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BIG-BANG NUCLEOSYNTHESIS REVISITED*

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ABSTRACT

The results of an improved calculation of the synthesis of elements during the high-temperature phase of the expansion of big-bang universes is presented. Adoption of the viewpoint (supported by recent evidence) that most of the observed deuterium and helium is of pregalactic origin allows very general constraints to be put on any cosmological model for their production. In fact, most models which differ appreciably from that of the most naïve (standard) big-bang are ruled out. Those standard big-bang models in which the present baryon density is (1–3) $\times 10^{-31}$ g cm^{-3} agree best with the probable pregalactic abundances of ^{2}H, ^{3}He, and ^{4}He, if the galactic production of ^{3}He is also assumed negligible. It is also shown on very general grounds that the present universal baryon density and average abundance (by mass) of primordial deuterium should be related by $\langle \rho_b(T = 2.7° \text{ K})\rangle \langle X(^2\text{H})\rangle \leq 2 \times 10^{-34}$ g cm^{-3}. Since no known galactic process can produce a deuterium abundance of $X(^2\text{H}) > 10^{-6}$, any detection of interstellar deuterium in this range could be of profound cosmological significance.

Subject headings: abundances — cosmology — gravitation — nuclear reactions

I. INTRODUCTION

Several recent developments have made it appropriate at this time to reinvestigate the synthesis of the elements during the early expansion of hot big-bang universes. These developments have all occurred since the initial detailed nucleosynthesis calculations of Peebles (1966*a*, *b*) and of Wagoner, Fowler, and Hoyle (1967) (hereafter referred to as WFH). They are listed below.

1) The discovery by Searle and Sargent (1972*a*) of two dwarf blue galaxies in which the abundance of helium is normal, while the abundances of the heavy elements (oxygen and neon) are significantly lower than normal (where "normal" refers to the solar neighborhood). The galaxies appear to be young systems, and to provide strong evidence for a universal pregalactic source of the helium.

2) The realization that no galactic process appears capable of generating the amount of deuterium which is thought to exist in the interstellar medium (Reeves *et al.* 1972). Indeed, while galactic cosmic rays can produce the observed abundances of the rare light elements ^{6}Li, ^{9}Be, ^{10}B, and possibly ^{11}B (Reeves, Fowler, and Hoyle 1970; Meneguzzi, Audouze, and Reeves 1971; Mitler 1970), they appear incapable of producing enough ^{2}H, ^{3}He, ^{4}He, and ^{7}Li. However, these are precisely the elements which can be formed in the early universe.

3) The increased evidence that the microwave background radiation is of primeval origin. Both narrow-band (Peebles 1971) and broad-band (Blair *et al.* 1971) measurements at wavelengths $\lambda \geq 0.8$ mm are consistent with the required blackbody spectrum, corresponding to a temperature $T = 2.7° \pm 0.1°$ K. The excess flux originally observed in the wavelength band $\lambda = 0.4$–1.3 mm (Pipher *et al.* 1971, and references cited therein) has not been seen in the recent flight of Houck *et al.* (1972). In addition, the extreme isotropy of the 2.7° K background (Peebles 1971; Boynton and Partridge 1972) is difficult to understand if its source is related to discrete objects.

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4) The improved knowledge of many of the cross-sections of the nuclear reactions involved in the element production, and an improved computer program.

Based on the above developments, in this paper we will take the viewpoint that most of the observed ^{2}H and ^{4}He (and possibly also ^{3}He and ^{7}Li) in the universe is of pregalactic origin. Since it will be seen that the simplest big-bang models can produce their required abundance, we feel no motivation to investigate more exotic element cookers in any detail. However, we will be able to derive some general constraints on *any* model.

The assumptions which define the class of models which we shall investigate in detail are the following.

1) There exists at each point in spacetime local Lorentz coordinate frames, in which all the laws of physics are expressed in their standard special-relativistic form. As a consequence, the strengths of the strong, electromagnetic, and weak interactions remain constant in time. However, this postulate only restricts the theory of gravitation to be a metric theory (Thorne and Will 1971).

2) The universe visible to the element of matter being studied was reasonably isotropic and homogeneous, at least with regard to its gravitational field. From assumption (1), it then can be shown that the metric is given by the Robertson-Walker form

$$ds^2 = -c^2dt^2 + R^2(t)[dw^2 + \sigma^2(w)d\Omega^2]\,, \tag{1}$$

where w is a comoving radial coordinate. This assumption would still allow sizable variations in the baryon density in most models, as long as the baryons comprised a small fraction of the total mass-energy density. The maximum size of any fluctuation is set by the observed isotropy of the microwave background radiation.

3) The universe expanded from temperatures $> T_0$, where T_0 is the lowest temperature at which the neutrinos can be in statistical equilibrium with the electrons and muons. (If the metric theory of gravitation is chosen to be general relativity, then $T_0 \simeq 10^{11}$ ° K.) Element production can only be affected by the properties of the universe at temperatures $T_0 > T > 10^8$ ° K.

4) Antibaryon (or baryon) annihilation has been complete within the element of matter being studied. For definiteness, we shall take the local baryon number to be positive.

5) All particles (in particular the neutrinos) are nondegenerate.

What can the present-day abundances of the elements tell us about the early universe? Consideration of the general process by which big-bang nucleosynthesis occurs makes it clear that the abundances produced depend *only* on the values of the following three quantities at $T \sim 10^9$ ° K, the temperature at which nucleosynthesis takes place: (1) the baryon mass density ρ_b (or the equivalent quantity h, to be defined in the following section); (2) the expansion rate $V^{-1}dV/dt$ of the volume element V of interest (containing a fixed number of baryons); (3) the neutron-proton ratio.

Thus, on the one hand, the abundance observations provide us with information about only these three properties of the correct model. However, on the other hand, we can predict the element production of any model not satisfying assumptions (2) and (5) as well. It is merely equal to the element production of that model within our class which has the same values of the above three quantities. We shall see that there is a one-to-one correspondence between the model parameters within our class and the above three quantities.

II. PROPERTIES OF THE MODELS

It is useful to consider the universe to consist of three types of matter during the epoch of interest. They are:

1) The strongly and electromagnetically interacting particles (nucleons, nuclei, photons, electrons, and positrons), which can be described as a perfect fluid.

2) The electron neutrinos (ν_e, $\bar{\nu}_e$), whose interaction is weak enough to let them expand freely with the universe, but strong enough to let them affect the neutron-proton ratio.

3) Those particles (ν_μ, $\bar{\nu}_\mu$, gravitons [?], scalarons [?], . . .) which are effectively noninteracting.

Within any metric theory, the evolution of the total mass-energy density ρ^* and pressure p^* of the perfect fluid is governed by the energy conservation equation

$$\frac{d}{dt}(\rho^* R^3) + \frac{p^*}{c^2}\frac{d}{dt}(R^3) = 0\,, \tag{2}$$

while the neutrinos and other noninteracting particles expand freely. The properties of the matter as a function of R or T_9 (the perfect-fluid temperature in units of 10^9 ° K) are thus the same as those given by WFH or Peebles (1971), for instance, to which the reader is referred.

In particular, conservation of baryons guarantees that the function $h(T_9)$ in the expression

$$\rho_b = hT_9{}^3 \tag{3}$$

for the baryon mass density remains constant except for a decrease by a factor 2.75 during pair annihilation if there is no other source of entropy. Its value preceding pair annihilation shall be designated by h_0. (The constant h employed by WFH refers to the value after pair annihilation.) We shall consider values of h_0, our first parameter, in the range $10^{-6} \leq h_0 \leq 10^{-1}$ for reasons to be seen later. Nucleosynthesis with larger values of h_0 (but with neutrino escape) has most recently been calculated by Wagoner (1971).

The expansion rate of the matter is determined by the (metric) theory of gravitation. Since its form is not too important, it will be sufficient to use the expansion rate

$$V^{-1}dV/dt = \xi(24\pi G\rho)^{1/2}\,, \tag{4}$$

with our second parameter ξ representing a correction factor to the general-relativistic expression. Since we take ρ to be the mass-energy density of known particles ($\rho = \rho^* + \rho_\nu$), ξ could be greater than unity even within general relativity if there were a primeval population of presently undetected particles.

The two parameters h_0 and ξ allow us to vary the first two basic factors affecting nucleosynthesis mentioned at the end of the preceding section. The third factor, the neutron-proton ratio at $T_9 \sim 1$, depends on the neutron-proton weak reaction rates as well as the expansion rate at temperatures above $T_9 \sim 1$. We can produce a sufficient range of values of this ratio by merely varying the effective weak interaction coupling constant, or equivalently the free-neutron mean lifetime

$$\tau_n = 926/C \qquad \text{seconds}\,, \tag{5}$$

even though a general modification of the weak reaction rates will not be of this form. We have normalized to the average half-life of 10.7 ± 0.1 minutes obtained from the recent measurements of Christensen *et al.* (1967) and Christensen *et al.* (1971). Details of these weak reaction rates are given in the Appendix. Our third parameter is thus the normalized rate coefficient C. It could differ from unity if the neutrino spectrum were not thermal due to spatial anisotropy or inhomogeneity, for instance, or if future experiments reveal a different neutron half-life. It will be sufficient to only consider values of ξ and C which do not differ greatly from unity.

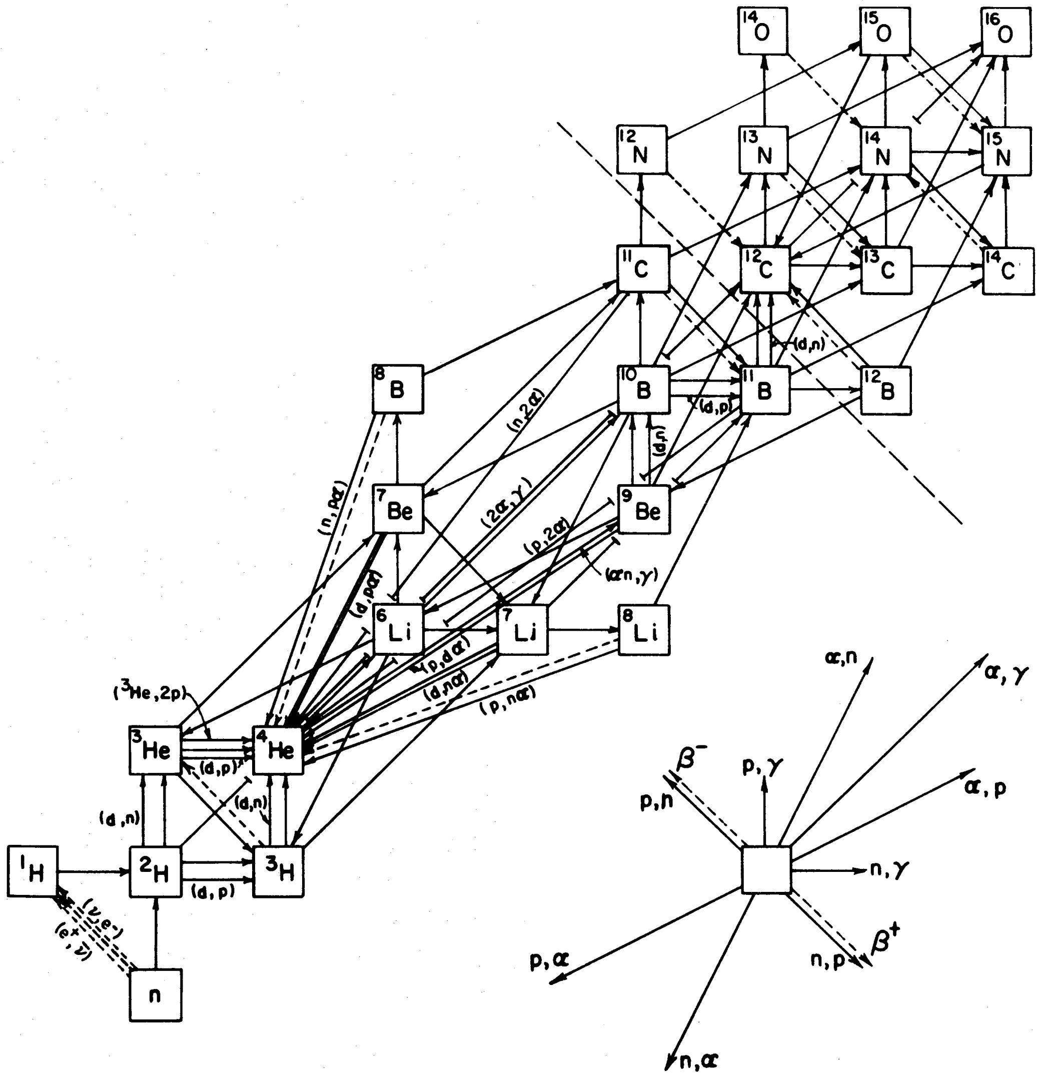

FIG. 1.—Diagram of all nuclear reactions included in the calculation. The exoergic directions are indicated by the arrows. The dashed line indicates that individual nuclear abundances were computed only for $A < 12$.

The nuclear reactions included in the present calculation are indicated in figure 1. Although the number of reactions in this range of atomic mass has been increased, those originally used by WFH still include most of the important ones (except for the production of ^{6}Li). New expressions for certain nuclear reaction rates are presented in the Appendix.

Details of the general method of computation of the abundances are given by Wagoner (1969) (mostly in the Appendices). The calculation is begun at a temperature of $T_9 = 100$, at which point the abundance of the nucleons is fixed by statistical equilibrium. The numerical errors in the final abundances produced by the computer program are less than 3 percent, with the larger abundances having the greater accuracy. Thus most of the uncertainties in the final abundances produced by a given model are due to uncertainties in the nuclear cross-sections. These are estimated to be less than 2 percent for ^{4}He, less than a factor of 2 for other nuclei of mass number $A \leq 7$, and somewhat greater for heavier nuclei. The rates of the important reactions ^{11}C +

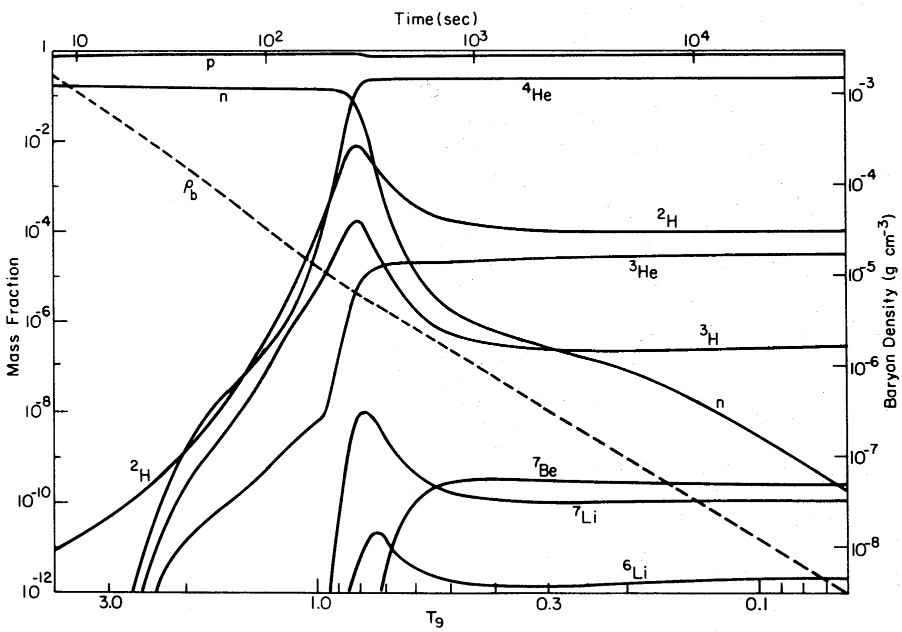

FIG. 2.—Evolution of nuclear abundances and baryon density (*dotted line*) during the expansion of a typical ($h_0 = 10^{-4.5}$) "standard" ($\xi = C = 1$) big-bang model.

$p \rightarrow \gamma + {}^{12}\mathrm{N}$ and ${}^{11}\mathrm{C} + \alpha \rightarrow p + {}^{14}\mathrm{N}$ are very uncertain since there are no laboratory data.

The evolution of the nuclear abundances and baryon density during the expansion of a typical "standard" ($\xi = C = 1$) big-bang model (also referred to as "canonical" or "naïve") is presented in figure 2. Abundances shall always be expressed in terms of mass fraction X. It is seen that the abundances "freeze out" at temperatures $0.8 \geq T_9 \geq 0.4$. Note also that in this model (as well as in all other models studied) the abundance of deuterium (^{2}H or d) is always greater than that of tritium (^{3}H or t). Thus it is a reasonable approximation to include only n, p, d, and α reactions in our calculation. (The only exception is ${}^3\mathrm{He} + {}^3\mathrm{He} \rightarrow 2p + {}^4\mathrm{He}$.)

III. COMPARISON OF COMPUTED AND OBSERVED ABUNDANCES

The final abundances (after all β-decays and electron captures) produced by the standard big-bang models are presented in tabular form in table 1 and in graphical form in figure 3, as a function of h_0. If h_0 was uniform and no entropy generation other than pair annihilation occurred, then the present average baryon density is related to h_0 by

$$\langle \rho_b(T = 2.7^\circ\ \mathrm{K}) \rangle = 7.15 \times 10^{-27} h_0 \quad \mathrm{g\ cm^{-3}}\,. \tag{6}$$

This relation has also been included in table 1 and figures 3–6. Table 1 can be compared directly with tables 3A and 3B of WFH, after remembering that $h_0 = 2.75h$ (WFH). The most significant differences are the inclusion of ^{6}Li, the reduction of ^{3}He and ^{7}Li by a factor of $\sim\frac{1}{2}$ for the larger values of h_0 (where mass 7 production occurs as ^{7}Be), and the much decreased production of ^{11}B, as compared to the WFH calculation. The reduction of $X({}^3\mathrm{He})$ and $X({}^7\mathrm{Li})$ is apparently due mostly to the increased destruction rates of ^{3}He, while that of $X({}^{11}\mathrm{B})$ seems to be due to its increased destruction rate by protons.

TABLE 1

Element Production in "Standard" Big Bang

log h_0	$\rho_b(T = 2.7°\ K)$ (g cm^{-3})	$X(^2H)$	$X(^3He)$	$X(^4He)$	$X(^6Li)$	$X(^7Li)$	$X(^{11}B)$	$X(A \geq 12)$
−6.00	7.15×10^{-33}	8.5×10^{-3}	3.6×10^{-4}	0.089	2.6×10^{-11}	2.0×10^{-9}	...	...
−5.75	1.27×10^{-32}	5.5×10^{-3}	2.8×10^{-4}	0.131	3.7×10^{-11}	3.0×10^{-9}	...	...
−5.50	2.26×10^{-32}	3.1×10^{-3}	1.9×10^{-4}	0.171	3.6×10^{-11}	2.8×10^{-9}	...	...
−5.25	4.02×10^{-32}	1.4×10^{-3}	1.1×10^{-4}	0.200	2.3×10^{-11}	1.5×10^{-9}	...	...
−5.00	7.15×10^{-32}	5.8×10^{-4}	6.7×10^{-5}	0.217	1.1×10^{-11}	5.0×10^{-10}	...	...
−4.75	1.27×10^{-31}	2.2×10^{-4}	4.3×10^{-5}	0.227	4.5×10^{-12}	2.2×10^{-10}	...	...
−4.50	2.26×10^{-31}	8.9×10^{-5}	2.8×10^{-5}	0.234	2.0×10^{-12}	3.4×10^{-10}	...	...
−4.25	4.02×10^{-31}	3.6×10^{-5}	1.8×10^{-5}	0.240	...	1.2×10^{-9}	...	...
−4.00	7.15×10^{-31}	1.3×10^{-5}	1.2×10^{-5}	0.246	...	3.5×10^{-9}	...	...
−3.75	1.27×10^{-30}	3.3×10^{-6}	8.5×10^{-6}	0.251	...	7.2×10^{-9}	...	...
−3.50	2.26×10^{-30}	3.9×10^{-7}	5.8×10^{-6}	0.255	...	1.2×10^{-8}	...	...
−3.25	4.02×10^{-30}	9.8×10^{-9}	4.1×10^{-6}	0.260	...	1.7×10^{-8}	...	...
−3.00	7.15×10^{-30}	1.2×10^{-11}	3.3×10^{-6}	0.265	...	2.5×10^{-8}	...	...
−2.75	1.27×10^{-29}	...	2.7×10^{-6}	0.270	...	3.8×10^{-8}	1.0×10^{-12}	2.4×10^{-12}
−2.50	2.26×10^{-29}	...	2.4×10^{-6}	0.275	...	6.0×10^{-8}	1.7×10^{-12}	1.0×10^{-11}
−2.25	4.02×10^{-29}	...	2.1×10^{-6}	0.280	...	9.4×10^{-8}	2.7×10^{-12}	5.0×10^{-11}
−2.00	7.15×10^{-29}	...	1.8×10^{-6}	0.284	...	1.5×10^{-7}	4.0×10^{-12}	2.5×10^{-10}
−1.75	1.27×10^{-28}	...	1.5×10^{-6}	0.289	...	2.2×10^{-7}	5.4×10^{-12}	1.2×10^{-9}
−1.50	2.26×10^{-28}	...	1.1×10^{-6}	0.294	...	3.0×10^{-7}	6.4×10^{-12}	5.4×10^{-9}
−1.25	4.02×10^{-28}	...	7.8×10^{-7}	0.299	...	3.7×10^{-7}	6.2×10^{-12}	2.1×10^{-8}
−1.00	7.15×10^{-28}	...	4.3×10^{-7}	0.304	...	3.7×10^{-7}	4.6×10^{-12}	6.5×10^{-8}

Note.—Three leaders (...) indicates $X < 10^{-12}$.

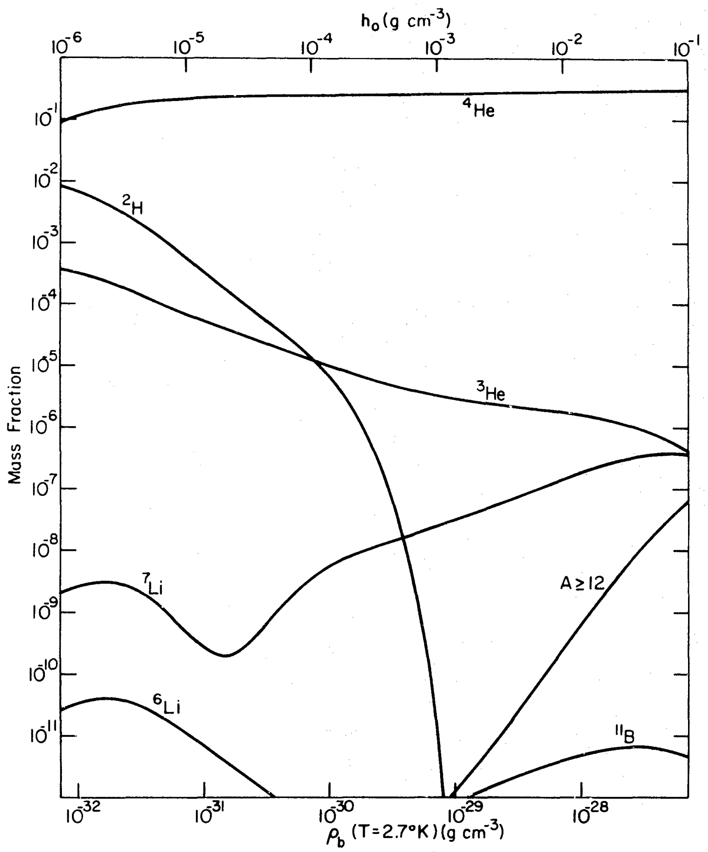

FIG. 3.—Final abundances produced by standard ($\xi = C = 1$) big-bang models, whose only parameter is h_0 (or present baryon density).

The final abundances produced by models expanding twice as fast and one-half as fast as the standard models are presented in figure 4. The change in some abundances (especially that of ^{11}B) is somewhat larger than might have been expected from such a change in expansion rate by only a factor of 2. (Some results for much larger and smaller values of ξ have been presented by Peebles 1966*b* and Wagoner 1967.) A similar plot for models with neutron-proton weak reaction rates twice as fast and one-half as fast as the standard models is presented in figure 5. The changes in most abundances are seen to be less than those in figure 4.

An exception is the helium abundance, which is shown in more detail in figure 6. The final abundance of ^{4}He depends mainly on the neutron-proton ratio at the temperature at which their weak reactions "freeze out" of equilibrium. Since this temperature is determined by the equality of the weak reaction rates and the expansion rate, it is not surprising that the helium production at a fixed h_0 depends only on the ratio ξ/C, for $h_0 \geq 3 \times 10^{-5}$ and $\xi \sim C \sim 1$. In fact, in this range of parameter values the helium mass fraction is given to good accuracy by the formula

$$X(^4\mathrm{He}) = 0.324 + 0.0195 \log h_0 + 0.380 \log (\xi/C) . \quad (7)$$

(All logs are base 10.) The helium production increases with increasing h_0 because element production begins at a higher temperature the higher the density, resulting in less time for the neutrons to decay.

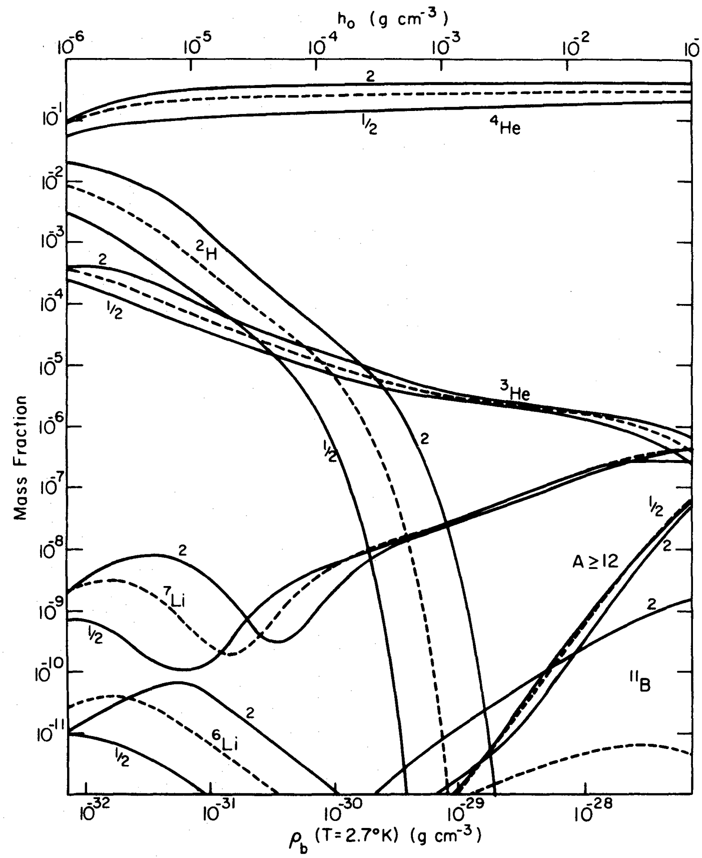

FIG. 4.—Final abundances produced by models with $\xi = \frac{1}{2}$, 2 and $C = 1$, compared with abundances produced by the standard model (*dashed curves*). The solid curves are labeled by the value of ξ used.

How would these results be modified in a nonuniform early universe? Within general-relativity theory, ideas concerning the structure of the universe during the epochs of interest here fall between two extremes (Peebles 1972). The extreme "reactionary" view pictures the early universe as completely homogeneous and isotropic, except for the small irregularities needed for galaxy formation. The main argument supporting this view is the fact that the universe is unstable against the growth of inhomogeneities, so that the approximate uniformity of the universe in the recent past (as inferred from the isotropy of the microwave background radiation) implies that the universe must have been even more uniform in the distant past. The extreme "revolutionary" view pictures the early universe as completely chaotic (Misner 1969). The main argument supporting this view is that in the reactionary picture, regions of the universe which have never communicated with each other must nevertheless have precisely the same history, and seemingly arbitrary initial conditions must be imposed.

In some chaotic models (Matzner and Misner 1972; Misner 1972), the universe becomes isotropic and homogeneous before nucleosynthesis begins, so that the extreme pictures predict similar element production. In other models, the matter which continually becomes visible to an observer as a completely chaotic universe expands will typically have a random velocity $\sim c$ relative to matter in his neighborhood (Rees 1972), although of course this could not have been true in the recent past. Relativistic turbulence leads to entropy generation via shock waves, resulting in a photon energy density $aT^4 \sim \rho_b c^2$ during nucleosynthesis. This means that for an appropriately defined average, $\langle h \rangle \sim 10 \langle T_9 \rangle$. The most obvious consequence is that no deuterium

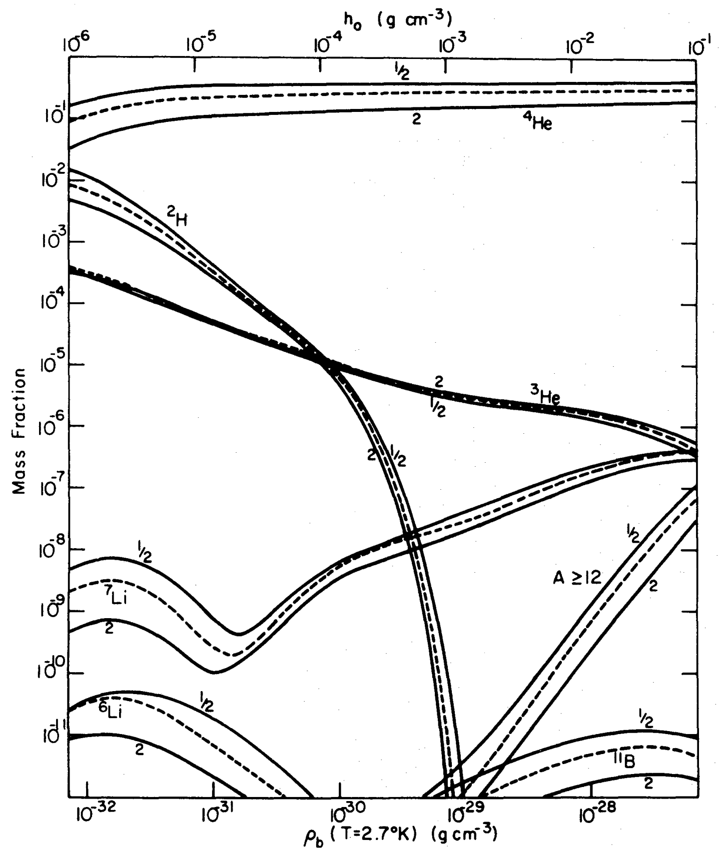

FIG. 5.—Final abundances produced by models with $C = \frac{1}{2}$, 2 and $\xi = 1$, compared with abundances produced by the standard model (*dashed curves*). The solid curves are labeled by the value of C used.

is produced for such large values of h, unless $\xi \gg 1$ (see fig. 4 and Wagoner 1969). Recall that because nucleosynthesis within any element of matter is only a function of h_0, ξ, and C, we can estimate element production in any model if the distribution of these quantities among all volume elements is given.

Let us next consider early universes which in some ways resemble the present one. That is, we allow variations in the baryon density ρ_b, but assume both the photon and neutrino temperatures to be uniform. Assumption (2) is then satisfied in any theory in which the total density (which is uniform) is the source of gravity. We also assume that $\xi \sim C \sim 1$, which we will see is probably necessary in any case to produce the observed amount of helium.

The average mass fraction of nucleus i produced within a given comoving volume V is

$$\langle X_i \rangle = \frac{\int X_i(\rho_b)\rho_b dV}{\int \rho_b dV} = \frac{\int X_i(h_0)h_0 f(h_0)dh_0}{\langle h_0 \rangle}, \tag{8}$$

where $f(h_0)dh_0$ is the fraction of the volume containing baryons whose evolution during nucleosynthesis was characterized by values of h_0 between h_0 and $h_0 + dh_0$. We then have the result

$$\langle X_i \rangle \langle h_0 \rangle = \int X_i(h_0)h_0 f(h_0)dh_0 \leq \text{Max.}\,(X_i h_0)\,, \tag{9}$$

which we shall employ later.

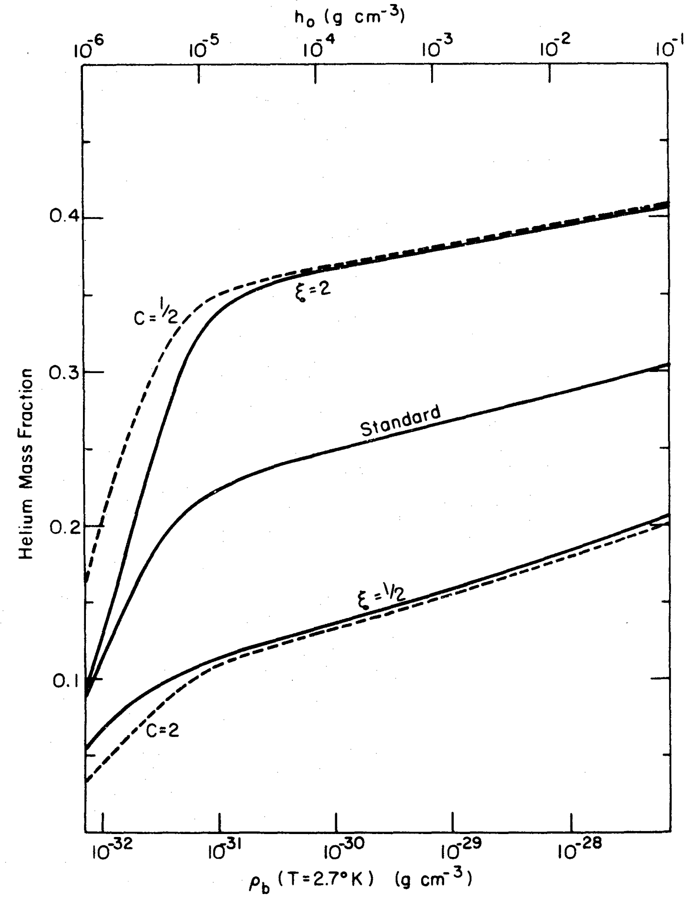

FIG. 6.—Comparison of the production of ^{4}He by the models referred to in figs. 3, 4, and 5.

The abundance data with which we shall compare these models are summarized in table 2. Column (2) lists the average abundances by mass observed in the locations indicated. Column (3) includes estimates of the contribution to the abundance of various processes occurring during the history of our Galaxy. The estimates of stellar destruction refer to the percentage of the interstellar gas which had been processed through stars to a temperature sufficient to destroy that nucleus. The large uncertainties in these estimates make it pointless to distinguish between the protosolar and present-day interstellar abundances. Use of the results in columns (2) and (3) then leads to the estimates of the required pregalactic abundance found in column (4). A more extensive summary of the abundances, origin, and galactic history of the light elements ($A \leq 11$) is given by Reeves *et al.* (1972).

The two major new results included in table 2 are:

1) The use of solar-wind and other abundance data by Geiss and Reeves (1972) and Black (1972) indicates that the protosolar deuterium abundance was less than the standard meteoritic and terrestrial value. The basic argument is that the present ^{3}He abundance in the solar wind represents an upper limit to that of the protosolar deuterium, which was completely converted into ^{3}He even in the outer layers of the Sun. The implied isotopic enrichment of deuterium appears possible through chemical reactions at low temperature occurring during the formation of the planetary system (Geiss and Reeves 1972).

2) The evidence from many sources now appears to support the view that there was

TABLE 2

ABUNDANCE DATA

Element (1)	Observed Mass Fraction and Location (2)		Galactic Production and Destruction (3)	Pregalactic Mass Fraction (4)
^{2}H.	2.3×10^{-4} (*a*)	Earth, meteorites	Solar-system fractionation enrichment factor = 2–10 (*b, c*)	$(0.3–5.0) \times 10^{-4}$
	$\leq 1.1 \times 10^{-4}$? (*d*)	Jupiter		
	$< 0.7 \times 10^{-4}$ (*e*)	Interstellar medium	Stellar destruction = 10–75% (*f, g*)	
^{3}He.	2.6×10^{-5} (*b, c*)	Gas-rich meteorites	Stellar production possible (*h*)	$\leq 1 \times 10^{-4}$
	$\leq 1.1 \times 10^{-4}$ (*b*)	Solar wind	Stellar destruction = 0–75% (*f, g*)	
	$< 1.1 \times 10^{-4}$ (*i*)	H II region		
^{4}He.	0.26–0.32 (*j, k*)	Interstellar medium and young stars	Stellar production = 0.01–0.04 (*f, g*)	0.22–0.32
	0.22–0.34 (*j, k*)	Nearby normal galaxies	Stellar destruction negligible	
	0.27–0.31 (*l*)	Dwarf blue galaxies		
^{6}Li.	4×10^{-10} [$= X(^{7}\mathrm{Li})/14.6$]	Earth, meteorites	Cosmic-ray production appears sufficient (*m*)	$\lesssim 10^{-9}$
^{7}Li.	5.5×10^{-9}	Meteorites	Cosmic-ray production appears insufficient (*m*)	$\leq 2 \times 10^{-8}$
	5×10^{-9} (*n*)	Stars without depletion	Stellar production possible (*o*)	
	$< 1.3 \times 10^{-8}$	Interstellar medium	Stellar destruction = 10–75% (*f, g*)	
^{9}Be.	1.3×10^{-10}	Meteorites	Cosmic-ray production appears sufficient (*m*)	$\lesssim 3 \times 10^{-10}$
	$< 4.6 \times 10^{-10}$	Interstellar medium		
^{10}B.	(3×10^{-10}) [$= X(^{11}\mathrm{B})/4.6$]	Meteorites	Cosmic-ray production appears sufficient (*m*)	$\lesssim 10^{-9}$
^{11}B.	(1.5×10^{-9})	Meteorites	Cosmic-ray production possibly sufficient (*m*)	$\lesssim 3 \times 10^{-9}$
	$\leq 1.5 \times 10^{-9}$ (*p*)	Sun		
$A \geq 12$.	1.5×10^{-2} (*a*)	Stellar photospheres	Stellar production sufficient	$\lesssim 10^{-5}$

REFERENCES.—(*a*) Cameron 1968; (*b*) Geiss and Reeves 1972; (*c*) Black 1972; (*d*) Beer *et al.* 1972; (*e*) Weinreb 1962; (*f*) Truran and Cameron 1971; (*g*) Talbot and Arnett 1972; (*h*) Iben 1967; (*i*) Predmore *et al.* 1971; (*j*) Danziger 1970; (*k*) Searle and Sargent 1972*b*; (*l*) Searle and Sargent 1972*a*; (*m*) Meneguzzi *et al.* 1971; (*n*) Zappala 1972; (*o*) Cameron and Fowler 1971; (*p*) Engvold 1970.

a large pregalactic abundance of helium (Searle and Sargent 1972*b*). The most striking piece of new evidence comes from the pair of dwarf blue galaxies mentioned in § I. In addition, detailed analyses of those blue halo stars with surface helium deficiencies indicate that they can no longer be regarded as evidence for a low pregalactic helium abundance (Baschek, Sargent, and Searle 1972).

How do the homogeneous standard big-bang models fare when compared with these estimates of pregalactic abundances? From tables 1 and 2 and figure 3 it is seen that any model with a present baryon density in the range $8 \times 10^{-32} \leq \rho_b \leq 5 \times 10^{-31}$ g cm^{-3} is consistent with the ^{2}H and ^{4}He abundances, although the resulting helium mass fraction of 0.22–0.24 is at the low end of the permissible range. They are all also consistent with the observed lower limit $\rho_b \geq 1.1 \times 10^{-31}(H_0/75)^2$ g cm^{-3} (Shapiro 1971), derived from the amount of matter visible in galaxies, taking into account the fact that the Hubble constant H_0 may be as low as 50 km s^{-1} Mpc^{-1} (Sandage 1968). However, if one also requires that the big bang be responsible for the ^{3}He, then only densities $\rho_b \leq 3 \times 10^{-31}$ in this range are allowable. Note that no models in this range can produce the observed amounts of any heavier elements; but that may not be an important objection, as they can be produced in other ways (with the possible exception of ^{7}Li).

Let us next consider models in which the expansion rate factor ξ and/or the weak-reaction rate factor C are unequal to unity, and not necessarily uniform. Inspection of figure 6 then would appear to rule out all models in which the average value of ξ or C is much different from unity, if one accepts a primordial helium mass fraction in the range 0.22–0.32. Local fluctuations in the temperature, for instance, will result in $\langle C \rangle \neq 1$ and helium production correspondingly different (Silk and Shapiro 1971; Ipavich 1972). The presence of strong magnetic fields ($B^2/8\pi \gg \rho c^2$), which would result in $\xi \gg 1$ and an effective $C \gg 1$, is also ruled out, since their effects on helium production do not cancel (Greenstein 1969; Matese and O'Connell 1970). In addition, there is no evidence for the presence of an intergalactic magnetic field of the required strength. Incidentally, the new upper limit on the $\bar{\nu}_e + e^-$ scattering cross-section (Gurr, Reines, and Sobel 1972) also rules out models in which the neutrinos remain thermalized for a longer period of time and thus affect the helium production slightly (Hecht 1971).

We therefore conclude that the average effective values of ξ and C do not differ from unity by more than a factor of ~ 2, allowing us to use figures 4 and 5 to estimate the production of the other elements in any big-bang model. This also implies that the photon and neutrino temperatures are reasonably uniform, since we see no reason why large variations in ξ and C would result in $\langle \xi \rangle \simeq \langle C \rangle \simeq 1$. Thus the big-bang models which would appear most likely include the possibility of large inhomogeneity only in h_0, the only remaining free parameter. Incidentally, most homogeneous anisotropic models (Hawking and Tayler 1966; Thorne 1967) are also ruled out by both the observed present-day isotropy of the microwave background radiation and the presence of neutrino viscosity at temperatures $10^{11}\,° \gtrsim T \gtrsim 10^{10}\,°$ K (Matzner and Misner 1972).

We can now apply equation (9) to obtain a limit on the average value of h_0 corresponding to any average primordial deuterium abundance. Assuming $\xi \leq 2$ and $C \geq \frac{1}{2}$ and using the results in figures 4 and 5, we obtain

$$\langle h_0 \rangle \langle X(^2\mathrm{H}) \rangle \leq \mathrm{Max.}\ [h_0 X(^2\mathrm{H})] \simeq 3 \times 10^{-8}\,. \tag{10}$$

Now $\langle X(^2\mathrm{H}) \rangle$ represents an average over all baryons, and thus could include a substantial contribution from unobserved matter (such as that trapped in black holes). If this matter had values of $h_0 \gtrsim 10^{-3}$, it would be devoid of deuterium. As it is unlikely that the ratio of unobserved to observed matter is greater than 10^2, we can use the result in table 2 to put a lower limit $\langle X(^2\mathrm{H})\rangle \gtrsim 3 \times 10^{-7}$ on the pregalactic universal abundance of deuterium. Interpreting this as a primordial abundance then leads to the result $\langle h_0\rangle \lesssim 10^{-1}$. This rules out those chaotic universe models discussed above in which $\langle h_0\rangle \sim 10$ during nucleosynthesis.

In such universes which generate significant entropy during their expansion, h will decrease between the time of element formation and the present. Thus the most general relation between h_0 and the present baryon density is

$$\langle \rho_b(T = 2.7^\circ\ \mathrm{K})\rangle \leq 7.15 \times 10^{-27}\langle h_0\rangle\ \mathrm{g\ cm^{-3}}\,. \tag{11}$$

We also note that any subsequent processing can only reduce the primordial component of the deuterium abundance. Then using equations (9), (10), and (11), we obtain the general and important inequality

$$\langle \rho_b(T = 2.7^\circ\ \mathrm{K})\rangle\langle X(^2\mathrm{H})\rangle \leq 7.15 \times 10^{-27} \int X_i(h_0)h_0 f(h_0)dh_0 \leq 2 \times 10^{-34}\ \mathrm{g\ cm^{-3}}\,, \tag{12}$$

relating the present baryon mass density and the observed primordial deuterium abundance in any big-bang model which produces the observed helium. It should be mentioned that the averages can be taken over any volume comoving with respect to the background radiation within which there is no change in baryon number.

If the minimum estimated pregalactic deuterium abundance in table 2 represented a universal value, then the present universal baryon density could be at most 7×10^{-30} g cm^{-3}. Within the class of Friedmann models ($\Lambda = p = 0$), such a density could not close the universe unless the Hubble constant $H_0 < 60$ km s^{-1} Mpc^{-1}.

In any case, the basic point which emerges here is that the abundance of deuterium may represent a significant source of cosmological information. Black (1971) has reached some conclusions similar to (but less general than) these, under the assumption that the ^{3}He is also of primordial origin.

A comparison of figures 4 and 5 with table 2 indicates that the only element in addition to ^{2}H, ^{3}He, and ^{4}He which may have been produced in a big bang is ^{7}Li (although ^{6}Li and ^{11}B remain possibilities if new cross-section data emerge). As a simple example of a model which could produce the observed amount of ^{7}Li, consider a universe in which a fraction α of the mass in baryons had a local density $h_0^{(b)}T_9^3$ before pair annihilation, with the remainder of the baryon mass at a lower density $h_0^{(a)}T_9^3$. Then from equation (9) we obtain

$$\langle X_i\rangle = (1 - \alpha)X_i(h_0^{(a)}) + \alpha X_i(h_0^{(b)})\,, \tag{13}$$

$$\langle h_0\rangle = \left[\frac{(1-\alpha)}{h_0^{(a)}} + \frac{\alpha}{h_0^{(b)}}\right]^{-1}. \tag{14}$$

With $\alpha \ll 1$, we can obtain possible agreement with the abundances of ^{2}H, ^{3}He, and ^{4}He if $h_0^{(a)} = \langle h_0\rangle = 10^{-5.0}$–$10^{-4.5}$, and can also match the abundance of ^{7}Li if $h_0^{(b)} \gtrsim 10^{-3}$. Then $\alpha = \langle X_{^7\mathrm{Li}}\rangle / X_{^7\mathrm{Li}}(h_0^{(b)})$.

IV. CONCLUSIONS

In interpreting the results of these calculations of element synthesis, it is important to remember that the local abundances of the elements produced in any big-bang model of the universe depend essentially only on the neutron-proton ratio, expansion rate, and baryon density at that time when the local temperature has dropped to $\sim 10^9$ °K. Thus even though our computer calculations were carried out within the framework of the assumptions listed in § I, we can at least estimate the abundances produced in most other models in which the above three numbers are given by properly choosing our parameters C, ξ, and h_0.

The present evidence appears to argue strongly for a primordial source of at least ^{2}H and ^{4}He. We have thus taken the viewpoint that their pregalactic mass fractions were in the ranges $3 \times 10^{-5} \leq X(^2\mathrm{H}) \leq 5 \times 10^{-4}$ and $0.22 \leq X(^4\mathrm{He}) \leq 0.32$. Our calculations then indicate that the following conclusions can be drawn.

The sensitivity of the helium abundance to the expansion rate and the weak reactions which govern the neutron-proton ratio rules out many models which differ from the standard one, as seen in figure 6. In particular, the early universe must have had the following properties if one accepts the requirement that $C \sim \xi \sim 1$.

1) The maximum temperature was $\gtrsim 10^{11}$ ° K, so that the neutron and proton abundances could achieve near-equality through statistical equilibrium. Note that this range does include the "initial" universal temperature of $\sim 10^{12}$ ° K suggested by Hagedorn (1965).

2) The neutrinos, photons, and electron-position pairs (and thus most of the mass-energy density) were distributed approximately homogeneously and isotropically. If they were not, the nucleon weak-reaction rates would have had a different functional dependence on the local temperature (of the photons and pairs).

3) The ratio of electron-lepton number to baryon number was (and is) $\lesssim 10^4/h_0$ in absolute value. Otherwise the resulting electron neutrino degeneracy would have produced an appreciably different neutron-proton equilibrium abundance ratio (in addition to increasing the expansion rate), as shown by WFH.

4) The expansion rate had the same value as predicted by general relativity with the assumption that most of the mass was not in the form of unknown particles. This also precludes the existence of a density of short-wavelength (less than horizon size) gravitons much greater than the density of photons. This of course has implications for the detection of gravitational waves, since for the horizon size of $\sim ct$ predicted by general relativity for the early universe, the average present-day density of gravitational waves of wavelength $\lambda \lesssim 10^{21}$ cm would then be $\rho_g \lesssim \rho_\gamma = 4.5 \times 10^{-34}$ g cm^{-3}. In addition, the same limit can be put on the density of a randomly oriented intergalactic magnetic field, since it also affects helium production, and evolves in the same way as the photons and gravitons.

5) The possibility that electron-neutrinos decay (Bahcall, Cabibbo, and Yahil 1972) while the neutron-proton weak reactions are in equilibrium cannot be ruled out until the interaction of the decay products with neutrons and protons is specified. Note that if they did not affect the interconversion of the nucleons, the value of C would effectively be $\simeq \frac{1}{2}$, while ξ would be virtually unaffected (since the decay products would still contribute to the energy density in the same way as the electron-neutrinos, whose rest mass of $\lesssim 60$ eV would still be much less than their energy).

6) Those "little bangs" ($\lesssim$ galactic-size big bangs) which can produce heavy-element abundances that resemble the minimum level observed in our Galaxy (Wagoner 1969) appear to be unlikely. This is because the production of the observed heavy-element ratios requires neutron-proton equality to be maintained throughout nucleosynthesis, resulting in most of the processed matter emerging in the form of ^{4}He. It would appear unreasonable to expect that the subsequent mixing of this material with the surrounding gas would result in the same observed abundance of ^{4}He in all galaxies.

7) It would also appear very unlikely that matter-antimatter symmetric universes could produce the observed amount of ^{4}He, since the thermodynamic conditions are so different in such models. The annihilation reactions also tend to produce an increased disintegration of nuclei back into nucleons.

The survival of the deuterium produced in any model requires either a low baryon density or a very rapid expansion. Accepting the requirement from the helium abundance that $C \sim \xi \sim 1$, we found that also matching the deuterium abundance probably requires $\langle h_0 \rangle \lesssim 10^{-1}$. This limit leads to the following conclusions.

1. Those universes in which the number of photons per baryon increases by more than a factor of $\sim 10^4$ (due to such processes as dissipation of turbulence in chaotic models) between the nucleosynthesis epoch and the present are ruled out. This is because h is inversely proportional to the photon-baryon ratio, and $\langle h(\text{today}) \rangle \gtrsim 10^{-5}$.

2. Those "little bangs" (considered by WFH) which do produce roughly the observed amount of ^{4}He would also be ruled out because they produce no deuterium.

It is rather remarkable that the most naïve (standard) big bang produces just those elements for which no galactic production mechanism has been found to be quantitatively successful. For instance, the completely homogeneous model in which the present baryon density is 2×10^{-31} g cm^{-3} produces mass fractions $X(^2\text{H}) = 1 \times 10^{-4}$, $X(^3\text{He}) = 3 \times 10^{-5}$, and $X(^4\text{He}) = 0.23$. The incorporation of a small fraction of the mass in higher-density regions would also produce agreement with the observed amount of ^{7}Li.

It is of critical importance to discover new evidence relevant to our fundamental hypothesis that at least the observed ^{2}H and ^{4}He are of primordial origin. It would seem that the interstellar deuterium abundance could be the most informative, since one could at least conceive of an early generation of pulsating massive stars being responsible for the ^{4}He (Talbot and Arnett 1971), while it would be difficult to attribute an interstellar abundance $X(^2\text{H}) > 10^{-6}$ to anything other than the big bang. As we have shown, a crucial test of whether there indeed was a hot big bang would be the observational confirmation of the relation (12) between the present baryon density and deuterium abundance.

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APPENDIX

NUCLEAR REACTION RATES

Although all the reactions indicated in figure 1, and their inverses, have been included in the calculation, many of those that have been added to the original network of WFH (mostly in connection with other calculations) do not have an appreciable effect on the final abundances produced. On the other hand, recent data have necessitated revision of many of the important rates included in the extensive compilation of Wagoner (1969). Most of these have been very kindly supplied by W. A. Fowler, G. R. Caughlan, and B. A. Zimmerman, who are preparing a revision of their previous compilation (Fowler, Caughlan, and Zimmerman 1967). These new reaction rates are listed in table A1, which employs the notation of Wagoner (1969).

A special case is the neutron-proton weak reactions (1), which incorporate several improvements introduced to increase their accuracy, since they are of such importance in determining the helium abundance. The first is the use of the new value of the neutron mean life τ_n given by equation (5) with the parameter $C = 1$. The second improvement is a more accurate approximation to the temperature dependence of the integrals given in Appendix B of WFH (for $\phi_\nu = 0$). These power series in T_9^{-1} are accurate to better than 1.4 percent for $\lambda_w(n)$ at all T_9, and for $\lambda_w(p)$ at $T_9 > 2$ (at which point this rate is unimportant anyway).

The third improvement is the approximate inclusion of Coulomb corrections. This is done by noting that at temperatures $T_9 \sim 10$, where the magnitude of these rates is of most importance, the important reactions are $n + \nu_e \rightleftharpoons e^- + p$ and $n + e^+ \rightleftharpoons \bar{\nu}_e + p$. Only the first pair contains a Coulomb correction factor, which may be approximated by the expression $\exp(\pi/137\langle\beta\rangle)$, where $\langle\beta\rangle c$ is the average velocity of the electron. At low temperatures ($T_9 \lesssim 1$), on the other hand, the important reaction is $n \to \bar{\nu}_e + e^- + p$, which contains the full Coulomb correction. Thus in reality the correction factors $F_n(T_9 \lesssim 1) = F_p(T_9 \lesssim 1) = 1$, since we normalize the rates to that of the free neutron. However, it is of sufficient accuracy to merely set $F_n(T_9) = F_p(T_9) = F(T_9 \sim 10) = 1 - \frac{1}{2}(\pi/137\langle\beta\rangle) \simeq 0.98$. We neglect radiative and other such corrections, which should be less than ~ 1 percent (Blin-Stoyle and Freeman 1970).

TABLE A1

RATES OF IMPORTANT REACTIONS WHICH DIFFER FROM THOSE GIVEN BY WAGONER (1969)

(1) $n + \nu_e \to e^- + p, \quad n + e^+ \to \bar{\nu}_e + p, \quad n \to \bar{\nu}_e + e^- + p$:

$$\lambda_w(n) = F_n\tau_n^{-1}(27.512Z^{-5} + 36.492Z^{-4} + 11.108Z^{-3} - 6.382Z^{-2} + 0.565Z^{-1} + 1)\ \mathrm{sec}^{-1},$$

$$F_n = 0.98, \quad Z = 5.930T_9^{-1}.$$

$n + \nu_e \leftarrow e^- + p, \quad n + e^+ \leftarrow \bar{\nu}_e + p, \quad n \leftarrow \nu_e + e^- + p$:

$$\lambda_w(p) = F_p\tau_n^{-1}(27.617Z^{-5} + 34.181Z^{-4} + 18.059Z^{-3} - 16.229Z^{-2} + 5.252Z^{-1})\exp(-qZ)\ \mathrm{sec}^{-1},$$

$$F_p = F_n, \quad q = 2.531.$$

(2) $^2\mathrm{H} + p \to \gamma + {}^3\mathrm{He}$:

$$[^2\mathrm{H}\,p]_\gamma = 2.65 \times 10^3\rho_b T_9^{-2/3}(1 + 0.112T_9^{1/3} + 1.99T_9^{2/3} + 1.56T_9 + 0.162T_9^{4/3} + 0.324T_9^{5/3})\exp(-3.72T_9^{-1/3})\ \mathrm{sec}^{-1}.$$

(3) $^2\mathrm{H} + d \to n + {}^3\mathrm{He}$:

$$[^2\mathrm{H}\,d]_n = 3.97 \times 10^8\rho_b T_9^{-2/3}(1 + 0.098T_9^{1/3} + 0.876T_9^{2/3} + 0.600T_9 - 0.0405T_9^{4/3} - 0.0706T_9^{5/3})\exp(-4.26T_9^{-1/3}).$$

(4) $^2\mathrm{H} + d \to p + {}^3\mathrm{H}$:

$$[^2\mathrm{H}\,d]_p = 4.17 \times 10^8\rho_b T_9^{-2/3}(1 + 0.098T_9^{1/3} + 0.518T_9^{2/3} + 0.355T_9 - 0.0104T_9^{4/3} - 0.0181T_9^{5/3})\exp(-4.26T_9^{-1/3}).$$

(5) $^3\mathrm{H} + d \to n + {}^4\mathrm{He}$:

$$[^3\mathrm{H}\,d]_n = 8.09 \times 10^{10}\rho_b T_9^{-2/3}(1 + 0.092T_9^{1/3} + 1.80T_9^{2/3} + 1.16T_9 + 10.5T_9^{4/3} + 17.2T_9^{5/3})\exp[-4.52T_9^{-1/3} - (T_9/0.386)^2] + 1.21 \times 10^9\rho_b T_9^{-3/2} \times \exp(-0.89T_9^{-1}).$$

(6) $^3\mathrm{He} + d \to p + {}^4\mathrm{He}$:

$$[^3\mathrm{He}\,d]_p = 6.67 \times 10^{10}\rho_b T_9^{-2/3}(1 + 0.058T_9^{1/3} - 1.14T_9^{2/3} - 0.464T_9 + 3.08T_9^{4/3} + 3.18T_9^{5/3})\exp[-7.18T_9^{-1/3} - (T_9/1.373)^2] + 2.59 \times 10^9\rho_b T_9^{-3/2}\exp(-3.30T_9^{-1}).$$

(7) $^3\mathrm{He} + {}^3\mathrm{He} \to 2p + {}^4\mathrm{He}$:

$$[^3\mathrm{He}^3\mathrm{He}]_{2p} = 5.96 \times 10^{10}\rho_b T_9^{-2/3}(1 + 0.034T_9^{1/3} - 0.199T_9^{2/3} - 0.047T_9 + 0.0316T_9^{4/3} + 0.0191T_9^{5/3})\exp(-12.28T_9^{-1/3}).$$

(8) $^3\mathrm{He} + \alpha \to \gamma + {}^7\mathrm{Be}$:

$$[^3\mathrm{He}\,\alpha]_\gamma = 6.33 \times 10^6\rho_b T_9^{-2/3}(1 + 0.033T_9^{1/3} - 0.350T_9^{2/3} - 0.080T_9 + 0.0563T_9^{4/3} + 0.0325T_9^{5/3})\exp(-12.83T_9^{-1/3}).$$

(9) $^3\mathrm{He} + n \to p + {}^3\mathrm{H}$:

$$[^3\mathrm{He}\,n]_p = 7.07 \times 10^8\rho_b(1 - 0.150T_9^{1/2} + 0.098T_9) + 1.29 \times 10^{11}\rho_b T_9^{-3/2}\exp(-20.61T_9^{-1}).$$

(10) $^4\mathrm{He} + 2\alpha \to \gamma + {}^{12}\mathrm{C}$:

$$[^4\mathrm{He}\,2\alpha]_\gamma = \rho_b^2 T_9^{-3}[2.13 \times 10^{-8}\exp(-4.41T_9^{-1}) + 5.31 \times 10^{-7}\exp(-27.43T_9^{-1})].$$

(11) $^4\mathrm{He} + \alpha + n \to \gamma + {}^9\mathrm{Be}$:

$$[^4\mathrm{He}\,\alpha\,n]_\gamma = 2.59 \times 10^{-6}\rho_b^2 T_9^{-2}(1 + 0.344T_9)^{-1}\exp(-1.07T_9^{-1}).$$

(12) $^6\mathrm{Li} + p \to \alpha + {}^3\mathrm{He}$:

$$[^6\mathrm{Li}\,p]_\alpha = 3.46 \times 10^{10}\rho_b T_9^{-2/3}(1 + 0.050T_9^{1/3} - 0.048T_9^{2/3} - 0.0165T_9 + 0.0016T_9^{4/3} + 0.0014T_9^{5/3})\exp[-8.41T_9^{-1/3} - (T_9/6.13)^2].$$

(13) $^7\mathrm{Li} + p \to \alpha + {}^4\mathrm{He}$:

$$[^7\mathrm{Li}\,p]_\alpha = 7.66 \times 10^8\rho_b T_9^{-2/3}(1 + 0.049T_9^{1/3} + 0.443T_9^{2/3} + 0.152T_9 - 0.149T_9^{4/3} - 0.130T_9^{5/3})\exp[-8.47T_9^{-1/3} - (T_9/30.07)^2] + 1.07 \times 10^{10}\rho_b T_9^{-3/2}\exp(-30.44T_9^{-1}).$$

TABLE A1—*continued*

(14) $^7\text{Li} + \alpha \rightarrow \gamma + {}^{11}\text{B}$:
$$[^7\text{Li}\,\alpha]_\gamma = 3.52 \times 10^8 \rho_b T_9^{-2/3}(1 + 0.022 T_9^{1/3}) \exp[-19.16 T_9^{-1/3} - (T_9/0.268)^2] + \rho_b[1.51 \times 10^3 T_9^{-3/2} \exp(-2.96 T_9^{-1}) + 1.33 \times 10^4 \exp(-4.29 T_9^{-1})] .$$
(15) $^7\text{Be} + \alpha \rightarrow \gamma + {}^{11}\text{C}$:
$$[^7\text{Be}\,\alpha]_\gamma = 3.61 \times 10^7 \rho_b T_9^{-2/3}(1 + 0.018 T_9^{1/3} + 1.71 T_9^{2/3} + 0.215 T_9 + 2.88 T_9^{4/3} + 0.919 T_9^{5/3}) \exp[-23.21 T_9^{-1/3} - (T_9/0.654)^2] + \rho_b T_9^{-3/2}[7.27 \times 10^4 \exp(-6.50 T_9^{-1}) + 1.82 \times 10^5 \exp(-10.24 T_9^{-1})] .$$
(16) $^7\text{Be} + p \rightarrow \gamma + {}^8\text{B}$:
$$[^7\text{Be}\,p]_\gamma = 4.34 \times 10^5 \rho_b T_9^{-2/3}(1 + 0.041 T_9^{1/3}) \exp(-10.26 T_9^{-1/3}) + 3.30 \times 10^3 \rho_b T_9^{-3/2} \exp(-7.31 T_9^{-1}) .$$
(17) $^7\text{Be} + d \rightarrow p + \alpha + {}^4\text{He}$:
$$[^7\text{Be}\,d]_p = 1.07 \times 10^{12} \rho_b T_9^{-2/3} \exp(-12.43 T_9^{-1/3}) .$$
(18) $^7\text{Be} + n \rightarrow p + {}^7\text{Li}$:
$$[^7\text{Be}\,n]_p = 6.77 \times 10^9 \rho_b(1 - 0.903 T_9^{1/2} + 0.218 T_9) .$$
(19) $^{11}\text{B} + p \rightarrow \alpha + 2\,{}^4\text{He}$:
$$[^{11}\text{B}\,p]_\alpha = 1.90 \times 10^{11} \rho_b T_9^{-2/3}(1 + 0.034 T_9^{1/3} + 1.63 T_9^{2/3} + 0.394 T_9 + 3.12 T_9^{4/3} + 1.91 T_9^{5/3}) \exp[-12.10 T_9^{-1/3} - (T_9/2.02)^2] + \rho_b T_9^{-3/2}[7.40 \times 10^6 \exp(-1.73 T_9^{-1}) + 8.15 \times 10^9 \exp(-7.18 T_9^{-1}) + 8.49 \times 10^9 \exp(-14.76 T_9^{-1})] .$$
(20) $^{11}\text{B} + p \rightarrow \gamma + {}^{12}\text{C}$:
$$[^{11}\text{B}\,p]_\gamma = 6.58 \times 10^7 \rho_b T_9^{-2/3}(1 + 0.034 T_9^{1/3} + 2.22 T_9^{2/3} + 0.535 T_9 + 7.13 T_9^{4/3} + 4.38 T_9^{5/3}) \exp[-12.10 T_9^{-1/3} - (T_9/0.239)^2] + \rho_b T_9^{-3/2}[7.89 \times 10^3 \exp(-1.73 T_9^{-1}) + 4.57 \times 10^5 \exp(-7.18 T_9^{-1}) + 2.78 \times 10^6 \exp(-14.76 T_9^{-1})] .$$
(21) $^{11}\text{C} + p \rightarrow \gamma + {}^{12}\text{N}$:
$$^*[^{11}\text{C}\,p]_\gamma = 2.04 \times 10^7 \rho_b T_9^{-2/3} \exp(-13.66 T_9^{-1/3}) + 1.08 \times 10^5 \rho_b T_9^{-3/2} \exp(-5.70 T_9^{-1}) .$$
(22) $^{11}\text{C} + n \rightarrow p + {}^{11}\text{B}$:
$$[^{11}\text{C}\,n]_p = 1.69 \times 10^8 \rho_b(1 - 0.048 T_9^{1/2} + 0.010 T_9) .$$

* Cross-section not measured.

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