

# On the Frequency of Type I and Type II Supernovae

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**Summary.** Using the recent statistical data collected by Tammann on supernovae, an attempt has been made to deduce the frequencies of those events which are relevant for stellar evolution. The main points inspiring the investigation are the separation of type I supernovae into two subtypes occurring respectively in young (SN Iy) and in old (SN Io) stellar populations, and the determination of the frequencies of SN II, SN Iy, and SN Io events per unit galactic mass and per stellar population type. The frequencies thus obtained are

used to determine, according to the Salpeter birthrate function, the stellar mass range to which these different types of events may be referred. For the case of the young population it turns out that the frequencies of both SN II and SN Iy events indicate a value of  $\sim 10 M_{\odot}$  as the lower limit to the initial main sequence mass range of the presupernovae.

**Key words:** supernovae

## § 1

Although observational data concerning supernovae outbursts are being more and more systematically collected, still our knowledge concerning their real frequency of occurrence has remained amazingly uncertain and incomplete. This may be immediately felt even at first sight just by looking at the large dispersion of the data, ranging, according to different writers, from one event every 350 years (Zwicky, 1965) to one event every 16 years (Kukarkin, 1965) per galaxy. The recent work of Tammann (1970), although limited to supernovae in Sb and Sc galaxies, has contributed to clarifying and interpreting the causes of such discrepancies. According to its conclusions, the large spread of uncertainty is mainly due to the fact that the crude data are by themselves devoid of a direct physical meaning, and it turns out that, when spiral Sb and Sc galaxies are divided into different luminosity groups, the supernovae frequency is strongly dependent on such a subdivision, being much higher for luminous galaxies than for fainter ones. This by itself is enough to show that a frequency of supernovae per galaxy cannot be interpreted as long as the galaxy type and group to which one refers is not specified, because any figure may be obtained just by changing the sample of galaxies considered. On the contrary, one may deduce frequencies per unit luminosity or per unit mass, which appear, not only for all supernovae, but also for their two more common subtypes SN I and SN II, to be fairly independent of the luminosity group both for Sb and Sc galaxies, although they turn out to be different for these two Hubble classes.

It is of course tempting to utilize the frequencies thus obtained, in order to try to deduce from them some

further information concerning more detailed characteristics of these events. This in fact turns out to be possible provided one integrates Tammann's data with some further information derived from other writers, concerning exact location of events in the Spirals, and relative frequencies in Ellipticals and So's in respect to Spirals.

From the present day statistics we know that supernovae of types SN I and SN II are related to quite different stellar populations. In fact, works aiming to give relative frequencies of these two types as a function of different Hubble classes of galaxies indicate this (Bertola and Sussi, 1965):

a) SN II supernovae appear only in spirals and irregular galaxies; moreover, in the first instance, their location is mostly on the outer spiral arms. This is a clear indication of their belonging to a young population I.

b) Supernovae appearing in Elliptical and So galaxies are only of type I. This bears out the suggestion that the SN I phenomenon occurs at least partly in an old disk population (rather than in population II, if we preserve this name strictly for low metal abundance stars).

These considerations are enough to indicate that supernovae outbursts must occur for quite different mass ranges and phases of stellar evolution; and that therefore, in order to derive relevant frequencies for the sake of theoretical interpretations, one has to refer to frequencies per unit mass and per given population type and not only per a complete galaxy.

The present contribution is therefore: first, an attempt to derive such figures, and in addition, some further speculations concerning the stellar progenitors of these

Table 1

Group	Type and lum. class	No. Gal. observed	$L_{pg} \times 10^8 L_{\odot}$	$M \times 10^{10} M_{\odot}$	No. total SN	No. SN 1959–1969	No. total SN I	No. total SN II
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1b	Sb <sub>I</sub> Sb <sub>I-II</sub>	11	194	16.4	7	2	2	3
2b	Sb <sub>II</sub> Sb <sub>II-III</sub>	32	77.3	6.5	4	(1.8)	0	2
3b	Sb <sub>III</sub>	22	28.1	2.4	1	0	0	0
Total		65			12	2	2	5
1c	Sc <sub>I</sub> Sc <sub>I-II</sub>	16	149	16.5	19	10	2	7
2c	Sc <sub>II</sub> Sc <sub>II-III</sub>	29	78.0	8.7	9	4	4	4
3c	Sc <sub>III</sub> Sc <sub>III-IV</sub>	25	34.0	3.8	4	2	1	2
4c	Sc <sub>IV</sub> Sc <sub>IV-V</sub>	38	9.4	1.0	2	2	0	0
Total		108			34	18	7	13
General total		173			46	20	9	18

events, as this may turn out to be an important clue for several theories concerning the advanced stages of stellar evolution. This, of course, can be achieved only according to a number of assumptions which are still quite disputable, but appear perhaps to be the most reasonable ones which may be formulated on the basis of our present knowledge.

Moreover, it has also to be stressed that even when referring to the data of Tammann one is faced with a rather drastic limitation of statistics, due to the short fiducial period of observation (extending only during the decade September 1959 to September 1969), during which observational losses may be considered as much smaller than for previous decades; so that the sample of events on which quantitative estimates have to be based is only a small fraction of the whole data presented in Tammann's work, and statistical errors become enormous in some cases. Owing to the absence of any more abundant and detailed statistical material on the subject, there are at present no means of avoiding these large uncertainties in the deductions which will be met in what follows. Therefore we shall limit ourselves to stressing once for all that many of the results are affected by errors of the order of 100%, without computing them systematically; only here and there, will the effects of changing some of the figures inside the error box be presented in order to keep in mind the level of indeterminacy which will affect some of the conclusions.

## § 2

As a starting point, Tammann's data related to the frequencies of SN of both types for spiral Sb and Sc galaxies, divided into several luminosity groups collected from different tables of his paper, are presented in Table 1. Column 1 states the names given to the different luminosity groups, and the corresponding types and luminosity classes are found in Column 2. Column 3 gives then the number of galaxies patrolled for each group, Column 4 their mean luminosity, and

Column 5 their mean mass value deduced from the average mass/luminosity ratios of Roberts (1969) for Sb and Sc galaxies; Column 6 the total number of SN observed for all galaxies of each group, and Column 7 the number of these for the same groups in the fiducial period September 1959–September 1969; Columns 8, 9 and 10 yield then the total number of SN I, SN II and SNU of unidentified type; and Columns 11, 12 and 13 the same numbers limited to the period September 1959 to September 1969. From these data, the average time interval between two SN events per galaxy, the number of supernovae per  $10^8 L_{\odot}$  and 100 years, and the number of SN per  $10^{10} M_{\odot}$  and 100 years may be deduced, and are given in Columns 14, 15 and 16. All these values are deduced from the data for the fiducial period 1959 to 1969, except those of line 2 B, for which the figures are calculated from the total number of supernovae observed, corrected by the factor  $\frac{140}{7 \cdot 45} = 0.444$  as explained in Tammann's paper. From the data of the last two columns we may then deduce mean values for all groups of Sb and Sc galaxies separately, thus getting:

	Sb	Sc
$\bar{N}$ of SN per $10^8 L_{\odot}$ per 100 years	0.0083	0.035
$\bar{N}$ of SN per $10^{10} M_{\odot}$ per 100 years	0.10	0.32

If we wish now to obtain separate frequencies for the two supernovae types, owing to the extreme paucity of data, nothing better can be done than try to use also the unidentified supernovae, statistically divided between type I and type II according to the observed relative ratios of these types. From the totals of Columns 8, 9, 11, 12, we get:

$$(a) \frac{\text{SN I}}{\text{SN II}} = \frac{6}{7} = 0.85$$

Table 1 (continued)

No. total SNu (10)	SN I 1959–1969 (11)	SN II 1959–1969 (12)	SNu 1959–1969 (13)	Time interval (years) between SN per gal. (14)	No. SN/10 <sup>8</sup> L <sub>⊙</sub> /100 y (15)	No. SN/10 <sup>10</sup> M <sub>⊙</sub> /100 y (16)
2	0	1	1	55	0.0094	0.11
2	0	(0.9)	(0.9)	(178)	(0.0072)	(0.087)
1	0	0	0	—	—	—
5	0	1	1			
10	2	5	3	16	0.042	0.38
1	3	1	0	72	0.0177	0.16
1	1	0	1	125	0.0235	0.21
2	0	0	2	190	0.0560	0.53
14	6	6	6			
19	6	7	7			

if we refer to the period 1959–1969 and

$$(b) \frac{\text{SN I}}{\text{SN II}} = \frac{9}{18} = 0.5$$

if we refer to the total data.

As the value of this ratio should not depend sensibly on the random choice of events used to determine it, there should be no strong reason to adopt value (a) rather than value (b), and, owing to the large discrepancy between the two determinations, all following calculations have been carried out with both values. Results are presented in Table 2 [corrected using (a)] and in Table 3 [corrected using (b)]. Columns 1 and 2 of both tables are thus obtained by adding to Columns 11 and 12 of Table 1, Column 13, statistically divided according to (a) or (b).

One may now try to obtain directly the frequency for SN II supernovae per unit mass of young population I to which they appear to belong entirely, provided we know the composition of the Sb and Sc galaxies. In this respect, our only source of information is our Galaxy, for which one may pick up the following two sets of data:

Galaxy models

Writers	% Masses of		
	Halo	Old disk	Young disk
Perek, L. (1962)	17	76	7
Oort, J. H. (1958)	23	67	10

Nowadays it is believed that our Galaxy is of a type intermediate between Sb and Sc. As not much is known concerning the amount of change of the ratios of the preceding table with Hubble type galaxy, nothing better is offered to us than a tentative assumption of the first line as indicative for Sb and the second one for Sc galaxies.

With these figures, the average amounts of old disk and respectively of young disk population may be calculated

from Table 1. Column 5, for each galaxy group, thus leading to the data of Tables 2 and 3, Columns 3 and 4. With these assumptions one can then deduce mean intervals between two SN II events per galaxy and the frequencies of SN II events per 10<sup>10</sup> M<sub>⊙</sub> and per 100 years. These figures are given in Columns 5 and 6. For Column 6, the values do not change too much with luminosity group, and one may deduce mean values for these frequencies for Sb and Sc galaxies of all groups. One obtains:

Number of Sn II per 10 <sup>10</sup> M <sub>⊙</sub> of young population matter per 100 years	Sb	Sc
Ratio (a)	1.09	1.57
Ratio (b)	1.21	1.81

§ 3

For SN I events, the analysis is somewhat more involved, as there are some indications that this phenomenon cannot be attributed to the single old disk population, as is sometimes done on account of its appearance in Elliptical and So galaxies. Owing to their presence in Spiral galaxies, from a detailed analysis of Bertola and Sussi (1965) concerning their location in Spirals, it turns out that in a total of 29 SNI events, 16 were situated in the outer arms, 4 in the central parts, and 9 in intermediate regions, a distribution completely different from the one of novae, practically present only in the central parts as any typical old disk population product. This strongly suggests that SN I outbursts may take place in two different stellar populations, due to two different explosion triggering mechanisms in different stellar structures and evolution stages, but giving rise to almost identical spectroscopic and luminosity behaviour. This, it may be observed, should by no means represent a new or unexpected situation; it has in fact frequently been pointed out (Rosino) that SN II luminosity curves and spectra are extremely analogous to those of novae, although the physical causes giving

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Table 2 (case a)

Group	Correct. $U \cdot 0.46 + \text{SN I}$	Correct. $U \cdot 0.54 + \text{SN II}$	Mass old pop. per gal. in $10^{10} M_{\odot}$	Mass young pop. per gal. in $10^{10} M_{\odot}$	Time interval between SN II per gal. (years)	SN II/ $10^{10} M_{\odot}/100 \text{ y}$	$\frac{y}{y+0} \text{ SN I}$	$\frac{0}{y+0} \text{ SN II}$	No. SN Iy/ $10^{10} M_{\odot}/100 \text{ y}$	No. SN Io/ $10^{10} M_{\odot}/100 \text{ y}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	
1b	0.46	1.54	18.5	1.15	71	1.21	0.35	0.11	0.28	0.008
2b	(0.41)	(1.39)	5.0	0.45	(230)	(0.97)	(0.31)	(0.10)	(0.22)	(0.006)
3b	0	0	1.82	0.17	—	—	—	—	—	—
Total	0.87	2.93	—	—	Mean	1.09	0.33	0.11	0.25	0.007
1c	3.38	6.62	11	1.65	24	2.50	2.57	0.81	1	0.046
2c	3.0	1.0	5.8	0.87	290	0.40	2.28	0.72	0.9	0.043
3c	1.46	0.54	2.5	0.38	460	0.57	1.11	0.35	1.2	0.056
4c	0.92	1.08	0.67	0.10	350	2.82	0.70	0.22	1.8	0.086
Total	8.76	9.24	—	—	Mean	1.57	6.66	2.10	1.2	0.058
General total	9.63	12.17	—	—	—	—	7.01	2.21	—	—

Table 3 (case b)

Group	Correct. $U \cdot 0.46 + \text{SN I}$	Correct. $U \cdot 0.54 + \text{SN II}$	Mass old pop. per gal. in $10^{10} M_{\odot}$	Mass young pop. per gal. in $10^{10} M_{\odot}$	Time interval between SN II per gal. (years)	SN II/ $10^{10} M_{\odot}/100 \text{ y}$	$\frac{y}{y+0} \text{ SN I}$	$\frac{0}{y+0} \text{ SN II}$	No. SN Iy/ $10^{10} M_{\odot}/100 \text{ y}$	No. SN Io/ $10^{10} M_{\odot}/100 \text{ y}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	
1b	0.33	1.67	12.5	1.15	60	1.32	0.25	0.08	0.20	0.0058
2b	(0.30)	(1.50)	5.0	0.45	(206)	(1.11)	(0.23)	(0.07)	(0.16)	(0.0044)
3b	0	0	1.82	0.17	—	—	—	—	—	—
Total	0.63	3.17	—	—	Mean	1.22	0.24	0.08	0.18	0.0051
1c	3	7	11	1.65	23	2.65	2.28	0.72	0.86	0.041
2c	3	1	5.8	0.87	290	0.40	2.28	0.72	0.90	0.043
3c	1.33	0.67	2.5	0.38	370	0.70	1.01	0.32	1.06	0.051
4c	0.67	1.33	0.67	0.10	285	3.50	0.51	0.16	1.35	0.063
Total	8.00	10.00	—	—	Mean	1.81	6.08	1.92	1.04	0.050
General total	8.63	13.17	—	—	—	—	6.33	2.00	—	—

rise to the phenomena are of an entirely different order of magnitude and certainly due to completely different physical causes. If, taking into account such a situation, and following a first suggestion in this sense of Bertola and Sussi (1965), one may venture to assume that SN I supernovae are a mixture not yet distinguished of two different types of events occurring in two different stellar populations, which may be indicated tentatively as SN Iy (young) and SN Io (old), one may try again to divide statistically the figures given by Bertola and Sussi (1965) in order to get a relative frequency of these two types. The simplest and crudest assumption will then consist in attributing the 4 events observed in central parts of the galaxies to the SN Io type, and the 16 in the arm regions to SN Iy events. While the first attribution is likely to be correct at least as a first approximation, the second one is certainly more questionable, as there may be chance superpositions of events which appear as projected on some arm. We shall not attempt to calculate any possible geometrical correction for this effect, and assume, as indicative for the ratio SN Iy/SN Io we are looking for, the crude figures from which an upper limit for the value of the ratio will be expected; if then one further divides the 9 remaining cases according to the ratio  $16/4 = 4$  one arrives for the relative ratio of the two subtypes at SN Iy/SN Io  $\approx 23/6$ . We may then further divide according to this ratio the SN I frequencies given in Tables 2 and 3, Column 1, and thus obtain the numbers of SN Iy and SN Io events (Columns 7 and 8). Referring these events then to the respective mass fractions of young and old disk populations present in Sb and Sc galaxies (Columns 3 and 4), as already done for SN II cases, we obtain for the frequencies of SN Iy and SN Io events per 100 years per  $10^{10} M_{\odot}$  of young or old population matter the data of Columns 9 and 10. The mean values for all luminosity groups of Sb and Sc galaxies are:

	Case a)		Case b)	
	SN Iy	SN Io	SN Iy	SN Io
Sb	0.25	0.007	0.18	0.005
Sc	1.2	0.058	1.04	0.050

We see that for what concerns the young population, for both SN II and SN Iy events, the frequency is higher for Sc than for Sb galaxies. This may be interpreted as giving evidence that the star birthrate function is higher for the first, in agreement with the general viewpoint of their more intense young population I activity. Instead the large difference in the productivity of the SN Io events between Sc's and Sb's galaxies related to their post-birthrate activity, may perhaps be understood if one keeps in mind that these old population supernovae appear in the central parts of the galaxy, where

more of them in the Sb than in the Sc case may be lost, owing to absorption.

These frequencies for SN Io events in Spirals may further be directly compared with those observed in Ellipticals and So galaxies, when we assume they occur in the same type of stars. From the work of Barbon (1968) we then get that 37 supernovae were observed out of a total of 1123 Spiral and Irregular galaxies, and 8 supernovae out of a total of 894 Elliptical and So's. Assuming as SN Io events all these 8 supernovae in Ellipticals and So's, and  $37 \cdot 0.105 = 3.9$  among those observed in the Spirals and Irregulars ( $0.105 = 2.1/20$  being the average percentage between case a) and case b) of SN Io's in the statistics on Spirals we have now analysed), we obtain, assuming equal birthrate function in Ellipticals and in old disk Spiral populations:

$$\frac{M_E \cdot 894}{0.7 M_S \cdot 1123} = \frac{8}{3.9}$$

0.7 being the average percentage of old disk population assumed for Spirals, and  $M_E$  and  $M_S$  the unknown mean masses of the Ellipticals and Spirals used by Barbon (1968); we thus get:

$$\frac{M_E}{M_S} \sim 1.8,$$

a not unreasonable figure bearing in mind our general information concerning masses of giant Ellipticals and Spirals, thus justifying the assumption of a similar birthrate function in the two cases. The same analysis made on Bertola and Sussi's (1965) data, giving 30 supernovae for 35 Elliptical and So galaxies, and 124 supernovae in 55 Spiral and Irregular galaxies, yields:

$$\frac{M_E}{M_S} \sim 2.6,$$

which is compatible with the preceding result.

#### § 4

As a last step, one may try to connect the frequencies we have obtained for all types of SN with the birthrate functions of the different stellar mass ranges in our Galaxy for the cases in which this function may be surmised to be sufficiently indicative for the corresponding figures in other galaxies, and deduce the mass, or the absolute magnitude range, of the stars undergoing supernovae explosions at the end of their lives, on the further assumption of equilibrium between deathrates and birthrates.

If  $\Psi(M) dM$  is the tabulated Salpeter birthrate function, that is the number of stars created in  $T \approx 6 \cdot 10^9$  years per cubic parsec in the magnitude interval  $dM$ ,  $N$  the total number of stars present per cubic parsec, and  $\bar{m}$  the average star mass of the population considered,

Table 4

	SN II	SN Iy
$n_y$	1.7	1.1
$M_{2v}$	- 3.7	- 4.0
$M_{2b}$	- 5.7	- 6.0
Mass	$\sim 13$	$\sim 15$

then the number  $n$  of stars created (or exploded) in 100 years per  $10^{10} M_\odot$  between magnitudes  $M_1$  and  $M_2$  is given by:

$$n = 100 \frac{\int_{M_1}^{M_2} \Psi(M) dM}{T} \frac{10^{10} M_\odot}{(N\bar{m})^2}.$$

We may try to apply this formula in the case of supernovae related to the young populations, as it does not, perhaps, appear too inconsistent to extend the data on birthrate functions obtained from the solar neighbourhood to the young populations of the arm regions of both Sb and Sc galaxies. Assuming then:

$$\bar{m} = M_\odot \quad N = 0.120 \text{ stars/pc}^3$$

one finds:

$$n_y = 1.15 \cdot 10^4 \int_{M_{1v} \sim -\infty}^{M_{2v}} \Psi(M) dM. \quad (1)$$

In Table 4 we report the mean explosion rates  $n_y$  of the two supernovae types of the young disk population (taking a mean value between cases a) and b) from the Sc data) considered in our previous analysis [Columns (6) and (9) of Tables 2 and 3], and the corresponding visual absolute magnitude  $M_{2v}$  for which the birthrate function

$$\int_{M_{1v} \sim -\infty}^{M_{2v}} \Psi(M) dM$$

satisfies relation (1) ( $M_{1v}$  being assumed as  $-\infty$ ). The corresponding values of the bolometric magnitudes (using the bolometric corrections for the spectral class B 1), and the corresponding mass values (in  $M_\odot$  units) obtained from the mass-luminosity relation are then given in the next columns.

It may thus be seen that the birthrates of all main sequence stars brighter than  $M_b = -5.7$  and respectively  $-6.0$  provide the death rates of SN II and respectively SN Iy events; and thus such absolute bolometric magnitudes correspond to a mass range of about  $13 M_\odot$ . Owing to the very steep decrease of the birthrate function with increasing mass, it would thus result that for young populations, only stars with masses higher than this limit should undergo the supernovae phenomenon. If the adopted frequencies of SN are still to be considered as lower limits owing to experimental losses, and to the fact that in Sc galaxies,

yielding most of the supernovae considered, the birthrate function may possibly be higher than in our galaxy, this lower limit of the mass range might perhaps be shifted down to about  $10 M_\odot$ . In any case it does not seem that it could be brought down as low as  $4 M_\odot$ , as is frequently assumed or deduced (Stothers, 1963). This is perhaps the widest discrepancy in the results of the present analysis with respect to those generally quoted. A more detailed subdivision of the mass ranges respectively responsible for the SN II and SN Iy events is of course outside the scope of the present investigation.

It may be mentioned, at this point, that it has been found (Barbon *et al.*, 1965) that a value of  $\sim 10 M_\odot$  corresponds also to the lower mass limit for the occurrence of outbursts triggered off by the Fe-He conversion mechanism (Hoyle and Fowler, 1960).

For SN Iy events, as they occur only in old populations, they must be related either to stars belonging to a mass range not much different from the solar one, or to delayed products of the evolution of larger mass stars, such as residual cores or white dwarfs. In the first case, the very low relative frequency of these events in respect to the abundance of stars corresponding to a solar mass value indicates that either a very restricted mass band or only a small fraction of all the stars pertaining to a larger mass range are subject to such final explosions. In the second case, one could try to deduce the mass range of the progenitors in the same way as for supernovae related to young populations, arriving thus, owing to their much lower frequency, at a still higher luminosity range ( $M_b \sim -6$ ) for the progenitors of these stars than for the young population cases. However, a specific value for these events would scarcely be indicative, because in this case the extrapolation of the birthrate data of the solar neighbourhood to populations older than several  $10^9$  years appears very little justified; if at these older times, the birthrate function was larger than it is now, the mass range corresponding to the observed frequencies of SN Iy events should be further increased.

It may further be remarked that, owing to the steepness of the birthrate function, the determination of the lower mass limit for stars undergoing supernovae explosions for young populations is strongly insensitive to such uncertainties of the data as are the choice of values (a) or (b) for the ratio of the two types of supernovae, or even the strong statistical fluctuations. Such a situation allows one to conclude that, even if most of the figures here deduced for the relevant frequencies will be somewhat modified when more extended and accurate statistical data is available, and more information concerning different stellar populations in outer galaxies is known, still one may expect with some confidence that the result on the stellar mass range responsible for young population supernovae will not be too much affected by these changes.

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## References

- Barbon, R. 1968, *Astron. J.* **73**, 1016  
 Barbon, R., Dallaporta, N., Perinotto, M., Sussi, M. G. 1965, *Mem. Soc. Astron. Ital.* **36**, 127  
 Bertola, F., Sussi, M. G. 1965, *Contrib. Oss. Astron. Univ. Padova* No. 176  
 Hoyle, F., Fowler, W. A. 1960, *Astrophys. J.* **132**, 565  
 Kukarkin, B. V. 1965, *Astrofiz.* **1**, 465  
 Oort, J. H. 1958, Dynamics and Evolution of the Galaxy, in so far as relevant to the Problem of the Populations, in *Stellar Populations*, Ed. O'Connell, North Holland Publishing Company, Amsterdam, p. 415  
 Perek, L. 1962, *Adv. Astron. & Astrophys.* **1**, 65, Ed. Z. Kopal  
 Roberts, M. S. 1969, *Astron. J.* **74**, 859  
 Rosino, L., private communication  
 Sandage, A. 1958, Luminosity functions of galactic cluster, globular clusters, and elliptical galaxies, in *Stellar Populations*, Ed. O'Connell, North Holland Publishing Company, Amsterdam, p. 75  
 Stothers, R. 1963, *Astrophys. J.* **138**, 1085  
 Tammann, G. A. 1970, *Astron. & Astrophys.* **8**, 458  
 Zwicky, F. 1965, *Stellar Structure (= Stars and Stellar Systems, Vol. 8)*, L. H. Aller and D. B. McLaughlin, Eds. The University of Chicago Press, Chicago, p. 385 and 410

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