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ABSOLUTE MAGNITUDES OF SEMI-REGULAR VARIABLES IN THE SOLAR VICINITY FROM STATISTICAL PARALLAXES

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SUMMARY

Absolute magnitudes of Semi-Regular and RV Tau variables have been determined from radial velocities and proper motions.

1. Photographic absolute magnitudes of M-type SRab variables vary with spectral type from $0^{m}\cdot0$ at about M4 to $+2^{m}\cdot5$ at M6 at mean brightness (Fig. 3).

2. Absolute magnitude of M-type SRa and b variables without emission is a function of period. M_{pg} at mean and maximum brightness may be expressed as $M_{pg}(\text{mean}) = 0\cdot6+3\cdot9\ (\log p-2\cdot0)$ and $M_{pg}(\text{max}) = 0\cdot1+2\cdot8\ (\log p-2\cdot0)$, where p is the period in days. These variables are fainter than the Miras at a given period (Figs 1 and 2).

3. SRab variables with $p \leqslant 200$ days showing emission lines fall on the extension of the period–absolute magnitude relation for Mira variables. There is no significant difference between SRab M and SRab Me variables for $p > 200$ days.

4. The mean absolute magnitude of N-type semi-regular variables has also been determined from differential galactic rotation. The combined mean value is $M_{pg}(\text{max}) = +0^{m}\cdot7\pm0^{m}\cdot4$.

5. SRd variables have $M_{pg}(\text{mean}) = -0^{m}\cdot9\pm0^{m}\cdot4$ and $M_{pg}(\text{max}) = -1^{m}\cdot7\pm0^{m}\cdot4$.

6. RV Tau variables have $M_{pg}(\text{mean}) = -0^{m}\cdot5\pm0^{m}\cdot6$ and $M_{pg}(\text{max}) = -1^{m}\cdot7\pm0^{m}\cdot6$.

The present results have been compared briefly with the absolute magnitudes of similar stars in clusters.

1. INTRODUCTION

The most recent determination of statistical parallaxes of semi-regular variables in the solar neighbourhood is that of Wilson (1942), whose work was essentially confined to those semi-regular and irregular variables of M-type spectra. Since then the late type variables have been re-classified according to the characteristics of their light curves and spectra (see *The General Catalogue of Variable Stars* (GCVS), Kukarkin *et al.* 1958, 1969). Of these only the statistical parallaxes of the more regular, Mira type, variables have so far been calculated (Osvalds & Risley 1961; Clayton & Feast 1969).

With a view to studying the kinematics and determining the statistical parallaxes of Semi-Regular (SR) and RV Tau variables, observational programs on radial velocities (Woolley & Penston 1967, private communication), on fundamental proper motions on the FK4 system (Blackwell & Lowne 1968) and on photographic absolute proper motions (Aslan 1972; Aslan, Clube & Russo 1973) were carried out at the Royal Greenwich Observatory. In this paper the mean absolute magnitudes from secular and statistical parallaxes are given for the following variables (in the notation of GCVS): (a) SRa, SRb variables of M type (without emission) as a function of period, (b) SRa, SRb variables of M type with emission as a function of period, (c) SR, SRa, SRb variables of N type, (d) SRd variables, and (e) RV Tau stars.

The following variables were excluded after a preliminary analysis of radial velocities and proper motions:

(i) SRc variables and other M-supergiants. SRc variables are late type stars of luminosity class I or II. M-type supergiants in general are Population I objects and are known to occur frequently in open clusters and associations (Bidelman 1943; Sharpless 1966; Stothers 1968; Humphreys 1970). The proper motions of SRc's are small, of the order of errors of measurement. On the basis of new spectral classifications and radial velocities by Humphreys (1970, and private communication), the following stars were also excluded: W Per (SRb), PZ Cas (SRa), EV Car (SRb), CK Car (SRb) and CL Car (SRb). According to the criteria of Kukarkin *et al.* (1958), these M-supergiants should be classified as SRc.

(ii) M-, Me-type semi-regular variables classified as SR in GCVS, due probably to lack of information or to peculiarities in the observed parameters. Although the majority of these may be SRa or SRb the ‘ group ’ as a whole is most likely a mixture. For example, some of the long period variables ($p > 400$ days) have properties similar to SRc's: low galactic latitudes, earlier spectral type, smaller light amplitudes, small radial velocities and proper motions such as UW Cam, IX Car, BM Sco, V485 Sco. IX Sco is in a region of a bright nebula and, according to Hoffmeister, is an eclipsing variable (see GCVS). In the new edition of GCVS (Kukarkin *et al.* 1971), BM Sco is classified as SRd; but its period of 850 days is too long and its deep minima are separated by 2×850 days, a property of RV Tau stars, but all RV Tau stars in GCVS have $p < 150$ days. According to Eggen (1961a) it is a possible member of the galactic cluster NGC 6405, and if so, Eggen gives $M_V = -3^m.3$, which makes it a supergiant like the SRc's. The radial velocities of 14 M-type SR variables with $p < 200$ days (all $p < 155$) gave $(U_0, V_0) = (-49 \pm 14, -3 \pm 15)$ km s^{-1}, where U is towards the galactic centre, V is in the direction of galactic rotation. The short period and small amplitude variables ($p < 50$ days, $\Delta m \leqslant 0^m.5$) VZ Cam, TV Psc, RR UMi are, according to Eggen (1966), members of the Hyades group (as reflected in the value of U_0 given above).

(iii) S-type variables. There are less than a dozen semi-regular variables of S type recorded in GCVS. As far as we know only one, GP Ori, of these has a radial velocity measured, and none has proper motions determined.

(iv) R-type stars. Among the carbon stars the N-type objects are, in their majority, concentrated in spiral arms, while the R-type stars are not concentrated in spiral arms (Blanco 1965). According to Vandervort (1958) and Dahn (1964) the N- and R-type stars have different mean motions and velocity ellipsoids. Also Sandford (1944), Gordon (1968) and others find different visual absolute magnitudes for N and R stars. The number of R-type semi-regular variables with radial velocities and proper motions (about half a dozen) is insufficient for a determination of the mean parallax.

2. OBSERVATIONAL MATERIAL

2.1 *Selection of stars*

The source of the variables is the second edition of *The General Catalogue of Variable Stars* (GCVS, Kukarkin *et al.* 1958). As a rule all the semi-regular variables and RV Tau stars in GCVS with $m_{\rm pg} \leqslant 11^{\rm m}\cdot0$ at maximum were selected. Approximate colour index corresponding to the spectral type was applied in cases when the magnitude in the catalogue was visual. The other relevant data, right ascension, declination, period, spectral type, were all taken from the GCVS. (Although carbon stars are also classified as C0 to C9, we have used the HD-classifications given in the catalogue.)

2.2 *Radial velocities*

Except in a few cases all the radial velocities were taken from a list kindly supplied by Sir Richard Woolley and Mrs M. J. Penston, which consisted of their own unpublished velocities for a large number of stars and those collected from the literature (Wilson 1953; Feast 1963; Smak & Preston 1965). The radial velocities for g Her and V453 Oph were taken from Kruszewski, Gehrels & Serkowski (1968) and Joy (1950), respectively; those for GT Pup and BR Tel were kindly supplied by Dr M. W. Feast before publication.

2.3 *Proper motions*

The two main sources of proper motions are the proper motions on the FK4 system by Blackwell & Lowne (1968) and the photographic proper motions determined by workers at the Royal Greenwich Observatory (Aslan *et al.* 1973). The other sources, with the number of stars in brackets are given below:

(a) Paloque *et al.* (1958, 1959, 1961) (14 stars). They determined proper motions of variable stars in the Paris Astrographic Zone on the PGC system. We have reduced the motions of the semi-regular variables to FK4 by FK4 = PGC + (GC − PGC) + (FK4 − GC). The corrections were taken from the GC (Boss 1937) and Brosche, Nowaki & Strobel (1964).

(b) Ölander *et al.* (1959) (nine stars). We have reduced the published relative proper motions to absolute using the mean parallaxes of the reference stars from number counts by a scheme outlined by Clube (1968).

(c) McCormick (Alden & Osvalds 1961; Osvalds & Risley 1961) (13 stars). Individual errors were not published; from the information given we have adopted $\epsilon_{\rm m} = \pm 0''\cdot003$/yr as the measuring error of a proper motion (see also Clayton & Feast 1969). The uncertainty due to cosmic dispersion was calculated from

$$\epsilon_{\rm c} = 1\cdot25\frac{\Sigma|\Delta\mu|}{\sqrt{N(N-1)}}$$

to give the total standard error $\epsilon = (\epsilon_{\rm m}^2 + \epsilon_{\rm c}^2)^{1/2}$ in either coordinate. The residuals $\Delta\mu_{\rm x}$, $\Delta\mu_{\rm y}$ for each reference star were taken from Alden & Osvalds (1961).

(d) Pavlovskaya (1968) (four stars). She gives proper motions of various variable stars reduced to absolute using 'statistical method'. A comparison of 14 RR Lyrae variables in common with Clube (1968) gave $\pm 0''\cdot0076$ and $\pm 0''\cdot0095$ as the standard error of one difference in RA and Dec, respectively. This implies $\pm 0''\cdot005$/yr, $\pm 0''\cdot007$/yr as the errors of Pavlovskaya's proper motions if Clube's errors are adopted (Pavlovskaya gives $\pm 0''\cdot002$ to $\pm 0''\cdot005$). We have adopted $\pm 0''\cdot006$/yr in both coordinates.

(e) Finally the proper motion of V453 Oph was taken from Clube (1968).

In calculating the mean parallaxes the proper motions on the FK4 system were first corrected for the errors in the precessional constants as recommended by

Fricke (1968), *viz.*: $\Delta n = +0''\cdot0040$, $\Delta k = -0''\cdot0020$/yr. In cases where there were more than one determination of an individual star's proper motion the weighted mean and its standard error were calculated in each coordinate, the weight being proportional to the inverse square of the standard error. Individual errors of the proper motions by Blackwell & Lowne were kindly made available by them (private communication). However, if a final proper motion component had a standard error greater than $0''\cdot010$/yr it was excluded from the calculation of mean parallaxes irrespective of the size of the proper motion component.

2.4 *Apparent magnitudes*

In determining the statistical parallaxes of a group of stars the choice of the apparent magnitudes should be such that the stars have as little spread in absolute magnitude as possible. Because of the character of their light curves the magnitudes at maximum or mean-light cannot easily be determined for the semi-regular variables. Further, unlike for a large number of Miras, no magnitudes at mean maximum are given in GCVS for the semi-regulars. We have therefore used the magnitudes at extreme maximum and extreme minimum given in the catalogue. For a substantial fraction of stars the visual magnitude, m_V, is given. For the majority of these stars no observed colours, even at random phases, are available. We have therefore applied average colours to reduce m_V to m_{pg} obtained from stars of the same spectral type. Using the spectral types in GCVS and *UBV* observations by Preston *et al.* (1963), Smak (1964), Eggen (1961a, b, 1966, 1967), Mendoza (1967) and Landolt (1966, 1967, 1968, 1969) the following mean colours were obtained: $B-V = 3^m\cdot6 \pm 0^m\cdot9$ (per star) for the N-type stars, $B-V = 1^m\cdot6$ for the M-type stars; the standard deviation about the latter being less than $0^m\cdot1$. For the SRd and RV Tau variables a linear relation between spectral type and the observed $B-V$ was used. There is no variation with subtype amongst the red variables (see also Plaut 1965). Some of the scatter is of course due to interstellar reddening. Since the variation of $B-V$ over one cycle is comparable to the variation of mean (observed) $B-V$ from star to star, the above colours were adopted even for the stars with observed colours.

More important is the reported presence of systematic errors in GCVS (Woolley *et al.* 1965; Van Herk 1965; Kinman 1965). These authors, using the photoelectric magnitudes of RR Lyrae variables, find that m_{pg} in GCVS is too bright by up to $0^m\cdot5$. Clube (private communication) found similar differences from southern RR Lyraes, with no dependence on colour or magnitude. Van Herk finds evidence for a dependence on declination. We have compared the photographic magnitudes of 17 RV Tau stars in GCVS with their photoelectric B magnitudes by Preston *et al.* (1963): $B(\max) - m_{pg}(\max) = +0^m\cdot51 \pm 0^m\cdot34$ (per star) and B(deep minimum) $- m_{pg}(\min) = -0^m\cdot13 \pm 0^m\cdot52$ (per star) with no evidence for a dependence on magnitude, colour or declination. These differences at maximum and minimum and those for a few other individual SR variables (Aslan 1972) are consistent with the fact that the catalogue magnitudes refer to the extreme maximum and minimum. The average difference $B(\text{mean}) - m_{pg}(\text{mean}) = +0^m\cdot24 \pm 0^m\cdot08$ from a sample of 17 stars is hardly significant, though it is in the same sense as the systematic correction found by the authors quoted above. Also, since the catalogue magnitudes are not on a uniform scale but rather taken from a variety of sources, the true correction to the magnitudes of semi-regulars may well differ from that obtained using RR Lyrae variables. We have therefore applied no corrections to the catalogue magnitudes except in one case: We have applied $-0^{m}\cdot 7$ to the catalogue magnitude of the SRd variable TY Vir after a comparison with photoelectric magnitudes by Eggen (1961b) and Preston & Wallerstein (1963).

3. CALCULATION OF THE MEAN PARALLAXES

3.1 *Data derived from radial velocities*

The mean motions and velocity dispersions given in Table I were based almost entirely on the radial velocities kindly supplied by Woolley & Penston (1967, private communication). (The kinematics of the M-type SR variables based on more radial velocities has been discussed recently by Feast, Woolley & Yilmaz 1972.) The solutions for the mean velocity components U_0, V_0, W_0 with respect to the Sun (towards the galactic centre, in the direction of galactic rotation and towards the north galactic pole, respectively) were carried out with and without the so-called K-term in the equations of condition. (The known supergiants and the stars designated as SR in the GCVS were excluded from groups 1 to 6. See Introduction.) Only for Me variables with $p \leqslant 200$ days (Group 1) was the K-term larger than its standard error: $(U_0, V_0, W_0, K) = (-23 \pm 19, -91 \pm 18, -27 \pm 17, +24 \pm 12)$ km s^{-1}. Because of the large uncertainties involved no significance may be attached to the value of K. As can be seen from the following table, the values of U_0, W_0 ($K = 0$ adopted) do not differ significantly from the reflex of the local solar motion obtained from other types of object. We have therefore adopted $U_0 = -10\cdot 1$, $W_0 = -6\cdot 2$ km s^{-1} (Feast & Shuttleworth 1965) and, having corrected the observed radial velocities for U_0, W_0, calculated the component V_0.

	U_0 (km s^{-1})	W_0 (km s^{-1})
Group 1	-28 ± 20	-23 ± 18
Groups 2–6 (weighted mean)	$-10\cdot 6 \pm 5\cdot 2$	$-12\cdot 0 \pm 5\cdot 7$
N-type stars	$-6\cdot 1 \pm 4\cdot 1$	$+1\cdot 6 \pm 8\cdot 4$
SRd variables	-20 ± 30	$+28 \pm 34$
RV Tau stars	-16 ± 24	-18 ± 66

In the reductions the velocity dispersions are also needed. For reasons given in Section 3.4 we have adopted the ratios $\sigma_u^2 : \sigma_v^2 : \sigma_w^2 = 3:2:1$ and calculated σ_w^2 from

$$\sigma_w^2 = \frac{\Sigma \Delta\rho^2 Q_3}{\Sigma Q_3^2} \tag{1}$$

where $\Delta\rho$ is the radial velocity corrected for the mean motion with respect to the Sun and $Q_3 = 3a_{13}^2 + 2a_{23}^2 + a_{33}^2$; a_{ij} being the direction cosines (see Murray 1961). Solution for σ_w in this way is necessary because of the adopted procedure for the statistical parallaxes in Section 3.4.

The values of σ_w, V_0 and the total mean motion S_0 are given in Table I for the adopted groups. Division into subgroups were effected by examining the period–amplitude, period–frequency distributions, apparent distribution in galactic coordinates, as well as the values of V_0 and σ_w.

M-type semi-regulars showing emission lines were considered separately. No difference between SRa Me and SRb Me variables was indicated by the radial velocity solutions. (In fact the number of stars is too small to carry out the analysis separately.) SRa M variables (without emission) with $p > 175$ days have been excluded from the analysis for the mean parallaxes because there were only three stars with radial velocities and seven stars with proper motions, all of which have periods exceeding 230 days. However, there are reasons for not combining these stars with the SRb variables.

All N-type SR, SRa, SRb variables, irrespective of period, have been combined into one group as no difference was found between SRa and SRb's nor was there a discernible variation of the mean motion with period for $p < 400$ days. Radial velocity solutions by Woolley & Penston (1967, private communication) as a function of period of carbon stars indicate that the mean motion with respect to the Sun for $p < 400$ days is not different from the reflex of the basic solar motion but they found that for the longer period group ($p > 400$ days, 13 stars) $(U_0, V_0, W_0) = (-13{\cdot}5 \pm 6{\cdot}5, -35{\cdot}3 \pm 7{\cdot}5, -35{\cdot}3 \pm 19{\cdot}2)$ km s^{-1} with a very small velocity dispersion, $\sigma_W = 6{\cdot}2$ km s^{-1} ($\sigma_U^2 : \sigma_V^2 : \sigma_W^2 = 3:2:1$ adopted), probably indicating some kind of group motion and/or deviation of the vertex. Excluding the R-type variables we obtain $(U_0, V_0, W_0) = (-18{\cdot}3 \pm 5{\cdot}1, \ -39{\cdot}6 \pm 6{\cdot}2, \ -41{\cdot}2 \pm 15{\cdot}8)$ with $\sigma_W = 8{\cdot}4 \pm 2{\cdot}2$ km s^{-1}. Correction of the radial velocities for the differential galactic rotation did not affect the figures much. There are only seven stars with proper motions in this group, of which only U Cam has $|\mu| > 0''{\cdot}015$/yr so that nothing can be said about them other than that the proper motions and the implied velocity dispersions *are* small. Inclusion or exclusion of these does not change the mean motion relative to the Sun significantly (Table III). We have therefore combined all N-type variables into a single group.

TABLE I

Data derived from radial velocities

Group	Type	Period (days)	No. of stars	V_0 (km s^{-1})	σ_W (km s^{-1})	S_0 (km s^{-1})
1	SRab, Me	$P \leqslant 200$	36	$-74{\cdot}3 \pm 16{\cdot}0$	$40{\cdot}0 \pm 5{\cdot}0$	75·2
2	SRab, Me	$P > 200$	17	$-18{\cdot}0 \pm 10{\cdot}3$	$21{\cdot}1 \pm 3{\cdot}5$	21·5
3	SRa, M	$P \leqslant 175$	19	$-33{\cdot}7 \pm 13{\cdot}5$	$24{\cdot}8 \pm 2{\cdot}5$	35·7
4	SRb, M	$P \leqslant 100$	33	$-40{\cdot}7 \pm 10{\cdot}5$	$23{\cdot}5 \pm 2{\cdot}7$	42·3
5	SRb, M	$100 < P \leqslant 200$	39	$-23{\cdot}5 \pm 6{\cdot}4$	$17{\cdot}4 \pm 2{\cdot}7$	26·3
6	SRb, M	$P > 200$	9	$-19{\cdot}4 \pm 11{\cdot}1$	$10{\cdot}7 \pm 3{\cdot}7$	22·7
7	SRab, N	All	61	$-10{\cdot}1 \pm 4{\cdot}9$	$14{\cdot}0 \pm 1{\cdot}2$	15·5
8	SRd	All	31	$-78{\cdot}1 \pm 31{\cdot}4$	67 ± 7	79
9	RV Tau	All	24	$-45{\cdot}3 \pm 33{\cdot}1$	42 ± 9	47

It is well known that RV Tau variables are a mixture of at least two populations. This may also be true in the case of SRd variables: about 50 per cent of SRd and 50 per cent of RV Tau variables are reported to have emission lines. When they are divided into ' emission ' and ' no emission ' groups different mean motions relative to the Sun result from radial velocities (Woolley, private communication). SRd variables give $V_0 = -154 \pm 51$ (17 emission stars), $V_0 = -12 \pm 18$ (14 no-emission stars). Although the well known halo SRd variable TY Vir was included in the first group the hydrogen emission is weak or absent (Preston & Wallerstein 1963). Also no emissions have been reported for CE Vir ($\rho = -75$ km s^{-1} and UU Her ($\rho = -131$ km s^{-1}), the latter being well observed by Joy (1952). The anomalous behaviour of TY Vir and CE Vir was noted by Woolley (private communication) and we have measured proper motions for them (Aslan *et al.* 1973). The radial velocity and proper motions of CE Vir ($\mu_\alpha = -0''{\cdot}041$, $\mu_\delta = +0''{\cdot}013$) with its apparent magnitude and high galactic latitude ($b = +58°$) indicate that it is a halo object. The median galactic latitudes of emission and no-emission SRd's are 18° and 20°, respectively.

Similarly, for RV Tau stars we have $V_0 = -99 \pm 43$ (11 emission stars), $V_0 = -3 \pm 27$ (13 no-emission stars) with RX Cap (-135 km s^{-1}) and V453 Oph (-95 km s^{-1}) in the second group. The median galactic latitudes are 12° and 7°, respectively.

It seems that neither the presence nor the strength of emission lines is a satisfactory criterion for the present purpose. Therefore only the analysis carried out irrespective of emission will be given here.

3.2 *Absorption corrections*

Parenago's formula $A_V = a_0\beta \operatorname{cosec} |b|\{1-\exp(-r(\sin|b|/\beta))\}$ has been used to calculate the galactic absorption of a star at a distance r (pc) and galactic latitude b. The constants a_0 and β, determined by Sharov (1964) for 118 regions covering the whole sky, had been put on punch-cards by Mrs Penston, who kindly allowed us to use them. A_V was calculated in the direction of each star for various distance moduli $m-M$. A_V corresponding to the uncorrected distance modulus based on the adopted absolute magnitude (see later) was then obtained by linear interpolation from an array $m-M+A_V$ versus A_V. A_V was then converted to the photographic absorption by $A_{pg} = 1{\cdot}33\, A_V$.

3.3 *Secular parallaxes*

If U_0, V_0, W_0 be the components of the mean motion, S_0, of a group of kinematically similar stars with respect to the Sun, the proper motion μ_j ($j = \alpha, \delta$) of a star with apparent magnitude m (corrected for absorption) and absolute magnitude M may be written as

$$47{\cdot}4\, \mu_{0j} = S_0\pi C_j + \pi(a_{1j}U' + a_{2j}V' + a_{3j}W') \tag{2}$$

where

$$\mu_{0j} = \mu_j \times 10^{0{\cdot}2m}, \quad \pi = 10^{0{\cdot}2M}$$

and

$$C_j = a_{1j}\frac{U_0}{S_0} + a_{2j}\frac{V_0}{S_0} + a_{3j}\frac{W_0}{S_0},$$

a_{ij} being the direction cosines between the galactic axes and the equatorial coordinates. U', V', W' are the components of the peculiar motion of the star. The mean absolute magnitude is determined by supplying the values of U_0, V_0, W_0 and S_0, which is equivalent to adopting the solar motion and the ant-apex, from radial velocities and solving for π by least squares. Assuming that the peculiar velocities U', V', W' are independent of distance and direction in the sky, and if W_{jk} is the weight associated with the observed reduced proper motion μ_{0jk} of the kth star, the two estimates of π from RA and Dec are given by

$$\bar{\pi} = \frac{47{\cdot}4 \sum\limits_k W_{jk}C_{jk}\mu_{0jk}}{S_0 \sum\limits_k W_{jk}C_{jk}^2}. \tag{3}$$

The combined solution is obtained by extending the summation to both components.

The weight W_j should contain not only the errors in the observed proper motions and apparent magnitudes but also the uncertainty due to the distribution of peculiar velocities amongst the variables themselves. It is given by

$$W_j^{-1} = \epsilon_j^2 10^{0\cdot4m} + \left(\frac{\bar{\pi}}{47\cdot4}\right)^2 (a_{1j}^2\sigma_u^2 + a_{2j}^2\sigma_v^2 + a_{3j}^2\sigma_w^2)$$

where ϵ_j is the standard error of the proper motion. The contribution due to errors in the apparent magnitudes is negligible compared to the errors in the proper motions. (But it should be noted that, if the errors of the apparent magnitudes are random the most likely parallax for a given absolute magnitude is obtained by multiplying $\bar{\pi}$ from equation (3) by the factor $10^{-(0\cdot2_m\sigma)^2/2\,\mathrm{Mod}}$ (Mod = 0·4343), where σ_m is the average standard error of an apparent magnitude. This, together with the dispersion in absolute magnitude, will be taken into account later in deriving the mean absolute magnitude.)

The values of σ_u, σ_v, σ_w were adopted from the radial velocities. Initially a value M_0 for the absolute magnitude was guessed. Having corrected the apparent magnitudes for absorption using M_0, and W_j being known for each star, $\bar{\pi}$ was calculated from equation (3). If M derived from $\bar{\pi}$ differed from M_0 by $1^m{\cdot}0$ or more the provisional solution was repeated with $M_0 = M$. Usually after one or two iterations a value for M_0 could be adopted. The final solution was carried out with this new value for M_0. The values of $\bar{\pi}$ at mean brightness, except for N-type variables for which the calculations were made at maximum brightness, are given in Table II. The values of $\bar{\pi}$ from proper motions in right ascension (RA) and in declination (DEC) are also given for comparison. Their errors *do not* include the uncertainty in the mean motion S_0, but in the combined solution it has been taken into account: The error of $\bar{\pi}$ (RA + DEC) was calculated from

$$\epsilon_\pi^2 = (\sigma_x^2 + \bar{\pi}^2\sigma_s^2)/(S_0^2 + \sigma_s^2),$$

where σ_x is the standard error associated with $\overline{\pi S}$ due to proper motions alone. For σ_s, the standard error of S, we have used the error of V_0 in Table I.

3.4 *Statistical parallaxes*

This consists in equating the dispersions from the proper motions and radial velocities. The true reduced peculiar proper motion of a star, $\Delta\mu_{cj}$, is given by the second term in equation (2). Under the usual assumptions each star provides an equation of condition for the dispersions σ_u^2, σ_v^2, σ_w^2, scaled by π^2, of the form

$$(47\cdot4)^2\,\Delta\mu_{cj}^2 = (a_{1j}^2\sigma_u^2 + a_{2j}^2\sigma_v^2 + a_{3j}^2\sigma_w^2)\,\pi^2. \tag{4}$$

But since π^2 is the quantity required it will be calculated from (4) by one-parameter least squares with the equations of condition of the form $Y = AX$ having equal weights. In this way the effect of large observational errors and large peculiar proper motions will also be reduced.

The dispersions can in principle be supplied from radial velocities but in practice unreasonable or even negative values (as were found in the case of RV Tau and Group 2 variables) may result for any of σ_u^2, σ_v^2, σ_w^2 due to a small number of stars in the sample and lack of uniform distribution. Therefore, following

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Woolley *et al.* (1965), we have adopted the ratios $\sigma_u^2:\sigma_v^2:\sigma_w^2$, which may be assessed for each sample from space motions by an iterative procedure. However we have used $\sigma_u^2:\sigma_v^2:\sigma_w^2 = 3:2:1$ throughout. Though they calculated the statistical parallaxes (of RR Lyrae stars) in a different manner, Woolley *et al.* found that the derived absolute magnitude was not usually sensitive to the adopted ratios. We have not tried different ratios to see the effect; but it is not likely for the errors in the adopted ratios to be very important as the ratios go into the weights and not directly into the calculated dispersions.

Before π^2 can be determined the observed proper motion dispersion must be corrected for errors in the proper motions and apparent magnitudes. Take first a number of stars at the same (true) apparent magnitude in a small area of the sky and let μ_{q0}, μ_{P0} and E_0 (in appropriate units) be the true parallactic motion in the direction concerned, peculiar proper motion and error in the observed proper motion, respectively; all reduced to (true) zero apparent magnitude. The mean square value of the observed reduced residuals $\Delta\mu_0 = \mu_0 - \bar{\mu}_0$ is $\overline{\Delta\mu_0^2} = \overline{\mu_0^2} - \bar{\mu}_0^2$, where $\mu_0 = (\mu_{q0} + \mu_{P0} + E_0)\, 10^{0\cdot 2\Delta m}$, Δm is the error in the apparent magnitude. If the peculiar motions and observational errors are random then $\bar{\mu}_0 = \mu_{q0} f$, $\overline{\mu_0^2} = (\mu_{q0}^2 + \overline{\mu_{P0}^2} + \overline{E_0^2}) f^4$ where $f = 10^{(0\cdot 2\sigma_m)^2/2\,\mathrm{Mod}}$ (see equation (7) below). From these one obtains

$$\Delta\overline{\mu_c^2} = \overline{\mu_{P0}^2} = f^{-4}\{\Delta\overline{\mu_0^2} - \overline{E_0^2} f^4 - \mu_{q0}^2(f^4 - f^2)\} \tag{5}$$

for the expected proper motion dispersion in the area concerned. An average value may be used for σ_m in f; but $f^4 = 10^{0\cdot 184\sigma_m^2} = 1\cdot 11$ and the last term in equation (5) is only about 5 per cent of μ_{q0}^2 for an extreme value of $\sigma_m = \pm 0^m\cdot 5$. This is negligible compared to the errors in the proper motions since

$$\mu_{q0} = \frac{S_0\bar{\pi}}{47{\cdot}4} C_j \sim C_j$$

for almost all the groups studied. We may also put $E^2 f^4 \simeq \epsilon^2 10^{0\cdot 4\mathrm{m}} \equiv \epsilon_0^2$ for individual stars, where ϵ is the standard error (in seconds of arc/yr) of the observed proper motion as before. Assuming that the peculiar proper motions are independent of direction and distance and the errors are random the equation of condition (4) for each star may be written as

$$(47{\cdot}4)^2(\Delta\mu_{0j}^2 - \epsilon_{0j}^2) = (a_{1j}^2\sigma_u^2 + a_{2j}^2\sigma_v^2 + a_{3j}^2\sigma_w^2)\,\pi^2$$

where

$$\Delta\mu_{0j} = \mu_{0j} - \frac{S_0\bar{\pi}}{47{\cdot}4} C_j$$

(we have set $f^{-4} = 1$, to be taken into account in Section 3.6). The solution for π^2 is

$$\overline{\pi^2} = \left(\frac{47{\cdot}4}{\sigma_w}\right)^2 \frac{\sum_k (\Delta\mu_{0jk}^2 - \epsilon_{0jk}^2)\, Q_{jk}}{\sum_k Q_{jk}^2} \tag{6}$$

where

$$Q_j = a_{1j}^2 \frac{\sigma_u^2}{\sigma_w^2} + a_{2j}^2 \frac{\sigma_v^2}{\sigma_w^2} + a_{3j}^2.$$

TABLE II

Mean parallaxes and absolute magnitudes

Group	$\overline{\pi}$(RA)* $\overline{\pi^2}$(RA)*	$\overline{\pi}$(DEC)* $\overline{\pi^2}$(DEC)*	$\overline{\pi}$(RA+DEC) $\overline{\pi^2}$(RA+DEC)	M_s (mean) M_d (mean)	ΔM	M_s (max) M_d (max)
	M, Me variables			m m	m	m m
1	0·89±0·37 1·21±0·39 32	0·63±0·30 1·99±0·64 31	0·73±0·27 1·60±0·55	−0·7±0·8 +0·5±0·4	0·0	−1·7±0·7 −0·6±0·4
2	2·18±0·92 7·98±2·55 16	6·83±2·29 11·01±3·48 16	4·18±2·05 10·70±4·37	+3·1±1·0 +2·6±0·4	0·0	+1·6±1·1 +1·3±0·5
3	2·68±1·53 6·74±2·75 15	1·53±0·56 0·93±0·37 15	1·97±0·97 3·96±1·71	+1·4±1·0 +1·5±0·5	−0·3	
4	0·20±0·49 2·97±0·73 31	1·11±0·36 2·16±0·61 31	0·74±0·34 2·56±0·70	−0·7±0·9 +1·0±0·3	−0·2	−1·2±0·8 +0·5±0·3
5	2·59±0·79 6·64±1·94 38	2·02±0·61 5·00±2·16 41	2·23±0·72 5·75±2·23	+1·7±0·7 +1·9±0·4	−0·3	+0·9±0·7 +1·4±0·4
6	0·04±0·90 7·06±5·49 11	1·52±0·94 29·6±11·8 10	1·00±1·22 21·0±15·1	0·0±2·0 +3·3±0·7	0·0	
7	N-type variables (at maximum brightness)					
	0·46±0·76 6·44±3·91 48	1·59±0·55 3·91±1·38 50	1·10±0·53 5·17±2·13			+0·2±1·0 +1·7±0·5
8	SRd variables					
	0·90±0·41 0·68±0·24 21	1·07±0·30 0·45±0·21 22	1·01±0·44 0·58±0·20	0·0±0·9 −0·6±0·4	−0·3	−0·9±0·9 −1·3±0·4
9	RV Tau variables					
	1·68±0·76 1·25±1·00 21	0·70±0·33 0·44±0·26 21	0·97±0·65 0·73±0·48	−0·1±1·3 −0·3±0·7	0·0	−1·1±1·4 −1·6±0·7

* The associated error does not include the uncertainty due to radial velocities.

For the combined solution the summation is extended to both components RA and Dec. The standard error associated with $\sigma_W{}^2\overline{\pi^2}$ is of order $(\sigma_W{}^2\overline{\pi^2})/\sqrt{N/2}$ (N = no. of equations of condition), but the formal value depends on the distribution of the stars in the sky. The values of $\overline{\pi^2}$ are given in Table II. The errors of $\overline{\pi^2}$ (RA) and $\overline{\pi^2}$ (DEC) do not include the uncertainty in the velocity dispersion from radial velocities but for that of the combined solution this has been taken into account as in the case of $\overline{\pi}$(RA+DEC).

3.5 *Absolute magnitude of N-type variables from galactic rotation*

Uncertainties in the proper motions and distances from the Sun are such that correction of the proper motions and radial velocities for the effects of differential galactic rotation before deriving the mean parallaxes is not warranted for any of the groups studied. But the N-type variables have a reasonable range in distance with a significant differential galactic rotation, so that meaningful mean absolute magnitude could be derived from radial velocities alone if a value of the Oort constant A is adopted.

The equation of condition including the first order differential galactic rotation term was written as

$$\rho = a_{13}U + a_{23}V + a_{33}W + A' 10^{0\cdot2(m-10)} \sin 2l \cos^2 b$$

where m is the apparent magnitude corrected for absorption and $A' = A10^{-0\cdot2M}$. Solving for A' instead of (Ar) avoids the complicated process of separating A and $\bar{M}$ from $\overline{Ar}$ since the distribution of distances must be taken into account. The results of the solutions are given in Table III.

TABLE III

*N-type variables**

	ρ (days) ⩽ 250	ρ ⩾ 300	All
U_0 (km s^{-1})	−6·8 ± 6·7	−4·3 ± 6·1	−6·1 ± 4·1
V_0 (km s^{-1})	−6·0 ± 8·3	−12·4 ± 7·1	−10·1 ± 4·9
W_0 (km s^{-1})	−1·1 ± 13·1	−0·1 ± 12·4	+1·6 ± 8·4
A' (km s^{-1} kpc^{-1})	+12·7 ± 6·7	+11·8 ± 5·3	+13·0 ± 3·8
No. of stars	31	25	61

* No variables between the periods 250 and 300 days.

Having determined A', the mean absolute magnitude (at maximum) is given by $M = 5 \log A/A' + 0{\cdot}23\sigma^2$, where σ is the standard error of one distance modulus (see Section 3.6 below). Assuming $A = 14{\cdot}3$ km s^{-1} kpc^{-1} (Feast & Shuttleworth 1965) as being the appropriate value for Population I objects, we obtain $M_{\mathrm{pg}}(\max) = +0^{\mathrm{m}}{\cdot}4 \pm 0^{\mathrm{m}}{\cdot}6$. The error was calculated from the error of A' only.

3.6 *Mean absolute magnitudes*

The mean absolute magnitudes, at mean brightness, from secular parallaxes, M_{s}, and from dispersion parallaxes, M_{d}, are given in Table II (column 5). Column 7 gives M_{s} and M_{d} at maximum brightness (the corresponding mean parallaxes are given for N-type stars only). Their errors were calculated as follows. The relation between $\overline{P^r}$ and $\bar{M}$ when $P_i = 10^{bM_i}$ and M_i is normally distributed is given by

$$\overline{P^r} = 10^{rb\bar{M} + (rb)^2 \sigma_M{}^2/2\,\mathrm{Mod}} \tag{7}$$

(see Van de Kamp 1967, p. 62). From this it follows that

$$\sigma_P{}^2 = \overline{P^2} - \bar{P}^2 = \bar{P}^2(10^{b^2\sigma_M{}^2/\mathrm{Mod}} - 1), \quad \text{or} \quad \sigma_M{}^2 = \frac{\mathrm{Mod}}{b^2} \log\left(1 + \frac{\sigma_P{}^2}{\bar{P}^2}\right).$$

If N is the size of the sample at hand, then

$$\epsilon_{\bar{M}}{}^2 = \frac{\sigma_M{}^2}{N} = \frac{\mathrm{Mod}}{b^2} \log\left(1 + \frac{\sigma_P{}^2}{N\bar{P}^2} + \ldots\right), \; (\sigma_P{}^2/\bar{P}^2 < 1)$$

Since $\sigma_P^2/N = \sigma_{\bar{P}}^2$ we have for the present purpose

$$\epsilon_{\bar{M}}^2 = \frac{\text{Mod}}{b^2}\log\left(1+\frac{\sigma_{\bar{P}}^2}{\bar{P}^2}\right).$$

The errors of M_s and M_d were calculated from this expression with $b = 0{\cdot}2$ for M_s with $\bar{P} = \bar{\pi}$ and $b = 0{\cdot}4$ for M_d with $\bar{P} = \overline{\pi^2}$. (Note that the variance of π^2 was not calculated from the variance of π). In general the accuracy of the absolute magnitudes is not related to the size of the mean motion in Table I because the stars with large V_0 also have large intrinsic velocity dispersions; the realistic error of V_0 depends on the distribution of the stars in the sky and is always larger than $\sigma_0/\sqrt{N}$, where σ_0 is the standard error of unit weight, which is dominated by the peculiar velocities.

ΔM given in column 6 of Table II is a correction to be applied to M_s and M_d. The reason for it is this: A significant fraction of the proper motions involved in the calculations were determined photographically and were reduced to absolute using the number count technique. It has been found that the number count technique gives too large a mean parallax for the reference stars and that the appropriate parallax appears to be that determined from the proper motions of the reference stars themselves (Aslan 1972). However, instead of repeating the calculations with the corrected proper motions the corrections to the mean absolute magnitudes were calculated exactly using the corrections to the proper motions.

TABLE IV

Mean absolute magnitudes

Group	M_{pg} (mean)	M_{pg} (max)	Median period (days)	Mean spectral type
1	−0·2±0·4	−1·2±0·3	145	M4·1 e
2	+2·5±0·4	+1·2±0·5	290	M6·0 e
3	+0·9±0·5	+0·3*	120	M5·1
4	+0·1±0·3	−0·4±0·3	75	M4·3
5	+1·3±0·3	+0·7±0·3	140	M5·0
6	+2·2±0·7	+1·4*	330	M5·7
7	+1·6*	+0·7±0·4	All	
8	−0·9±0·4	−1·7±0·4	All	
9	−0·5±0·6	−1·7±0·6	All	

* Not calculated but deduced by considering the mean amplitude of the stars concerned.

Finally M_s and M_d should be corrected for the dispersion in absolute magnitude and errors in the apparent magnitudes, which have been ignored so far. The corrected values of M_s and M_d are $M_s = M_s$ (Table II)$-0{\cdot}23\sigma^2$ and $M_d = M_d$ (Table II)$-0{\cdot}46\sigma^2$, which follow from equation (7) by replacing σ_M with σ, the standard error of one distance modulus. We have adopted σ(app. mag) $= \pm 0^m{\cdot}5$, σ(absorption) $= \pm 0^m{\cdot}3$, σ(dispersion in M) $= \pm 0^m{\cdot}5$ for all the groups except the N-type variables, for which $\sigma = \pm 0^m{\cdot}9$ was used on the basis of a discussion of absolute magnitudes of carbon stars in binary systems by Gordon (1968).

Since M_s and M_d are not strictly independent, they were given weight according to their inverse standard errors instead of according to the inverse squares in calculating the final mean absolute magnitudes in Table IV. This procedure reduces the weight of M_d by a factor of about 2 relative to that of M_s since the errors of

$M_{\rm d}$ are decidedly smaller. The final absolute magnitudes given in Table IV are for a limiting apparent magnitude of $m_{\rm pg} \sim 11^{\rm m}$ at maximum and not for unit volumes in space.

4. DISCUSSION

4.1 *M-type variables*

The absolute magnitudes at mean and at maximum brightness are plotted against the logarithm of median period in Figs 1 and 2, respectively. The open circles denote M-type semi-regulars with emission, filled circles denote those without emission. The crosses represent Mira variables taken from Clayton & Feast (1969) for comparison, their visual absolute magnitudes being shifted by $B\text{-}V = 1^{\rm m}\cdot 6$. Also plotted are the red variables in the globular cluster 47 Tuc (Arp, Brueckel & Lourens 1963) at the mean periods 202 days (plus sign, three stars), 160 days (open triangle, two stars) and 50 days (filled triangle, three stars). The mean periods and the apparent magnitudes were taken from Arp *et al.* For the apparent distance modulus $m-M = 13\cdot 3 \pm 0\cdot 3$ by Feast (1965) was used. The spectra of these 47 Tuc variables have been discussed by Feast & Thackeray (1960). They observed two of the three 50-day variables: they did not detect any emission features in their spectra.

As seen in Figs 1 and 2, SRab Me variables with $p \leqslant 200$ days (group 1) and the 160-day SR Me variables in 47 Tuc fall on the 'Mira sequence', while the non-

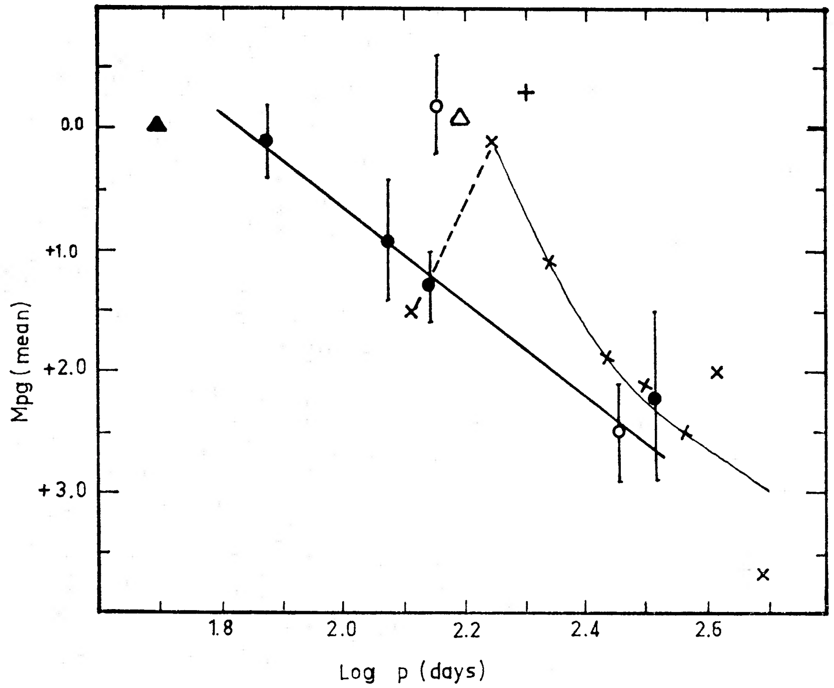

FIG. 1. *Absolute magnitude at mean brightness as a function of period. The points with error bars are semi-regular (SR) variables from Table IV. Open circles, SRab Me variables; filled circles, SRa M or SRb M variables; crosses, Mira variables from Clayton and Feast. The plus sign, open triangle and filled triangle denote, respectively the long period Me variables, SR Me variables and the SR M variables in 47 Tuc from Arp* et al. *The straight line is a least squares-fit to the field SR variables excluding Group 1.*

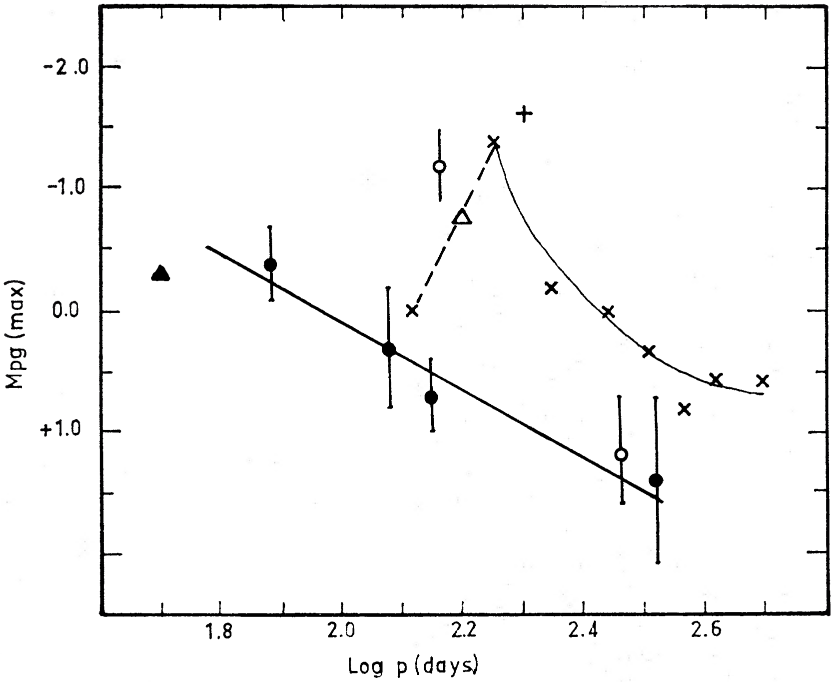

FIG. 2. *Absolute magnitude at maximum brightness as a function of period. Symbols are as in Fig. 1.*

emission semi-regulars and group 2 define a separate sequence, which is approximately a straight line. Least squares-fit to the field variables gives M_{pg}(mean) = 0·6 + 3·9 (log p − 2·0) (solid line, Fig. 1), each point receiving weight proportional to its inverse square error. The formal errors are about ±0^m·4 in the zero point and ±0·2 in the slope. At maximum we have M_{pg}(max) = 0·1 + 2·8 (log p − 2·0) (solid line, Fig. 2).

Within the errors there are no significant differences between the field and 47 Tuc variables. Including the point for the 50-day variables in 47 Tuc the solutions give M_{pg}(mean) = 0·75 + 3·3 (log p − 2·0) and M_{pg}(max) = 0·15 + 2·3 (log p − 2·0).

The H–R diagram for the M-type semi-regular variables at mean brightness is shown in Fig. 3. The abscissae are the mean spectral types of the groups concerned (Table IV). Spectrum variation is reported in GCVS only for a few stars with emission lines. In such cases the average spectral type was used in calculating the group means. (For the majority of stars no range in spectral type is given in GCVS: according to the spectroscopic survey of Joy (1942) the variation of spectral type in stars with weak or no emission is less than one subdivision.)

The mean spectral types of Mira variables at mean maximum are given as a function of period by Keenan (1966). Their positions in the H–R diagram at mean maximum are shown in Fig. 4 (crosses). The semi-regulars are also plotted using the same spectral types as in Fig. 3. Since the spectral variation in semi-regulars is small, their positions relative to the Miras should be systematically correct to within 0·5 subdivision as far as the abscissae are concerned.

The semi-regular and long period variables in 47 Tuc are also plotted in Figs 3 and 4. Their spectral types, near maximum light, are from Feast & Thackeray (1960). The 50-day variables have small amplitudes so that a large difference between the spectral types at maximum and mean light is not expected. On the other hand 160-day variables may be later than shown in Fig. 3 at mean light as they have larger amplitudes.

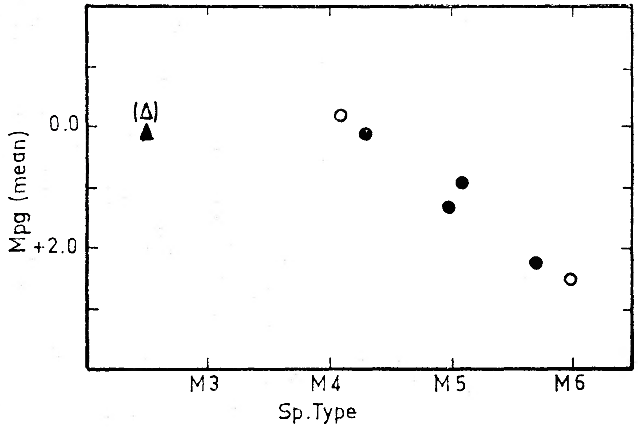

FIG. 3. *Positions of SR variables in the H–R diagram at mean brightness. Symbols are as in Fig.* 1.

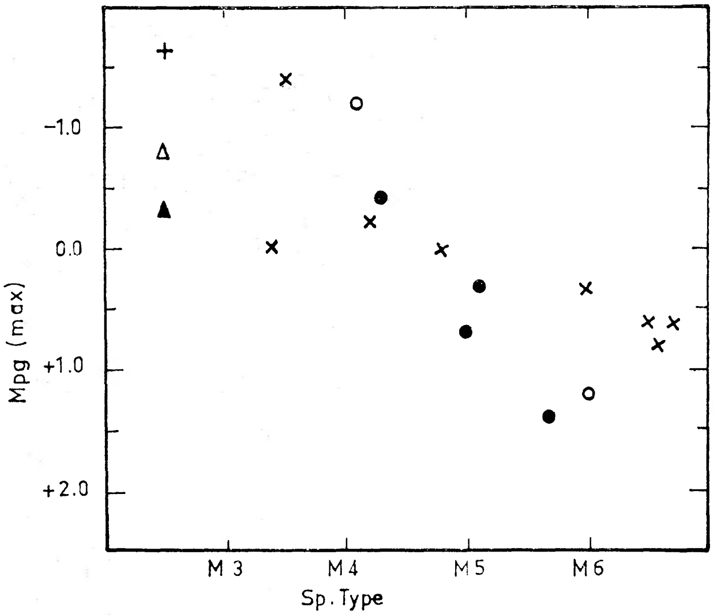

FIG. 4. *Positions of SR and Mira variables in the H–R diagram at maximum brightness. Symbols are as in Fig.* 1.

4.2 *SRd and RV Tau variables*

It is of interest to compare the present results with the absolute magnitudes of similar stars in globular clusters. The mean visual absolute magnitude of SRd-like variables in globular clusters at maximum is $M_V = -3^m{\cdot}0$, derived by Feast (1965) from 12 stars. Applying a colour index of $(B-V)_0 = 1{\cdot}3$ obtained from the photometry of similar stars in the field, SX Her and TY Vir, by Preston & Wallerstein (1963) we get $M_{pg}(\max) = -1^m{\cdot}7$. This is the same as the mean absolute magnitude of field SRd variables given in Table IV. As suggested by Feast (1965), the SRd variables do extend the period–luminosity relation of Mira variables to shorter periods (not indicated in Figs 1 and 2).

As for the RV Tau variables in globular clusters, we obtain $M_{\mathrm{pg}}(\mathrm{max}) = -2^{\mathrm{m}}{\cdot}0$ using the apparent magnitudes and distance moduli given by Gaposchkin & Gaposchkin (1965). This compares well with $M_{\mathrm{pg}}(\mathrm{max}) = -1^{\mathrm{m}}{\cdot}7$ for the field variables from the statistical parallaxes.

4.3 *N-type variables*

The observed B–V colours of N-type variables vary from star to star by up to 3^{m} or more (e.g. Mendoza 1967). Observations concerning the variation of B–V in individual stars are lacking. Nevertheless we may apply an average colour to the photographic absolute magnitude in Table IV with a view to comparing the resultant visual absolute magnitude with other independent determinations. The mean colour of N-type semi-regulars, uncorrected for reddening, is $B\text{–}V = 3^{\mathrm{m}}{\cdot}6$ (Section 2.4). The average value of the photographic absorption corrections applied in deriving the mean parallaxes is $0^{\mathrm{m}}{\cdot}75$, which, however, corresponds to an average reddening of $0^{\mathrm{m}}{\cdot}2$ in B–V. Therefore $M_{\mathrm{pg}} = +0^{\mathrm{m}}{\cdot}7$ (Table IV) is equivalent to a visual absolute magnitude of $-2^{\mathrm{m}}{\cdot}7 \pm 0^{\mathrm{m}}{\cdot}4$.

This is compared in Table V with the visual absolute magnitudes of N-type stars in binary systems and clusters by Gordon (1968). Of his 13 N-type stars six are semi-regular (SR, SRa or SRb) and two are irregular (Lb) variables. The mean values for the variables (eight stars) and non-variables (five stars) were calculated using Gordon's individual absolute magnitudes (at maximum in the case of variables). Exclusion of the two irregular variables does not change the figures. The dispersions are 1·25 times the average residual from the means. For one SR variable, HN Aur, Gordon gives $M_{\mathrm{V}}(\mathrm{max}) = -0^{\mathrm{m}}{\cdot}1$ on the basis of its being a member of NGC 1664. This value, as he notes, is somewhat fainter than one would expect.

TABLE V

Absolute visual magnitudes of N-stars

Field variables (this paper)	$M_{\mathrm{V}}(\mathrm{max}) = -2^{\mathrm{m}}{\cdot}7 \pm 0^{\mathrm{m}}{\cdot}4$
Variables in clusters and binaries	$M_{\mathrm{V}}(\mathrm{max}) = -2{\cdot}6 \pm \frac{1{\cdot}1}{\sqrt{8}}$ or $-2{\cdot}9 \pm \frac{0{\cdot}7}{\sqrt{7}}$ (without HN Aur)*
Non-variables in clusters and binaries	$M_{\mathrm{V}} = -0{\cdot}5 \pm \frac{1{\cdot}3}{\sqrt{5}}$ or $-1{\cdot}0 \pm \frac{0{\cdot}8^{*}}{\sqrt{5}}$
Carbon stars in LMC	$M_{\mathrm{V}} = -3{\cdot}0 \pm 0{\cdot}5$ (per star)

* See text.

For two non-variable stars, HD 209596 and BD+48°3827, Gordon finds $M_{\mathrm{V}} = -0^{\mathrm{m}}{\cdot}8$ or $+2^{\mathrm{m}}{\cdot}2$ and $M_{\mathrm{V}} = -0^{\mathrm{m}}{\cdot}4$ or $+2^{\mathrm{m}}{\cdot}1$, respectively, on the basis of alternative luminosity classes assigned to the companions. The second figure in Table V is obtained when only the brighter values are considered.

The mean visual absolute magnitude of carbon stars in Large Magellanic Cloud is also quoted in Table V from Gordon, who considers the stars involved to be of type N.

Sanford (1944) found $M_V = -2^m{\cdot}3 \pm 0^m{\cdot}3$ for 171 N-type stars from differential galactic rotation. For the Oort constant A he used $+17{\cdot}7$ km s^{-1} kpc^{-1}. Using $A = +14{\cdot}3$ and allowing for an assumed dispersion in absolute magnitude and an average error in apparent magnitude as in Section 3.5, Sanford's value becomes $M_V = -2^m{\cdot}6$. (This may be compared with $M_V = -3^m{\cdot}0$ for the semi-regulars from the differential galactic rotation alone.) About two-thirds of Sanford's stars are variable, for which he used magnitudes at maximum brightness.

Table V shows that there is good agreement between the two determinations of M_V for the variables, while the non-variables do not appear to be brighter than the variables at mean light. (Gordon's maximum and minimum magnitudes for variables give M_V(mean) $= -1^m{\cdot}6 \pm 0^m{\cdot}4$ or $-1^m{\cdot}9 \pm 0^m{\cdot}4$ with or without HN Aur, respectively). However, Sanford's value of $M_V = -2^m{\cdot}6$ mentioned in the previous paragraph is not inconsistent with Table V if the constant stars are really fainter than the variables at maximum. It would be useful to carry out the statistical analysis for variables and non-variables separately. It would also be useful to consider the Mira type carbon stars separately from the smaller amplitude semi-regular and irregular variables to see if there are any differences in absolute magnitudes as is the case for the M-type stars (see Figs 1 and 2).

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