. Selection effects can produce an apparent correlation between both D and p_V and distance from the Sun, since objects that are small and/or of low albedo will have been observed preferentially at small values of a. The large correlation coefficient for $\log D$ with $\sin i$

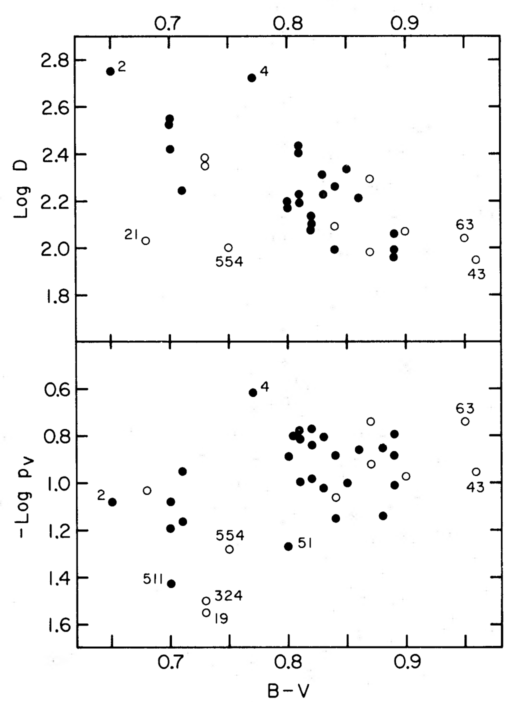

FIG. 5.—Plot of the $(B - V)$ color index against diameter and geometric albedo for the asteroids discussed in this paper. The largest and two smallest asteroids are omitted from the upper plot. Open circles indicate cases where the color index has not been measured directly (see Zellner *et al.* 1974). Those objects for which no colors are available—140 Siwa, 471 Papagena, and 747 Winchester—have been omitted. As discussed in the text, a division into three groups of minor planets is evident.

is heavily influenced by the very large inclination of 2 Pallas, but even without this object the correlation may be significant. Finally, the coefficients in the table show a correlation between $B - V$ and a. This relationship has been noted before for a larger sample of minor planets (e.g., Chapman *et al.* 1973), however, and it is not an appropriate topic to pursue in the present paper.

The most interesting correlations that can be examined on the basis of new information reported here are between color and the diameters and albedos of the asteroids. The correlation coefficients for these relationships are the most significant of those given in table 4. In figure 5 both log D and log p_V are plotted as a function of $B - V$. Both plots show that most of the asteroids observed fall into a distinct group characterized by $0.8 < B - V < 0.9$, $0.08 < p_V < 0.18$, and $D < 250$ km. Zellner (1973) finds that these same objects in general have a minimum visual linear polarization of $0.5 < P_{\min} < 1.0$ percent, and the nine of these also observed spectrophotometrically by Chapman *et al.* (1973) have a steeply sloping reflectivity from 0.3 to 0.8 μ and in most cases an absorption band near 0.95 μ. All of these observations are consistent with a silicate composition for the asteroids in this class, which appears to be the predominant one for the larger main-belt asteroids.

A second, less well defined, group of minor planets in figure 5 consists of the objects with $B - V < 0.74$ and $p_V < 0.11$. This group includes each of the five largest objects except 4 Vesta. Chapman *et al.* (1973) describe the spectral reflectivity of four of the five asteroids from this group that they observed as flat or bluish with no absorption bands evident, and Zellner (1973) shows that these asteroids generally have deeper linear polarization minima. Johnson *et al.* (1974) have extended the spectrophotometry of two of these—1 Ceres and 2 Pallas—into the PbS region of the infrared and conclude that their entire reflection spectra are similar to those of carbonaceous chondrites. It thus appears that asteroids in this second group are probably carbonaceous in mineralogy, as has also been suggested for several individual objects by Chapman and Salisbury (1973) and Johnson and Fanale (1973).

Very few of the asteroids plotted in figure 5 fall into neither of the major classes—silicate and carbonaceous—described above. The most outstanding exception is 4 Vesta, which has the highest albedo of any asteroid by a substantial margin, falls into the gap between the two groupings in the $B - V$ color index, and is known (McCord, Johnson, and Adams 1970; Chapman *et al.* 1973) to have a unique spectrum. The two reddest objects—43 Ariadne and 63 Ausonia—appear to lie outside the main group of silicate asteroids. But their $B - V$ colors have not been measured directly on the *UBV* photometric system and may be suspect. In particular, recent observations of 43 Ariadne by Johnson and Matson (1973) suggest that this object may be bluer by more than 0.1 mag than the value plotted. Of the three darkest objects, 747 Winchester does not have a measured color and is not plotted, while the $B - V$ indices for both 324 Bamberga and 19 Fortuna have been obtained only indirectly from spectrophotometric studies. McCord and Chapman (1974) have reported dramatic differences between the spectral reflectivity curves they measured for 19 Fortuna in 1972 and 1973. The color index for this asteroid indicated in figure 5 is derived from their 1973 spectrophotometry, which showed a spectrum very similar to that of 324 Bamberga. Had I used instead the color obtained in 1972, 19 Fortuna would have fallen alone in the lower right-hand part of figure 5.

Superficially, the two major classes of asteroids appear to be similar to the chondritic and carbonaceous classes of meteorites. The presence of asteroids that have a carbonaceous mineralogy makes an asteroidal origin for this type of meteorite more plausible, in distinction to the cometary origin sometimes suggested (cf. Anders 1971*a*, *b*). Neither this study nor the spectrophotometric investigation by Chapman *et al.* (1973) has yet identified a group of asteroids that have metallic surfaces and might be related to the nickel-iron meteorites.

d) Asteroid Densities

Masses for 1 Ceres and 2 Pallas have been determined by Schubart (1974) and for 4 Vesta by Hertz (1968). For each of these three objects the diameters given in this paper are in excellent agreement with those obtained by Zellner (1973) and other recent workers, so that they may be used with confidence. Table 5 summarizes the masses and diameters of these three asteroids and gives the corresponding densities. The error bars were computed on the assumption of a 5 percent uncertainty in the diameters. The density of 4 Vesta is consistent with a bulk composition of silicate minerals; that of 1 Ceres is too low for silicates but is consistent with a composition similar to that of the carbonaceous chondrites; and that of 2 Pallas is insufficiently well established to distinguish between carbonaceous and silicaceous mineralogy. All three densities are low enough to exclude any major metallic component in these asteroids.

e) Comments on Individual Asteroids

The first asteroid, 1 Ceres, is notable for its large size, which puts it in a class by itself. Several recent determinations of its size are in good agreement, and the low albedo and low density discussed here, together with its spectral reflectivity (Chapman *et al.* 1973), appear to place its surface in the mineralogical class of the carbonaceous chondrites, although there

TABLE 5

DENSITIES OF ASTEROIDS

Name	Mass (g)	D (km)	ρ(g cm^{-3})
1 Ceres...	$(1.17 \pm 0.06) \times 10^{24}$	1025 ± 50	2.1 ± 0.3
2 Pallas...	$(2.59 \pm 0.80) \times 10^{23}$	560 ± 30	2.8 ± 0.9
4 Vesta...	$(2.39 \pm 0.24) \times 10^{23}$	525 ± 25	3.1 ± 0.5

remain important differences in the details of the spectra (Johnson and Fanale 1973; Chapman and Salisbury 1973).

The unique properties of 4 Vesta are well known; however, it is worth noting that the albedo reported here for this asteroid is entirely consistent with its identification by McCord *et al.* (1970) with the basaltic achondrite meteorites.

Chapman *et al.* (1973) noted that 16 Psyche has an unusual spectrum, being red but lacking any absorption bands, and Johnson and Fanale (1973) have suggested that this is one of the few asteroids with a spectrum consistent with iron-nickel composition. The albedo of 0.08 determined in this paper is much lower than that found for particles of the iron-nickel meteorite (Odessa) measured by Johnson and Fanale, although without further information on how texture and particle size affect the reflectivity of such surfaces, this result cannot exclude the possibility that 16 Psyche is metallic.

In terms of size and albedo, the two very dark asteroids 19 Fortuna and 324 Bamberga are seen in this study to be virtually identical. Their spectra may also be similar, but as discussed earlier McCord and Chapman (1974) report discordant results from their spectrophotometry of 19 Fortuna. In addition, neither of these asteroids has been the object of broad-band photometric studies.

Johnson and Matson (1973) suggest from five-color photometry that 43 Ariadne may have a spectrum similar to that of ordinary chondritic meteorites. The albedo found here (0.11) is lower than would be expected from such an identification; however, this asteroid is not one of the best observed in my list, and it is probably wise to defer judgment until more observations have been made. In contrast, the other asteroid from this sample that has been identified as having a spectrum similar to one L-type chondrite (Chapman and Salisbury 1973), 192 Nausikaa, has an albedo of $\sim$0.18, the second highest albedo in the sample. This value is in good agreement with the laboratory reflectivities for these types of meteorites listed by Chapman and Salisbury.

V. CONCLUSIONS

In this paper the radiometric method of measuring the albedos and sizes of small airless objects, which has been developed by several workers during the past few years, is applied for the first time to a statistically significant number of asteroids. The largest asteroids are revealed by this study to be 1 Ceres, 2 Pallas, 4 Vesta, 10 Hygiea, and 511 Davida. The highest albedo ($p_V = 0.24$) belongs to 4 Vesta; the darkest asteroids observed in this study are 19 Fortuna, 324 Bamberga, and 747 Winchester, all with $p_V \simeq 0.03$. Although the geometric albedos are relatively well determined by this method, there is no way to derive from these measurements either the Bond albedos or the mean phase integrals of these asteroids.

A comparison of the derived diameters and albedos with photometrically measured colors reveals three distinct classes of objects. The largest group is those with red color and albedos in the lunar range (0.09–0.18); they presumably have surfaces composed of silicate minerals, although it is clear that within this class there exists a wide variety of mineralogical types. The second group, which includes most of the largest objects, is characterized by more neutral color and lower albedo. Their surfaces may be similar in composition to the carbonaceous chondrites. The range in albedo within this class is large, from 0.11 down to 0.03. The third class has but a single known member, 4 Vesta, an object of high albedo and intermediate color. Spectrophotometry has shown that 4 Vesta has a surface similar in composition to the basaltic achondrite meteorites. Masses are known for 4 Vesta and for the two largest members of the carbonaceous class; when combined with the diameters derived here, these masses give mean densities for all three between 2.1 and 3.1, consistent with the compositions suggested above. The most accurately determined density, 2.1 ± 0.3 for 1 Ceres, is also the lowest of the three.

Infrared radiometry is but one of several new techniques being applied to physical study of the asteroids. Many of the conclusions presented in this paper are similar to those reached independently by Zellner *et al.* (1974) from their polarimetric investigation of a slightly smaller sample of objects. Mineralogical information for nearly 100 asteroids is being obtained from 24-filter spectrophotometry by Chapman, McCord, and their co-workers (e.g., Chapman *et al.* 1973; McCord and Chapman 1974). A synthesis of many of the new results from spectrophotometry, radiometry, and polarimetry is in preparation by Chapman *et al.* (1974). Taken together, these studies are beginning to reveal the broad outlines of the sizes, albedos, and mineralogy of the main-belt asteroids and of their relationship to the meteorites. A great heterogeneity of surface materials is indicated, suggesting that upon closer examination the asteroids will turn out to be as individual in their physical characteristics as are the satellites and the planets.

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