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On the Mass Estimations of Elliptical Galaxies

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ABSTRACT

An effect of the assumption of the stellar motion type on mass estimations from the dispersion of radial velocities for an elliptical galaxy is considered. Theoretical expressions are obtained and calculations are made for the ratio of the dispersion of radial velocities, measured inside the diaphragm with radius x, to the total dispersion of radial velocities as a function of the radius of the diaphragm under assumption of circular motions and oscillations.

1. Introduction

The difficulties with mass determination for elliptical galaxies are well known. In contrast to spiral galaxies, most of elliptical ones do not show apparent evidences of rotation, and their mass is determined by the virial theorem using a dispersion of radial velocities of stars in the central regions of the galaxy. Obtained in such a way a mass to luminosity ratio, M/L, for E-galaxies considerably exceeds the estimates of M/L for spirals. The knowledge of masses of elliptical galaxies is of special importance for giving a more precise definition of the mean density of matter in the Universe, for determining dynamical conditions in pairs, groups and clusters of galaxies and, at last, for comprehention of evolutional difference between the E- and S-galaxies.

The purpose of the present paper is to clear up the matter taking into account how the predominant type of motion of the stellar population influences the mass estimations of elliptical galaxies.

Let us consider consecutively the stages of mass determination of a galaxy by the virial theorem using the directly measurable magnitudes.

2. An expression for the E-galaxy mass

The following premises are adopted:

(a) The stage of the galaxy is stationary, *i.e.* the virial theorem

$$2T+U=0 \tag{1}$$

may be applied, where T and U are kinetic and potential energies of the system, respectively;

(b) The galaxy is spherically symmetrical;

(c) The optical thickness of the galaxy is negligible;

(d) The mass-to-luminosity ratio is constant along the galaxy radius;

(e) Mass distributions in the galaxy when projected on celestial plane is described by Vaucouleurs' law (Vaucouleurs 1953)

$$f(x)=\mu^8/(4\cdot 7!)\,x\exp(-\mu x^{1/4}), \tag{2}$$

where $\mu=7.6692494$, and the distance x is expressed in portions of the "effective" radius x_e of the galaxy (inside of which half of the total luminosity of the galaxy is included).

Under these assumptions the kinetic energy of the galaxy may be written as

$$T=\frac{1}{2}ME\{v^2\}=\frac{3}{2}ME\{y^2\}, \tag{3}$$

where M is the total mass of the galaxy, $E\{v^2\}$ and $E\{y^2\}$ are the dispersion of spatial and radial velocities, respectively. The potential energy U may be conveniently expressed through $h(z)$ — the mass density distribution in projection on an arbitrary axis z passing across the center of the galaxy. According to Ambartsumian (1939)

$$U=-\frac{1}{2}\gamma M^2\int\limits_0^\infty h^2(z)\,\mathrm{d}z. \tag{4}$$

Using the connection between $h(z)$ and $f(x)$ in the form

$$h(z)=\frac{2}{\pi}\int\limits_z^\infty\frac{f(x)\,\mathrm{d}x}{\sqrt{x^2-z^2}} \tag{5}$$

we have

$$U=-a\gamma M^2/x_e, \tag{7}$$

where γ is the constant of gravitation, and dimensionless parameter a is given by

$$a=x_e[\mu^{16}/8\pi^2(7!)^2]\int\limits_0^\infty\Big[\int\limits_0^\infty\exp\{-\mu(s^2+z^2)^{1/8}\}\,\mathrm{d}s\Big]^2\mathrm{d}z. \tag{7}$$

Calculations give $a = 0.3358$*. Substituting the expressions for T and U in Eq. (1) we obtain the mass of the galaxy

$$M = 3\gamma^{-1}\cdot a^{-1} x_e \cdot E\{y^2\}. \tag{8}$$

The total dispersion of radial velocities of stars in the galaxy, $E\{y^2\}$, is unknown; from observations one obtains only the total dispersion of radial velocities, $\sigma^2(x)$, in the central region of the galaxy with radius x (Minkowski, 1962). Denoting the conditional dispersion of radial velocities of stars at a distance x by $E\{y^2|x\}$, we have

$$\sigma^2(x) = \int\limits_0^x E\{y^2|s\} f(s)\,ds \Big/ \int\limits_0^x f(s)\,ds \equiv \beta(x)\cdot E\{y^2\}, \tag{9}$$

where $\beta(x)$ is a dimensionless factor of transition from the observed value $\sigma^2(x)$ to $E\{y^2\}$.

Thus, the final expression for the mass through the directly measured values will be

$$M = 3\gamma^{-1}\cdot a^{-1}\cdot x_e\cdot \beta^{-1}(x)\cdot\sigma^2(x). \tag{10}$$

3. Determination of $\beta(x)$

It follows from the data (Karachentsev *et al.* 1972*a*, 1972*b*) that distribution of the dispersion of radial velocities $E\{y^2|x\}$ along the radius of the spherically symmetric gravitating system depends on the type of motion of its members very strongly. In the case of strictly circular motions we obtain

$$\beta_c(x) = \frac{\gamma M}{2E\{y^2\}} \int\limits_0^x s^3 \int\limits_s^\infty \frac{\Phi(r)\,\Phi'(r)\,dr}{r^4\sqrt{r^2-s^2}}\,ds \Big/ \int\limits_0^x f(s)\,ds, \tag{11}$$

where $\Phi(r) \equiv \int\limits_0^z F(s)\,ds$ — is the integral law of the distribution of spatial distances of stars from the center.

The case when all stellar tracks pass exactly via the centre of the galaxy (unharmonic oscillations) is given by

$$\beta_0(x) = \frac{\gamma M}{2E\{y^2\}} \int\limits_0^x t \int\limits_t^\infty s^{-2}[t^{-1}\cdot\arccos(t/s) - $$

$$- s^{-2}\sqrt{s^2-t^2}]\cdot\Phi(s)\,\Phi'(s)\,ds\,dt \Big/ \int\limits_0^x f(s)\,ds. \tag{12}$$

* According to graphic integration of Poveda (1958) $\alpha = 0.32$.

Table 1.

Factor β for E-galaxy mass determination as a function of radius x for two types of stellar motions.

x	$\int_0^x f(s)ds$	$\beta_c(x)$	$\beta_o(x)$
0.00	0.0000000	0.0000	∞
0.05	0.0319320	0.5833	2.1699
0.10	0.0719674	0.7971	1.5569
0.15	0.1107452	0.9235	1.3182
0.20	0.1471590	1.0064	1.1905
0.25	0.1811078	1.0641	1.1115
0.30	0.2127297	1.1057	1.0584
0.35	0.2422202	1.1365	1.0207
0.40	0.2697771	1.1596	0.9928
0.45	0.2955841	1.1771	0.9715
0.50	0.3198059	1.1904	0.9550
0.55	0.3425887	1.2005	0.9418
0.60	0.3640616	1.2082	0.9312
0.65	0.3843386	1.2138	0.9226
0.70	0.4035205	1.2180	0.9155
0.75	0.4216966	1.2208	0.9096
0.80	0.4389465	1.2227	0.9047
0.85	0.4553414	1.2237	0.9005
0.90	0.4709447	1.2241	0.8970
0.95	0.4858137	1.2239	0.8941
1.00	0.5000000	1.2233	0.8915
2.00	0.6900056	1.1797	0.8791
3.00	0.7880664	1.1370	0.8843
4.00	0.8465793	1.1059	0.8913
5.00	0.8845549	1.0835	0.8982

Eqs. (11) and (12) have been obtained from Eq. (9) with using expressions for $E\{y^2|s\}$, given in (Karachentsev *et al.* 1972*a*, *b*), by simple changing of order of integration.

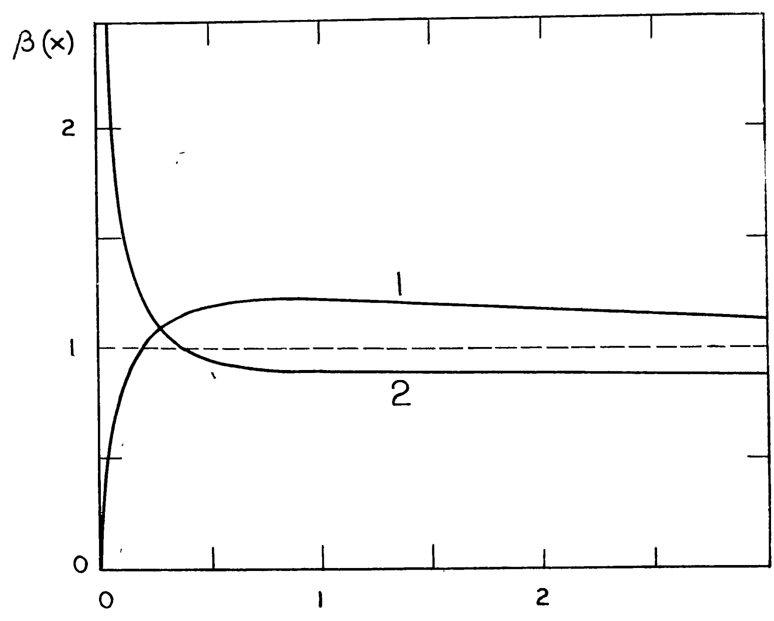

Fig. 1. The curves $\beta(x)$ for circular motions (1) and oscillations (2); x is expressed in portions of the "effective" radius x_e. Dashed line indicates the case of constant dispersion of stellar radial velocities along the E-galaxy radius.

The calculations using Eqs. (11) and (12) have been performed numerically with a computer. The results are presented in Table 1 and in Fig. 1. The second column of the table gives the values of $\int_0^x f(s)\,ds$ — the portions of the galaxian luminosity inside the diaphragm with radius x.

Certainly, the infinitely large value $\beta_0(0)$ should be explained first of all by the fact that Eq. (2) cannot be applied at zero point. Vaucouleurs (1953) indicates a limit to application of the mass distribution law $x/x_e > 0.03$.

The authors would like to emphasize that although circular motions and oscillations both are extreme cases which apparently cannot be purely realized, however the real cases of motions occupy an intermediate position between them and therefore the corresponding values $\beta(x)$ will take place between $\beta_c(x)$ and $\beta_0(x)$, given in Table 1.

4. Discussion

The measurements of dispersion of stellar radial velocities in central parts of galaxies have been performed using diaphragms with radius from 0.06 x_e to 0.33 x_e (Minkowski 1962). As it is seen from our data, a possible error in mass estimations caused by ignoring the stellar motions types does not exceed $\pm 25\,\%$ for $x > 0.20$. However, for smaller values of diaphragm the uncertainty in mass estimations becomes significant and increases with decreasing of diaphragm; *e.g.*, under the assumption of circular motions, and oscillations, mass estimations for $x = 0.05$ differ by a factor of 3.72.

The data obtained allow to conclude that to determine the mass of E-galaxy the optimal value for the radius of the diaphragm is $x \simeq 0.3$. Factor β for such values is practically independent of the stellar motion type.

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