Hu 1335: The 87-year orbit by Knipe (1962) gave a poor representation, and is ruled out by Worley's measurement. Despite the large changes in the elements, the dynamical parallax and the masses are unchanged.

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Comets and nongravitational forces. V.

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The problem of the variation of the nongravitational forces with heliocentric distance is considered. Calculations are presented for nine short-period and five long-period comets, the variation of the forces with distance being determined from a law based on the vaporization rate of water snow. Results obtained earlier are modified to conform to the new law, and the relative values of the forces on different comets are interpreted. The effect of emissivity of the cometary nucleus on the vaporization rate is also discussed. Particular attention is paid to the matter of deriving "original" and "future" orbits of long-period comets when nongravitational forces are taken into account.

ONE of the shortcomings of our calculations concerning the nongravitational forces on comets (Paper II, Marsden 1969; Paper III, Marsden 1970; Paper IV, Marsden and Sekanina 1971; see also Yeomans 1971) is that the expression adopted for the variation of the forces with heliocentric distance r was selected essentially at random. In our defense, we should point out that, when we embarked on the project, it was not our desire to favor any particular cometary model: we merely assumed that, since the effects of the nongravitational forces on the motion of a given comet seemed generally to be very regular, it was reasonable to approximate the forces by some continuous function. The numerical results for a number of short-period comets then showed that the influence of the forces diminished very substantially with increasing r.

We expressed the nongravitational acceleration components (Paper II) as

$$F_i = G_i f(r) \quad (i=1, 2, 3), \tag{1}$$

where $i=1$ for radial component (positive outward from the Sun), $i=2$ for the transverse component (in the orbit plane and positive toward the direction 90° ahead of the comet in true anomaly), and $i=3$ for the component perpendicular to the orbit plane. The G_i were written

$$G_i = A_i \exp(-B_i \tau) \quad (i=1, 2, 3), \tag{2}$$

where the A_i and B_i are constants, and τ is the time from some initial epoch. (It was sufficient to assume generally that $A_3 = B_1 = 0$.) We adopted

$$f(r) = r^{-3} \exp(-r^2/2), \tag{3}$$

r being measured in astronomical units, for this represented the observations at least as well as any of the other possibilities considered. For the unit of mass, we took the solar mass, and although the unit of time was generally 40 days, τ was measured in units of 10^4 days.

While the results obtained do not *prove*, with full mathematical rigor, that the sandbank model is incorrect, we feel that they do speak considerably in favor of the icy-conglomerate model; and taken in conjunction with purely physical reasoning, such as the large gas-to-dust ratio observed in comets (Whipple 1961)—very graphically portrayed in the tremendous ultraviolet hydrogen halos observed surrounding the bright comets of 1970 (e.g., Code *et al.* 1972)—they suggest that it is virtually certain that a cometary nucleus must bear some resemblance to an icy conglomerate, and that the ice is basically water ice (Delsemme 1971).

Accordingly, it is important to consider the question of the r-variation of the nongravitational forces (i.e., forces of reaction to the output of matter from a comet) in terms of the icy-conglomerate model. Soon after the model was introduced (Whipple 1950), Delsemme and Swings (1952) suggested the existence of solid hydrates in comets. The theory of sublimation of cometary ices was subsequently developed by Squires and Beard (1961), Huebner (1965), and Delsemme (1966). Delsemme and Miller (1971) computed the vaporization heat of methane clathrates and found it practically identical with that of water ice. While clathrates are chemically more complex than water, their dynamical effects are essentially the same.

Since the incident solar radiation is peaked at around 5000 Å, the total energy absorbed by the surface of a cometary nucleus depends primarily on the absorptivity κ_{vis} in the visible. On the other hand, since the temperature of a cometary nucleus at modest heliocentric distances is typically several tens to several hundreds of degrees Kelvin, the nucleus radiates mainly in the infrared (between 10 and 100 μ), and the total energy radiated depends primarily on the nuclear emissivity ϵ_{inf} in the infrared. Delsemme and Miller (1971) have constructed curves giving the rate of vaporization of various volatile materials from a rapidly rotating cometary nucleus as a function of r. They assumed that the Bond albedo was 0.1, both for absorption in the visible (i.e., $\kappa_{\text{vis}}=0.9$) and for reradiation in the infrared (i.e., $\epsilon_{\text{inf}}=0.9$). The second author of the present paper suggested that the vaporization flux Z for water snow can be expressed empirically by

$$Z=Z_0 g(r), \tag{4}$$

where

$$g(r)=\alpha\left(\frac{r}{r_0}\right)^{-m}\left[1+\left(\frac{r}{r_0}\right)^{n}\right]^{-k}. \tag{5}$$

The factor α is chosen such that $g(1)=1$, and hence Z_0 is the vaporization flux at heliocentric distance 1 A.U. At our request, Delsemme and Delsemme (1971) have compared Eq. (4) with the vaporization curves and found $Z_0=3\times10^{17}$ molecules cm^{-2} sec^{-1}, while for the remaining constants in Eq. (5) they gave $r_0=2.808$ A.U., $m=2.15$, $n=5.093$, and $k=4.6142$ (or $nk=23.5$); then the normalizing constant $\alpha=0.1113$. For $0.1<r<4.0$ A.U., Eq. (5) represents the actual vaporization-equilibrium data within $\pm5\%$.

Delsemme (1973) has pointed out that $g(r)$, or more specifically, the Delsemme–Miller curve for water snow, corresponds closely, in the case of P/Schwassmann-Wachmann 2, to $f(r)$; in fact, if multiplied by appropriate constants, it falls near the mean of $f(r)$ and several rather similar expressions that had been found to give satisfactory residuals for this comet; see our Paper II. The corresponding vaporization curves for CO, CO_2, CH_4, and NH_3, on the other hand, essentially follow an inverse-square law (out to beyond the comet's aphelion distance, at least), and this gave completely unacceptable residuals. As it happens, the ratio $f(r)/g(r)$ varies by only some $\pm15\%$ for $1.8<r<4.5$ A.U., and hence for most of the short-period comets it matters little whether one uses $f(r)$ or $g(r)$ in Eq. (1). For small r, the function $g(r)$ approximates an inverse square, and the ratio $f(r)/g(r)$ becomes significantly different (it is some 10 times greater for $0.4<r<0.6$ A.U. than for $1.8<r<4.5$ A.U.). The principal conclusions from our earlier calculations using $f(r)$ for comets of small perihelion distance (P/Encke, P/Honda–Mrkos–Pajdušáková, P/Brorsen) are not completely inconsistent with what would follow if $g(r)$ had been used, however, for such comets spend relatively little time near the Sun. We should also point out that the calculations basically determine the transverse component A_2 of the nongravitational force; this involves the lag angle between the subsolar meridian and the point of effective mass ejection; we have found that this lag angle seems generally to be small, so it may well be valid to utilize $g(r)$ in Eq. (1), but the question of dependence of the lag angle on r requires further investigation.

One very good reason for now adopting $g(r)$ instead of $f(r)$ is that it enables us to make a more meaningful comparison of the nongravitational parameters for different comets. If the icy-conglomerate model is accepted, the nongravitational parameters represent the relative mass-loss rates. A difficulty arises because the anisotropy factor λ (cf. Sekanina 1969; Paper IV, Table XI) is usually unknown, but it is obviously far less satisfactory to compare different values of A_1 and A_2 using $f(r)$, an expression selected largely by chance; we have previously also modified the $f(r)$ figures so that they "correspond roughly" to an inverse-square law, but since the forces are now known generally to be completely incompatible with an inverse-square law, this procedure is clearly unacceptable.

I. COMPUTATIONS USING THE $g(r)$ LAW

In this section, new calculations of nongravitational parameters are presented, $g(r)$ and the associated constants replacing $f(r)$ in Eq. (1) and the equations of motion. Equation (2) has proved to be of somewhat limited use, and the present solutions have been made over short enough time spans that the $G_i=A_i (i=1, 2)$ are constant. Values of B_2 can be derived, if desired, by comparison of the values of A_2 fitted to different observational intervals. A time unit of 10^4 days is used in the definition of A_1 and A_2, the maximum values of which then become of order unity; if the unit of time is taken as 1 day, these values should be divided by 10^8. Results are discussed here for nine short-period comets, but in order to conserve space the nongravitational parameters (and their mean errors) are listed together in Table I, the comets being mentioned in order (more or less) of decreasing values of $|A_2|$. The orbital elements are omitted completely; these can generally be found in the new *Catalogue of Cometary Orbits* (Marsden 1972a), as well as in the

TABLE I. Nongravitational parameters for short-period comets (r_0=2.808 A.U.).

				$f(r)$ law		$g(r)$ law			
Comet	q	e	Interval	A_1	A_2	A_1	A_2	B_2	Epoch
Brooks 2	2.0	0.5	1889–1904			+3.61±0.38	−0.3269±0.0010		
	2.0	0.5	1896–1910			+2.93±0.38	−0.2863±0.0026		
	1.9	0.5	1925–1940			−0.54±0.43	−0.1893±0.0006		
	1.8	0.5	1946–1961			+1.12±0.11	−0.1911±0.0046		
Schwassmann-	2.1	0.4	1929–1955			+1.01±0.04	−0.1972±0.0005		
Wachmann 2	2.1	0.4	1929–1968	+1.53	−0.1719	+1.4	−0.158	+0.3	1961.7
	2.2	0.4	1941–1969			+1.12±0.03	−0.1688±0.0004		
Tempel-Swift	1.1	0.7	1869–1908	+0.04	−0.0459	+0.1	−0.113	+0.2	1880.9
d'Arrest	1.2	0.7	1851–1870			0.00	+0.1038±0.0011		
	1.3	0.6	1870–1897			−0.50±0.13	+0.0960±0.0006		
	1.3	0.6	1890–1910			+0.79±0.24	+0.0937±0.0008		
	1.3	0.6	1910–1943			−0.47±0.10	+0.0957±0.0011		
	1.3	0.6	1910–1943	−0.12	+0.0576	−0.2	+0.099		
	1.4	0.6	1923–1951			+0.08±0.05	+0.1014±0.0002		
	1.4	0.6	1923–1964	−0.03	+0.0608	0.0	+0.100	+0.1	1943.8
	1.4	0.6	1943–1964			−0.24±0.12	+0.0961±0.0002		
	1.3	0.6	1950–1971			+0.21±0.04	+0.0989±0.0006		
	1.3	0.6	1950–1971	+0.08	+0.0595	+0.1	+0.108		
Wirtanen	1.6	0.5	1948–1967	−0.20	−0.0715	−0.2	−0.088		
Biela	0.9	0.7	1806–1832	+0.28	−0.0250	+0.9	−0.085		
	0.9	0.8	1826–1846	+0.39	−0.0254	+1.3	−0.089		
	0.9	0.8	1832–1852	+0.36	−0.0260	+1.2	−0.094		
Brorsen	0.6	0.8	1846–1868	+0.02	−0.0116	+0.1	−0.066		
	0.6	0.8	1846–1873	+0.03	−0.0115	+0.2	−0.066	−0.7	1857.2
	0.6	0.8	1857–1873	+0.12	−0.0147	+0.6	−0.087		
	0.6	0.8	1868–1879	+0.17	+0.0224	+0.9	+0.134		
Daniel	1.5	0.6	1937–1964	+0.87	+0.0545	+1.1	+0.073		
Whipple	2.5	0.4	1933–1964	+0.61	−0.0621	+0.6	−0.063		
Forbes	1.5	0.6	1929–1948			−0.14±0.18	+0.0733±0.0028		
	1.5	0.6	1929–1961	+0.29	+0.0215	+0.4	+0.029	+1.2	1961.6
	1.5	0.6	1942–1961			+0.65±0.06	+0.0497±0.0006		
Finlay	1.0	0.7	1886–1906			+0.53±0.03	+0.1266±0.0001		
	1.0	0.7	1906–1926			−0.03±0.04	+0.0118±0.0008		
	1.1	0.7	1926–1960			+0.49±0.02	−0.0026±0.0001		
	1.1	0.7	1926–1960	+0.15	−0.0009	+0.4	−0.003		
	1.1	0.7	1953–1967			+0.26±0.07	+0.0255±0.0007		
	1.1	0.7	1953–1967	+0.09	+0.0100	+0.2	+0.026		
Honda-Mrkos-	0.6	0.8	1948–1964	−0.01	−0.0064	0.0	−0.042		
Pajdušáková	0.6	0.8	1948–1969	+0.02	−0.0063	+0.1	−0.041	−0.4	1954.0
Borrelly	1.4	0.6	1904–1968	+0.09	−0.0265	+0.1	−0.041		
Schaumasse	1.2	0.7	1911–1928	+0.32	−0.0170	+0.6	−0.036		
	1.2	0.7	1944–1960	+0.18	−0.0191	+0.4	−0.040		
Comas Solá	1.8	0.6	1926–1944			+1.04±0.12	+0.0015±0.0013		
	1.8	0.6	1935–1953			+0.67±0.06	+0.0402±0.0012		
	1.8	0.6	1943–1962			+0.74±0.05	−0.0173±0.0011		
	1.8	0.6	1951–1970			+0.72±0.06	−0.0737±0.0011		
Giacobini-Zinner	1.0	0.7	1900–1947	+0.05	+0.0113	+0.1	+0.034	−0.1	1901.0
	1.0	0.7	1913–1960	+0.07	+0.0109	+0.2	+0.032	−0.2	1913.8
Arend	1.8	0.5	1951–1967	−0.12	−0.0251	−0.1	−0.026		
Tuttle	1.0	0.8	1858–1899			+0.32±0.03	+0.0131±0.0001		
	1.0	0.8	1912–1967			−0.04±0.03	+0.0131±0.0000		
Faye	1.7	0.6	1843–1881	+0.60	+0.0103	+0.7	+0.012		
	1.7	0.6	1858–1896	+0.56	+0.0061	+0.6	+0.007		
	1.7	0.6	1888–1925	+0.29	+0.0104	+0.3	+0.012		
	1.6	0.6	1910–1948	+0.38	+0.0069	+0.5	+0.008		
	1.6	0.6	1932–1970	+0.26	−0.0004	+0.3	0.000		
Encke	0.3	0.8	1927–1967	0.00	−0.0005	0.0	−0.006	+0.8	1957.8
	0.3	0.8	1947–1967	0.00	−0.0005	0.0	−0.006		

TABLE I (*continued*)

Comet	q	e	Interval	$f(r)$ law A_1	$f(r)$ law A_2	$g(r)$ law A_1	$g(r)$ law A_2	B_2	Epoch
Pons-Winnecke	0.8	0.7	1858–1875	+0.03	−0.0020	+0.1	−0.008		
	0.8	0.7	1858–1886			+0.27±0.03	−0.0072±0.0001		
	0.9	0.7	1875–1898			−0.01±0.05	−0.0021±0.0002		
	1.0	0.7	1892–1915			+0.01±0.02	+0.0008±0.0001		
	1.1	0.7	1933–1951			+0.01±0.02	+0.0024±0.0002		
	1.2	0.7	1939–1964	−0.01	+0.0006	0.0	+0.001		
Grigg-Skjellerup	0.9	0.7	1922–1942			−0.04±0.01	−0.0010±0.0000		
	0.9	0.7	1942–1961			+0.03±0.04	−0.0025±0.0000		
Tempel 2	1.4	0.6	1873–1915	+0.03	+0.0012	+0.1	+0.002		
	1.3	0.6	1904–1946	+0.02	+0.0010	0.0	+0.002		
	1.3	0.6	1915–1956	−0.02	+0.0007	0.0	+0.001		
	1.4	0.6	1930–1967	−0.02	+0.0005	0.0	+0.001		

Note

The nongravitational parameters of the following comets (of three or more apparitions) are too small to be detected: Schwassmann-Wachmann 1 (perihelion distance $q=5.5$ A.U., eccentricity $e=0.1$); Oterma ($q=3.4$, $e=0.1$); Ashbrook-Jackson ($q=2.3$, $e=0.4$); Johnson ($q=2.3$, $e=0.4$); Neujmin 3 ($q=2.0$, $e=0.6$); Reinmuth 1 ($q=2.0$, $e=0.5$); Reinmuth 2 ($q=1.9$, $e=0.5$); Harrington-Abell ($q=1.8$, $e=0.5$); Tempel 1 ($q=1.6$, $e=0.5$); Neujmin 1 ($q=1.5$, $e=0.8$); Arend-Rigaux ($q=1.4$, $e=0.6$).

Royal Astronomical Society's report on the comets of 1971 (Marsden 1972b).

All but two of the comets, P/Brooks 2 and P/Grigg-Skjellerup, have been discussed to some extent in earlier papers of this series. The calculations in which the first two authors have been involved were made on the CDC 6400 computer at the Smithsonian Astrophysical Observatory. The third author has a completely independent set of programs and access to a faster computer, the IBM 360/91 at the Goddard Space Flight Center. He has made the computations from the twentieth-century observations of P/Finlay and P/Tuttle, and he ran the 1858 orbit for P/Tuttle back to the beginning of the nineteenth century. There are slight differences between the planetary masses and coordinates used in the Smithsonian and the Goddard calculations, but several comparisons have been made (particularly for P/Finlay, but also in the case of the third author's earlier work on P/Giacobini–Zinner), and the agreement for both orbital elements and nongravitational parameters is excellent.

It is possible to use the nongravitational parameters obtained previously with the $f(r)$ law to derive reasonably accurate values for the parameters corresponding to the $g(r)$ law. This can be achieved by multiplying the $f(r)$ parameters by

$$\left(\frac{10^4}{40}\right)^2 \frac{1}{\beta} \frac{\int_{-\pi}^{\pi} r^\nu f(r) dv}{\int_{-\pi}^{\pi} r^\nu g(r) dv},$$

where the initial factor allows for the difference in time unit, $1/\beta = f(1) = 0.6065$, and v is the true anomaly. The exponent in the weighting factor r^ν should clearly be 1 in the case of the transverse component A_2 (cf. Sekanina 1973); for the radial component A_1, we have adopted $\nu=2$, but in view of the greater uncertainty of this component, the value of ν is not critical. The $f(r)$ parameters given previously (Papers II, III, and IV; Yeomans 1971) are also included (though having been multiplied by 10^4) in Table I, together with the approximate $g(r)$ figures derived from them by the above procedure. In some cases, the same comets (and even the same observations) have been processed using both laws, and the $g(r)$ values of A_2 derived using the $f(r)$ law are seen to be in excellent agreement with those obtained directly from the $g(r)$ law, even when the planetary perturbations are quite large.

P/Brooks 2

Dubyago (1950, 1956), who paid considerable attention to the orbit of this comet, established that there was a substantial secular acceleration. Since the perihelion distance is rather large (1.8–2.0 A.U.), it therefore seemed probable that A_2 would be particularly large (and negative). In fact, P/Brooks 2 consistently has the largest values of A_1 and A_2 for any short-period comet. Discovered in 1889, this comet has been missed at only two of its 11 returns (those of 1918 and 1967). There was an approach to within 0.08 A.U. of Jupiter in 1922, and we have therefore avoided any runs that involve observations made both before and after this time. Stumpff (1972) has discussed the problems associated with this approach to Jupiter, as well as with the prediscovery approach of 1886 (0.001 A.U.). Inspection of Table I shows that A_2 has decreased (numerically) from −0.3 before the 1922 encounter with Jupiter to −0.2 afterward. The radial parameter A_1 seems to be the usual order of magnitude larger (and positive), except in the case of the 1925–1940 solution. The mean residuals of the four solutions are

1″.90, 1″.65, 1″.48, and 1″.04, respectively, illustrating the general improvement in accuracy of modern observations.

P/Schwassmann-Wachmann 2

Since this comet was discussed in great detail in Paper II, it is certainly appropriate to consider it now in terms of the $g(r)$ law. Two fits have been made from overlapping sets of five apparitions, 1929–1955 and 1941–1969, the mean residuals being 1″.04 and 0″.94, respectively. The result $A_2=-0.2$ is exceeded (numerically) only by that of P/Brooks 2. Comparison of the two values of A_2 suggests that $B_2 \simeq +0.3$, as found before. The observational material was not quite the same as that utilized in Paper II, but the results clearly support Delsemme's (1972a) deduction that the $g(r)$ law would be very suitable for this comet. (This matter will be discussed further in Section II.)

P/d'Arrest

This is another comet we have investigated before, but our earlier calculations (Paper III) were limited to the four apparitions 1923–1924, 1943, 1950–1951, and 1963–1964. A slightly positive value of $B_2(+0.1)$ was obtained, and since this comet has been mentioned as the possible objective of a space probe, it is important to confirm that B_2 cannot be negative; for, as discussed in Paper IV, there is circumstantial evidence that comets of negative B_2 are also subject to "erratic" behavior, and there is a distinct possibility that such comets will completely disperse into meteoroids in the relatively near future. In this present study, we have therefore utilized observations spanning the whole observational history of P/d'Arrest (1851–1971).

Three-apparition fits using the $g(r)$ law, 1923–1951 and 1943–1964, give $A_2=+0.1$ and apparently confirm that $B_2 \simeq +0.1$ also. The 1943–1964 solution yields residuals of up to 1′.5 in 1970, and fits to the observations during 1950–1971, and also during 1910–1943, suggest that B_2 may in fact be negative; however, these two fits are influenced by close approaches to Jupiter (minimum separations 0.50 A.U. in 1920 and 0.41 A.U. in 1968).

The spans 1870–1877–1890, 1877–1890–1897, and 1890–1897–1910 are all free from encounters with Jupiter. Unfortunately, the 1877 observations (only seven in number) are all discordant, so the first two spans were replaced by a single fit to observations in 1870, 1890, and 1897. These older undisturbed solutions indicate $B_2 \simeq +0.04$, but it is hard to reconcile this with A_2 during 1870–1910 and the larger values during 1923–1964. The solution 1851–1870 involves another approach to Jupiter (0.35 A.U. in 1861), and in order to improve the determinancy, A_1 was *assumed* to be zero. All the orbits using nineteenth-century observations give mean residuals of some 2″.5 to 3″.0.

We conclude that A_2 is really remarkably constant for as long as 120 years. P/d'Arrest may perhaps be reasonably safe from erratic behavior, but the approaches to Jupiter certainly affect the precision of the predictions; as an example, consider various predicted times for the next perihelion passage:

1976 August		
12.8401 E.T.	Fit 1950–1971, $g(r)$ law	
12.8947	1950–1971, $f(r)$	
12.9599	1923–1951, $g(r)$	
13.0069	1923–1964, $f(r)$	$B_2=+0.1$.
13.0352	1943–1964, $g(r)$	

Particularly disturbing is the fact that the perihelion times differ by more than 0.05 day when $g(r)$ and $f(r)$ fits are made to the same observations.

P/Forbes

The result in Paper II gave an unusually large value for $B_2(+1.2)$, although since it was derived from only four apparitions, it is necessarily open to some suspicion. Three-apparition fits during 1929–1942–1948 and 1942–1948–1961 give $A_2=+0.07$ and $+0.05$, respectively, suggesting that $B_2 \simeq +0.8$. Observations at the 1974 return should make it possible to decide whether this value is meaningful or whether the comet is erratic.

P/Finlay

In Paper I (Marsden 1968) it was tentatively established, from observations at the four apparitions since 1926, that this comet has a secular deceleration, possibly amounting to 0.06 day per (period)². Our more extensive investigation, which also utilizes observations at the four earlier apparitions, going back to 1886, shows that this comet must be classed as erratic.

The first solution listed in Table I indicates that A_2 is quite large. The other solutions all give much smaller values for A_2, and in the case of the 1926–1960 fit, A_2 has the opposite sign. There is some slight instability in the 1906–1926 solution, due mainly to the particular selection of observations utilized in 1906, and an alternative calculation gives $A_1=+0.09\pm0.03$, $A_2=+0.0155\pm0.0007$. The 1906–1926 run is affected by an approach to Jupiter (within 0.45 A.U.) in 1910, and there was also an approach (to 0.60 A.U.) in 1957.

A particular reason for believing this comet to be erratic is our complete failure to obtain a satisfactory solution from the 1893, 1906, and 1919 apparitions. The formal result gave $A_2=-0.002\pm0.001$, but there were systematic trends in the residuals amounting to well over 20 sec.

P/Comas Solá

Inspection of the various values of A_2 suggests that this comet may also be erratic. Furthermore, although the residuals of purely gravitational fits to three apparitions were really quite small (see Paper I), we have been unable to derive satisfactory nongravitational fits to more than three apparitions. Another confirmation of the anomalously large value of A_2 during 1951–1970 is that, while the first five apparitions indicated that P/Comas Solá had a very slight secular deceleration, amounting to about 0.01 day per (period)^2, the (gravitational) 1969 predicted perihelion time required correction by some -0.08 day.

P/Tuttle

As shown in Paper I, this comet has a secular deceleration of 0.09 day per (period)^2. Although the revolution period is as long as 14 years, we have had no difficulty making nongravitational fits to observations at four apparitions. In fact, as Table I shows, A_2 seems to be precisely constant, and it might well be possible to link all eight apparitions of the nineteenth and twentieth centuries. Another fit, to the three apparitions 1858, 1871–1872, and 1885, is practically identical with that extending to 1899. On running this solution back to the time of Méchain's observations of the comet, we obtained a perihelion time of 1790 January 31.332 E.T., which yields residuals of up to 4 min; if a correction of $+0.017$ day is applied, the maximum residuals are reduced to 2 min. It is clear that the nongravitational parameters of P/Tuttle have been virtually unchanged for nearly two centuries.

P/Pons-Winnecke

In Paper III, we showed that this comet had a rather significant secular acceleration during the nineteenth century but has a very small secular deceleration now. We refrained from investigating the interval around the turn of the century on account of the close approaches to Jupiter every 12 years. These repeated encounters have not been so troublesome as we had feared, however, and the first three direct $g(r)$-law solutions listed in Table I have been obtained from overlapping sets of four apparitions during 1858–1915; the solutions are not perfect, having mean residuals of some 3 sec–4 sec (and the residuals are scarcely improved if only three apparitions are linked), but they clearly show the progressive change from negative to positive values of A_2. We have not made any solutions during 1915–1933, partly because of the minimum approach to Jupiter in 1918 (0.35 A.U.), and also because of the extremely close approach to the Earth (0.04 A.U.) in 1927. The fit to observations 1933–1951 is not so good as we would wish, the mean residual of 2″.26 being high for modern photographic observations; but it suggests that the trend toward increasingly positive A_2 has been maintained.

P/Grigg-Skjellerup

Sitarski (1964, 1966) has made a careful analysis of the observations since 1947 and has shown that this comet has a small secular acceleration. The comet has been observed at every return since 1922, however, and there seems to be some value in attempting to analyze these earlier observations too. It has been possible to represent the observations at five apparitions, and the results, from 1922–1942 and 1942–1961, suggest that B_2 may be negative; however, since the values of A_2 are themselves extremely small, there is no significance in the formal result $B_2 \simeq -1$. A close approach to Jupiter in 1964 (to 0.33 A.U.) makes it difficult to obtain a meaningful result that incorporates also the observations in 1966 (which in any case are but two in number, both having been obtained on the same night), but an orbit fitted to the observations in 1956–1957, 1961, and 1966, *assuming* the nongravitational parameters of 1942–1961, gave a predicted perihelion time in 1972 that required correction by -0.01 day, which would seem to confirm that B_2 is negative.

An orbit fitted to the observations in 1922, 1927, and 1932 alone (but essentially identical with that incorporating also the observations in 1937 and 1942) was run back to the time of Grigg's discovery of the comet, the calculated perihelion time then being 1902 July 3.509 E.T. Grigg's observations are too rough to establish a meaningful correction to this: if one uses all six of them, the correction becomes $+0.100$ day, but the first two residuals are rather larger than the others (20 min on 23 July); the remaining four observations (27 July–3 August) can be satisfied within 8 min, and they then indicate instead a correction of -0.005 day. This new calculation confirms Merton's (1927) masterly study of the identity of comets 1902 II (Grigg) and 1922 I (Skjellerup), a problem that was extremely difficult in the days of hand calculation on account of the comet's approach to within only 0.17 A.U. of Jupiter in 1905 (and furthermore, the work was done before the 1927 observations were available to help define the comet's revolution period); Merton's 1902 values for the argument of perihelion and longitude of the ascending node are each in error by some 3°, but this largely cancels out in their sum, the longitude of perihelion.

II. EFFECT OF VARYING THE ALBEDO

The numerical constants of Eq. (5) were derived on the assumption that both the visible and the infrared albedos were 0.1. It is important to examine how the constants would be modified if different values were used for the albedos. A Bond albedo $A_{\rm vis}=0.1$ in the visible may be reasonable for water snow heavily contaminated by dust. For pure water snow, however, $A_{\rm vis} \simeq 0.7$, while for grains of methane clathrate, the vaporization rate of which is comparable with that of water snow, Wenger (1969) has found that $A_{\rm vis}$ can be as high as 0.9. There is also uncertainty in the infrared albedo $A_{\rm inf}$, although except for metals, macroscopic structures of most substances have $A_{\rm inf}<0.5$ in the far infrared. The situation is more complex for microscopic particles (see Section IV).

We have calculated, directly from the energy-balance equation, the vaporization rate Z for water snow, assuming several values of $A_{\rm vis}$ and $A_{\rm inf}$ between

0 and 0.99. It was found that, in a $\log Z:\log r$ plot, all the vaporization curves have essentially the same shape, and they may be brought into coincidence by changing Z_0 and r_0 (while keeping m, n, and k constant); see Fig. 1 for a few examples. Whenever $A_{\mathrm{vis}}=A_{\mathrm{inf}}$, $r_0 \simeq 2.8$ A.U., while in general,

$$r_0 \simeq 2.8\left(\frac{1-A_{\mathrm{vis}}}{1-A_{\mathrm{inf}}}\right)^{\frac{1}{2}} \text{ A.U.} \tag{6}$$

A more precise analysis, in which Eq. (5) was fitted by least squares to the vaporization flux resulting from solution of the energy-balance equation at heliocentric distances for which $10^5 \leq Z \leq 10^{20}$ molecules cm^{-2} sec^{-1} (which for $A_{\mathrm{vis}}=A_{\mathrm{inf}}$ means $0.1 \lesssim r \lesssim 7.0$ A.U.), gives values of r_0 consistent within $\pm 1\%$ of the formula

$$r_0=C(1-A_{\mathrm{vis}})^{\frac{1}{2}}(1-A_{\mathrm{inf}})^{-\gamma}, \tag{7}$$

where $C=2.828$ A.U. and $\gamma=0.555$. The dependence of r_0 on A_{vis} and A_{inf} is illustrated in Fig. 2. It is possible to justify this value of γ on physical grounds. The energy-balance equation may be written in the form

$$r_0=\frac{1}{2T_0^2}\left(\frac{Q_0}{\sigma}\right)^{\frac{1}{2}}\left(\frac{1-A_{\mathrm{vis}}}{1-A_{\mathrm{inf}}}\right)^{\frac{1}{2}}\left(1+\frac{E_{\mathrm{v}}}{E_{\mathrm{r}}}\right)^{-\frac{1}{2}}, \tag{8}$$

where T_0 is the vaporization-equilibrium temperature, Q_0 is the solar constant, σ is the Stefan-Boltzmann constant, and E_{v} and E_{r} are the fractions of the absorbed solar energy used for vaporization and reradiation, respectively. If $E_{\mathrm{v}} \ll E_{\mathrm{r}}$, the radiative-equilibrium temperature for $C \approx 2.8$ A.U. is about 167 K. For water snow, in the vicinity of 167 K,

$$\frac{E_{\mathrm{v}}}{E_{\mathrm{r}}}=0.0295\left(\frac{T_0}{167}\right)^{32.5}(1-A_{\mathrm{inf}})^{-1}. \tag{9}$$

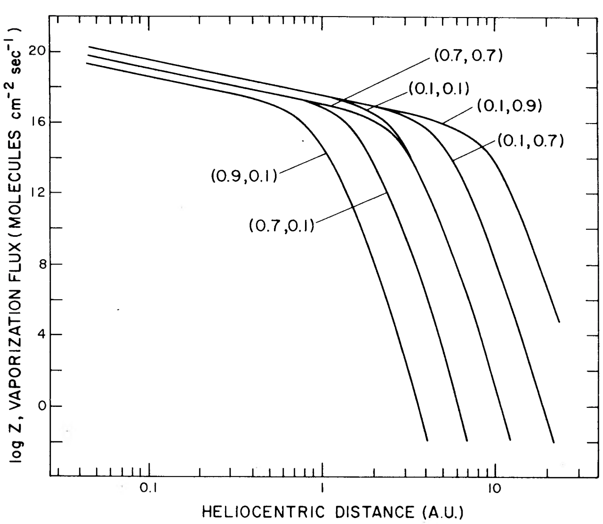

FIG. 1. Vaporization flux of water snow as a function of heliocentric distance. Each curve is characterized by the visual albedo A_{vis} (the first figure in the parentheses) and the infrared albedo A_{inf} (the second figure).

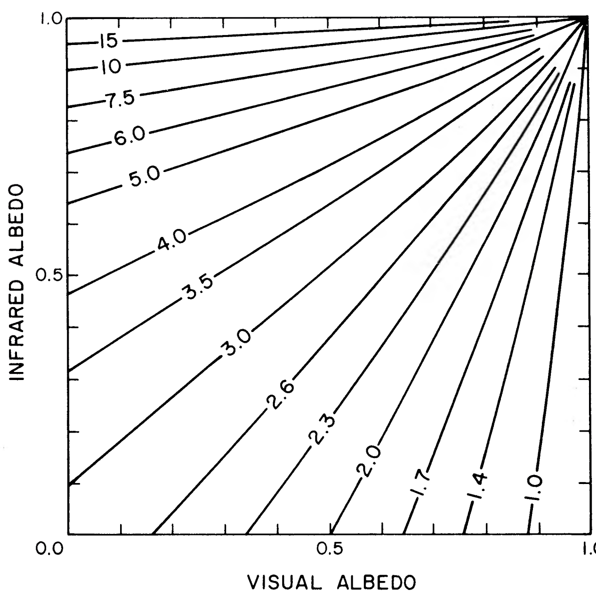

FIG. 2. The scaling distance r_0 for water snow, as a function of the visual albedo A_{vis} and the infrared albedo A_{inf}.

To keep this ratio constant, we require

$$T_0 \sim (1-A_{\mathrm{inf}})^{1/32.5}.$$

From Eq. (8) it then follows that $r_0 \sim (1-A_{\mathrm{inf}})^{-0.561}$, giving good agreement with the empirical value of γ. In order that $C=2.828$ A.U., we must have

$$T_0=165.8(1-A_{\mathrm{inf}})^{1/32.5}\mathrm{K}$$

and $E_{\mathrm{v}}/E_{\mathrm{r}}=0.023$. For water snow, the scaling distance r_0 is thus the distance at which the solar energy spent in reradiation is about 40 times that spent in vaporization.

The least-squares analysis also confirms that the exponents m, n, and k vary only slightly with the albedos. Specifically, m tends to decrease with increasing A_{inf}; n increases with A_{inf}, particularly for large A_{vis}; and the product nk also tends to increase with increasing A_{inf}. The total ranges are $2.08<m<2.16$, $4.5<n<5.3$, and $24<nk<27$. Although not of any consequence as far as orbital calculations are concerned, the dependence of Z_0 on albedo is of some theoretical interest. It can be approximated by

$$\log(\alpha Z_0)=16.56+\log(1-A_{\mathrm{inf}}) -0.13A_{\mathrm{inf}}^2-0.10A_{\mathrm{vis}}^3. \tag{10}$$

The study of nongravitational accelerations in the orbits of comets requires knowledge of the momentum exerted by the vapor flux on the cometary nucleus, rather than of the flux itself. The momentum depends on the thermal velocity of the molecules, which is determined by the equilibrium temperature T, the r-dependence of which is known from the solution to the energy-balance equation. For water snow, $T \sim r^{-\frac{1}{2}}$ at distances governed by radiative equilibrium and

TABLE II. Mean residuals for different values of r_0.

Comet \ r_0(A.U.)	0.8	1.6	2.808	4.0	10.0
P/Brooks 2	1″.05	1″.05	1″.04	1″.05	1″.60
P/Schwassmann-Wachmann 2	0.94	0.93	0.94	1.03	2.25
P/Tuttle	2.01	1.69	1.68	1.66	~8

$T \sim r^{-0.075}$ when vaporization-equilibrium dominates. Consequently, the empirical Eq. (5) is also applicable to variations in T, the values of m and nk becoming 0.075 and 0.425, respectively; r_0 is again given by Eq. (7). Least-squares fits to the curves for the momentum have confirmed that the parameters of Eq. (5) are hardly changed when one goes from vaporization flux to vaporization momentum.

Calculations with Different Values of r_0

For practical calculations on the orbits of comets, the effect of changing the albedos is thus a simple change in the value adopted for r_0. We have made additional computations for three of the comets considered in the previous section, assuming the values $r_0=0.8$, 1.6, 4.0, and 10.0 A.U. As can be seen from Fig. 2, $r_0=1.6$ A.U. corresponds to $A_{\rm vis}\gtrsim 0.7$, while $r_0=0.8$ A.U. corresponds to $A_{\rm vis}\gtrsim 0.9$. The value $r_0=4.0$ A.U. corresponds to $A_{\rm inf}\gtrsim 0.5$, while $r_0=10.0$ A.U., corresponding to $A_{\rm inf}\gtrsim 0.9$, would seem to be completely outside the range of physical possibility.

The comets for which these additional computations have been made are P/Brooks 2, P/Schwassmann-Wachmann 2, and P/Tuttle, and in each case the observations utilized were those from the most recent of the time intervals shown in Table I. The mean residuals for these fits (including those for $r_0=2.808$ A.U.) are shown in Table II. It is quite clear that $r_0=10.0$ A.U. is unacceptable (and in the case of P/Tuttle, it was difficult to get this solution to converge); this large value of r_0 implies that $g(r)$ would differ but little from r^{-2} over the whole orbits of these comets (except for P/Tuttle near aphelion), so these new results serve to confirm our earlier conclusions concerning the unacceptibility of an inverse-square law. With $r_0=0.8$ A.U., the fit to observations is significantly inferior for P/Tuttle, and with $r_0=4.0$ A.U., it is somewhat so for P/Schwassmann-Wachmann 2. The values $r_0=1.6$ and 2.808 A.U. are both well within the range of acceptability for all three comets.

Applicability to Other Volatile Substances

Inspection of the vaporization curves for snows other than water snow indicates that the parameters m, n, and k of Eq. (5) are essentially independent of the material, and that it is only r_0 that changes. Indeed, it can also be shown that Eq. (6) is approximately applicable for other substances. The clue to this is provided by Eq. (8), which shows that $r_0 \sim T_0^{-2}$. The ratio $E_{\rm v}/E_{\rm r}$ depends primarily on L/T_0, where L is the vaporization heat (in calories per mole) of the volatile substance. Keeping $E_{\rm v}/E_{\rm r}$ constant therefore requires keeping L proportional to T_0, and following Eq. (8), we obtain C for a substance of vaporization heat L as

$$C = C_{\rm H_2O}\left(\frac{L_{\rm H_2O}}{L}\right)^2 \simeq \frac{4.0\times 10^8}{L^2}\,\text{A.U.} \qquad (11)$$

This gives for C, or the coefficient in Eq. (6), the values 7.7 A.U. in the case of NH_3, 11 A.U. in the case of CO_2, about 75 A.U. for CH_4, and 140–180 for CO and N_2. We have checked Eq. (11) by calculating the vaporization curves directly for these other molecules; the resulting values of r_0 are in extremely good agreement with those determined from Eqs. (6) and (11). The value $r_0=4.0$ A.U., which can be regarded as the maximum value compatible with the observations, thus implies that the visible albedo would have to be greater than 0.75 if the comet consisted of NH_3, greater than 0.85 if CO_2, and practically unity if CH_4, CO, or N_2.

III. LONG-PERIOD COMETS

In Papers II and III, we discussed at some length the question of detectability of nongravitational effects in the motions of long-period, single-apparition comets, and we concluded that the residuals for comets 1960 II (Burnham) and 1957 III (Arend-Roland) could be significantly improved by solving for them. Contrary to the situation with the short-period comets, it is basically A_1, not A_2, that is determined; and we also found it preferable to use an inverse-square law, rather than the $f(r)$ law. This is not unreasonable, for these comets both have small perihelion distances q (0.5 and 0.3 A.U., respectively), distances at which most of the solar energy is expended in vaporization, and where $g(r)$ is basically an inverse square. It was indicated that consideration of nongravitational terms made it possible to conclude that these comets did not necessarily have orbits that were originally hyperbolic. In this section, we shall discuss this matter further and apply the $g(r)$ law, not only to these two long-period comets, but also to others.

1960 II (Burnham)

A $g(r)$-law fit, assuming $r_0=2.808$ A.U., to the 37 observations by Roemer *et al.* (1966) gives residuals that are essentially the same as those from the inverse-square fit (see Paper III, Table I), the mean value again being 1″.39. As with the short-period comets, solutions have also been made assuming other values of r_0, and in some cases we have made solutions for A_1 alone, forcing A_2 to be zero. The mean residuals for the various runs are shown in Table III. For $r_0\gtrsim 3$ to 4 A.U., they seem to converge to a constant value, and it is only for these cases that neglecting A_2 gives tolerable results. If one solves for A_2, it is clear that the results are satisfactory when $r_0\gtrsim 1.5$ A.U.

1957 III (Arend-Roland)

For this comet also, the residuals from the $g(r)$ fit ($r_0=2.808$ A.U.) are practically identical (mean value $1''.37$) with those from the inverse-square fit (to 150 selected observations) presented in Paper III. Even if $r_0=0.8$ A.U., the mean residual increases only to $1''.41$; while if A_2 is neglected, it increases to $1''.43$ for $r_0=2.8$ A.U., and to $1''.48$ for $r_0=1.6$ A.U.

1970 II (Bennett)

We had anticipated that the spectacular comet 1970 II (Bennett) would, with $q=0.5$ A.U., be a good candidate for showing nongravitational effects. This indeed turned out to be the case: a gravitational solution from 391 observations over a 10-month arc gave systematic residuals with mean value $1''.86$. A nongravitational solution with $r_0=2.808$ A.U. reduced the mean residual to $1''.28$; a solution with $r_0=0.8$ A.U. gave practically identical results, and the same was true when A_2 was ignored. There is, of course, no question of an original hyperbolic orbit for this comet, for the revolution period is well established as somewhat less than 2000 years.

1971 V (Toba)

This is another comet for which the purely gravitational solution indicates that the original orbit was hyperbolic. There do seem to be systematic trends in the residuals, however; a solution, more recent than that in the new *Catalogue of Cometary Orbits*, and based on 113 observations over a $4\frac{1}{2}$-month arc, gives a mean residual of $1''.60$.

It is perhaps surprising that a single-apparition comet with q as large as 1.2 A.U. should show the effects of nongravitational forces in its motion. Nevertheless, nongravitational solutions reduced the mean residual to something less than $1''.4$ for all values of r_0 tested, and whether or not A_2 was considered.

1944 I (van Gent–Peltier-Daimaca)

This comet ($q=0.9$ A.U.) was under observation for only two months, but if nongravitational effects are ignored, both the osculating and the original orbits turn out to be substantially hyperbolic (Marsden and Van Biesbroeck 1963)—the original value all the more so, since the derivation of this quantity in the cited reference is incorrect.

There were severe difficulties with the observations of this comet, particularly during the first half of December 1943, when the comet passed only some 0.25 A.U. from the Earth; but to ignore all the observations after 7 January 1944 (when in fact they extended for another 17 days and were mutually quite consistent), as was done in deriving the parabolic orbit listed in the *Catalogue of Cometary Orbits*, is not really a satisfactory solution to the problem.

A new, general least-squares calculation from 28 carefully chosen observations covering the whole interval of observation gives a mean residual of $1''.39$, and under the circumstances this fit is excellent. However, the original value of the reciprocal semimajor axis $1/a$ is hyperbolic to some 14 times the mean error of its determination.

TABLE III. Mean residuals for comet 1960 II.

r_0(A.U.)	Solution for A_1 and A_2	Solution for A_1 alone
0.8	$2''.49$	
1.2	1.87	
1.6	1.48	$2''.31$
2.2	1.45	
2.8	1.39	1.85
3.4	1.42	
4.0	1.42	1.58
10.0	1.42	1.58

Even though the arc is so short and the mean residual satisfactory, we did attempt nongravitational solutions from the same observations. The changes in the mean residuals were negligible, but these solutions showed that the osculating orbit could be substantially less hyperbolic.

Derivation of Original Values of $1/a$

In the earlier papers, we implied that it was sufficient, even in the case of a nongravitational orbit solution, to derive the original value of $1/a$ (referred to the barycenter of the solar system) from an osculating value simply by subtracting the quantity u_b tabulated by Everhart and Raghavan (1970) for the 400 or so long-period comets observed between 1800 and 1970. [Figures for more recent comets, and for different osculation epochs, have also been calculated by Everhart and are tabulated by Marsden (1971, 1972b)]. It is now desirable to study this matter in more detail.

Any perturbation in $1/a$ can be obtained by integrating the standard equation given by the method of variation of arbitrary constants, namely,

$$\frac{d}{dt}\left(\frac{1}{a}\right)=-\frac{2}{(\mu p)^{\frac{1}{2}}}\left(Re\sin v+S\frac{p}{r}\right), \tag{12}$$

where μ is the product of the solar mass and the gravitational constant, p is the semilatus rectum of the orbit, e is the eccentricity, and R and S are the radial and transverse components of the perturbative force. In the case of the perturbation due to the nongravitational force,

$$R=A_1g(r)$$
$$S=A_2g(r), \tag{13}$$

and by making use of the transformations

$$(\mu p)^{\frac{1}{2}}dt=r^2dv=\frac{p}{e\sin v}dr, \tag{14}$$

it follows that the *radial* contribution δ_R to the perturbation in $1/a$, from aphelion to the time when the comet's radius vector is r, is given by

$$\delta_R = -\frac{2A_1}{\mu}\int_Q^r g(r)dr, \tag{15}$$

where Q is the aphelion distance. The function $g(r)$ is given, of course, by Eq. (5), and it is possible to integrate it analytically in terms of the hypergeometric function. We have

$$\int g(r)dr = -\frac{\alpha r_0}{m-1}\left(\frac{r}{r_0}\right)^{-(m-1)}\left[1+\left(\frac{r}{r_0}\right)^n\right]^{-(k-1)}$$

$$\times F\left\{1, 1-k-\frac{m-1}{n}; 1-\frac{m-1}{n}; -\left(\frac{r}{r_0}\right)^n\right\}, \tag{16}$$

where F denotes the hypergeometric function, with the standard arrangement of arguments. When $r \gg r_0$, it is preferable to write the result

$$\int g(r)dr = -\frac{\alpha r_0}{m-1}$$

$$\times\left(\Lambda + \frac{m-1}{m+nk-1}\left(\frac{r_0}{r}\right)^{m+n-1}\left[1+\left(\frac{r_0}{r}\right)^{-n}\right]^{-(k-1)}\right.$$

$$\left.\times F\left\{1, 1+\frac{m-1}{n}; 1+k+\frac{m-1}{n}; -\left(\frac{r_0}{r}\right)^n\right\}\right), \tag{17}$$

where

$$\Lambda = \frac{\Gamma\left(1-\dfrac{m-1}{n}\right)\Gamma\left(k+\dfrac{m-1}{n}\right)}{\Gamma(k)}. \tag{18}$$

TABLE IV. The quantity $10^6(\delta_R/A_1)$.

r(A.U.)	$10^6(\delta_R/A_1)$	r(A.U.)	$10^6(\delta_R/A_1)$
0.1	820	1.1	24.2
0.2	353	1.2	19.3
0.3	210	1.3	15.3
0.4	142	1.4	12.0
0.5	103	1.6	7.2
0.6	78	1.8	4.0
0.7	60	2.0	2.1
0.8	48	2.2	1.0
0.9	38	2.6	0.2
1.0	30	3.0	0.02

We note that the first term in Eq. (17) is independent of r. Hence, after expansion of the hypergeometric function, Eq. (15) becomes in the limit as $Q\rightarrow\infty$,

$$\delta_R = \frac{2A_1}{\mu}\frac{\alpha r_0}{m-1}\left[\left(\frac{r}{r_0}\right)^{-m+1} - \Lambda + k\frac{m-1}{n-m+1}\left(\frac{r}{r_0}\right)^{n-m+1} + O\left(\frac{r}{r_0}\right)^{2n-m+1}\right], \tag{19}$$

if $r<r_0$, and

$$\delta_R = \frac{2A_1}{\mu}\frac{\alpha r_0}{m+nk-1} \times\left[\left(\frac{r_0}{r}\right)^{m+nk-1} + O\left(\frac{r_0}{r}\right)^{m+n(k+1)-1}\right], \tag{20}$$

if $r>r_0$. Substituting the numerical values given earlier for m, n, and k and the appropriate value of μ, we

TABLE V. The quantity $10^6(-\delta_S/A_2)$.

r(A.U.) \ q(A.U.)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.6	1.8	2.0	2.2	2.6	3.0
0.1	1463																			
0.2	228	656																		
0.3	105	179	407																	
0.4	61	98	142	288																
0.5	40	62	85	116	218															
0.6	28	42	56	73	96	173														
0.7	20	30	40	50	63	81	141													
0.8	15	22	29	36	44	54	68	116												
0.9	11	17	22	27	32	38	46	58	98											
1.0	9	13	17	20	24	28	33	40	49	82										
1.1	7	10	13	15	18	21	25	29	34	42	69									
1.2	5	8	10	12	14	16	18	21	24	29	35	59								
1.3	4	6	7	9	10	12	14	16	18	21	24	30	49							
1.4	3	4	6	7	8	9	10	12	13	15	17	20	25	41						
1.6	2	3	3	4	4	5	6	6	7	8	9	10	11	13	28					
1.8	1	1	2	2	2	3	3	3	4	4	4	5	5	6	8	17				
2.0	0	1	1	1	1	1	1	2	2	2	2	2	3	3	3	5	10			
2.2	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	2	2	5		
2.6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
3.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Note

It is assumed that the osculation epoch is at or before perihelion. If the epoch is after perihelion, calculate instead $2(\delta_S)_q-\delta_S$, where $(\delta_S)_q$ is the value at perihelion $(r=q)$ and δ_S is the value at the same r before perihelion.

TABLE VI. Results for long-period comets.

Comet	r_0 (A.U.)	A_1	A_2	$10^6(1/a)_{\mathrm{osc}}$	$10^6(\delta_R+\delta_S)$	$10^6(\Delta/A_1)$	$10^6(1/a)_{\mathrm{orig}}$
1960 II	(Epoch 1960 April 16.0; $r=0.8$ A.U.)			-596 ± 23	($10^6u_b=-463$)		-133
	1.611	$+5.5\pm0.3$	-1.6 ± 0.2	$+110\pm39$	$+1005$		-432
	1.611	$+3.9\pm0.3$	0.0	-175 ± 40	$+138$	$+73$	$+150$
	2.808	$+5.8\pm0.3$	-1.4 ± 0.2	-59 ± 33	$+807$		-403
	2.808	$+4.8\pm0.3$	0.0	-209 ± 31	$+237$	$+31$	$+17$
	4.0	$+4.6\pm0.3$	0.0	-237 ± 26	$+267$	$+20$	-41
	10.0	$+4.6\pm0.3$	0.0	-237 ± 26	$+325$	$+7$	-99
1957 III	(Epoch 1957 May 2.0; $r=0.7$ A.U.)			-780 ± 6	($10^6u_b=-682$)		-98
	0.8	($+2.5\pm0.3$	$+0.4\pm0.04$)*	-756 ± 14	-132		$+58$
	1.611	$+1.3\pm0.1$	0.0	-641 ± 16	$+57$	$+63$	-16
	2.808	$+1.7\pm0.2$	$+0.1\pm0.02$	-667 ± 19	$+6$		$+9$
	2.808	$+1.9\pm0.2$	0.0	-609 ± 17	$+108$	$+32$	-35
1970 II	(Epoch 1970 April 4.0; $r=0.6$ A.U.)			$+6907\pm5$	($10^6u_b=-439$)		$+7346$
	2.808	$+1.9\pm0.1$	0.0	$+7094\pm7$	$+135$	$+27$	$+7398$
1971 V	(Epoch 1971 March 30.0; $r=1.3$ A.U.)			-972 ± 39	($10^6u_b=-679$)		-293
	1.611	$+13\ \pm2$	0.0	-566 ± 54	$+33$	$+29$	$+80$
	2.808	$+5.0\pm0.5$	-1.4 ± 0.6	-685 ± 53	$+138$		-144
	2.808	$+5.1\pm0.5$	0.0	-596 ± 47	$+84$	$+57$	-1
	10.0	$+4.6\pm0.5$	0.0	-605 ± 45	$+176$	$+41$	-102
1944 I	(Epoch 1944 January 11.0; $r=0.9$ A.U.)			-3030 ± 204	($10^6u_b=-175$)		-2855
	2.808	$+8.6\pm1.6$	0.0	-1204 ± 408	$+342$	$+173$	-1371
1964 VI	(Epoch 1964 June 14.0; $r=0.6$ A.U.)			$+8128\pm15$	($10^6u_b=-12$)		$+8140$
	2.808	$+1.0$	0.0	$+8216\pm14$	$+71$	$+17$	$+8157$
1963 I	(Epoch 1963 April 1.0; $r=0.7$ A.U.)			$+10477\pm3$	($10^6u_b=-927$)		$+11404$
	2.808	$+1.0$	0.0	$+10564\pm4$	$+65$	$+22$	$+11426$
1968 I	(Epoch 1968 March 5.0; $r=1.7$ A.U.)			$+500\pm7$	($10^6u_b=-277$)		$+777$
	2.808	$+3.0$	0.0	$+541\pm6$	$+16$	$+8$	$+802$
1962 VIII	(Epoch 1962 December 2.0; $r=2.1$ A.U.)			$+4889\pm1$	($10^6u_b=-48$)		$+4937$
	2.808	$+3.0$	0.0	$+4906\pm1$	$+4$	$+4$	$+4950$

* Times 10^{-3}.

obtain $\Lambda=1.6549$, and Eqs. (19) and (20) become

$$\delta_R=A_1\alpha r_0\left[\frac{58.77}{(r/r_0)^{1.15}}-97.26+79.09(r/r_0)^{3.943}+O(r/r_0)^{9.036}\right]\times10^{-6}\ \text{A.U.}^{-1}\qquad(21)$$

and

$$\delta_R=A_1\alpha r_0[2.74(r_0/r)^{24.65}+O(r_0/r)^{29.74}]\times10^{-6}\ \text{A.U.}^{-1}.\qquad(22)$$

It is clear that, even when r is only slightly greater than r_0, δ_R is essentially zero, while nearer the Sun δ_R has the sign of A_1.

As for the corresponding contribution δ_S from the *transverse* component of the nongravitational force, it follows from Eqs. (12), (13), and (14) that this is given by

$$\delta_S=-\frac{2A_2}{\mu}\int_{-\pi}^{v}rg(r)dv.\qquad(23)$$

We shall not attempt to integrate this analytically but point out merely that, since $rg(r)>0$, δ_S is of the opposite sign to A_2. (This is reasonable since a change in reciprocal semimajor axis has the same sign as one in mean motion, and as has been remarked in previous papers, a positive value of A_2 corresponds essentially to a secular deceleration and a negative one to a secular acceleration.)

The original value of $1/a$ then follows from the osculating value by

$$(1/a)_{\mathrm{orig}}=(1/a)_{\mathrm{osc}}-\delta_R-\delta_S-u_b,\qquad(24)$$

where the value of r required in the expression for δ_R [Eqs. (21) or (22)] is that at the epoch of osculation, and the upper limit v of the integral in the expression for δ_S [Eq. (23)] is the true anomaly at this epoch. Similarly, the future (barycentric) value of $1/a$ is given by

$$(1/a)_{\mathrm{fut}}=(1/a)_{\mathrm{osc}}-\delta_R+\delta_S'+u_a,\qquad(25)$$

where u_a is also tabulated by Everhart and Raghavan (1970) for the comets of 1800–1970, and δ_S' is an expression analogous to Eq. (23), except that the limits of integration are v to $+\pi$.

Numerical values of δ_R/A_1 and $-\delta_S/A_2$ (assuming $r_0=2.808$ A.U.) are given in Tables IV and V, respectively. The latter quantity is expressed as a function of r and q, and it is assumed that the epoch of osculation is at or before perihelion. It follows that

$$\delta_S'=2(\delta_S)_q-\delta_S,\qquad(26)$$

where $(\delta_S)_q$ represents the value of δ_S at perihelion $(r=q)$. If the epoch of osculation is after perihelion, δ_S' should be used instead of δ_S in Eq. (24) and δ_S instead of δ_S' in Eq. (25).

Results for Individual Comets

In the upper part of Table VI, we list the relevant results for the five comets discussed earlier in this section. The first line for each comet gives the epoch of osculation and heliocentric distance (in parentheses), the osculating $1/a$ (and its mean error) from the gravitational solution, the quantity u_b (in parentheses), and $(1/a)_{\rm orig}=(1/a)_{\rm osc}-u_b$. The other entries for each comet represent nongravitational solutions, the various values of r_0, A_1, and A_2 (and their mean errors) being specified (the solutions $A_2=0.0$, with no error indicated, mean that A_2 was assumed to be zero); the corresponding values of $(1/a)_{\rm osc}$ are given, together with the quantity $\delta_R+\delta_S$ necessary for deriving $(1/a)_{\rm orig}$ when nongravitational terms are included. The penultimate column gives, for the cases where $A_2\equiv 0$, the ratio Δ/A_1, where Δ is the difference between $(1/a)_{\rm orig}$ in the nongravitational and in the gravitational solutions. (At this stage we might point out that the normalizing factor α depends, of course, on r_0, and that it has the values 1105, 0.5297, 0.05097, and 0.007080 for $r_0=0.8$, 1.611, 4.0, and 10.0 A.U., respectively.)

Inspection of the results obtained for the three comets for which more than one nongravitational solution is listed in Table VI shows considerable dispersion in the values of $(1/a)_{\rm orig}$. One is forced to conclude that it is impossible to establish the original value of $1/a$ for an individual comet, and sometimes it is far from clear whether the original orbit was elliptical or hyperbolic. The effect of A_2 (through δ_S) on $(1/a)_{\rm orig}$ is by no means insignificant, and although in the case of comet 1960 II the values of $(1/a)_{\rm osc}$ obtained when solutions are made for A_2 are more elliptical than those obtained when A_2 is assumed to be zero, the negative contribution from δ_S more than counteracts it; an original elliptical orbit is possible if $A_2\equiv 0$ and $r_0\lesssim 3$ A.U., although Table III shows that the mean residual becomes tolerable only if $r_0\gtrsim 3$ to 4 A.U. For comet 1957 III, it seems plausible that the original orbit could have been elliptical; and the same is also true for comet 1971 V, provided that r_0 is not too large, for in this case it is certainly unnecessary to solve for A_2.

The nongravitational solution for comet 1944 I has had a rather dramatic effect on $(1/a)_{\rm orig}$, and if A_1 is supposed to be even larger, one can doubtless force the original orbit to be elliptical. If $A_1=+13$, for example, $(1/a)_{\rm orig}$ becomes -0.000463, and although such a value of A_1 is much larger than what has been directly determined for any comet (for $r_0=2.8$ A.U.), the resulting mean residual of 1″.55 is certainly not unacceptable in this case.

It seems to be true that, provided A_2 is not significantly negative, nongravitational solutions do make the original orbits more elliptical than they would be if the nongravitational terms were ignored. This is connected with the fact that A_1 is positive, but the underlying cause is not obvious. We have also established the same fact for the four comets listed in the bottom portion of Table VI. Direct solutions for nongravitational terms are not significant in these cases, but this does not mean that these comets are not subject to nongravitational forces. The figures tabulated for these comets have resulted from *assuming* small positive values of A_1 (+1.0 for comets 1964 VI and 1963 I, +3.0 for the more distant comets 1968 I and 1962 VIII); in no case was there a really deleterious effect on the mean residual—the largest change being from 1″.16 to 1″.30 for comet 1963 I—and for two of the comets the mean residual was slightly diminished.

IV. DISCUSSION AND CONCLUSIONS

The results presented in this paper show that the assumption that cometary nuclei consist to a large extent of water snow, and that the absorbed solar radiation partly vaporizes this snow and is partly reradiated according to the theory developed, in particular, by Delsemme, provides a very satisfactory quantitative explanation of many of the nongravitational effects observed in the motions of comets. Specifically, the observations are consistent with the theoretical sudden drop in vaporization rate somewhere between 1 and 4 A.U. from the Sun. It is not possible to set very firm bounds on this distance (and hence on the albedo of a comet), but it seems unlikely that the visible Bond albedo can be greater than ~0.8.

Values of the radial and transverse components A_1 and A_2 of the resulting reactive force have been directly calculated for 9 short-period and 5 long-period comets, and the corresponding figures have been indirectly derived from earlier work on 14 other comets. For most of the short-period comets, A_1 (in the particular units utilized, notably a time unit of 10^4 days; and assuming $r_0=2.808$ A.U.) is between +0.1 and +1.0, while—as found also in earlier papers in this series—A_2 is either positive or negative and generally an order of magnitude smaller. In the case of long-period comets (and also for $r_0=2.808$ A.U.), small values of A_1 are not detectable, but there are four reasonably certain examples with A_1 in the range +2 to +5; and except in the case of comet 1960 II, A_2 seems to be indeterminate.

The individual values of A_1 (and A_2) for the short-period comets have been found to range over at least $2\frac{1}{2}$ orders of magnitude. We reiterate that this is on the assumption that $r_0=2.808$ A.U. If r_0 is smaller, the range becomes much larger. With $r_0=0.8$ A.U., for example, the results for a comet of $q=2.5$ A.U. would be increased by a factor of 10^{10} over those for a comet of $q=0.5$ A.U. Table VII gives the logarithms of

TABLE VII. Logarithms of the multiplicative factors for A_1 and A_2.

q(A.U.) \ e	$r_0=0.8$ A.U. 0.4	$r_0=0.8$ A.U. 0.8	$r_0=1.611$ A.U. 0.4	$r_0=1.611$ A.U. 0.8	$r_0=4.0$ A.U. 0.4	$r_0=4.0$ A.U. 0.8	$r_0=10.0$ A.U. 0.4	$r_0=10.0$ A.U. 0.8
0.5	−2.26	−2.28	−0.09	−0.06	0.00	−0.04	+0.01	−0.10
1.0	+0.62	+0.56	+0.28	+0.23	−0.09	−0.08	−0.12	−0.21
1.5	+4.06	+4.04	+1.14	+1.12	−0.23	−0.19	−0.38	−0.38
2.0	+6.53	+6.52	+2.56	+2.55	−0.46	−0.45	−0.78	−0.74
2.5	+8.11	+8.11	+3.81	+3.81	−0.91	−0.90	−1.45	−1.38

the factors by which the nongravitational parameters should be multiplied in order to convert from $r_0=2.808$ A.U. to $r_0=0.8$, 1.611, 4.0, and 10.0 A.U. for selected values of q and e.

It is obvious that the nongravitational acceleration of the nucleus of a comet depends principally on the momentum of the outgoing gas per unit surface area per unit time. It is clear, however, that the nongravitational parameters must also be inversely proportional to the radius and density of the nucleus and that they must depend on the rotation rate.

If the radius of the nucleus had a dominant influence on the values of A_1 and A_2, we should expect the upper part of Table I to be populated by small comets and the lower part by large ones. Comparison with the photometric data on the radii of cometary nuclei indicates that this is not the case. According to Roemer (1966), P/Schwassmann-Wachmann 2 is one of the largest of the periodic comets, yet it is the second entry in Table I; and several of the comets near the end of the table are of average size only.

The density of the nucleus does not seem to be a significant factor either. For a cometary nucleus in the form of an agglomerated snowball, the average density should increase with size because of the increasing compression of the snow in the center. The effect of the density on the nongravitational parameters would thus merely tend to reinforce the effect of the radius. If the nucleus is inhomogeneous in structure, however, as in the core-mantle model (Sekanina 1971), small nuclei that have lost their volatile shells could be more dense than much larger agglomerated snowballs.

In any case, it is hard to believe that, among the typical short-period comets, the combined effect of radius and density could account for variations in A_1 and A_2 much larger than 1 to $1\frac{1}{2}$ orders of magnitude. Of course, if the albedo varies significantly from comet to comet, the photometric radii may be greatly in error. But if the product of radius and density has an important effect on the nongravitational acceleration, then the "erratic" comets—suggested in Paper IV as being both smaller and less dense than other comets—would have to be among the first entries in Table I, whereas in fact most of them are near the middle of the table.

A cometary nucleus rotating infinitely fast would have constant surface temperature, and since the evaporation would be perfectly isotropic, there would be no nongravitational effects. Obviously this model is useful only as an approximation for a nucleus rotating fairly rapidly, where the temperature difference ΔT between the warmest and the coldest spots on the surface is very small. The observed dynamical effect depends on the degree of anisotropy in the total momentum of the outgoing gas, the anisotropy factor for a rapidly rotating nucleus being

$$\lambda = -\frac{\pi}{4} i \frac{J_1(i\theta)}{J_0(i\theta)}, \tag{27}$$

where $J_0(i\theta)$ and $J_1(i\theta)$ are the Bessel functions of the first kind, $i=\sqrt{-1}$, and for water-snow vaporization

$$\theta = \left(\frac{53}{T_{\text{av}}}\right)^2 \Delta T. \tag{28}$$

Here T_{av} is the average surface temperature. At $T_{\text{av}}=150$ K, λ has the values 0.10, 0.19, and 0.28 for $\Delta T=2$, 4, and 6 K, respectively. At $T_{\text{av}}=200$ K, λ is 0.055, 0.11, and 0.16, respectively, for the same three amplitudes ΔT. The nongravitational parameter A_1 for a rapidly rotating, water-snow nucleus of radius 1 km and density 0.5g cm^{-3}, and with $A_{\text{vis}}=A_{\text{inf}}=0.1$ (i.e., $r_0=2.8$ A.U.), would be approximately 1.2 λ, which is of the order of magnitude of the data in Table I. Delsemme's (1972) calculations for a model consisting of a nonrotating nucleus—a convenient approximation for slowly rotating nuclei—suggest that the nongravitational acceleration would be rather larger ($A_1 \simeq 1$ for the case considered above).

Considerably smaller values of A_1 (and A_2) that cannot be explained by larger radii and higher densities can be due to "slow" subsurface vaporization of volatiles via diffusion through a matrix of depleted solid material, rather than to direct (surface) vaporization. In evolutionary terms, this leads to the core-mantle model, with the eventual progressive decrease in the nongravitational acceleration and the comet ultimately becoming asteroidal; the model has been described semiquantitatively for P/Encke (Sekanina 1972) and is presumably applicable also to P/Tempel 2.

Values of A_1 much greater than unity could arise if the comets contain a sizable proportion of substances more volatile than water snow. This would also imply a substantial increase in r_0, but for the long-period comets, this is not precluded (see, in particular, Table III). Delsemme (1966) has pointed out that most of the other potential cometary volatiles (CH_4 in particular) will have been lost after one or two passages near the Sun, however, so while this explanation is attractive for the four comets discussed that have practically hyperbolic orbits, it can scarcely be applicable to comet 1970 II (which has presumably made a considerable number of passes within 1 A.U. of the Sun), unless free CH_4 (for example) becomes exposed at or near the surface in the process of vaporization.

There would seem a possibility that the presence of substances more volatile than water snow can perhaps explain the large nongravitational parameters of P/Brooks 2 and P/Schwassmann-Wachmann 2, which have only recently been perturbed in by Jupiter from orbits of much larger perihelion distance. At its discovery, P/Brooks 2 was truly a "new" comet, for it is unlikely that it had ever been previously much within the orbit of Jupiter. Fractionation of the volatile materials by distillation during the initial close approaches (within, say, 2.5 A.U.) of a comet to the Sun would cause a sharp decrease in the nongravitational acceleration (Delsemme 1972), and such decreases are in fact observed for these two comets. (It is extremely difficult to explain these decreases by vaporization via diffusion, for the mass-loss rates are much too high.) However, this mechanism would require r_0 to be unacceptably large.

Another process leading to a secular decrease in the nongravitational acceleration is a systematic increase in the visible albedo of the nucleus. There is plenty of evidence that the surface layer of a new comet is rich in dust. The presence of fine, low-reflectivity dust particles, acting as local sources of radiation, would have caused the sublimation rate to be higher, for much the same reason that holes in a snowfield first appear in the vicinity of stones. As the surface layer of snow, heavily contaminated by the particles, is being vaporized, the albedo would start increasing.

There is also an effect of surface emissivity in the infrared. Becklin and Westphal (1966) found that solid particles in the coma of comet 1965 VIII had color temperatures higher than the blackbody temperature. Maas *et al.* (1970) found a similar effect in the case of comet 1970 II. It is therefore likely that the presence of fine dust embedded in the icy matrix on the surface of a cometary nucleus may decrease the effective emissivity. Again, the removal of such a heavily contaminated layer would lead to a subsequent drop in the production rate. It is conceivable that the decrease in the nongravitational activity of P/Brooks 2 and P/Schwassmann-Wachmann 2 can be interpreted by the combined effect of variations in A_{vis} and A_{inf}. To demonstrate how powerful this process can be, we point out that an increase in A_{vis} from 0.1 to 0.5 and a simultaneous decrease in A_{inf} from 0.5 to 0.1 results in a drop in the production rate of water snow at a heliocentric distance of 2 A.U. by a factor of 22, this being accompanied by a systematic decrease in r_0 from 4.0 to 2.1 A.U.

More cases of comets with erratic behavior have been found; and to those listed in Paper IV, Table XII, we must now add P/Finlay and P/Comas Solá (and just possibly, also P/Grigg-Skjellerup). The erratic behavior of P/Comas Solá is of some significance, because the perihelion distance of this comet is 1.8 A.U. (and before an approach to Jupiter in 1912, it was 2.2 A.U.), whereas all the other erratic comets (except for the uncertain case of P/Forbes) have $q<1.2$ A.U. That comets of large q can also be erratic tends to favor an external cause, such as that discussed at length in Paper IV.

In the case of the long-period comets, a positive value of A_1 has been found to have the effect of making the original orbit more elliptical, although only a tiny negative value of A_2 can cancel this out. With the help of the Everhart–Raghavan quantities u_b and u_a, as well as the nongravitational perturbations δ_R and δ_S, it is possible to derive "nongravitational" original and future values of $1/a$. We do want to stress, however, that the results for individual comets, particularly when solutions are made for A_2, may be meaningless (cf. Table VI). It *appears* that we might have to accept that comet 1960 II originated outside the solar system, although we cannot exclude the possibility that *part* of the effect we have attributed to nongravitational forces is merely due to a departure of the center of light from the center of mass; the latter phenomenon is particularly troublesome for comets such as this (and 1944 I, for that matter) that come close to the Earth. Alternatively, comet 1960 II may have experienced the erratic behavior to which several of the short-period comets are prone.

The systematic change in $1/a$ due to the nongravitational forces is well illustrated by the column Δ/A_1 of Table VI. Although Δ/A_1 seems to be invariably positive, there is little pattern to the differences from comet to comet; indeed, there is some dispersion among the values from different solutions for the same comet. We suspect that Δ/A_1 must somehow be related to the distribution of the observations, and there is perhaps some indication of this in the slight correlation of Δ/A_1 with the mean error of $(1/a)_{\mathrm{osc}}$; if the last third of the observed arc for comet 1963 I is ignored, $10^6\Delta/A_1$ increases to +53, and the mean error of $(1/a)_{\mathrm{osc}}$ is increased in proportion. But why a large *positive or negative* observational uncertainty should always result in a large *positive* Δ/A_1, seems rather obscure.

Since positive and negative values of A_2 seem to be equally probable (at least in the case of the short-period comets), while A_1 is predominantly positive, Δ/A_1 must in some way represent an average reduction in the size of the Oort cloud. If one regards the size of the cloud to be generally established from comets whose $1/a$ values were determined with an average mean error of $\pm 30\times10^{-6}$ A.U.$^{-1}$ (Oort 1950), and if, following Hamid and Whipple (1953), an average long-period comet has—in our units—an A_1 of $+0.3$, it would seem that the average $(1/a)_{\rm orig}$ should be increased, owing to the neglected nongravitational effects, by some $+20\times10^{-6}$ A.U.$^{-1}$. This would have the effect of reducing the radius of the outer part of the Oort cloud from the commonly accepted figure of 200 000 A.U. to some 70 000 A.U.

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