Astron. & Astrophys. 24, 337–355 (1973)

Black Holes in Binary Systems. Observational Appearance

N. I. Shakura
Sternberg Astronomical Institute, Moscow, U.S.S.R.

R. A. Sunyaev
Institute of Applied Mathematics, Academy of Sciences, Moscow, U.S.S.R.

Received June 6, 1972

Summary. The outward transfer of the angular momentum of the accreting matter leads to the formation of a disk around the black hole. The structure and radiation spectrum of the disk depend, mainly on the rate of matter inflow $\dot{M}$ into the disk at its external boundary. The dependence on the efficiency of mechanisms of angular momentum transport (connected with the magnetic field and turbulence) is weaker. If $\dot{M}=10^{-9}-3\cdot 10^{-8}\frac{M_{\odot}}{\text{year}}$ the disk around the black hole is a powerful source of X-ray radiation with $h\nu \sim 1-10$ keV and luminosity $L\approx 10^{37}-10^{38}$ erg/s. If the flux of the accreting matter decreases, the effective temperature of the radiation and the luminosity will drop. On the other hand, when $\dot{M}>10^{-9}\frac{M_{\odot}}{\text{year}}$ the optical luminosity of the disk exceeds the solar value. The main contribution to the optical luminosity of the black hole arises from reradiation of that part of the X-ray and ultra-violet energy which is initially produced in the central high temperature regions of the disk and which is then absorbed by the low temperature outer regions. The optical radiation spectrum of such objects must be saturated by broad recombination and resonance emission lines. Variability, connected with the character of the motion of the black hole, with gas flows in a binary system and with eclipses, is possible. Under certain conditions, the hard radiation can evaporate the gas. This can counteract the matter inflow into the disk and lead to autoregulation of the accretion.

If $\dot{M}\gg 3\cdot 10^{-8}\frac{M_{\odot}}{\text{year}}$ the luminosity of the disk around the black hole is stabilized at the critical level of $L\approx 10^{38}\frac{M}{M_{\odot}}\frac{\text{erg}}{\text{s}}$. A small fraction of the accreting matter falls under the gravitational radius whereas the major part of it flows out with high velocity from the central regions of the disk. The outflowing matter is opaque to the disk radiation and completely transforms its spectrum. In consequence, at the supercritical regime of accretion the black hole may appear as a bright, hot, optical star with a strong outflow of matter.

Key words: black holes – binary systems – X-ray sources – accretion

The black hole (collapsar) does not radiate either electromagnetic or gravitational waves (Zeldovich and Novikov, 1971). Therefore, it can be found only due to its gravitational influence on the neighbouring star or on the ambient gas medium (the gas must accrete with the release of large amount of energy (Salpeter, 1964; Zeldovich, 1964)).

Many papers have suggested searching for collapsars in binary systems. It is often considered that the collapsar should appear as a "black" body which practically does not influence the total radiation of the system. In this paper, the attention of the reader is drawn to the case where the outflow of matter from the surface of the visible component and its accretion by the black hole should lead to an appreciable observational effect. In the system with an outflow of matter $\frac{dM}{dt}=\dot{M}>10^{-12}\frac{M_{\odot}}{\text{year}}$, the luminosity of the disk around the black hole formed by the accreting matter can be comparable and even exceed the luminosity of the visible component. In a typical case most of the radiation is emitted in the spectral range of $h\nu \sim 100-10^{4}$ eV. However, as will be shown below, the optical and ultra-violet (responsible for the formation of a Strömgren region) luminosities are also high. Therefore, it is entirely possible that black holes are among the optical objects, soft X-ray sources and the harder X-ray sources now being intensively investigated. The radiation connected with accretion by black holes in binary systems has, in fact, distinctive features. However, they are not as astonishing as is usually assumed; the black holes may be hidden among known objects.

Truly "black" objects may be found only in remote binary systems typified by a weak stellar wind from the visible component.

I. The General Picture

Up to 50% of stars are in binary systems (Martynov, 1971). A sufficiently massive ($M > 2M_\odot$) star, being a component of the binary system, is able to evolve up to the moment when it loses stability and to collapse[1]). In this case, it is possible that an appreciable number of binary systems will not be destroyed and the stars will remain physically bound. These statements are, of course, controversial. However if we recall that the total number of stars with $M > 2M_\odot$ which have existed in the Galaxy is of the order of 10^9 (Zeldovich and Novikov, 1967), then it becomes clear that the number of binary systems including a black hole might be very large (up to $10^6 - 10^8$).

The outflow of matter from a star's surface – the stellar wind – is evidently one of the main properties of stars. The rate of mass loss depends upon the type of star and varies from $2 \cdot 10^{-14} \frac{M_\odot}{\text{year}}$ for the Sun up to $10^{-5} \frac{M_\odot}{\text{year}}$ for the nuclei of planetary nebulae, Wolf-Rayet stars, MI supergiants and O-stars of the main sequence (Pottasch, 1970). In binary systems, an additional strong matter outflow connected with the Roche limiting surface is possible. At a definite stage of evolution, for example after leaving the main sequence, the star begins to increase in size and after the Roche volume is filled, there is an intensive outflow of matter, mostly through the inner lagrangian point (Martynov, 1971).

What will be the consequences of the existence of a black hole in a binary system if matter flows strongly outwards from the visible star? Some fraction of the matter flowing out from the normal star must fall into the sphere of influence of the gravitational field of the black hole, accrete to it, and finally fall within its gravitational radius (Fig. 1). If the matter undergoes free radial infall (if it was initially at rest and there was no magnetic field), the cold matter accretes to the black hole without any energy release or observational effect (Zeldovich and Novikov, 1971). However, in a binary system, the matter flowing out from the normal star and falling on the black hole has considerable angular momentum relative to the latter, which prevents free fall of the matter. At some distance from the black hole centrifugal forces are comparable to gravitational ones and the matter begins to rotate in circular orbits. The matter is able to approach the gravitational radius only if there exists an effective mechanism for transporting angular momentum outward.

The magnetic field, which must exist in the matter flowing into the disk, and turbulent motions of the

[1]) It is possible that $M_{\min}$ considerably exceeds $2M_\odot$ (Zeldovich and Novikov, 1971).

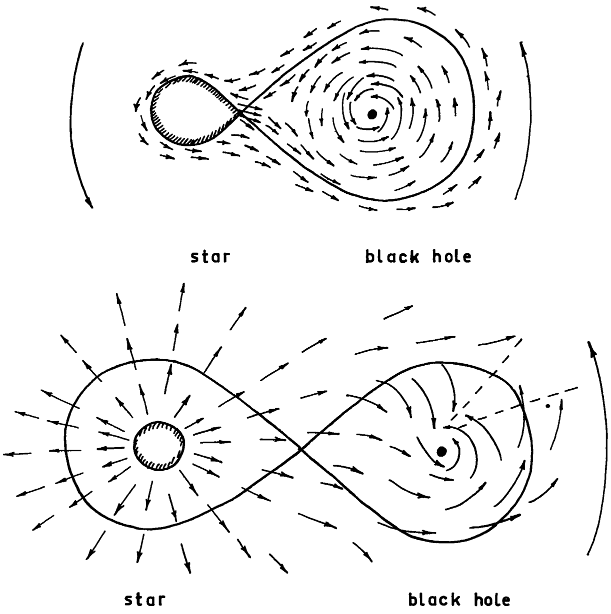

Fig. 1. Two regimes of matter capture by a collapsar: a) a normal companion fills up its Roche lobe, and the outflow goes, in the main, through the inner lagrangian point; b) the companion's size is much less than Roche lobe the outflow is connected with a stellar wind. The matter loses part of its kinetic energy in the shock wave and thereafter, gravitational capture of accreting matter becomes possible

matter enable angular momentum to be transferred outward. The efficiency of the mechanism of angular momentum transport is characterized by parameter $\alpha = \frac{v_t}{v_s} + \frac{H^2}{4\pi \varrho v_s^2}$ where $\frac{\varrho v_s^2}{2} = \frac{3}{2} \varrho \frac{kT}{m_p} + \varepsilon_r$ is the thermal energy density of the matter, ε_r is the energy density of the radiation, v_s is the sound velocity and v_t the turbulent velocity. In part II below we show that $\alpha \leqq 1$. The most probable model is that of accretion with formation of a disk around the black hole (Prendergast, 1960; Gorbatsky, 1965; Burbidge and Prendergast, 1968; Lynden-Bell, 1969; Shakura, 1972; Pringle and Rees, 1972). The particles in the disk, due to friction between adjacent layers, lose their angular momentum and spiral into the black hole[2]). Gravitational energy is released during this spiraling. Part of this energy increases the kinetic energy of rotation and the other part turns into the thermal energy and is radiated from the disk surface. The total energy release and the spectrum of the outgoing radiation are determined mainly by the rate of accretion, i.e. by the rate inflow of matter into the disk[3]). The basic parameter is the

[2]) The disks formed by the matter flowing out from the second component are some times observed around one of the stars in ordinary binary systems (Kraft, 1963).

[3]) The efficiency α of the angular momentum transport mechanism is assumed to be constant along the disk in our calculations (v_t and H are varied in accordance with the change of ϱv_s^2). The observational appearance of the disk (spectrum of its radiation and the effective temperature of the surface) do not strongly depend on the chosen value of α. However, at supercritical regime, this dependence becomes dominant.

value of the flux of matter $\dot{M}_{cr}$ at which the total release of energy in the disk $L=\eta \dot{M} c^2$ is equal to the Eddington critical luminosity $L_{cr}=10^{38}\frac{M}{M_\odot}\frac{\text{erg}}{\text{s}}$, characterized by the equality of the force of radiation pressure on the completely ionized matter and of the gravitational forces of attraction to the star (η is the efficiency of gravitational energy release, in the case of Schwarzschild's metric $\eta \simeq 0.06$, in a Kerr black hole η can attain 40%). For a black hole of mass M, the critical flux is given by $\dot{M}_{cr}=3\cdot 10^{-8}\frac{0.06}{\eta}\frac{M}{M_\odot}\frac{M_\odot}{\text{year}}$. This is no particular reason for considering a rate of accretion

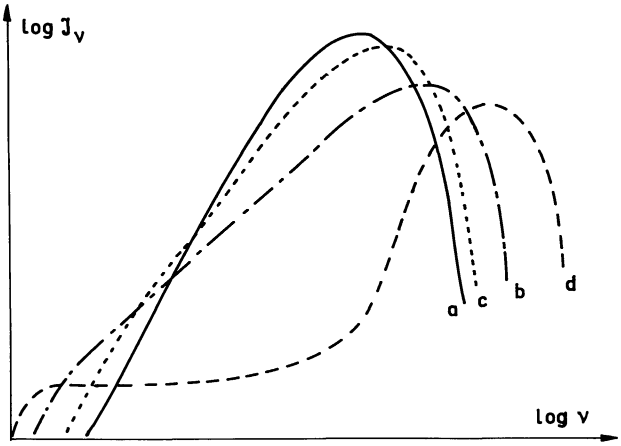

Fig. 2. Characteristic local spectra of radiation formed in the disk a) the black body spectrum $Q=bT^4$. b) the radiation spectrum of an isothermic, homogeneous medium where the main contribution to the opacity comes from scattering $Q=\text{const}\sqrt{n}\,T^{2.25}$. c) the same in an isothermal, exponential atmosphere: $Q=\text{const}\,T^{2.5}$. d) the spectrum formed as a result of comptonization $Q=\text{const}\,T^4$. The intensities are normalized so that the energy flux of radiation Q is the same in all four cases. The change of effective temperature of the radiation is clearly seen

exactly equal to $\dot{M}_{cr}$. A subcritical rate of accretion to the disk is possible as well as an inflow of the matter to the disk many times exceeding the critical value.

At essentially subcritical fluxes $\dot{M}=10^{-12}-10^{-10}\frac{M_\odot}{\text{year}}$ the luminosity of the disk is of the order of $L=10^{34}-10^{36}\frac{\text{erg}}{\text{s}}$.

Maximal surface temperatures are of the order of $T_s=3\cdot 10^5-10^6\,^\circ\text{K}$ in the inner regions of the disk where most of the energy is released. This energy is radiated mainly in the ultraviolet and soft X-ray bands, which are inaccessible to direct observations[4]). The local radiation spectrum of the disk is formed in its upper layers and depends on the distance to the black hole and the distribution of matter along z-coordinate. The possible forms of the local spectrum reduce to four characteristic distributions (Fig. 2). An integral spectrum (Fig. 3) is determined from the expression $J_\nu=2\pi\int F_\nu(R)R\,dR$.

For the case of disk accretion, a weak dependence of the radiation intensity on frequency $\nu^{-1\div+1/3}$ at $h\nu<kT_{max}$ is typical. As a result, the optical luminosity of the black hole may be appreciable. Estimations show (see, part II, §3) that, for black holes with $M=10\,M_\odot$, even if $\dot{M}=10^{-9}\frac{M_\odot}{\text{year}}$, we may expect the optical luminosity to be of the order of the solar value.

In fact, the optical luminosity can be much higher. It arises from reradiation of the hard radiation of the hot central regions of the disk by the outer layers. The thickness of the disk increases with distance from the black hole (Fig. 4) This is why the outer regions of the disk effectively absorb the X-ray radiation from the

[4]) Black holes radiating in the soft X-ray and hard ultra-violet bands may significantly contribute to the galactic component of the soft X-ray background and to the thermal balance of interstellar medium. Their radiation must ionize and heat neutral interstellar hydrogen.

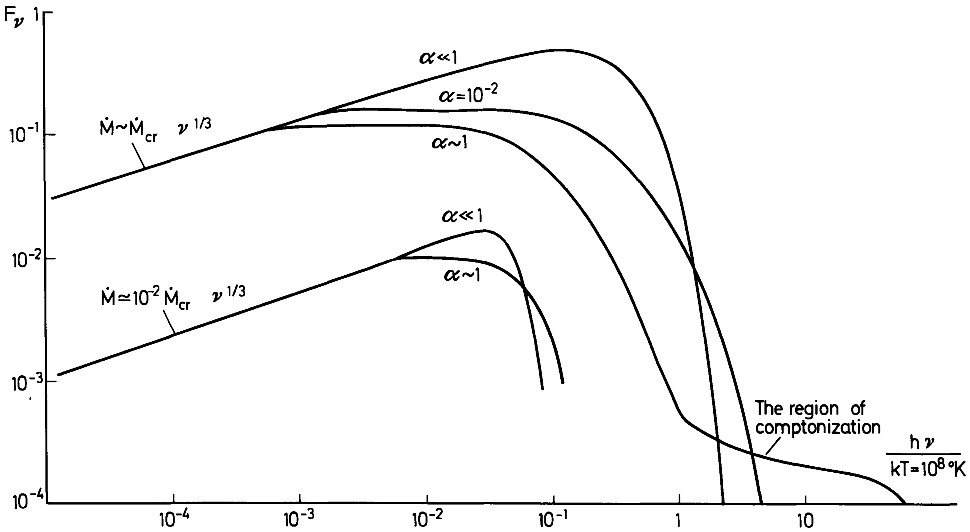

Fig. 3. The integral radiation spectrum of the disk, computed for different $\dot{M}$ and α

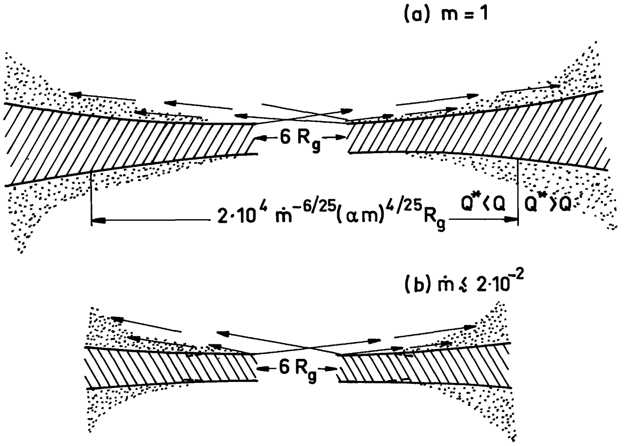

Fig. 4. The thickness of the disk as a function of the distance to the black hole: a) $\dot{M} = \dot{M}_{cr}$. b) $\dot{M} < 10^{-2}\dot{M}_{cr}$. In the central zone, $R < 3R_g$ newtonian mechanics is not applicable. Trajectories of X-ray and ultra-violet quanta which lead to evaporation and heating of the matter in the outer regions of the disk are shown by the arrows. The corona formed by the hot, evaporated matter is denoted by dots

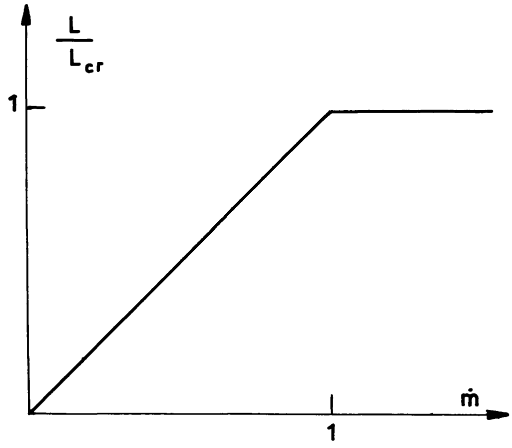

Fig. 5. Dependence of luminosity of the disk around the collapsar on the flux of matter entering its external boundary

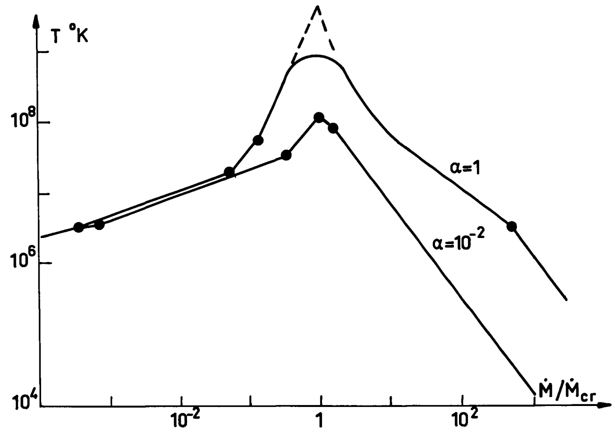

Fig. 6. Maximum effective temperature of radiation of the disk as a function of the flux of matter inflowing into it for different values of the efficiency α of the mechanisms of angular momentum transfer

central regions of the disk and reradiate the absorbed energy in the ultra-violet and optical spectral bands. Thus, from 0.1 – 10% of the total luminosity of the disk can be reradiated (see part III). The hard radiation must be reradiated both in the lines of the different elements and in the continuum.

Strong recombinational fluorescence of hydrogen must be observed with no apparent ionization source and there are possibly also lines of helium and highly ionized heavy elements. All these lines must be broad because the matter in the disk has large rotational velocities ($\gtrsim$ 100 km/s). The density of matter in the disk is high and forbidden lines should be absent.

Considerable ultra-violet luminosity of the disk can lead to the formation of Strömgren region which distinguishes a black hole from normal optical stars with similar optical luminosity. In certain conditions the hard radiation of the central regions of the disk can heat the matter in the outer regions up to high temperatures and evaporate the disk, decreasing the inflow of matter into the black hole. Such an autoregulation of accretion can essentially influence the luminosity of the disk around the black hole.

In a close binary system, a significant part of the X-radiation of the black hole can hit the surface of the normal star (Shklovsky, 1967) and be reradiated by its atmosphere, which can lead to an unusual optical appearance in such a system. This effect is observed now in the HZ Her = Her X1 system. The hemisphere of the optical component turned to X-ray source is three times brighter than opposite one and has a different spectral class (Cherepashchuk *et al.*, 1972; Lyutiy *et al.*, 1973)

When the rate of accretion increases, the luminosity grows linearly and the effective temperature of radiation rises (Figs. 5, 6). At fluxes $\dot{M} = 10^{-9} - 10^{-8}\,\frac{M_\odot}{\text{year}}$ the black hole is found to be a powerful X-ray star with luminosity $L = 10^{37} - 10^{38}\,\frac{\text{erg}}{\text{s}}$ and an effective temperature of radiation $T_r = 10^7 \div 10^8$ °K. The star radiates also in the optical and ultra-violet spectral bands.

Aperiodic variability of certain properties such as fluctuations of brightness, principally connected with the variability of infalling matter flux and its non-homogeneity, distinguish collapsars which radiate due to disk accretion of matter. In remote systems, the collapsar at the perigee of its orbit gets into the more dense part of the matter flux flowing out from the visible component. Therefore, the periodic variability of luminosity (non-sinusoidal in the general case!) should be expected. We also note the possibility of eclipses of the radiation of the central source by the disk if its plane does not coincide with the plane of rotation of the system. Such an orientation of the disk occurs, for example, when matter flows through the inner lagrangian point from the asynchronously rotating star, whose axis of rotation is inclined to the plane of rotation of the system. Taking into account the eclipse of the X-ray source by the adjacent star, the number of eclipses amounts to 3 per period of rotation. If the plane of the disk coincides with the plane of the system, then only the disk can be occulted or darkened whereas a thin disk covers only an insignificant fraction of the stellar surface. However, the most characteristic property of a black hole in a close binary system is its X-ray radiation. The detection of compact X-ray stars of mass $M > 2 \cdot M_{\odot}$ in binary system will be the proof of the existence of the black holes in the Galaxy.

For a neutron stars we can estimate (in order of magnitude) that the energy release of infalling matter per gramm is the same as for a black hole

$$\left(\eta \sim \frac{GM}{R_{ns}C^2} \sim 10 - 20\%\right)$$

However, accretion on a neutron star in a binary system has its own peculiarities.

In the case of a neutron star without a magnetic field, the disk extends to its surface. Therefore the disk radiates one half of the entire energy released at the accretion. The other half is radiated by the surface of the neutron star. Accretion to a rotating neutron star whose magnetic field does not coincide in direction with the axis of rotation can lead to the phenomenon of an X-ray pulsar (Schwartzman, 1971 a; Pringle and Rees, 1972) and can explain the pulsations of Her X 1 and Cen X 3.

At a subcritical rate of flow of matter into the Roche lobe of the black hole, we may assume that most of the inflowing matter is accreted. At a supercritical value of inflow of matter into the Roche lobe of a black hole or a neutron star[5]) (there is no major difference here) there should be a qualitatively different picture. The region surrounding the black hole is apparent where an effective outflow of matter under the influence of radiation pressure takes place. The outflow begins from a radius close to that at which the forces of radiation pressure and gravitation, pressing the matter to the plane of disk, are comparable. Only the critical flux of the matter can go under the radius $R_0 = \frac{6GM}{C^2}$.

The integrated luminosity of such an object is limited by the value of the critical Eddington luminosity, and the band of the electromagnetic spectrum in which most of the energy is radiated depends strongly on the density of the outflowing matter. The density of the matter is, in turn, a function of $\frac{\dot{M}}{\dot{M}_{cr}}$ and the efficiency α of mechanism of the transport of angular momentum, which determines velocity of the outflowing gas. If the flux $\dot{M}$ exceeds insignificantly the critical value and $\alpha \sim 1$, the radiation of the disk is reradiated by the outflowing gas practically without changing its spectral properties, i.e. the object is a source of X-ray radiation as before. If $\frac{\dot{M}}{\dot{M}_{cr}}$ increases and α decreases the opacity of the outflowing matter grows, and the radiation is re-emitted as quanta of smaller energy. If $\frac{\dot{M}}{\dot{M}_{cr}} \gtrsim 3 \cdot 10^3 \left(\frac{\alpha M_{\odot}}{M}\right)^{2/3}$ the black hole turns into a bright optical star. The smaller is the parameter α, the greater the effective radius of the radiating envelope and the smaller is the effective temperature of the radiation.

By virtue of the fact that the angular momentum of the ejected matter is conserved relative to the axis rotation of the disk a strongly anisotropic picture of matter outflow can be observed. The hot plasma is ejected with high velocity in a narrow cone about the axis of rotation. The optical depth of the outflowing gas in this cone is not great and, at a specific orientation of the binary system relative to the observer, the X-ray radiation of the black hole together with the optical should be observed.

The observational appearance of the black hole in a strongly supercritical regime of accretion can be characterized as follows: the luminosity is fixed at the Eddington critical limit $L_{cr} \approx 10^{38} \frac{M}{M_{\odot}} \frac{\text{erg}}{\text{s}}$; most of the energy is radiated in the ultra-violet and optical regions of the spectrum; in the upper, rarefied layers of the outflowing matter, broad emission lines are formed. In consequence, at the supercritical regime of accretion the black hole may appear as a hot, optical star. There is a strong mass outflow with velocities $v \sim \alpha \cdot 10^5 \left(\frac{\dot{M}_{cr}}{\dot{M}}\right)^{1/2} \frac{\text{km}}{\text{s}}$ and the star is surrounded by a colder disk where the accreting matter enters the collapsar. Eclipses of the black hole by the normal component are possible as well as eclipses of the star by the matter flowing out from the black hole. The latter is opaque due to Thomson's scattering to large distances from the black hole ($R_T \sim 10^{10} - 10^{12}$ cm). In the radiorange, the hot, outflowing matter becomes opaque far from the binary system. It can be a source of an appreciable thermal radiation with a smooth dependence of intensity on frequency ($J_\nu \sim \nu^{2/3}$).

In the radio range (this relates also to subcritical accretion) non-thermal radiation mechanisms connected with the existence of magnetic fields (which may achieve $H \sim 10^5 - 10^7$ Gauss) and beams of fast outflowing particles can also appear (Lynden-Bell, 1969).

Apparently, the "quiet" disk, radiating only due to thermal mechanisms, can really exist at low values of the parameter α. If $\alpha \sim 1$, the new important effects (connected with turbulent convectivity; plasma turbulence; reconnection of magnetic field lines through neutral points, leading to solar type flares; the acceleration of particles) and non-thermal radiation can appear. The

[5]) Critical accretion to the neutron star leads to its collapse in $\sim 3 \cdot 10^7$ years.

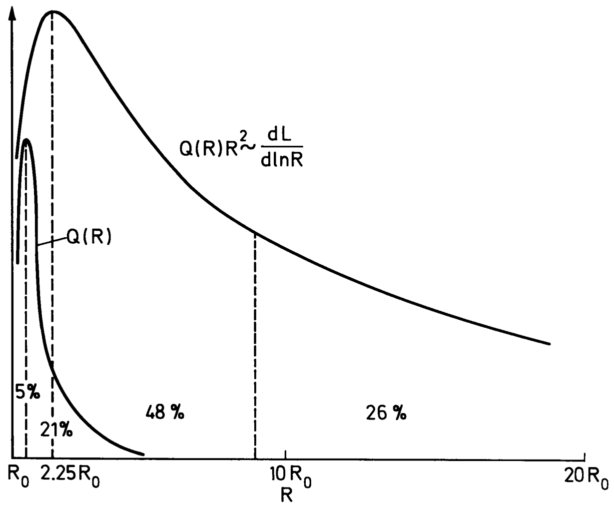

Fig. 7. The luminosity of the surface unit of the disk as a function of the radius. The function $Q(R)\,R^2$ is proportional to the luminosity of the ring with radius R and $\Delta R \sim R$. The numbers illustrate the contribution of the corresponding regions to the integral luminosity of the disk

flares and hot spots on the rotating disk surface must lead to the short term fluctuations of radiation flux in some spectral bands. The variability may have a stochastic (Schwartzman, 1971b) and/or quasiperiodic nature (Sunyaev, 1972). Quasiperiod of this fluctuations must be of the order of rotational period

$$t \sim 2\frac{\pi R}{v_\varphi} \sim 6\cdot 10^{-4}\left(\frac{R}{3R_g}\right)^{3/2}\left(\frac{M}{M_\odot}\right)\,\mathrm{s}$$

and depends on the distance of the hot spot from the collapsar. According to the Fig. 7, the main part of the energy released is radiated from the region with $6R_g < R < 30R_g$. At $M \sim 10\,M_\odot$ the corresponding quasiperiods $t \sim 2\cdot 10^{-2} \div 0.2$ s are of the order of observed in Cyg X1. Minimal period of the radiation of disk with the hot spot in the Kerr gravitational field of the rotating black hole is 8 times less, than in the Schwarzschild field of the non-rotating one with equal mass (Sunyaev, 1972).

II. The Theory of Disk Accretion at Subcritical Regime

We consider a disk around the black hole in the case of the existence of an effective mechanism of outwards transport of angular momentum. All calculations are carried out using newtonian mechanics. It is necessary to take into account the effects of general relativity only in the region $R < 3R_g$. Near the black hole at $R < 3R_g$ stable circular orbits are not possible (Kaplan, 1949), and the motion acquires a radial character without transport of angular momentum. In the last stable orbit the binding energy is given by $0.057\,m_0c^2$ and it must escape before the particle reaches the radius $R = 3R_g$. The energy is also radiated by matter moving in a zone with $R_g < R < 3R_g$ but the contribution of this zone to the total luminosity of the disk does not appreciably alter the estimate of the energy released. In our calculations we shall assume that $0.06\,c^2$ energy per unit mass of infalling matter is released and that the total luminosity of the disk is $L_0 = 0.06\,c^2\dot{M}$. The radius $R_0 = 3R_g$ should be taken as the inner boundary of the disk, because, that in the range $R < R_0$ the matter falls without any external observable effect.

1. Mechanisms of Angular Momentum Transfer

In a differentially rotating medium, tangential stresses between adjacent layers, which are connected with existence of a magnetic field, turbulence and molecular and radiative viscosity are the mechanisms of transport of angular momentum. In the conditions of interest to us the role of molecular viscosity is negligibly small and cannot lead to disk accretion; neither can angular momentum transport by means of radiation (which itself is the consequence of accretion).

Magnetic fields exist in stars. They must also occur in the matter flowing out from the normal star and then accreting to the black hole. Depending on the initial conditions, the method of matter compression and on the degree of ordering of the magnetic field, the energy of the latter can be less than the thermal energy of matter $\varepsilon = 3\varrho\frac{kT}{m_P} + bT^4 = \varrho\frac{v_s^2}{2}$, as well as sufficiently can exceed it, reaching the value $\frac{H^2}{8\pi} \sim \varrho\frac{GM}{R}$.

In this last case, the stresses $\left(w_{r\varphi} = \left|\frac{H_r \times H_\varphi}{8\pi}\right|\right)$ and the efficiency of angular momentum transport are so high, that radial accretion will occur (Bisnovaty-Kogan and Sunyaev, 1971; Schwartzman, 1971a).

The magnetic field in the gas flowing into the disk may have a regular structure at the distances comparable to the outer radius of the disk. But as always div $\boldsymbol{H} = 0$, the radial component of the magnetic field must have alternating sign. If alternation of the sign of the magnetic field takes place in the disk, then the differential rotation rapidly leads to division of the large magnetic loops into smaller ones. Hence, within the disk, the field is most likely to be chaotic and of small scale. In this case, because of plasma instabilities, reconnection of the magnetic field lines in regions of opposite polarity field, etc..., the energy of the magnetic field in the disk evidently cannot exceed the thermal energy of the matter. The latter is characterized by the velocity of sound v_s

$$\frac{H^2}{8\pi} < \varrho\frac{v_s^2}{2} \quad \text{and} \quad w_{r\varphi} \sim -\varrho v_s^2\left(\frac{H^2}{4\pi\varrho v_s^2}\right). \tag{1.1}$$

Now, in the absence of complete theory of turbulence on one hand, and of some observational check of existence of turbulence in the disks on the other hand, we may only assume its presence. In a differentially rotating medium with a distribution of angular momentum increasing outwards, matter is stable with respect to small shifts preserving the angular momentum. But the theory of small perturbations, being linear, can give, at best, only the condition of loss of stability of a laminar flow. The energy criterion (Vasjutinsky, 1946), which takes into account non-linearity, i.e. interaction between the turbulent pulsations at high Reynolds numbers, leads to the conclusion that in differential rotation a self-perpetuating turbulence is always possible regardless of the angular momentum distribution. Experiment (Taylor, 1937) also shows that there exists the boundary behind which fluid, rotating between two coaxial cylinders, develops turbulence (even in the case of rotation of the outer cylinder, the inner one being at rest) although conditions of stability relative to the distribution had been applied. The results obtained for an incompressible fluid are not, of course, completely applicable to the gaseous disk, but the conclusion concerning perpetuation of turbulence developed in the disk, where the Reynolds numbers are especially large, apparently remains valid. We must also bear in mind that additional sources of turbulence connected with release of gravitational energy and transfer of the radiative flux to the surface layers exist in the disk. In this case, the maximal scale of the turbulent cell l is probably of the order of the disk thickness z_0.

In scales comparable with the radius R, turbulence is homogeneous and isotropic. To describe the average motions in the presence of such turbulence one may use the formulae obtained for laminar flows on replacing the molecular viscosity by the turbulent one $\eta_t = \varrho v_t l$. For tangential stresses, we have

$$w_{r\varphi} = \eta_t R \frac{d\omega}{dR} \sim -\eta_t \frac{v_\varphi}{R} \sim -\varrho v_s^2 \frac{v_t}{v_s}$$

where the disk thickness $z_0 \sim R\frac{v_s}{v_\varphi}$. Here, v_t is the turbulent velocity, v_φ is the circular keplerian velocity. Thus the efficiency of two of the most important mechanisms of angular momentum transport connected with the magnetic field (which always is present in astrophysical conditions) and turbulence (whose existence in the disk is less definite)

$$-w_{r\varphi} \sim \varrho v_s^2 \left(\frac{v_t}{v_s}\right) + \varrho v_s^2 \left(\frac{H^2}{4\pi\varrho v_s^2}\right) = \alpha \varrho v_s^2 \qquad (1.2)$$

can be characterized by only one parameter, α. In the case of a turbulent mechanism $\alpha < 1$ always. For $\alpha > 1$ turbulence must be supersonic and leads to rapid heating of the plasma and to $\alpha \leqq 1$. For magnetic transport of angular momentum in the disk, it is likely that $\alpha < 1$. For a wide range of the initial conditions, it is possible that $\alpha \ll 1$. In addition, the parameter α can (and must) be a function of the disk radius[6]). Below, we point out that in the wide range

$$10^{-15}\left(\frac{\dot M}{\dot M_{cr}}\right)^2 < \alpha < 1$$

the structure of the disk is not essentially changed. This result allows us to compute the external appearance of the disk, the character of energy release, the radiation spectrum etc..., without deciding upon the mechanism of angular momentum transfer, an exact account of which is difficult.

2. The Structure of the Disk

To a first approximation, the matter in the disk may be assumed to rotate in circular keplerian orbits

$$v_\varphi = \sqrt{\frac{GM}{R}}, \quad \omega = \sqrt{\frac{GM}{R^3}}. \qquad (2.1)$$

The friction between the adjacent layers, connected with the existence in the disk of turbulence and chaotic, small scale magnetic fields, leads to the loss of the angular momentum of the particles. A radial component of velocity appears and the particles spiral inward to the black hole

$$\frac{u_0 d\omega R^2}{dt} = -u_0 v_r \frac{d\omega R^2}{dR} = \frac{1}{R}\frac{d}{dR} W_{r\varphi} R^2. \qquad (2.2)$$

Here, $u_0 = 2\int_0^{z_0} \varrho dz$ is the surface density of matter in the disk, $W_{r\varphi}$ is the stress between adjacent layers. According to (1.2)

$$W_{r\varphi} = 2\int_0^{z_0} w_{r\varphi} dz = -\alpha u_0 v_s^2$$

In stationary conditions, $v_r < 0$ and $\dot M = 2\pi u_0 v_r R$ = const and integrating (2.2) we obtain

$$\dot M \omega R^2 = -2\pi W_{r\varphi} R^2 + C. \qquad (2.3)$$

Practically the whole angular momentum is transported outward and only a small part $\sim \sqrt{\frac{3R_g}{R_1}}$ of the initial angular momentum falls together with the matter into the black hole (R_1 is the outer radius of the disk). The constant in (2.3) is determined by the condition, that $W_{r\varphi} \simeq 0$ on the last stable orbit ($R_0 = 3R_g$ in the Schwarzschild gravitational field of black hole or a neutron star with $R_s < R_0$, or the corresponding Kerr metric value for rotating black hole). To describe the loss of stability one must make a consequent general relativistic theory, which is worked through in Novikov and Thorne (1972) lectures.

In the case of nonrotating black hole const. in (2.3) equals to $C = \dot M \omega(R_0) R_0^2$ and

$$\dot M \omega \left[1 - \left(\frac{R_0}{R}\right)^{1/2}\right] = 2\pi\alpha u_0 v_s^2. \qquad (2.4)$$

In the case of a neutron star with $R_s > R_0$ and without magnetic field the condition $W_{r\varphi} \simeq 0$ is fulfilled practically at the surface: during the slow decrease of R, the

[6]) The dependence $\alpha(R)$ for the turbulent mechanism may be approximately evaluated from the experimental data (Taylor, 1937) on supposing the scale of the turbulence to be equal to the width of the channel between the cylinders. At the periphery of the disk, where $Z_0/R \ll 1$, the coefficient may be of the order of unity; in the vicinity of a black hole in the supercritical regime, when the accretion picture is sphereazied $Z_0/R \sim 1$ and the coefficient $\alpha \sim 10^{-3}$.

angular velocity ω first increases according to the Kepler law and then suddenly goes practically to zero in a thin layer of rotating gas supported by pressure. The maximum of ω is nearly equal to the last keplerian value $\sqrt{\frac{GM}{R_s^3}}$ at R_s and const. in (2.3) equals to $\dot{M}\omega(R_s)R_s^2$ because $W_{r\varphi}(R_s)\simeq 0$.

A selfconsistent axialsymmetric picture uses injection of matter at some R_1. Part of matter situated at R_1 falls on the star but one must imagine also matter flow outward R_1, taking away the excess of an angular momentum.

In a direction perpendicular to the plane of the disk the normal component of the gravitational force of the star is balanced by the sum of the gradients of the gas, radiation turbulent and magnetic pressures. The equation of hydrostatic equilibrium gives the half-thickness of the disk.

$$z_0 = \frac{v_s}{v_\varphi} R . \tag{2.5}$$

In losing their angular momentum the particles also lose their gravitational energy. Part of the latter goes to increasing the kinetic energy of rotation and the other part is converted into thermal energy and can be radiated from the surface of the disk. The forces leading to the angular momentum transfer in a rotating system are also inducing the energy flow equal to $-2\pi W_{r\varphi}R^2\omega$. In keplerian motion with ω increasing inward and $W_{r\varphi}<0$, the energy flow is directed outward. The rate of the energy dissipation in the ring between R_2 and R_3 has a term equal to divergence of this energy flow. Collecting all terms, one obtains the energy flux, radiated from surface unit of the disk in unit of the time

$$\begin{aligned} Q &= \frac{1}{2} W_{r\varphi} R \frac{d\omega}{dR} \\ &= \frac{1}{4\pi R}\frac{d}{dR}\left[\dot{M}\left(\frac{v_\varphi^2}{2}-\frac{GM}{R}\right)-2\pi R^2 W_{r\varphi}\omega\right] \\ &= \frac{3}{8\pi}\dot{M}\frac{GM}{R^3}\left\{1-\left(\frac{R_0}{R}\right)^{1/2}\right\}. \end{aligned} \tag{2.6}$$

At $R \gg R_0$, R_s the energy flux is equal to $\frac{3}{8\pi}\frac{GM}{R^3}\dot{M}$ and the release of energy between the radii R_2 and R_3 is equal to $L=4\pi\int QRdR=\frac{3}{2}\dot{M}GM\left(\frac{1}{R_2}-\frac{1}{R_3}\right)$. This value is increased three times as compared with the release of gravitational energy in the same region. The unexpected energy is actually released from gravity at much smaller radii, and then is transported outward mechanically by the shear stresses before being converted into heat[7]). In the contrary at $R-R_0 \ll R$ the energy flux, decreases (going to zero) near the last stable orbit (Novikov and Thorne, 1972). The maximum of $Q(R)$ takes place at $R=1.36\ R_0$. The main contribution to integral luminosity of the disk $L(R_0)$ gives the region with $R=2.25\ R_0$, where $Q(R)R^2$ has a maximum. At a given flux Q, the energy density of radiation inside the layer with the surface density u_0 is determined by the relation

$$\varepsilon = \frac{3}{4}\frac{Q}{c}\sigma u_0 = \frac{9}{32\pi}\dot{M}\frac{GM}{R^3}\frac{\sigma u_0}{c}\left[1-\left(\frac{R_0}{R}\right)^{1/2}\right] \tag{2.7}$$

where σ is the opacity of the matter. In the conditions considered, the main contribution to the opacity comes from Thomson scattering on free electrons of cross-section 6.65 $10^{-25}\,\text{cm}^2$ and free-free absorption for which $\sigma_{ff}=0.11\,T^{-7/2}n\,\frac{\text{cm}^2}{\text{g}}$ (Zeldovich and Rayzer, 1966). Inside the disk, which is optically thick with respect to the "true" absorption, $\tau=\sigma_{ff}u_0$, or $\tau^*=\sqrt{\sigma_T\sigma_{ff}}u_0$, if $\sigma_T>\sigma_{ff}$, there exists complete thermodynamic equilibrium and the energy density of radiation is equal to $\varepsilon=bT^4$. This last expression, the dynamic Eq. (2.4) and the equation of energy balance (2.7) form a closed system of equations. Upon solving them, we find the distribution of surface density of matter $u_0(R)$ and the temperature $T(R)$ along the radius of the disk as functions of the mass flux $\dot{M}$, the mass of the black hole M and the efficiency α of the angular momentum transport mechanism.

In the general case of rotating black hole the releasing gravitational energy is transferred mechanically from the relativistic region $\frac{1}{2}R_g<R<3R_g$ into the nonrelativistic one $R>3R_g$ and dissipate there. Changing the value of constant in Eq. (2.3) it is easy to take into account this contribution to Q at $R>3R_g$. According to (2.6) in the case of nonrotating black hole the energy flux from a surface unit at first increases with approaching to collapsar, reaches the maximum at $R=1{,}36\ R_0$ and then decreases. Only 5% of the total disk luminosity is radiated at $R<1.36R_0$. Therefore we consider this region only schematically.

The disk may be considered to be composed of a number of distinct parts:

a) the radiation pressure is dominant and $v_s^2=\frac{\varepsilon}{3\varrho}$. In the interaction of matter and radiation, electron scattering on free electrons plays the main rôle;

b) the pressure is determined by the gas pressure and $v_s^2=\frac{kT}{m_p}$; electron scattering gives still the main contribution to the opacity;

c) the speed of sound is given by $v_s^2=\frac{kT}{m_p}$ and the opacity is determined by free-free absorption and other mechanisms. Two regions c) are the extensive outermost and the very narrow closest to the black hole. In the region of the maximal energy flux the radiation

[7]) Dr. K. S. Thorne directed our attention to this increase and to the importance of the last term in square brackets in the Eq. (2.6).

pressure is dominant. Two regions b) are intermediate between a) and c).

It is convenient to introduce nondimensional parameters

$$m=\frac{M}{M_\odot},\quad \dot{m}=\frac{\dot{M}}{\dot{M}_{cr}}=\frac{\dot{M}}{3\cdot 10^{-8}\frac{M_\odot}{\mathrm{yr}}}\times\left(\frac{M_\odot}{M}\right),$$

$$r=\frac{R}{3R_g}=\frac{1}{6}\frac{Rc^2}{GM}=\frac{M_\odot}{M}\frac{R}{9\,\mathrm{km}}.$$

Let us consider the region a) $P_r \gg P_g$, $\sigma_T \gg \sigma_{\rm ff}$. The half-thickness of the disk, corresponding to (2.5) and (2.7), is

$$z_0[\mathrm{cm}]=\frac{3}{8\pi}\frac{\sigma_T}{c}\dot{M}(1-r^{-1/2})=3.2\cdot 10^6\dot{m}m\,(1-z^{-1/2}) \tag{2.8}$$

i.e. in this region at $r \gg 1$ the disk has a constant thickness along the radius, whose value depends only on the flux of accreting matter. The result (2.8) is expected in view of the fact that z_0 is determined by equating the force of the radiation pressure $F \sim Q \sim \dot{M}R^{-3}$ to the component of the gravitational force, normal to the plane of the disk which is also proportional to R^{-3}. The maximal ratio $\frac{z_0}{R}=\dot{m}$ is reached at $r=2.25$.

Substituting $v_s^2=\frac{\varepsilon}{3\varrho}$ into (2.3) and bearing in mind that $u_0=2\varrho_0 z_0$, we obtain from (2.3) and (2.7)

$$u_0\left[\frac{\mathrm{g}}{\mathrm{cm}^2}\right]=\frac{64\pi}{9\alpha}\frac{c^2}{\sigma^2}\frac{1}{\omega\dot{M}(1-r^{-1/2})} \tag{2.9}$$
$$=4.6\alpha^{-1}\dot{m}^{-1}r^{3/2}(1-r^{-1/2})^{-1},$$

$$\varepsilon\left[\frac{\mathrm{erg}}{\mathrm{cm}^3}\right]=2\frac{c}{\sigma}\omega=2.1\cdot 10^{15}\alpha^{-1}m^{-1}r^{-3/2}, \tag{2.10}$$

$$\left.\begin{aligned}
n[\mathrm{cm}^{-3}]&=\frac{u_0}{2m_p z_0}\\
&=4.3\cdot 10^{17}\alpha^{-1}\dot{m}^{-2}m^{-1}r^{3/2}(1-r^{-1/2})^{-2}\\
v_r\left[\frac{\mathrm{cm}}{\mathrm{s}}\right]&=\frac{\dot{M}}{2\pi u_0 R}\\
&=7.7\cdot 10^{10}\alpha\dot{m}^2 r^{-5/2}(1-r^{-1/2})\\
H[\mathrm{Gauss}]&\leqq\sqrt{\frac{4\pi}{3}\alpha\varepsilon}=10^8 m^{-1/2}r^{-3/4}.
\end{aligned}\right\} \tag{2.11}$$

Assuming, that the disk is optically thick with respect to the "true" absorption, i.e. the relation $\varepsilon=bT^4$ is satisfied, we find from (2.10) the temperature of the plasma and of the radiation inside the disk

$$T=2.3\cdot 10^7(\alpha m)^{-1/4}r^{-3/4}\,{}^\circ\mathrm{K}. \tag{2.12}$$

For a plasma with $\sigma_T \gg \sigma_{\rm ff}$ a "true" optical depth with respect to absorption is determined as: $\tau^*=\sqrt{\sigma_T\sigma_{\rm ff}}u_0$ i.e.

$$\tau^*=8.4\cdot 10^{-5}\alpha^{-17/16}m^{-1/16}\dot{m}^{-2}\cdot r^{-93/32}(1-r^{-1/2})^{-2}.$$

From conditions that $\tau^*>1$, one finds that the disk is opaque if

$$r>25\,\alpha^{34/93}\dot{m}^{64/93}m^{2/93}(1-r^{-1/2})^{64/93} \tag{2.13}$$

and that the assumption of local thermodynamic equilibrium inside the disk is justified. The inequality (2.13) is only an upper limit. When $y=\frac{kT}{m_e c^2}\tau_T^2>1$ Compton scattering plays the dominant rôle in the formation of the radiation spectrum because of the Doppler shift in the frequency of the bremsstrahlung quanta (Kompaneets, 1956; Illarionov and Sunyaev, 1972). In this case an equilibrium, black body spectrum is formed when

$$\tau_T\tau_{\rm ff}\ln^2\frac{2.35}{x_0}\gtrsim 1. \tag{2.14}$$

Here

$$\left.\begin{aligned}
x_0&=\frac{h\nu_0}{kT}=3\cdot 10^5\frac{n^{1/2}}{T^{9/4}}\sqrt{g(x_0)};\\
g(x_0)&=\frac{\sqrt{3}}{\pi}\ln\frac{2.35}{x_0}.
\end{aligned}\right. \tag{2.15}$$

ν_0 is a frequency near which the rates of the free-free and Compton processes are comparable. The factor $A=\frac{3}{4}\ln^2\frac{2.35}{x_0}=2.5\,g^2(x_0)=L_c^-/L_{\rm ff}^-$ characterizes the relation of the Compton energy losses with the bremsstrahlung losses. In the physical conditions of interest to us the factor A ranges from 10 to 300. Using condition (2.14), it is easy to show that, even, at $\alpha\sim 1$ and $\dot{m}\sim 1$ local thermodynamic equilibrium exists inside the disk up to $R\approx 10R_0$.

A high temperature $T \gg 10^7\,{}^\circ$K of the matter near the black hole corresponds to narrow intervals in the values of $\alpha\sim 1$ and $\dot{m}\sim 1$. The energy losses of the plasma due to radiation are limited by the low rate of production of photons due to free-free processes. Synchrotron radiation of thermal electrons in the magnetic field and plasma radiation at the Langmuire frequency are additional sources of quanta. Comptonization of the low frequency radiation leads to an increase in the energy losses as well as to a decrease in the plasma temperature (Gnedin and Sunyaev, 1972). These mechanisms of quanta production are of importance in the above mentioned conditions because, at $\alpha\sim 1$ and $r\sim 1$, the gyrofrequency $\nu_H=\frac{eH}{2\pi m_e c}=\frac{e}{2\pi m_e c}\sqrt{4\pi\alpha\varrho v_s^2}$ and Langmuire frequency $\nu_{pe}=\sqrt{\frac{e^2 n}{\pi m_e}}$ exceed ν_0. Estimations show that nowhere in the disk the temperature exceed $T\simeq 10^9\,{}^\circ$K.

Correlations similar to (2.8 – 2.14) can be also obtained for the remaining parts of the disk. For regions b) and c) we give only the final expressions; their derivation is similar to the one given above for the region a)

b) $P_g \gg P_r, \quad \sigma_T \gg \sigma_{ff}$

$$\begin{aligned} u_0 &= 1.7 \cdot 10^5 \alpha^{-4/5} \dot{m}^{3/5} m^{1/5} r^{-3/5} (1-r^{-1/2})^{3/5} \\ T &= 3.1 \cdot 10^8 \alpha^{-1/5} \dot{m}^{2/5} m^{-1/5} r^{-9/10} (1-r^{-1/2})^{2/5} \\ z_0 &= 1.2 \cdot 10^4 \alpha^{-1/10} \dot{m}^{1/5} m^{9/10} r^{21/20} (1-r^{-1/2})^{1/5} \\ n &= 4.2 \cdot 10^{24} \alpha^{-7/10} \dot{m}^{2/5} m^{-7/10} r^{-33/20} (1-r^{-1/2})^{2/5} \\ \tau^* &= \sqrt{\sigma_{ff}\sigma_T}\, u_0 = 10^2 \alpha^{-4/5} \dot{m}^{9/10} m^{1/5} r^{3/20} (1-r^{-1/2})^{9/10} \\ v_r &= 2 \cdot 10^6 \alpha^{4/5} \dot{m}^{2/5} m^{-1/5} r^{-2/5} (1-r^{-1/2})^{-3/5} \\ H &\leqq 1.5 \cdot 10^9 \alpha^{1/20} \dot{m}^{2/5} m^{-9/20} r^{-51/40} (1-r^{-1/2})^{2/5} . \end{aligned} \tag{2.16}$$

The boundaries between the regions a) and b) lie on the radii

$$\frac{r_{ab}}{(1-r_{ab}^{-1/2})^{16/21}} = 150(\alpha m)^{2/21} \dot{m}^{16/21} . \tag{2.17}$$

From condition (2.17) we find that region a) of the disk exists only if

$$\dot{m} \gtrsim \frac{1}{170} (\alpha m)^{-1/8} \tag{2.18}$$

c) $P_r \ll P_g, \quad \sigma_{ff} \gg \sigma_T$

$$\begin{aligned} u_0 &= 6.1 \cdot 10^5 \alpha^{-4/5} \dot{m}^{7/10} m^{1/5} r^{-3/4} (1-r^{-1/2})^{7/10} \\ T &= 8.6 \cdot 10^7 \alpha^{-1/5} \dot{m}^{3/10} m^{-1/5} r^{-3/4} (1-z^{-1/2})^{3/10} \\ z_0 &= 6.1 \cdot 10^3 \alpha^{-1/10} \dot{m}^{3/20} m^{9/10} r^{9/8} (1-r^{-1/2})^{3/20} \\ n &= 3 \cdot 10^{25} \alpha^{-7/10} \dot{m}^{11/20} m^{-7/10} r^{-15/8} (1-r^{-1/2})^{11/20} \\ \tau &= \sigma_{ff} u_0 = 3.4 \cdot 10^2 \alpha^{-4/5} \dot{m}^{1/5} m^{1/5} (1-r^{-1/2})^{1/5} \\ v_r &= 5.8 \cdot 10^5 \alpha^{4/5} \dot{m}^{3/10} m^{-1/5} r^{-1/4} (1-r^{-1/2})^{-7/10} \\ H &\lesssim 2.1 \cdot 10^9 \alpha^{1/20} \dot{m}^{17/40} m^{-9/20} r^{-21/16} (1-r^{-1/2})^{17/40} \end{aligned} \tag{2.19}$$

The boundaries between regions b) and c) lie near

$$r_{bc} = 6.3 \cdot 10^3 \dot{m}^{2/3} (1-r_{bc}^{-1/2})^{2/3} . \tag{2.20}$$

Our approximation is valid in the case $v_r \ll v_\varphi$. Therefore the formulae given above are valid only at $r-1 > 10^{-6} \cdot \alpha^{8/7} \dot{m}^{3/7}$. At $r-1 < 10^{-6}$ more complicated consideration is necessary. In addition any negligibly small energy flux from the relativistic region strongly influences the physical conditions in the vicinity of R_0, without any influence upon the conditions in the region $R > \frac{49}{36} R_0$, where the main part of the energy released is radiated and our consideration is applicable. Below we consider only this region. Analysing the formulae obtained above, we note the weak dependence of the thickness of the disk on the efficiency α of the mechanism of angular momentum transport. When α decreases, the surface density of the disk increases rapidly and the radial velocity of motion drops, but the disk thickness grows only as $\alpha^{-1/10}$ and is comparable with the radius only if $\alpha \sim 10^{-15} \dot{m}^2$. Disk accretion is in fact realized at sufficiently weak turbulence or at small values of the magnetic field. At extremely small α, nuclear reactions can give some contribution to the energy release and neutrino processes can influence energy losses. We would also like to point out the weak variation of the optical depth τ^* for the "true" absorption in the region c) on changes in the accretion rate $\dot{M}$, i.e. there the disk is opaque, as a rule.

2a. *Disk Structure along the Z-Coordinate*

In the direction perpendicular to the disk plane, hydrostatic equilibrium exists. The pressure gradient is balanced by the component of the gravitational attraction normal the disk plane (selfgravitation of the disk is negligibly small):

$$\frac{1}{\varrho} \frac{dP}{dz} = -\frac{GM}{R^3} z . \tag{2.21}$$

The equation (2.21) together with the equation of energy balance

$$\frac{1}{\varrho} \frac{dq}{dz} = \frac{3}{4\pi} \frac{GM}{R^3} \frac{\dot{M}}{u_0} \left[1 - \left(\frac{R_0}{R}\right)^{1/2}\right] . \tag{2.22}$$

and the equation of radiative transfer:

$$\frac{c}{3\sigma\varrho} \frac{d\varepsilon_r}{dz} = -q(z) \tag{2.23}$$

form a closed system of equations. Solution of this system determines the distribution of physical quantities along the Z-coordinate. It is easy to integrate Eq. (2.22)

$$q = 2Q \frac{u(z)}{u_0} \tag{2.24}$$

where

$$Q = \frac{3}{8\pi} \frac{GM}{R^3} \dot{M} \left[1 - \left(\frac{R_0}{R}\right)^{1/2}\right] \quad \text{and} \quad u(z) = \int_0^z \varrho(z) dz .$$

In regions a) and b) electron scattering dominates the opacity and, in (2.23), one may assume σ to be equal to $\sigma_T = 0.4 \frac{\text{cm}^2}{\text{g}}$. Postulating thermodynamic equilibrium inside the disk, we obtain from (2.22) and (2.23):

$$T^4 = T_c^4 \left[1 - \frac{3Q\sigma_T u_0}{cbT_c^4} \left(\frac{u}{u_0}\right)^2\right] .$$

Near the surface of the disk, $Q = \frac{cb}{4} T_s^4$ and $T_c^4 = T_s^4 \left[1 + \frac{3}{16} \sigma_T u_0\right]$. Therefore for an opaque disk $\left(\frac{3}{16} \sigma_T u_0 \gg 1\right)$ we obtain

$$T(u) = T_c \left[1 - 4\left(\frac{u}{u_0}\right)^2\right]^{1/4} . \tag{2.25}$$

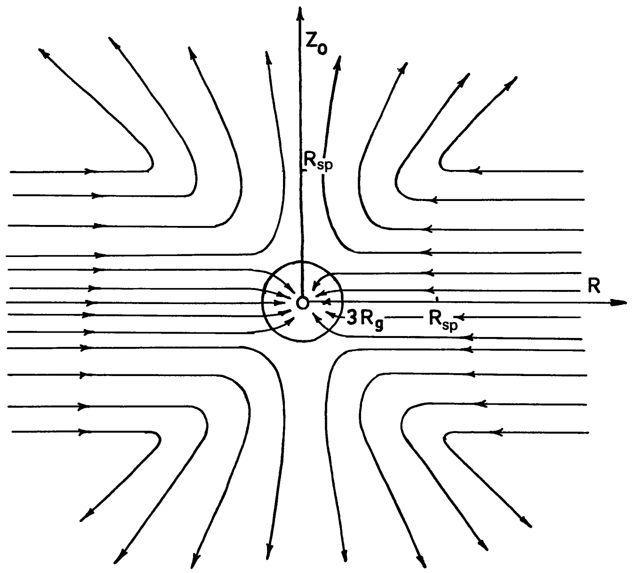

Fig. 8. Lines of matter flow at supercritical accretion (the disk section along the Z-coordinate). When $R < R_{sp}$ spherization of accretion takes place and the outflow of matter from the collapsar begins

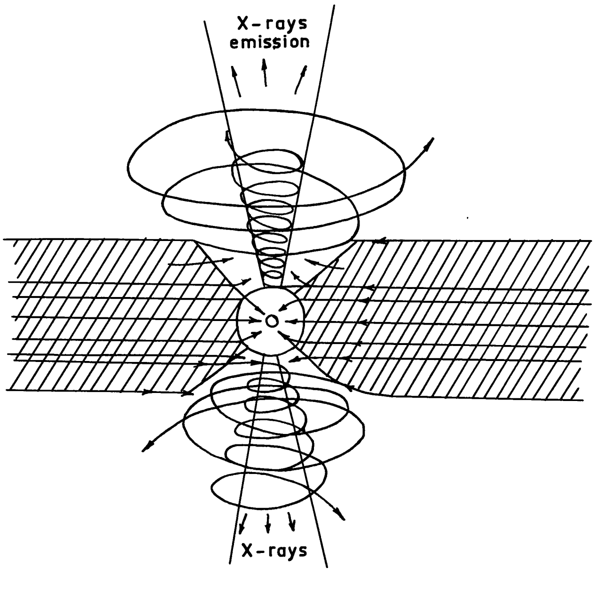

Fig. 9. The outflow of the matter from the collapsar at the supercritical regime of accretion

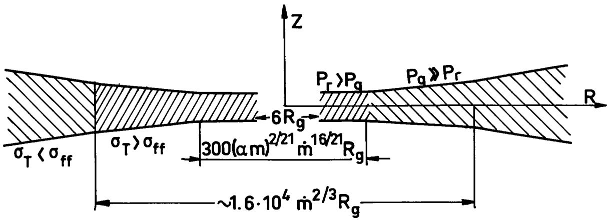

Fig. 10. The regions of disk having different physical conditions

The temperature does not vary much at low

$$u = \int_0^z \varrho dz \ll \frac{u_0}{2} = \int_0^\infty \varrho dz$$

and correspondingly at low optical depths $\tau = \sigma_T u(z)$ and low Z. Therefore, the disk structure may be characterized by a central temperature depending only on the coordinate R, and the dependence on the coordinate Z may be neglected. However, the outgoing radiation spectrum, formed in the upper layer of the disk is strongly dependent on the density and temperature distribution along the coordinate z.

In the region a) (closest to the collapsar) radiation pressure $P_r = \frac{\varepsilon}{3}$ dominates. According to (2.21) and (2.23)

$$q(z) = \frac{c}{\sigma_T} \frac{GM}{R^3} z. \tag{2.26}$$

Furthermore, $q(u) = 2Q \frac{u}{u_0}$ and $\varrho = \frac{du}{dz}$, and therefore the disk must be homogeneous with a sharp (depending only on the temperature of the plasma, the turbulent and magnetic pressure) decrease of matter density at $z > z_0$.

In regions b) and c) the gas pressure $p = \varrho \frac{kT}{m_p}$ dominates.

For $\frac{u}{u_0} \ll \frac{1}{2}$, the temperature in the disk is practically constant (2.25), and the density decreases according to a gaussian curve $\varrho = \varrho_0 \exp\left[-\left(\frac{z}{z_0}\right)^2\right]$. With increasing z and u, the temperature rapidly decreases and, according to (2.21), the density drops more rapidly.

In zone b), the outgoing radiation spectrum is formed at the depth defined by the condition $\tau^* = \int_u^\infty \sqrt{\sigma_T \sigma_{ff}} \cdot du \sim 1$. At $z > z_1$ the plasma temperature is practically constant. Therefore, according to (2.25), at $z > z_1$ we can assume the density profile

$$\varrho = \varrho(z_1) \exp\left(-\frac{z}{H_0}\right) \tag{2.28}$$

where $H_0 = \frac{R^3 kT(z_1)}{GM m_p z_1}$. The numerical solution of the system of equations (2.21 ÷ 2.23) showed that because of the rapid decrease of the temperature at $z > z_0$ for any conditions $z_1 \approx 1.2 - 1.5 z_0$. In the estimates below we shall assume $z_1 \simeq z_0$.

3. *Radiation Spectrum of the Disk*

a) Local Radiation Spectrum

The spectrum shape formed at the disk surface depends on its structure and temperature (which was calculated in the previous section) and, therefore, on the distance to the black hole. The local spectrum of the thermal radiation in the conditions of interest to us may be one of three typical distributions (Fig. 2): a Planck distribution (in the outer regions of the disk), a Wien distribution (in the inner regions) and a spectrum of radiation which has passed through the medium with scattering presumably playing a rôle in the opacity (intermediate region of the disk).

In the outer $r > 800\,\alpha^{4/57}\mathrm{m}^{-46/57}\dot{\mathrm{m}}^{37/57}$ regions[8]), where free-free processes (as well as free-bound processes and absorption in the lines of heavy elements broadened by the gas pressure) give the main contribution to the opacity a planckian spectrum of radiation

$$F(x)=B(x)=\frac{2\pi h}{c^2}\left(\frac{kT}{h}\right)^3\frac{x^3}{e^x-1}, \quad \text{where} \quad x=\frac{h\nu}{kT} \tag{3.1}$$

is formed on the disk surface (more exactly, at a depth of $\tau_{ff}\approx 1$). The corresponding flux of energy is equal to

$$Q=\int F(x)dx=\frac{c}{4}bT_s^4\,\frac{\mathrm{erg}}{\mathrm{cm}^2\,\mathrm{s}}$$

In the intermediate region

$800\,\alpha^{4/57}\mathrm{m}^{-46/57}\dot{\mathrm{m}}^{37/57} > r > 25\times\alpha^{2/9}\dot{m}^{2/3}$

where in the opacity, Thomson scattering dominates, thermal equilibrium exists only where the "true" optical depth is large ($\tau_{ff}\tau_T > 1$). At the surface, where at sufficiently high frequencies $\varkappa_\nu \ll \sigma_T m_P$, the outgoing radiation spectrum is distorted. In the case of a homogeneous medium with a sharp boundary (Shakura, 1972; Felten and Rees, 1972)

$$F(x)=\sqrt{\frac{3\varkappa(x)n}{\sigma_T m_P}}\,B(x)\sim \mathrm{const}\,\sqrt{n}\,T^{5/4}\frac{x^{3/2}e^{-x}}{(1-e^{-x})^{1/2}} \tag{3.2}$$

and $Q=1.8\cdot 10^{-4}\sqrt{n}\,T^{2.25}\,\frac{\mathrm{erg}}{\mathrm{cm}^2\,\mathrm{s}}$.

In the case of exponential varying atmosphere $n=n(z_1)e^{-z/H_0}$ according to Zeldovich and Shakura (1969) the emerging spectrum has an appearance

$$F(x)=\left(\frac{3\varkappa(x)}{\sigma_T^2 m_P^2 H_0}\right)^{1/3}B(x) \sim \mathrm{const}\,H_0^{-1/3}T^{11/6}x^2\frac{e^{-x}}{(1-e^{-x})^{1/3}} \tag{3.3}$$

$$Q=1.3\cdot 10^4 H_0^{-1/3}T^{17/6}\,\frac{\mathrm{erg}}{\mathrm{cm}^2\,\mathrm{s}}\sim T^{2.5}.$$

In (3.2) and (3.3) $\varkappa(x)=\dfrac{4.1\cdot 10^{-23}(1-e^{-x})}{T^{7/2}x^3}\,\mathrm{cm}^5$ is the coefficient of free-free absorption (Zeldovich and Rayzer, 1966). The optical depth due to Thomson scattering of the layer where $\tau^*=\sqrt{\tau_T\tau_{ff}}=1$ is equal to $\left(\dfrac{\sigma_T m_P}{3\varkappa(x)n}\right)^{1/2}$ in the case of a homogeneous medium and to $\left(\dfrac{\sigma_T^2 m_P^2 H_0}{3\varkappa(x)}\right)^{1/3}$ in the case of exponentially varying atmosphere.

In the inner part of the disk $r < 25\times\alpha^{2/9}\dot{m}^{2/3}$, the processes of comptonization effect strongly the shape of the emitted spectrum. The radiation spectrum formed in the layer with $y > 1$ has a Wien distribution (Illarionov and Sunyaev, 1972)

$$F(x)\sim x^3e^{-x}, \quad Q=\frac{cd(r)}{4}T^4, \quad d(r)\ll b. \tag{3.4}$$

Due to the dominant role of Compton processes in a) nowhere in the disk is the radiation spectrum that of an optically thin plasma

$$F(x)=\varkappa(x)B(x)\sim e^{-x},$$

$$Q=1.4\cdot 10^{-27}T^{1/2}n^2z_0\,\frac{\mathrm{erg}}{\mathrm{cm}^2\,\mathrm{s}}.$$

At the same time, processes connected with the existence of magnetic fields and turbulence can lead to acceleration of the particles and generation of non-thermal radiation at low frequencies (Lynden-Bell, 1969).

b) Distribution of the Surface Temperature along the Disk

The local radiation energy flux Q is determined by the gravitational energy release $\dfrac{3}{8\pi}\dfrac{GM}{R^3}\dot{M}\left(1-\left(\dfrac{R_0}{R}\right)^{1/2}\right)$ in the disk. This equation gives the surface temperature of matter as a function of radius (Fig. 12) corresponding

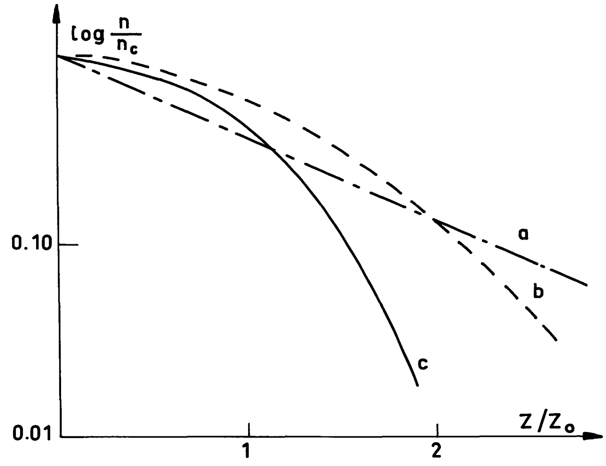

Fig. 11. The profile of the density of the matter in the disk along the Z-coordinate: a) the plane isothermal atmosphere $n\sim\exp-(Z/Z_0)$, b) gaussian atmosphere $n\sim\exp-\left[\frac{1}{2}\left(\frac{Z}{Z_0}\right)^2\right]$, c) real profile of density

[8]) This boundary is closer to the collapsar than that given above r_{bc} because the surface temperature is less than the central value.

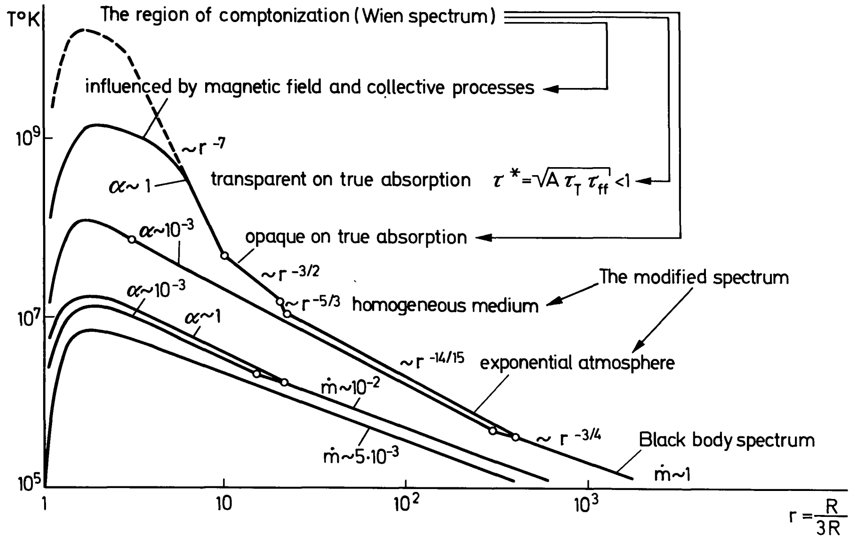

Fig. 12. Distribution of temperature along the radius of the disk if $\dot{M}$ and the parameter α are different

to an effective radiation temperature and mean energy of the outgoing quanta. In the outer regions of the disk

$$T_s = 3 \cdot 10^7 m^{-1/4} \dot{m}^{1/4} r^{-3/4} (1 - r^{-1/2})^{1/4} \,{}^\circ\mathrm{K}. \tag{3.5}$$

As the local radiation spectrum is planckian, then $\bar{x} = \dfrac{\int F_\nu d\nu}{\int \frac{F_\nu}{h\nu} d\nu} = 2.7$. In the intermediate region, according to (3.3), we obtain $\bar{x} = 1.66$ and

$$T_s = 10^8 \alpha^{1/75} \dot{m}^{28/75} m^{-19/75} r^{-141/150} (1 - r^{-1/2})^{28/75} \,{}^\circ\mathrm{K}. \tag{3.6}$$

The intermediate region of the disk also extends over the region with homogeneous matter density in which radiation pressure is dominant. Then, according to (3.2) $\bar{x} = 1.2$ and

$$T_s = 1.4 \cdot 10^9 \alpha^{2/9} \dot{m}^{8/9} m^{-2/9} r^{-5/3} (1 - r^{-1/2})^{8/9} \,{}^\circ\mathrm{K}. \tag{3.7}$$

In the inner region $\bar{x} = 3$. The boundary of this region, determined by condition $y = \dfrac{kT(z_1)}{m_e c^2} \tau_T^2(z_1) > 1$, was estimated assuming the atmosphere to be homogeneous. In this way we also found[9])

$$\begin{aligned} T_s &= 1.4 \cdot 10^9 A^{-2/9} \alpha^{2/9} \dot{m}^{8/9} m^{-2/9} r^{-5/3} (1 - r^{-1/2})^{8/9} \\ &\simeq 5 \cdot 10^8 \alpha^{1/5} \dot{m}^{4/5} m^{-1/5} r^{-3/2} (1 - r^{-1/2})^{4/5} \,{}^\circ\mathrm{K}. \end{aligned} \tag{3.8}$$

In the region $\tau^*(u_0) = \sqrt{\tau_{ff} \tau_s} < 1$ we obtain

$$T_s = 10^{14} \alpha^{12/5} \dot{m}^{24/5} r^{-36/5} (1 - r^{-1/2})^{24/5}. \tag{3.9}$$

[9]) Here and below the approximation $A \approx 40(T/10^8)^{1/2}$ if $T < 4 \cdot 10^8$ °K and $A = 10^2 (T/10^9)^{1/3}$, if $T > 4 \cdot 10^8$ °K is used, the dependence on density being neglected.

c) An Integral Spectrum of the Outgoing Disk Radiation

The spectral distribution of the radiation from the whole disk is obtained by integrating the local spectrum

$$I_\nu = 4\pi \int_{R_0}^{R_1} F_\nu[T_s(R)] \, R dR, \tag{3.10}$$

where R_1 is the external boundary of the disk. For a local planckian spectrum and dependence $T_s(R)$ of the form (3.5), we obtain for $\nu \ll \dfrac{kT_0}{h}$

$$I_\nu = \frac{16\pi^2 R_0^2 h}{c^2} \left(\frac{kT_0}{h}\right)^{8/3} \nu^{1/3}. \tag{3.11}$$

The spectral index of radiation $\gamma = \dfrac{\nu d \ln I_\nu}{d\nu}$ is equal to $\gamma = \dfrac{1}{3}$ for a spectrum of the form (3.1) and dependence of temperature on radius of the form (3.5) (Lynden-Bell, 1969; Shakura 1972). It is easy to find γ in the case of $r \gg 1$, when the factor $(1 - r^{-1/2})$ is of no importance:

$\gamma = 0.07$ for spectrum (3.3) and temperature (3.6),

$\gamma = 0.04$ for spectrum (3.2) and temperature (3.7),

$\gamma = -1/3$ for spectrum (3.4) and temperature (3.8),

$\gamma = -1$ for spectrum (3.4) and temperature (3.9).

If $h\nu \gg kT_{\max}$ the radiation spectrum falls exponentially. The computed resulting radiation spectrum of the disk is given in Fig. 3. At these computations the factor $(1 - r^{-1/2})$ was taken into account.

In the optical region of the spectrum for a wide range of initial conditions a spectrum of the form of (3.11) is established. The optical luminosity of the disk (3000Å < λ < 10000Å) is equal to

$$L_{\rm opt} = \int_{\nu_1}^{\nu_2} L_\nu d\nu \simeq 10^{35} m^{4/3} \dot{m}^{2/3} \frac{\rm erg}{\rm s} \tag{3.12}$$

For a black hole of $M = 10 M_\odot$ and even if $\dot{M} \simeq 2 \cdot 10^{-11} \cdot \frac{M_\odot}{\rm year}$ one may expect the optical luminosity to be of the order of the solar value with a spectrum unusual for a star. In fact the optical luminosity may be somewhat higher than given by (3.12) because a part of the hard radiation flux of the disk is reradiated by the cold periphery.

III. Influence of the Hard Radiation of the Black Hole on the Outer Parts of the Disk and the Visible Component

1. Reradiation of the Hard Radiation by the External Regions of the Disk

From the theory of disk accretion, it follows that, at $r > 150 (\alpha m)^{2/21} \dot{m}^{16/21}$ the previously plane disk begins to thicken according to the law $z_0 \sim r^{21/20}$ and, for $r > 6 \cdot 10^3 \dot{m}^{2/3}$ as $z_0 \sim r^{9/8}$. Additional thickening may occur due to the increase of the ratio z_1/z_0 with radius. The real form of the disk is like a saucer (Fig. 4) and so its external regions catch a certain part L^* of the X-ray radiation of the inner regions L_0

$$L^*/L_0 \simeq \left(\frac{z_1(R_{\max})}{R_{\max}}\right)^2 \simeq \left(\frac{1.5\, z_0(R_{\max})}{R_{\max}}\right)^2 = 10^{-4} \dot{m}^{3/10} (\alpha \cdot m)^{-1/5} r_{\max}^{1/4} \tag{4.1}$$

Here, $R_{\max}$ is the external boundary of the disk. The disk radiates as a plane surface, and therefore the ratio L^*/L_0 is proportional to the square of the angle subtended by the external regions of the disk at the central zone. In the case of a spherical source at the centre of the disk (a collapsar at a supercritical regime of accretion or a hot neutron star, for example), the ratio L^*/L_0 is proportional to this angle $\frac{z_1}{R_{\max}}$. As the disk is optically thick to the X-ray radiation, its surface must absorb and reradiate a significant part of the incident flux of radiation $Q^* = \frac{1}{2\pi R} \frac{dL^*}{dR} = \frac{L_0}{8\pi R^2} \left(\frac{L^*}{L_0}\right)$. At $r > 10^4 (\alpha m)^{4/25} \dot{m}^{-6/25}$ the flux, incident on the disk surface, exceeds the gravitational energy release inside the disk $Q \approx \frac{3}{8\pi} \frac{GM}{R^3} \dot{M}$.

The incident X-ray radiation is absorbed in photoionizing the heavy elements. After absorption, the elementary processes of radiation of softer quanta, partial absorption of these quanta as well as thermalization of the photoelectrons, etc... lead to important effects. Heating of the matter is accompanied by an outflow of the hot gas: the disk thickness and the absorbed fraction of the hard radiation of the central regions of the disk both increase. The effective thickness of the disk is determined by the boundary of transparency to the incident radiation. Such a swelling of the disk will take place or as long as it is evaporated completely (see discussion below), or if such density and temperature profiles across the outer layers of the disk establish, that they result in complete reradiation of the incident flux of energy. The density of matter must drop rapidly with distance from the plane of symmetry. The temperature at $z > z_0$ must also decrease as Z increases providing there a radiative transfer of the gravitational energy released inside the disk. Then the absorption of the X-ray radiation leads to an increase in temperature with increasing Z (these layers are transparent with respect to free-free absorption but opaque to resonance lines, photoionizing quanta, Thomson scattering, etc). The density and temperature profiles can be determined from the numerical solution of the system of equations of thermal balance, radiative transfer and hydrostatic equilibrium of the matter with the variable temperature in the gravitational field of the black hole. In turn, the profiles of density and temperature determine the fraction of the hard radiation reprocessed in the outer regions of the disk. Estimates show that this fraction $\frac{L^*}{L_0} = \left(\frac{z^*}{R}\right)^2$ can greatly exceed the value (4.1) given by the simple theory of disk accretion which does not take into account the external heating of the disk. The system of equations mentioned above can be written in the first approximation in form: $\frac{z^*}{R} = \frac{v_s}{v_\varphi} = \sqrt{\frac{kT}{m_p} \frac{R}{GM}}$ (as a consequence of the equation of hydrostatic equilibrium), $\frac{R}{z^*} \sigma_{\rm eff}\, nR \sim 1$ (the condition for absorption of the hard radiation). If the temperature $T > 10^6$ °K and the hard radiation strongly influences the ionization balance of the plasma, one may put $\sigma_{\rm eff} \simeq \sigma_T = 6.65 \cdot 10^{-25}\,{\rm cm}^2$ in order of magnitude. $T^* = f\left(\frac{L_0}{nR^2} \frac{z^*}{R}\right) = f\left(\frac{L_0 \sigma_{\rm eff}}{R}\right)$ is the equation of thermal balance. If $\xi = \frac{L_0}{nR^2} \frac{z^*}{R} = \frac{L_0 \sigma_{\rm eff}}{R} > 10^4$ as is shown in the appendix, $T^* \sim T_{\rm eff}$ and one can use the results of the computations of Tarter *et al.* (1969) to determine the temperature if $\xi < 10^4$. The factor $\frac{z^*}{R}$ in the definition of ξ arises from the dependence of the radiation flux from the plane surface on direction ($\sim \cos\theta$). We always obtain $\xi > 10^{-1}$ and $T^* > 10^5$ °K for $10^{-3} L_{\rm cr} < L < L_{\rm cr}$ and $R < 10^{12}$ cm.

Therefore,

$$\frac{L^*}{L_0}=\left(\frac{z^*}{R}\right)^2=\frac{kT}{m_p}\frac{R}{GM}$$
$$=5\cdot 10^{-2}\left(\frac{T^*}{10^5\,{}^\circ\mathrm{K}}\right)\left(\frac{R}{10^{12}\,\mathrm{cm}}\right)\frac{M_\odot}{M}.$$

Note that σ_{eff} increases rapidly if $\dot{M}$ and the effective temperature of the hard radiation decrease. This leads to an increase in ξ and T^* tends to T_{eff}. It would appear that at least $0.1\div 10\%$ of the total energy released in the source must be reprocessed in the outer regions of the disk.

Fluorescence occurs, in general, in the lines of heavy elements and hydrogen. The hard quanta are able to penetrate so far into the disk that only quanta of energy less than the ionization potential of hydrogen are able to escape. Therefore a significant part (up to 10%) of the reprocessed energy leaves in the form of recombination radiation and resonance lines in the optical range of the spectrum[10]). Most of the energy reradiated is in the ultra-violet and soft X-ray bands and forms an appreciable Strömgren Zone.

2. *Autoregulation of Accretion*

Absorption of the hard radiation of the central source by the matter falling into the disk as well as by the matter on the disk periphery and its subsequent heating can lead to evaporation of this matter. It can also lead to a decrease in the accretion rate. The decrease in the accretion rate leads to a decrease in the hard radiation flux which heats the accreting matter. Such a regime of accretion will be established when the quantity of matter entering the disk is controlled by the flux of radiation emitted. The efficiency of autoregulation is determined, firstly, by the rate of mass loss from the adjacent star, by the type of outflow, by the degree to which the rotation is non-synchronous and by other parameters of the binary system. One can differentiate between two limiting cases: a) for a star filling only a small part of Roche volume matter flows out uniformly from the whole surface with a velocity greater parabolic one for this star $v_P(R_2)$ and b) outflow from a star exceeding its limiting volume takes place through the inner lagrangian point with a velocity much smaller than $v_P(R_2)$. Below we consider autoregulation for the case a) $v_{es}>v_P$ outflow is analogous to the usual "stellar wind" as in the case of single stars. Only some fraction of the outflowing matter will be captured by the black hole. In the vicinity of a collapsar, $R^*=\frac{GM_1}{v_{es}^2}$ a non-spherical shock wave appears (Salpeter, 1964) where matter loses a significant part of its kinetic energy and is gravitationally captured by the collapsar. The captured matter possesses angular momentum. Therefore, the disk is formed inside the sphere of radius R^*. If we know the density of the outflowing gas near radius of capture $n(R^*)=n(R_2)\left(\frac{R_2}{R_{12}}\right)^2$ one can easily express the rate of accretion in terms of the velocity of outflowing matter and parameters of the binary system

$$\dot{M}_{\mathrm{ac}}=\dot{M}_{\mathrm{out}}\left(\frac{M_1}{M_1+M_2}\right)^2\left(\frac{v_P(R_{12})}{v_{es}}\right)^4,$$
$$v_P=\sqrt{\frac{G(M_1+M_{22})}{R_{12}}}$$

where indices 1, 2 and 12 correspond to a black hole, a normal star and a binary system respectively. Such a picture of accretion is typical of remote binary systems. The thermal velocity of the particles v_s at the radius of capture may exceed v_{es}, as a result of heating by X-ray radiation. In this case, the rate of inflow of matter to the disk will drop abruptly $\dot{M}'_{\mathrm{ac}}\simeq\dot{M}_{\mathrm{ac}}\left(\frac{v_{es}}{v_s}\right)^3$. The characteristic times of gas cooling and heating by the central X-ray source are small compared with the time of free fall of the particles at the radius of capture. Therefore, results obtained for a stationary plasma can be used in computations of the accreting gas temperature and its ionization balance. In this case the gas temperature is determined by the effective radiation temperature T_{eff} and the parameter $\xi=\frac{L_0}{nR^{*2}}$. As $L_0\simeq 0.06c^2\cdot\dot{M}_{\mathrm{ac}}=0.06\cdot 4\pi\varrho v_{es}R^{*2}c^2$, then $\xi=2\cdot 10^{-3}v_{es}$ if $v_{es}>v_s$ or $\zeta=2\cdot 10^{-3}v_s$ if $v_{es}<v_s$. For $v_{es}>100\,\frac{\mathrm{km}}{\mathrm{s}}$, we have $\zeta>10^4$ and conditions in which Compton processes of heating and cooling play the leading role in the energy balance of the plasma (see Appendix).

In the limit of large ξ, when T is of the order of the effective temperature T_{eff} of the X-ray radiation, one can easily find the limiting rate of accretion above which it is necessary to take into account the heating: $v_{es}^2=v_s^2=\frac{kT_{\mathrm{eff}}}{m_P}$ and

$$\dot{M}^*=0.06\dot{M}_{\mathrm{cr}}\left(\frac{v_{es}}{100\,\frac{\mathrm{km}}{\mathrm{s}}}\right)^8\left(\frac{M_1}{M_\odot}\right)^2 \tag{5.3}$$

When $\dot{M}<\dot{M}^*$ the rate of accretion grows linearly with increasing rate of outflow from the visible star but, as soon as $\dot{M}>\dot{M}^*$, we have $\dot{M}_{\mathrm{ac}}\sim\dot{M}_{es}^{8/11}$. For the remote systems due to the geometrical factor, the rate of accretion itself becomes small and the additional influence of autoregulation results the black holes in these systems appearing as optical and ultra-violet objects of low luminosity only.

[10]) The spectral width of these lines must be of the order of

$$\frac{\Delta v}{v}\sim\frac{v_\varphi(R)}{c}\sim 3\cdot 10^{-4}\left(\frac{10^{12}\,\mathrm{cm}}{R}\right)^{1/2}\frac{M}{M_\odot}.$$

3. The Influence on the Normal Component

A considerable fraction of the hard radiation of the black hole may be captured by the surface of the normal component leading to unusual spectral effects (Shklovsky, 1967). Part of the energy absorbed must be transformed into thermal energy giving an additional outflow from the star surface. Another part of the absorbed energy is transformed into softer quanta in this way increasing the optical luminosity of the hemisphere of the normal star which is turned to the black hole. Due to the rotation of the whole system, the effects observed in the optical range will be unusual. At the moment when the black hole is situated between the observer and its companion, the optical spectrum has the characteristics of a gas with emission lines which are formed in the disk corona as well as in the companion's atmosphere. The absorption lines belonging to the colder side of the companion will be seen clearly expressed during total or partial eclipses of the black hole. Non-synchronous rotation of the companion and excentricity of the orbit lead asymmetry in the phases of emission and absorption spectra. The recent computations confirm this qualitative picture (Basko and Sunyaev, 1973), and show that sufficient part of X-ray energy flux absorbed by the surface of the visible component is reradiated in the optical continuum, increasing the optical luminosity of the system and leading to its variability.

We note the (somewhat improbable) possibility that, in such binary systems the rate of accretion increases to the Eddington critical value as the result of the additional outflow of matter caused by the heating of the surface of the normal star by the radiation of the black hole. The necessary condition for such a catastrophic increase is that the additional outflow should exceed the initial outflow. This becomes possible only in close binaries with

$$\frac{R_{12}}{R_2} < \left(\frac{\eta}{\gamma}\right)^{1/2} \frac{c}{v_{es}} \frac{M_1}{M_1+M_2} \left[\frac{v_P(R_{12})}{v_{es}}\right]^2$$

where γ is the part of the radiation energy absorbed turn into the kinetic energy of the matter flowing out and η is the efficiency of gravitational energy release.

IV. Supercritical Regime of Disk Accretion

At the critical Eddington luminosity $L_{cr} = \frac{4\pi GM}{\sigma_T} = 10^{38} \frac{M}{M_\odot} \frac{\text{erg}}{\text{s}}$ the force of radiation pressure on the electrons $f_1 = \frac{q\sigma_T m_P}{c}$, balances the gravitational attraction of protons and nuclei $f_2 = \frac{GMmp}{R^2} \frac{R}{R}$. Therefore, in spherical accretion the flux of matter cannot exceed $\dot{M}_{cr} = \frac{L_{cr}}{\eta c^2}$ at any distance from the star if even there exist conditions favourable for the formation of a supercritical ($\dot{M} > \dot{M}_{cr}$) flux (sufficiently great density and low temperature of the matter).

The infall of matter within a narrow sector is a peculiarity of the disk accretion. During infall, the matter moves in a slowly twisting spiral in the plane perpendicular to the direction of the angular momentum of the matter flowing into the disk. The step of the spiral is determined by the efficiency of the mechanisms of angular momentum transfer. The energy release in the disk is proportional to $R^{-3}\left[1-\left(\frac{R_0}{R}\right)^{1/2}\right]$. Radiation diffuses mainly across the disk. There is no qualitative difference between the subcritical and supercritical regimes of accretion in the outer regions; the rate of energy release there is less than the critical luminosity and light pressure does not prevent infall of the matter.

A number of developments is then possible. One can imagine that periodically a supercritical flux of matter falls towards the centre of gravitation and then powerful flares of radiation reject the matter and destroy the disk completely. During accretion to the collapsar it is possible but improbable that a significant fraction of the matter flux falls into the black hole. In this case, it follows that the radiation flux is only of the order of the critical value and most of the matter and radiation are dragged into the black hole without any observable effect. It is more probable that a stationary state is established with the luminosity close to the critical value and with outflow of most of the infalling matter beyond the system (see also Schwartzman, 1971c). The value of the established luminosity may few times differ from the value of the critical luminosity, computed for the spherically symmetric case. Now let us discuss this picture in more detail.

Approaching the black hole, the energy release and the force of light pressure grow rapidly. According to (2.8), if $\dot{m} = \frac{\dot{M}}{\dot{M}_{cr}} > 1$, near the radius of spherization

$$R_{sp} \approx \frac{9}{4} 10^6 m\dot{m} (\text{cm}); \qquad r_{sp} \approx \frac{9}{4} \dot{m} \tag{7.1}$$

the thickness of the disk becomes of the order of distance to the black hole. Further infall of matter will lead to the immediate outflow of a fraction of it from the region $R < R_{sp}$.

The outflowing matter is accelerated by the difference of the forces of gravitational attraction and light pressure. In the situation under discussion, there are only two characteristic velocities: the parabolic velocity $\sqrt{2}v_\varphi(R_{sp})$ and the radial velocity of the matter in the central plane of the disk $v_r = \alpha v_\varphi(R_s)$. Apparently, autocontrol is realised leading to a velocity of outflow $v_{out} \simeq \sqrt{2} v_\varphi \sqrt{\frac{L-L_{cr}}{L_{cr}}}$ close to v_r. A decrease in the velocity of the outflowing matter relative to v_r leads to the arrival of a considerable fraction of the inflowing matter in the region with $R < R_s$, increasing the luminosity and, consequently the velocity of the outflowing matter. If v_{out} exceeds v_r both the matter inflow to the region $R < R_{sp}$ and the luminosity decrease and also decreases v_{out}. On the other hand, if $R > R_{sp}$ the outflowing matter is subjected to the light pressure not only of the disk region which is left but to the integral radiation of the whole disk. The latter can logarithmically exceed the critical luminosity $\frac{L - L_{cr}}{L_{cr}} \simeq \ln \dot{m} \simeq \ln r_{sp}$. This must lead to the increase of v_{out} to $v_\varphi(r \gg r_{sp}) \ll v_\varphi(r_{sp})$. Evidently, even at small $\alpha < 10^{-2}$ the velocity of the outflow may not be less than $(0.01 \div 0.1)\, v_\varphi(r_{sp})$.

The same relation $v_{out} = \alpha v_\varphi(R)$ exists for matter flowing out from any radius $R < R_{sp}$, but here the dependence on $\alpha(R)$ is dominant. The lines of matter flow are illustrated in Fig. 8, not taking into account rotation. In fact, the matter flows out in a spiral (Fig. 9). The closer to the black hole is the matter, the greater is the velocity of the outflowing matter. However, the fraction of this matter $\dot{M}(R < R_{sp}) = \frac{R}{R_{sp}} \dot{M}_0$ in the flux $\dot{M}_0$, inflowing to the external boundary of the disk decreases.

If the angular momentum is preserved relative to the disk axis, the matter is not able to enter the cylindrical region of radius less than the radius of outflow. The result is a strongly anisotropic outflow of matter at an angle θ relative to the axis of disk rotation. In a small cone near the axis, hot matter is ejected from the region close to $R \sim \frac{9}{4} R_0 \sim 7 R_g$ with high velocity and hard X-ray radiation is emitted. When θ increases, the velocity of the outflow decreases rapidly and, in most of the cone (and most of the outflowing matter), $v_{es} \sim \alpha v_\varphi(R_{sp})$. The spectrum of radiation emitted by this region depends upon the rate of matter inflow to the disk and on the parameter α determining the volume emission measure of the outflowing gas and the degree of reradiation of the hard radiation of the disk to the softer quanta. When $\dot{m} \gg 1$ and $\alpha(r_{sp}) \ll 1$ (the latter is rather probable because, near r_{sp}, the degree of compression of the matter with the frozen magnetic field is small) the outflowing matter may be opaque far from the collapsar. In this case, the collapsar may be a bright optical object. Depending on the orientation of the system, the hard or soft X-ray radiation may also be observed.

Note that, if $\dot{m} \gg 1$ and $\alpha \ll 1$, the total spherization of the accretion picture may emerge – the fast particles will be thermalized by their interaction with the main mass of the outflowing matter. The radiation pressure and an effective exchange of angular momentum between adjacent layers of the outflowing matter can also lead to spherical symmetry. In this case, no X-ray radiation can exist. We now give a rough estimate of the spectrum of the radiation (and its effective temperature) formed in a large fraction of the outflowing matter.

The density of the matter flowing out with the constant velocity $v_{out}(r_{sp})$ changes according to the law

$$n(r) = n(r_{sp}) \left(\frac{r_{sp}}{r}\right)^2,$$

where

$$n(r_{sp}) = 1.3 \cdot 10^{19} \alpha^{-1} \dot{m}^{-1/2} m^{-1} (\mathrm{cm}^{-3}) \tag{7.2}$$

can be found from formulae (2.11) with $r = r_{sp} = \frac{9}{4}\dot{m}$ and $(1 - r^{-1/2}) = 1/3$. The same expression can be found from the relation $\dot{M} = 2\pi m_p n(R_{sp}) \times v_{out}(R_{sp})\, R_{sp}^2 z_0$. The comptonization plays an important role in this exchange of energy between the matter and radiation. At $\dot{m} < 10^3 \alpha^{34/29} A^{-16/29} m^{2/29}$, the outflowing matter is not able to reprocess the hard radiation of the disk. Therefore, the temperature of the outgoing radiation is defined by formulae (3.7 – 3.9) replacing r by $r_s = \frac{9}{4}\dot{m}$ and $(1 - r^{-1/2})$ by 1/3. At larger rates of accretion, the radiation spectrum is formed in the outflowing matter. The condition $A\tau_{ff}(R_{eff})\, \tau_T(R_{eff}) \sim 1$ determines the layer where a black body spectrum is formed. The outgoing radiation spectrum is distorted because $\tau_T \gg \tau_{ff}$. Its energy density is

$$\tau_T(R_{eff}) = \int_R^\infty \sigma_T n(R)\, dR = 30 \alpha^{-1} \cdot \dot{m}^{1/2} \left(\frac{R_{sp}}{R_{eff}}\right)$$

times less than bT^4. Using the condition

$$L_{cr} = 10^{38} m \frac{\mathrm{erg}}{\mathrm{s}} = \frac{4\pi R_{eff}^2}{\tau_T(R_{eff})} b T_{eff}^4 \frac{c}{4}$$

one can easily determine an effective radiation temperature and the radius R_{eff} of the radiating envelope

$$\begin{aligned} R_{eff} &\simeq 3 \cdot 10^2 \dot{m}^{51/22} m^{10/11} \alpha^{-17/11} A^{8/11} (\mathrm{cm}) \\ T_{eff} &\simeq 2 \cdot 10^{10} \dot{m}^{-15/11} m^{-2/11} \alpha^{10/11} A^{-6/11}\ {}^\circ\mathrm{K}. \end{aligned} \tag{7.3}$$

The temperature T_{eff} is practically constant in a wide region with $R > R_{eff}$. At $\dot{m} < 4 \cdot 10^7 \alpha^{2/3} m^{1/9}$ the radius at which the quanta undergo their last scattering, $R(\tau_T = 1) = 9 \cdot 10^7 \dot{m}^{3/2} m \alpha^{-1}$ cm exceeds greatly R_{eff}. If $R(\tau_T = 1) \leq R_{eff}$, the outgoing radiation spectrum is planckian. The parameter y is greater than unity and, owing to the Compton process, outgoing radiation spectrum has a Wien distribution unless $\dot{m} < 3 \cdot 10^3 \alpha^3 m^{-1/2} \cdot A^{-3/2}$. Note that the apparent radius at eclipses of the normal star by the matter flowing out from collapsar is $R(\tau_T = 1)$.

In the optical (low-frequency range at the discussed temperatures T_{eff}) spectral range, the radius near which the envelope becomes opaque exceeds R_{eff} considerably. Putting $\tau_T \tau_{ff}(\nu, R_{opt})$ equal to unity, we find

$$R_{opt} \simeq 10^7 \alpha^{-3/4} \left(\frac{10^6\ {}^\circ\mathrm{K}}{T}\right)^{3/8} \left(\frac{10^{15}\ \mathrm{Hz}}{\nu}\right)^{1/2} \dot{m}^{9/8} m^{3/4} (\mathrm{cm})$$

and the total optical luminosity of collapsar

$$L_{\rm opt} = \int_0^{\nu_0} \frac{4\pi R_{\rm opt}^2}{\tau_T(R_{\rm opt})} \frac{2\pi k T_{\rm eff}}{c^2} \nu^2 d\nu = 3 \cdot 10^{30} \left(\frac{10^6\,{}^\circ{\rm K}}{T}\right)^{1/8} \cdot \left(\frac{\nu_0}{10^{15}\,{\rm Hz}}\right) \left(\frac{\dot{m}^{3/2} m}{\alpha}\right)^{5/4} \frac{\rm erg}{\rm s} \simeq 10^{30} \left(\frac{\nu_0}{10^{15}\,{\rm Hz}}\right)^{3/2} \left(\frac{\dot{m}^{3/2} m}{\alpha}\right)^{15/11} \frac{\rm erg}{\rm s} \tag{7.4}$$

is weakly dependent on T. The optical spectrum is unusual $F_\nu \sim \nu^{1/2}$. In fact, the other sources of opacity will strongly influence the optical spectrum of the object. The spectrum will be saturated with emission lines, which will determine the excess of optical luminosity over (7.4). The emission lines must be broad with $\frac{\Delta\lambda}{\lambda} \sim \frac{v_{es}(R_{sp})}{c} \sim \alpha \frac{v_\varphi(R_s)}{c}$.

For $\alpha \sim 1$, direct exportation of radiation closed in the matter may be important in the narrow cone of angles where the matter flows out with high velocity. If $v_{\rm out} \sim v_\varphi(7R_g) \sim 10^5 \frac{\rm km}{\rm s}$, the time for radiation exportation is comparable with the time of diffusion of radiation $t \sim \frac{7R_g}{c} \tau_T \sim \frac{200 R_g}{c}$. Therefore the existence of a flux of fast particles ejected from the region $R \sim 7R_g$ is of independent interest and should lead to specific effects: the formation of shock waves and X-ray radiation being emitted far from the collapsar. Evidently, in case when $\dot{m} \gtrsim 3 \cdot 10^3 \left(\frac{\alpha}{m}\right)^{2/3}$ neither this flux of fact particles nor the radiation of the slowly $v_{\rm out} \sim \alpha v_\varphi(R_{\rm sp})$ outflowing matter with $T_{\rm eff} \sim 10^5\,{}^\circ$K can strongly influence the rate of inflow of the matter in the outer boundary of the disk. When $\dot{m} > 3 \cdot 10^3 \left(\frac{\alpha}{m}\right)^{2/3}$ and $\dot{m} > 1$, the collapsar must appear as a bright optical and ultra-violet object with high mass loss (much like Wolf-Rayet stars) and possible X-ray radiation. It is surrounded by a thin disk of cold, inflowing matter in contrast to normal stars. The same situation exists when there is supercritical accretion to a neutron star in a close binary system. Only the masses and corresponding luminosities are different.

V. Turbulence, Magnetic Fields and Temporal Characteristics of the Disk Radiation

If $\alpha \sim 1$, new, important effects connected with plasma turbulence and reconnection of magnetic field lines, through neutral points, must appear. In addition, short-term fluctuations of the radiation flux, connected with these effects, must take place.

So, for example, the flux of energy $Q' \sim \frac{\varrho v_t^3}{2}$ transferred by turbulence from the inner layer of the disk to its surface, is α^2 times less than the radiative energy flux $Q \simeq \frac{3}{8\pi} \frac{GM\dot{M}}{R^3}$. Such convection must lead to granulation of the disk surface, analogous to that observed on the Sun and of scale size of the order of z_0. Therefore when there is disk accretion and associated turbulence, chaotic fluctuations of the observed radiation flux $\frac{\delta I}{I} \sim \frac{Z}{R} \alpha^2 \sim \dot{m}\alpha^2$ and of its spectrum may be expected. The typical time of these fluctuations is equal to $\Delta t_f \sim \frac{Z_0}{v_t} \sim \frac{R}{\alpha v_\varphi} \sim 10^{-4} \frac{m}{\alpha} r^{3/2}$ s. The fluctuations of the hard radiation should be the strongest, because the turbulence takes out to the disk surface high temperature clumps of plasma. In the low-frequency (optical) spectral bands, which are emitted mainly by the outer regions of the disk, the typical time of fluctuations exceeds 1 minute.

Analogous irregular activity may also be caused by chaotic magnetic fields. Note that it is difficult to receive the giant strength of the magnetic fields $H \sim 10^8$ Gauss $\left(\text{which are consequence of an assumption of equipartition } \frac{H^2}{8\pi} \sim \varrho v_s^2\right)$ in the vicinity of a collapsar as the result of a simple compression of the magnetized gas flowing out from the normal star. The maximum matter density in the disk at $\alpha \sim 1$ and $\dot{m} \sim 1$ are $n \sim 10^{20}\,{\rm cm}^{-3}$, i.e. only $4 \div 6$ orders of magnitude larger than the matter densities in stellar atmospheres having magnetic field strengths $H \simeq 1 - 10$ Gauss. It is possible that the magnetic field strength in the disk is many orders of magnitude less than 10^8 Gauss. Then α is also small.

Equipartition could be the consequence of amplification of the magnetic field in the differentially rotating matter of the disk. Simultaneously, the magnetic field is partitioned into small loops of scale less than z_0. The loops with opposite directions of the magnetic field can approach under the influence of turbulent pulsations. Therefore, current sheets are formed and flares of the solar type can occur. However, the energy release may be much greater (Shakura, 1972a). The acceleration of the particles by the mechanism of Syrovatsky, which may be essential to accretion to a massive ($M > 10^6 M_\odot$) black hole (Lynden-Bell, 1969), is doubtful because the gas density is large. However, it is not excluded. The typical time of aperiodic fluctuations, connected with the flares greatly exceeds the time $\Delta t_f \sim 10^{-4} \frac{m}{\alpha^{1/2}} r^{3/2}$ s determined by the Alfven velocity and by the scale of the magnetic field $\sim z_0$. To within an order magnitude it is equal to the time $\Delta t_f \sim \frac{3R_g}{c} \sim 10^{-4}$ms deduced by Schwartzman (1971a) from different reasoning. The amplitude of the fluctuations of the integral radiation flux depends mainly on the structure of the magnetic field and drops rapidly when α decreases. Apparently, it cannot exceed $\frac{\delta I}{I} \sim \frac{Z}{R} \frac{H^2}{4\pi \varrho v_\varphi^2} \sim \alpha \left(\frac{Z_0}{R}\right)^3$. However, in certain spectral bands, this amplitude can be of the order of unity, because the spectra of thermal radiation and the spectra of flares are different.

At $H > 10^6$ Gauss the hot ($T \sim 10^8$ °K) plasma emits synchroton radiation with superposed resonances of cyclotron frequency $v_s = \frac{seH}{2\pi m_e c}$ (with $s \sim 10$) provides the black body intensity in this overlapping due to the Doppler-effect resonances and leads to the partial polarization of integral radiation of the disk in the optical and infra-red spectral bands (Gnedin and Sunyaev, 1972).

Appendix

The Compton Effect and the Thermal Balance of the Gas in the Vicinity of X-Ray Sources

In the vicinity $\left(\zeta = \frac{L_x}{R^2 n} > 10^4 \div 10^5 \frac{\text{erg cm}}{\text{s}}\right)$ of X-ray sources with $T_{\text{eff}} \sim 10^6 - 10^8$ °K, all elements up to neon which determine the energy balance of colder plasmas are completely ionized (Tarter *et al.*, 1969). In such conditions the stationary value of the temperature of the plasma is determined by the processes of Compton exchange of energy between radiation and free electrons and bremsstrahlung.

Equating, in turn, the rate of electron cooling due to Compton processes $P_c^- = \frac{4\sigma_T \varepsilon k T}{m_e c^2}$ and free-free radiation $P^- = 10^{-27} \sqrt{T} n$ to the rate of heating by Compton mechanism $P_c^+ = \frac{\sigma_T h}{m_e c} \int_0^\infty \varepsilon_v v \, dv$ (Levich and Sunyaev, 1971), we find for the temperature of the gas

$$T = \beta T_{\text{eff}} \quad \text{at} \quad \xi > \zeta_{\text{cr}} = \frac{3.3 \cdot 10^7}{(\beta T_{\text{eff}})^{1/2}}$$

when cooling is associated with Compton-effect, and $T = \beta T_{\text{eff}} (\zeta/\zeta_{\text{cr}})^2$ at $\zeta < \xi_{\text{cr}}$ when cooling is due to bremsstrahlung.

The coefficient β depends on the spectrum of radiation of the source: for a black body spectrum, $\beta = 0.94$, for a Wien distribution, $\beta = 1$, and for the spectrum of the optically thin plasma, $\beta = 0.25$. If $\zeta < 10^4 \div 10^5$, the energy losses in lines of the heavy elements begin to influence the thermal balance of the plasma and calculations are very complicated (Tarter *et al.*, 1969).

The authors wish to thank A. F. Illarionov, D. Ya. Martynov, L. R. Yangurasova and Ya. B. Zeldovich for consultations and the participants of the 1972 winter school on astrophysics in Arhyz for numerous discussions.

References

Basko, M. A., Sunyaev, R. A. 1973, preprint, *Astrophysics Space Science* (in press).
Bisnovaty-Kogan, G. S., Sunyaev, R. A. 1971, *Astron. Zh.* **48**, 881.
Cherepashchuk, A. M., Efremov, Yu. N., Kurochkin, N. E., Shakura, N. I., Sunyaev, R. A. 1972, *Inform. Bull. Var. Stars* N 720.
Felten, J. E., Rees, M. J. 1972, *Astron. & Astrophys.* **17**, 226.
Gnedin, Yu. N., Sunyaev, R. A. 1973, *Monthly Notices Roy. Astron. Soc.* (in press).
Gorbatsky, V. G. 1965, *Proceedings of Leningrad University Observatory* **22**, 16.
Illarionov, A. F., Sunyaev, R. A. 1972, *Astron. Zh.* **49**, 58.
Kaplan, S. A. 1949, *JETP* **19**, 951.
Kompaneets, A. S. 1956, *JETP* **31**, 876.
Kraft, R. 1963, *Advances in Astron. & Astrophys.* ed, Kopal, Z. **2**, 43.
Levich, E. V., Sunyaev, R. A. 1971, *Astron. Zh.* **48**, 461.
Lynden-Bell 1969, *Nature* **223**, 690.
Lyutiy, V. M., Sunyaev, R. A., Cherepashchuk, A. M. 1972, preprint 1973, *Astron. Zu.* **50**, 3.
Martynov, D. Ya. 1971, Course of General Astrophys., Nauka, Moscow.
Novikov, I. D., Thorne, K. S. 1972, Lectures in the summer school on the black holes, Les Houches, France.
Pottasch, S. R. 1970, in "*Cosmic gas dynamics*". IAU Symposium 31, Reidel, Dordrecht.
Prendergast, K. H. 1960, *Astrophys. J.* **132**, 162.
Prendergast, K. H., Burbidge, G. R. 1968, *Astrophys. J.* **151**, L 83.
Pringle, J. P., Rees, M. J. 1972, *Astronomy Aph*, **21**, 1.
Salpeter, E. E. 1964, *Astrophys. J.* **140**, 796.
Shakura, N. I. 1972a, *Astron. Zh.* **49**, 921.
Shakura, N. I. 1972b, *Astron. Zh.* **49**, 652.
Shklovsky, I. S. 1967, *Astrophys. J.* **148**, L 1, Astron. J. **44**, 930.
Schwartzman, V. F. 1971a, *Astron. Zh.* **48**, 479.
Schwartzman, V. F. 1971b, *Astron. Zh.* **48**, 438.
Schwartzman, V. F. 1971c, Thesis, Moscow University.
Sunyaev, R. A. 1972, *Astron. Zu.* **49**, 1153.
Tarter, C. B., Tucker, W. H., Salpeter, E. E. 1969, *Astrophys. J.* **156**, 943.
Taylor, G. I. 1937, *Proc. Roy. Soc. A.*, **157**, 546.
Wasiutinski, J. 1946, Studies in Hydrodynamics and Structure of Stars and Planets, Oslo.
Zeldovich, Ya. B. 1964, *Doclady Acad. Sci.*, U.S.S.R., **155**, 67.
Zeldovich, Ya. B., Novikov, I. D. 1971 The theory of the gravitation and stars evolution, Nauka, Moskow.
Zeldovich, Ya. B., Raizer, Yu. P. 1966, Physics of shock waves and high-temperature hydrodynamic phenomena, Nauka, Moscow.
Zeldovich, Ya. B., Shakura, N. I. 1969, *Astron. Zh.* **46**, 225.

N. I. Shakura
Sternberg Astronomical Institute
Moscow – U.S.S.R.

R. A. Sunyaev
Institute of Applied Mathematics
Academy of Sciences
Moscow – U.S.S.R.