

An unsuccessful search for a planetary companion of Barnard's star* (BD + 4°3561)

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The parallax, proper motion, and acceleration in position of Barnard's Star have been determined from 241 Allegheny and Van Vleck Observatory photographic plates. The measurements were reduced by means of a variant of a rigorous overlap type method previously proposed by one of us. (Eichhorn 1960). Despite relatively small rms errors, several test failed to confirm van de Kamp's published orbit. The weighting system was carefully studied from the adjustment residuals. In essence, the weight of a Thaw or Van Vleck plate is proportional to the number of exposures.

INTRODUCTION

IN 1916, E. E. Barnard announced the discovery of a star, in the constellation of Ophiuchus, with a proper motion of more than ten arcsec per year. Trigonometric parallaxes soon showed that this object is the Sun's second nearest neighbor; it has since acquired the name of "Barnard's star." Its astrometric parameters have been investigated by many observers, notably van de Kamp (1969), who suggests the presence of a perturbation with a period of approximately 25 yr. This perturbation would most probably be caused by a planet-like companion. The period P , total mass $M+m$, and relative semimajor axis $a+A$ of the system are linked by the revised version of Kepler's third law

$$(m+M)P^2 = (A+a)^3 \quad (1)$$

while the masses of M and m of the primary and companion, respectively, and the semimajor axes A and a , respectively, of their orbits with respect to the system's center of gravity are connected by the equation

$$M/m = a/A. \quad (2)$$

In these two equations, only P is directly observable; A can be calculated from the perturbation, that is the amplitude of the periodic deviation from an accelerated great circle motion, and the parallax. The unknown quantities in these two equations are M , m , and a , and clearly show that one of these quantities must be known before a unique solution is theoretically possible. Spectrum and absolute magnitude provide a good clue to the value of M , so that m and a can be computed. Elimination of m from Eqs. (1) and (2) results in a fourth order equation for a , which has as one solution $a=-A$, leading to $m=-M$, which is physically meaningless. The reduced equation of the

third order has a negative discriminant and therefore exactly one real solution, which can be obtained from Cardano's formula and is given by

$$a = \left[\frac{A^3}{27} + \frac{MP^2}{2} + \frac{\sqrt{3}}{18} (4A^3MP^2 + 27M^2P^4)^{\frac{1}{2}} \right]^{\frac{1}{3}} + \left[\frac{A^3}{27} + \frac{MP^2}{2} - \frac{\sqrt{3}}{18} (4A^3MP^2 + 27M^2P^4)^{\frac{1}{2}} \right]^{\frac{1}{3}} - \frac{2A}{3}. \quad (3)$$

m is then easily obtained from Eq. (2) by inserting the value of a . With van de Kamp's (1969) values $A=0.0498$ A.U., $M=0.15\odot$, $P=25$ yr inserted into (3), one obtains $a=4.51$ A.U. and from (2), $m=0.0017\odot$, in good agreement with the values given by van de Kamp in his above quoted paper, which were obtained from an approximate formula.

The accuracy of van de Kamp's results may be estimated by assuming M , A , and P to be uncorrelated, so that

$$\sigma_a^2 = \left(\frac{\partial a}{\partial M} \right)^2 \sigma_M^2 + \left(\frac{\partial a}{\partial A} \right)^2 \sigma_A^2 + \left(\frac{\partial a}{\partial P} \right)^2 \sigma_P^2,$$

and an analogous formula for σ_m^2 , where σ_a^2 and σ_m^2 are the variances of a and m , respectively. From (1) and (2) we find

$$D^2 \sigma_m^2 = [3(a+A)^2(m+M)]^2 \sigma_A^2 + [3(a+A)^2 - P^2 m]^2 \sigma_M^2 + [2Pm(m+M)]^2 \sigma_P^2, \quad (4)$$

and

$$D^2 \sigma_a^2 = [3a(a+A)^2 - P^2 M]^2 \sigma_A^2 + [P^2(a+A)]^2 \sigma_M^2 + [2Pa(m+M)]^2 \sigma_P^2, \quad (5)$$

with $D=3a(a+A)^2+P^2m$.

From (4) and (5) we obtain from van de Kamp's data, assuming $\sigma_A=0''.003/0.55=0.0054$ A.U., $\sigma_P=2$ yr, and $\sigma_M=0.05\odot$, that $\sigma_a=0.56$ A.U. and $\sigma_m=0.00042\odot$.

* This paper is based on the dissertation of one of us (G. G.), submitted to the Graduate Faculty of Arts and Sciences in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

TABLE I. The relative positions, proper motions, and parallaxes of 19 stars in the region of Barnard's star on the system of the FK4.

No.	m_v	CI	Spec	π_{abs}	rms error π	R. A. 1950	rms error α	μ	rms error μ	Dec. 1950	rms error δ	μ'	rms error μ'	Series
1	9 ^m 2	0 ^m 7	F0	+0 ^o .004	0 ^o .0235	17 ^h 53 ^m 33 ^s .1038	10	+0 ^o .00144	4	4 ^o 32'29.885	15	+0 ^o .0009	6	R
2	8.0	1.2	K2	0.046	0.0125	17 53 43.7779	5	-0.00018	2	4 50 22.110	22	+0.0015	8	R
4	8.8	1.5	K5	0.000	0.0289	17 54 01.2351	13	-0.00040	5	4 13 49.633	29	-0.0123	9	R
5	10.5	0.4		-0.001	0.0016	17 54 15.4442	1	-0.00016	1	4 47 48.413	5	-0.0140	1	AV
6	11.0	0.2	A7	0.007	0.0024	17 54 16.1821	1	-0.00034	1	4 18 25.523	6	-0.0097	1	A
7	8.2	-0.1	A0V	0.002	0.0075	17 54 18.5190	4	-0.00063	4	4 22 56.854	30	-0.0091	10	R
8	8.8	0.2	B9	-0.001	0.0030	17 54 19.5430	2	-0.00013	1	4 50 31.651	8	-0.0171	3	V
10	10.2	0.6	G0V	0.006	0.0022	17 54 56.0472	1	-0.00076	1	4 36 24.266	7	+0.0249	1	AV
11	8.9	-0.1	B8V	0.003	0.0023	17 55 15.3486	2	-0.00037	1	4 26 39.143	7	-0.0130	2	V
12 ^a	9.5	1.7	dM5	0.549	0.0025	17 55 22.7088	2	-0.05026	1	4 33 15.448	9	10.2735	1	AV
13	11.3	0.1	A3V	0.007	0.0036	17 55 23.6763	1	-0.00033	1	4 42 35.279	7	-0.0182	1	A
14 ^b	10.0	0.6		0.001	0.0312	17 55 23.6436	16	-0.00911	6	4 07 08.717	7	-0.1983	5	R
15	10.5	0.2	A2	-0.001	0.0017	17 55 28.1116	1	-0.00017	1	4 15 06.178	6	-0.0025	1	AV
16	10.6	2.2	M1III	+0.004	0.0041	17 55 53.2154	3	-0.00054	2	4 36 22.463	10	-0.0120	4	V
17 ^c	9.6	1.2	dK5	0.019	0.0029	17 55 56.0627	2	-0.00449	1	4 27 55.974	11	-0.0544	2	AV
18	9.7	0.5	F2IV	0.004	0.0021	17 56 11.9164	1	-0.00088	1	4 46 36.484	6	-0.0502	1	AV
20	11.2	0.2	A2V	0.005	0.0028	17 56 27.6024	1	-0.00010	1	4 35 45.383	6	-0.0149	1	A
21	10.6	1.1	K0	-0.002	0.0041	17 56 59.8711	3	-0.00046	2	4 35 55.102	10	-0.0103	3	V
22	9.5	1.9	K2	0.006	0.0980	17 57 07.5042	35	-0.00044	16	4 48 28.794	27	-0.0193	10	R

^a Barnard's Star.^b Ross 539.^c Kuiper 84.

The least significant unit of each rms error coincides with the least significant unit of the corresponding quantity.

Thus it appears that the values derived by van de Kamp are affected by rather moderate uncertainties.

One must, however, consider that the formal uncertainty of van de Kamp's results as shown by the figures above probably does not give a true picture of the actual uncertainties of his results. Data obtained from long focus astrometric investigations are notorious for errors caused by hidden, unrecognized sources which are revealed only by comparison of the results obtained by the analysis of material from at least two independent sources. An examination by Wagman (unpublished) of the residuals of Barnard's star based on a preliminary Allegheny solution showed no convincing argument for a significant perturbation. Thus, we decided to measure and reduce the plates in such a way that virtually no reasonable additional effort—by contemporary standards—could yield results of higher accuracy from the plates which were used in the present investigation.

I. MATERIALS

Two hundred and forty-one plates carrying a total of 610 exposures were measured for the present investigation. Eighty of these (210 exposures) had been obtained on the 20-in. refractor of the Van Vleck Observatory (V). Thirty-two of the V plates had been exposed in the period from 1923 to 1928, 46 from 1943 to 1954, and two in 1971. The remaining 161 one plates (400 exposures) were obtained on the 30-in. Thaw refractor of the Allegheny Observatory (A) in a series spanning the periods from 1916 to 1918 and 1943 to 1971, with isolated plates taken in 1934 and 1937. A list of plates, showing the date of exposure and the number of exposures was given by one of us in a more

detailed description of this work (Gatewood 1972 Table VII).

It was impracticable to choose the same set of reference stars for both V and A series of plates. This is so because the sizes of the fields of view are different and also because differences in the relative image blackening occur since the Allegheny telescope is a photographic refractor, and the Van Vleck telescope is a visual one. A different set of reference stars was therefore adopted for the V and A series. The positions and proper motions on the FK4 system of these stars are contained in Table I. They were newly derived as part of the results obtained in the course of the work described in this paper. The reductions will be described below.

The rectangular coordinates of the images were measured at the Strand Automatic Measuring Machine (SAMM) of the U.S. Naval Observatory, which was described by Strand (1971). He and several other investigators have found a significant increase in the accuracy of results of photographic astrometric data when the coordinates of the images were measured by means of a measuring process from which human judgment is excluded. Other obvious advantages of the SAMM are the speed and the ease with which the measuring can be done. The plates were measured in the direct and the reverse position, and a comparison of the results showed the measuring rms error of a mean of the direct and reverse measurement of a coordinate of one image to be 1.8 μ .

Seven of the A plates ("reference plates"), uniformly distributed over the time span covered by the series, were chosen to establish a system of positions and

proper motions of the reference stars. These plates were measured in direct and reverse positions on the Mann 422 measuring machine of the Allegheny Observatory. The rms error of a measured coordinate of one image in this case turned out to be 2.3μ . The ratio of the rms errors of automatically and manually measured coordinates—on the same plate material—is thus close to 3:4 corresponding to a ratio in weight of about 16:9, in good agreement with the numbers reported earlier by Riddle (1970).

II. REFERENCE SYSTEM

We had decided that this paper should also give the positions and proper motions of a sufficiently large number of stars in the neighborhood of Barnard's star. This was done for the purpose of enabling those investigators who work with another telescope to combine their results with ours in a manner which was, for example, used in an investigation of the binary ADS 11632 by Eichhorn and Alden (1960). Although for this intended purpose it is sufficient to give accurate relative positions of the set of stars which might serve as reference stars, we decided nevertheless to derive positions and proper motions on a system as close to that of the FK4 as possible. Since Barnard's star appears as $11^m.3$ on the (photographic) A and as $9^m.5$ on the (visual) V plates, the reference stars had to be chosen over a sufficiently wide magnitude interval. The size of the field of the Van Vleck refractor (0.9° diameter) and the fact that it was then judged unwise to measure a field of more than about 135 mm sq on the USNO Strand SAMM, and the fact that the SAMM cannot measure an image whose diameter is over 250μ , also had an influence on the selection of the reference stars. The positions of 11 of the 19 stars measured on the reference plates and listed in Table I are also listed in the AGK3, and of these 11, the positions of eight are listed in the SAOC. Both the AGK3 and SAOC catalogs are ostensibly on the FK4 system. For those stars which are listed in both the AGK3 and SAOC, positions and proper motions from both catalogs were combined to obtain reference positions for the reduction of the measurements made on the seven reference plates. There are significant systematic differences between the positions and proper motions of the eight stars common to the AGK3 and the SAOC. These, in the sense SAOC minus AGK3, are $\alpha = -0.009$, $\mu = +0.0009$ per year, $\delta = -0''.18$, and $m = -0.003$ per year. The systematic differences in positions are different than those derived by Scott and Smith (1971) between the AGK3R and SAOC, both of which are ostensibly on the FK4 system. For this region their values are, in the sense SAOC minus AGK3R, $\alpha = -0.024$ and $-0''.08$ (1950). However, the accuracy of the process of "putting a catalog on a fundamental system" has been considerably overestimated in the past, as is discussed in some detail

by Eichhorn (1974). Since there was no obvious reason why the system of the AGK3 or that of the SAOC should have been given preference, we adopted the straight means of the AGK3 and the SAOC values of the astrometric parameters for these eight stars as standard positions and proper motions. The mean of the astrometric data of these stars from the two catalogs seems likely to be closer to the FK4 system than those from either catalog.

The data which resulted from the combination of the AGK3 and the SAOC were used to compute reference positions for all stars measured on the A plates. The reductions were based on an affine transformation between standard and measured coordinates, that is the conventional six linear plate constant model. The small magnitude term previously detected on Thaw plates (Gatewood and Fredrick 1971) was neglected because of its small effect and the nonhomogeneous nature of the reference material.

The standard coordinates of the 19 reference stars thus computed from the measurements on each of the seven plates were then converted to right ascension and declinations. These were subjected to a linear adjustment from which first approximations for the zero epoch (1950) positions and proper motions of the 19 reference stars were found. Their proper motions are small enough (with the exception of that of Barnard's star) so that the spherical coordinates may be regarded as linear functions of time within the time interval in question.

Following the first step of the procedures called for in the plate overlap method (Eichhorn 1960), the data thus obtained were used to compute positions of all but three of the 19 stars. These three are Nos. 12, 14, 17, all of which have measurable parallaxes. The others were used as reference positions for finding second approximations to the plate constants of the reference plates. These plate constants were then used, as before, to compute new approximations to the star's positions and proper motions at epoch 1950. After this point, no further iterations were made.

III. REDUCTIONS OF THE MEASUREMENTS

The magnitudes communicated in Table I were derived from measurements of the diameters of their images on three each of the A and the V plates. The rms error of the magnitudes is $0^m.2$, that of the color indices, $0^m.3$.

Stars Nos. 5, 6, 10, 13, 15, 18, and 20 from Table I were chosen as reference stars for the A series. The magnitude range of this set of reference stars is $1^m.2$; the average photographic magnitude is $0^m.3$ brighter than that of Barnard's star. The positions and proper motions found from the reference plates were used to compute reference positions for each plate epoch. Again, a simple transformation was assumed to properly model the conversion of measured to standard coor-

dinates. Different plate parameters were determined for every exposure, no constraints were imposed on the reduction parameters for even those exposures that were placed on the same piece of glass.

The solutions were iterated as suggested by Eichhorn and Jefferys (1971); the situation here is very similar to their Case 1. The process had converged with the third iteration. The most noteworthy difference between the procedure followed here and that suggested by Eichhorn and Jefferys is that we solved for the parallaxes of the reference stars and the parallax star simultaneously.

It is clear that this process will lead to parallaxes which are relative to the average of all parallaxes determined simultaneously. The reason for this is that the parallax of each star involved in the reductions could be increased or decreased by any amount without thereby causing a measurable effect in the relative positions. The problem of determining the parallaxes of *all* stars simultaneously (without even one known reference star parallax) therefore leads to a system of equations which is singular. As pointed out in Eichhorn and Jefferys' paper, this does not prevent iterations of the type carried out by us from converging.

The average of the parallaxes thus found should converge to the average of the parallaxes enforced on the reference positions, but these were, of course, computed without considering any effects of annual parallax, so that the average of the parallaxes of the reference stars, which we found in the course of the iterations, is equal to zero. Thus, as mentioned in Sec. V, we have corrected the parallaxes found by us by the additive constant $+0''.003$ to make them absolute.

As we shall report in Sec. IV, detailed investigations did not suggest that the iterations should be repeated with unequal weights and the results of the last iteration were therefore adopted as the best available ones from the A plates.

Two hundred ten exposures were available on 80 V plates. Stars 5, 8, 10, 11, 15, 16, 18, and 21 from Table I served as reference stars. The visual magnitudes of these stars have a range of $1^m.8$ and are on the average $0^m.5$ brighter than Barnard's star. The measurements on the V plates were reduced in almost the same way as those on the A plates, except that we allowed for linear magnitude equation and linear coma (proportional to magnitude and distance from the plate center) in the model for the relationship between standard and measured coordinates on the V plates. During a preliminary reduction, no restriction was put on the magnitude of the coma coefficients on the various plates. An investigation of the values obtained showed that their scatter around their average value of $-0.08 \mu/\text{mag mm}$ was no larger than could have been expected on the basis of their formal rms errors. The three subsequent solutions, which were required before convergence was reached, were performed with mea-

sured coordinates that were corrected to remove the coma effect, using the above mentioned value.

IV. INVESTIGATION OF THE WEIGHTS

When the plates were measured, the quality of the images was judged by assigning a quality number from 2 (poor) to 5 (excellent) on the basis of their visual appearance. It is an established custom in long focus astrometry to use these judgments as a basis for assigning weights to all measurements on images obtained during the same exposure. This procedure is backed by years of experience, cf. van de Kamp (1967). However, it was felt that since we had obtained our image coordinates at the SAMM, a weighing system which was established for visual measurements made on hand operated machines might not hold.

From the reductions, we obtain an estimate for the rms error of a measurement for any one exposure, separately in the x and the y coordinate, from the adjustment residuals of the reference star coordinates.

If there is a significant correlation between the rms errors in x and y which refer to the same exposure, we may conclude that different exposures do indeed produce images that yield coordinates of different quality. The coefficient of correlation was only 0.15 for the A plates and 0.37 for the V plates, from which we concluded that the rms errors of the measurements as established from the definitive adjustment residuals are either not a reliable indicator of weight, or that the coordinates obtained from measuring the various exposures should all be used with the same weight. We decided to adopt the latter alternative.

It was also somewhat surprising that the correlations of the rms errors with the above described numbers which characterize the image qualities, were insignificantly small, although they had the expected sign; for the A plates, -0.15 in x and -0.04 in y ; for the V plates, -0.14 in x and -0.12 in y .

We also investigated the correlations of the quality numbers and then the rms errors (which were computed without including the coordinate residual of Barnard's star), with the absolute value of the coordinate residual of Barnard's star, both in x and y and both for the A and V plates. These correlations had the expected sign in seven out of eight cases, but were insignificantly small.

It is a standard procedure in astrometry to place several exposures of the same plate, with a space of one or two millimeters between the images of the same star. Evidence accumulated so far indicates that the errors of the measurements of the coordinates of the same stars obtained from neighboring images on the same plate are somewhat correlated. Therefore, the weight of the average of n coordinates measured from neighboring images of the same star is usually taken to be less than n times the weight of a measured coordinate of an isolated exposure. According to van de Kamp

TABLE II. Weight as a function of number of images.

n	W_A	W_V
1	1.06	0.53
2	2.08	0.94
3	2.87	2.12
4	2.84	1.79
5	5.18	3.08

(1967), the average of the measurements on four good images on one plate is only two times the weight of the measurement on one good image.

In order to decide how the weight of an exposure varies depending on how many exposures the plate carries, we proceeded as follows. The averages of the adjustment residuals, taken over all images of the same star on the same plate, were formed for the x and the y coordinates and grouped according to the number of exposures on any particular plate. The variances of these averages should be inversely proportional to the number of images that were combined into the averages. (The x and y residuals were combined for this purpose.) For more easily interpretable numbers, we have converted the variances to weights, assuming that the rms error corresponding to unit weight is $0''.03$, which is indeed very close to the rms error of a coordinate obtained from measuring a single image on an A plate. Table II gives the results, for the A plates as well as for the V plates. From a weighted adjustment of these data, with the weight being proportional to the total number of exposures on which each is based, we find

$$W_A(n) = (0.97 \pm 0.04) W_A(1)$$

and

$$W_V(n) = (0.94 \pm 0.08) W_V(1).$$

These slopes, given with their rms errors are indistinguishable from the value 1, which they would have if the weights were precisely proportional to the number of images.

Based on these investigations, we decided that the attribution of different weights to different exposures would not be justified within the separate solutions for the material from the A and the V plates. The results of the reductions, described in Sec. III which were carried out without the attribution of any weights, were therefore regarded as definitive.

It is quite remarkable that the weights show this behavior which is in significant contrast with the experience of other investigators. In particular, the linear relationship between weight and number of neighboring exposures has important repercussions on the observation technique, since it suggests that one plate with five exposures will yield as much information as five plates with one exposure each, which are more expensive and more cumbersome to obtain and measure.

At this time, with the somewhat scanty material at hand, any attempt to explain the discrepancy between

our results and those of other investigators would be speculative. Further studies are planned as more series of Allegheny plates are measured at the SAMM; these results will be reported on later.

V. RESULTS AND CONCLUSIONS

Table I lists our definitive astrometric parameters of the stars involved in this investigation referred to orientation and epoch 1950. Images of the stars 1, 2, 4, 7, 14, and 22 were measured on the reference plates only. The values for the rms errors are in units of the least significant figure of the quantity to which they refer. These rms errors are strictly internal.

The external rms errors of the astrometric parameters in Table I are hard to assess. The systematic difference against the FK4 system cannot be established from the data at hand. In order to have an estimate of the accuracy of the listed rms errors, we have not combined the A plate results with the V plate results in an overlap type reduction. Table III gives the systematic differences between the A and the V data for the stars common to both systems, as well as the dispersion of the random differences between the estimators for the same numbers obtained from the A plate and V plate reductions. These numbers indicate very good agreement of the values of parallax and proper motion found from the Allegheny and Van Vleck series. We conclude that the parallaxes are at least as accurate as indicated by their rms errors while the rms error of the proper motions may be underestimated by a factor of 1.4. The external errors of the positions, on the other hand, are on the average about 2.7 times larger than the computed internal ones. One possible contributing factor to this discrepancy could be the following: while the simple models for the relationship between the plate coordinates of a star's image and its standard coordinates are quite satisfactory for differential work, such as parallax and proper motion studies, they are not sufficient to account for all of the metric field characteristics of one or both of the telescopes involved.

As long as these unmodeled imaging characteristics remain constant, their only effect is to decrease the accuracy of the positions found from a study such as this. However if they should change during the time interval during which a long series of plates was exposed, they would produce an effect which might be interpreted as a nonlinear trajectory of the parallax star.

In the present study, the stability of the two telescopes involved was ascertained from the agreement of the data derived from material obtained at them.

TABLE III. Comparison of A and V plate reduction data.

	$\Delta\alpha$	$\Delta\mu$	$\Delta\delta$	$\Delta\mu'$	$\Delta\pi$
A-V	$-0''.013$	$-0''.00016$	$-0''.003$	$-0''.00036$	$-0''.0015$
$\sigma(A-V)$	0.024	0.00077	0.031	0.00057	0.0037

TABLE IV. Parallax, proper motion components and secular acceleration terms for Barnard's star. (Referred to the epoch and orientation of 1950.0)

	π	μ	$\Delta\mu$	μ'	$\Delta\mu'$
A	0".5429	-0".75174	-0".000088	10".27349	0".001210
rms	35	15	12	15	14
error					
V	0.5461	-0.75081	-0.000073	10.27350	0.001292
rms	35	36	35	49	0
error					37

Soon it should be possible to study the imaging characteristic of a telescope as a function of time against an astrometric standard region, which is being prepared by one of us. (See Gatewood and Fredrick 1971.)

To determine the correction of relative to absolute parallax, the distance modulus of each reference star was computed from its spectral type and magnitude. This was then compared with its relative parallax. From these comparisons, an average correction of 0".003 was found and applied to the relative parallaxes to yield the absolute parallaxes listed in Table I. Our star 17, BD+40°3562, is a known double star (Kuiper 84) with a parallax of about 0".020. This investigation shows a periodic pattern in the residuals. One of us (GG) plans the publication of the data pertaining to this star in a separate paper.

Some of the astrometric parameters, namely parallax, proper motion and acceleration term of Barnard's star are listed, with their internal rms errors, in Table IV. These values are consistent with each other within their rms errors. Even without consulting the rms errors, it is obvious that the A data are more accurate than the V data if we consider that we must have $\mu'/\mu = \Delta\mu'/\Delta\mu$. This relationship is satisfied for the A data to three figures, for the V data to only one figure. It is somewhat conspicuous that the rms error of the parallax is the same for the results of the A and the V series. This is explained by the fact that all V plates were taken at large parallax factors, while the A plates were taken without paying special attention to the parallax factor.

From the well known first order relationship $\Delta\mu = -2.05 \cdot 10^{-6} V\mu\pi$ (where the parallax π is given in seconds of arc, the radial velocity V in kilometers per second, the proper motion μ in seconds of arc per year and the acceleration $\Delta\mu$ in units of seconds of arc per year per year) which is valid for μ and $\Delta\mu$ as well as for μ' and $\Delta\mu'$, we can calculate the radial velocity V . From the μ data of the A solution and the parallax we find $V = -106 \pm 1$ km/sec, well in agreement with the observed values which show the rather wide range from -40 to -128 km/sec (Wilson 1953).

From a study of the rms errors of the estimators in Table IV, it is evident that a comprehensive investigation of the proper motion components of Barnard's star as functions of time could yield an "acceleration parallax" which is more accurate than the available

mean of the trigonometric parallaxes, once a good value of the star's radial velocity becomes available. Since the proper motions always refer to a frame of reference stars, the positions and especially the proper motions of these reference stars would have to be known on a uniform and well defined system before the data communicated by the various observers could be meaningfully combined in a comprehensive investigation of the acceleration terms.

This again points out the need for publishing the positions (and proper motions) of the reference stars in astrometric investigations, as is done in this paper. The coordinates of the images of the reference stars have to be measured accurately anyway, and the computation of these stars' positions and, when indicated, proper motions and even relative parallaxes, is only a matter of writing the appropriate computer program. Broadly speaking, the value of positions for investigations into stellar kinematics increases with the square of the time elapsed since the epoch. The value of a published proper motion, on the other hand increases its value for the determination of an acceleration term only linearly with time. It is therefore difficult to appreciate the reasons of some authors for not publishing the relevant positions but only the immediate target results of their investigations, such as the parallax, proper motion, and acceleration of the parallax star.

We now turn to the planetary companion hypothesized by van de Kamp (1969). For some reason, almost all of the perturbation observed by van de Kamp occurs in the x positions, thus we will concern ourselves only with these. Figure 1 is a plot of the normal points formed from the average of our positions within each one-tenth phase of the proposed orbit. The dashed line represents the positions computed from this orbit while the straight line represents the motion of an unperturbed star. The weight of a normal point is represented by its size. The range in weights of each size dot is, small: 12-24, medium: 35-47, large: 73-127, the 610 individual observations may be obtained in punched card form from the author.

Using the individual observations as was the practice during all computations, we next computed by standard techniques the semimajor axis of the photocentric orbit α . Assuming the published orbital elements, we find $\alpha = 0".004 \pm 0".003$. The value given by van de Kamp is $0".028 \pm 0".003$. Thus the weight of this study, though based on only 241 plates, is equal to the one cited above.

The coefficient of correlation of the variation of our positions from a nonperturbed motion to those variations predicted by van de Kamp is 0.06. We also note that if we subtract the predicted variation from each observation, we increase, not decrease, the sum of the squares of the residuals.

Finally, an attempt was made to improve the published orbit by a standard differential correction

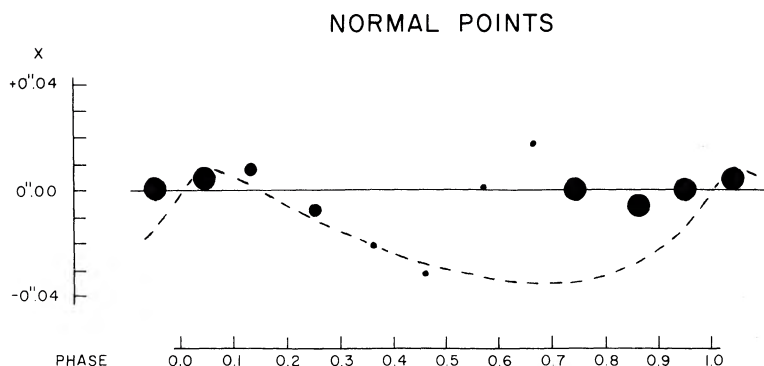


FIG. 1. Normal points.

technique. However, the elements changed radically on the first step and by the second iteration e had exceeded 1.0. Thus we conclude, with disappointment, that our observations fail to confirm the existence of a planetary companion to Barnard's star.

The following are speculations, perhaps idle, on the failure of our data to confirm the perturbation found by van de Kamp.

van de Kamp's reduction technique is strictly differential and based on the use of dependences. Some of the various disadvantages of using dependences in work of this type were previously pointed out by Eichhorn and Jefferys (1971). In addition to those, there are the following points to consider in the use of stars with large proper motions that were observed over a long period of time.

We have already mentioned that the metric properties of the optical system of any one telescope may and almost certainly will not remain precisely constant over an extended period of time. This can be checked only by measuring stars that are fairly well distributed over the field. A sufficient number of reference stars which match the target star's magnitude and color may not be available and thus introduce another unavoidable factor of uncertainty into the results.

It is also remarkable that van de Kamp's perturbation is mostly in the x coordinate, while the proper motion is mostly in declination. The optical system certainly does not follow the geometry of the gnomonic projection in complete rigor. This means that the image of a great circle in the sky will only very nearly be a straight line. Since the motion of Barnard's star is mainly "up and down," the deviation of the actual curved path of the star's image from the implicitly assumed straight-line path would then produce spurious effects in the x motion, either in the form of an acceleration or possibly a periodic motion.

Whatever perturbation there may be, is on the verge of significance. There are perhaps similar instances in the past, when astrometric investigations have suggested the reality of actually unreal things. Van Maanen's discovery of the rotation of a galaxy and the

measurements of the parallaxes of several stars before Bessel's are examples of this.

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